A Model of Sales, Prices, and Liquidity in the Housing Market

Gabriel Ehrlich*

University of Michigan

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-PRELIMINARY-

Abstract

I embed a search and matching model of the housing market into a DSGE framework and use it to address three main questions. First, what model of search and price determination best describes the housing market? Second, can a general equilibrium model generate the observed correlations between housing market variables? Third, what shocks have driven the historical behavior of the housing market, especially the recent boom and bust in house prices? I show that a model of competitive search is more likely conditional on the data than a model featuring random search and bargaining. Simulated data from the model qualitatively matches the correlations between key housing market time series. Finally, estimation results imply that the recent housing boom and bust were associated with a large increase and subsequent decrease in the pool of eligible buyers, in addition to expectations of higher future productivity that turned out not to be realized.

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1 Introduction

Historically, house prices have been positively correlated with sales volumes and starts, which have been negatively correlated with housing market liquidity and the vacancy rate. However, despite a growing literature examining the housing market through the lens of search and matching models, it remains unclear how well a dynamic general equilibrium model featuring search and matching frictions in the housing market can replicate these patterns. Ideally, a model that is able to replicate housing market dynamics successfully would also improve our inference concerning what shocks drive the housing market, and more specifically, what accounts for the recent boom and bust cycle in the U.S. housing market.

Figures 1 and 2 illustrate some of the key housing market time series using U.S. data. Figure 1 shows detrended GDP, house prices, and home sales, while Figure 2 shows housing starts, months' supply (the inventory of homes listed for sale divided by the number of sales in a month), and the homeowner vacancy rate over the period 1982Q3-2010Q4.[1] Several empirical regularities, summarized in Table 1, emerge from the figure. First, house prices, sales volumes, and vacancies are much more volatile than GDP, while housing starts and months' supply are more volatile still. Second, GDP, prices, sales, and starts all comove positively, while months' supply and vacancies comove negatively with sales and starts. Ideally, a theory of search and matching in the housing market would account for many of these empirical regularities.

A substantial literature has examined search frictions in the housing market both empirically and theoretically. Rosen and Topel (1988) find that time to sale has a large effect on new construction in the U.S. over the period 1963-1983. Following Wheaton's (1990) seminal model, a number of papers have taken a search and matching approach to the housing market. Recently, Diaz and Jerez (2009) and Head et al. (2010) have

[1] All series are in logs and have been linearly detrended and seasonally adjusted. Please see section 3.2, Observed Data Series, for details of the series' construction.

examined different models of search and price determination in this context. Aqeel (2009) and Magnus (2010) both illustrate the difficulty of matching the joint dynamics of key housing market time series such as prices, starts, and vacancies.

Identifying what shocks drive the housing market has been a topic of significant research interest recently, but no consensus has emerged. Some authors, such as Iacoviello and Neri (2010) and Wheaton and Nechayev (2008) offer fundamentals-based explanations. Others, such as Kahn (2008) and Lambertini et al. (2010) emphasize households' learning process concerning the economy. Finally, many authors, such as Case and Shiller (2003) and Piazzesi and Schneider (2009) argue that unrealistic expectations of future price appreciation were a key driver in the housing boom, implying that the recent boom and bust in prices was an irrational bubble.

Thus, there appear to be a number of open questions in the area of housing search. First, can a search and matching model of the housing market match the observed behavior of housing prices, sales, and construction over the business cycle? Second, what model of search and price determination best describes the housing market? Third, what types of shocks drive fluctuations in the housing sector?

I embed a search and matching model of the housing market into a DSGE framework and use it to adress these questions. I consider both random search and bargaining and competitive search models, and compare the two statistically. I consider the role of several shocks in the model: housing productivity shocks, consumption productivity shocks, and "eligible buyer" shocks meant to represent changes in financing conditions. Crucially, I allow for "news shocks" to consumption productivity, in which agents anticipate changes in productivity ahead of time. I estimate the parameters of the shock process using the Metropolis-Hastings algorithm and the historical shocks hitting the housing market using the Kalman filter. Finally, I simulate data from the estimated model and compare it to the historical data.

My model replicates most of the key patterns in the U.S. housing market qualitatively.

Price, sales, and starts are positively correlated, while prices comove stronlgy with GDP. Furthermore, starts are negatively correlated with months' supply of housing. Contrary to the data, starts are negatively correlated with GDP, although this correlation is not statistically significant. Data simulated from the competitive search model matches the historical correlations better than data from the random search model.

I compare the random and competitive search models in terms of their posterior marginal densities. The higher likelihood of the competitive model constitutes "decisive evidence" in its favor according to the guidelines of DeJong and Dave (2007). Therefore, I use the competitive search model as my baseline specification, although I report estimation results for both models.

I estimate the historical shocks that hit the economy over the last thirty years using the Kalman filter. Estimates from the competitive search model imply that the housing boom from 1997 to 2006 had two primary causes, a large increase in the fraction of eligible buyers, and expectations that productivity in the consumption sector would rise quickly in the future. These patterns reversed in the subsequent housing bust, as the fraction of eligible buyers fell and agents became more pessimistic about future productivity. Furthermore, the anticipated improvements to productivity turned out not to be realized over the period of the housing bust.

2 Model

I consider an economy in which households value a perishable consumption good and a durable housing good. Households invest the consumption good to create two types of physical capital, consumption capital and housing capital. Two representative, perfectly competitive firms rent capital and employ labor to produce the consumption and housing goods. Trade in the consumption good occurs on a frictionless spot market, but trade in the housing good is subject to search and matching frictions.

2.1 Households

Following Merz (1995), I assume there is a measure-one continuum of households, each comprising a 'large family' with a continuum of members. The members of the family pool income and consumption, but live in different dwellings[2]. At any given time some members of the household are satisfied with their dwelling, while others are dissatisfied. Household i maximizes the objective function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big(\underbrace{(x\tilde{C}_{it}^{\lambda} + (1-x)\tilde{N}_{it}^{\lambda})^{\frac{1}{\lambda}}}_{\substack{\text{flow utility from} \\ \text{consumption and} \\ \text{housing}}} - \underbrace{\frac{\sigma}{2} E_{it}^2 \gamma_t \tilde{B}_{it}}_{\substack{\text{disutility} \\ \text{from} \\ \text{search} \\ \text{effort}}} - \rho \underbrace{\frac{\tilde{L}_{it}^{1+\frac{1}{\mu}}}{1+\frac{1}{\mu}}}_{\substack{\text{disutility} \\ \text{from} \\ \text{labor} \\ \text{effort}}} \Big) \tag{1}$$

where x is a preference parameters that governs the household's relative taste for consumption and housing, ρ is a parameter that governs the households distaste for labor, C_{it} is household i's consumption in period t, N_{it} is the fraction of family members who live in a well-matched house in period t ('Non-traders'), E_{it} is the household's search effort in period t, B_{it} is the fraction of family members who do not live in a well-matched house in period t ('Buyers'), and γ_t is the fraction of Buyers who are able to search in a given period ('Active Buyers'). λ is the coefficient of relative risk aversion and μ is the Frisch elasticity of labor supply. The basic idea of the households' flow utility function is that households value consumption and housing, but dislike labor and expending effort searching for housing; the disutility from search effort is proportional to the number of buyers but convex in the effort exerted per active buyer. There is population growth in the model; variables marked with a ˜ have been divided by the number of agents per household. In effect, the household's preferences are over per-agent variables rather than household totals.

[2] This assumption is equivalent to assuming a complete markets allocation.

The household maximizes its utility subject to the constraints:

$$
\begin{aligned}
\underbrace{C_{it}}_{\text{consumption}} &= \underbrace{W_t L_{it}}_{\substack{\text{labor}\\\text{income}}} + \underbrace{R_t^C K_{it}^C + R_t^H K_{it}^H +}_{\text{capital income}} + \underbrace{q(\theta_t, E_t) P_t^R S_{it}}_{\substack{\text{revenue from}\\\text{selling houses}}} \\
&\quad - \underbrace{P_t^W Y_{it}^H}_{\substack{\text{money}\\\text{spent on}\\\text{new}\\\text{houses}}} - \underbrace{I_{it}^C - I_{it}^H}_{\substack{\text{investment}\\\text{in capital}}} - \underbrace{f(\theta_t, E_t, E_{it}) P_t^R \gamma_t B_{it}}_{\substack{\text{money spent on existing}\\\text{houses}}} \qquad (2)
\end{aligned}
$$

$$K_{it+1}^C = (1-\delta)K_{it}^C + I_t^C - \frac{\chi^C}{2}\Big(\frac{K_{it+1}^C - (1+g)K_{it}^C}{(1+g)K_{it}^C}\Big)^2 K_{it}^C \qquad (3)$$

$$K_{it+1}^H = (1-\delta)K_{it}^H + I_t^H - \frac{\chi^H}{2}\Big(\frac{K_{it+1}^H - (1+g)K_{it}^H}{(1+g)K_{it}^H}\Big)^2 K_{it}^H \qquad (4)$$

Equation 2 is the household's budget constraint. There are two types of capital in the economy, consumption sector capital, K^C, and housing sector capital, K^H, which command rental rates R^C and R^H respectively. The households hold the economy's productive capital stock directly; investment in consumption capital is denoted I^C and investment in housing capital is denoted I^H. P^W is the price at which the household buys newly built houses (the 'wholesale' price) and Y^H is the quantity of new houses purchased, so $P_{it}^W Y_{it}^H$ is the amount spent on newly built housing. P^R is the price of already existing housing (the 'retail' price), S is the number of existing houses the household markets for sale ('Sellers'), and $q(\theta_t, E_t)$ is the probability that a seller will meet a buyer given θ_t, the ratio of buyers to sellers in the housing market, and E_t, the average search effort expended by buyers. Later I will impose an assumption that a sale will occur any time a buyer meets a seller, so that $q(\theta_t, E_t)P_t^R S_{it}$ is the amount the household spends on already existing housing. Finally, $\gamma_t B_{it}$ is the number of buyers the household has on the market and $f(\theta_t, E_t, E_{it})$ is the probability that a buyer will meet a seller, so $f(\theta_t, E_t, E_{it})P_t^R \gamma_t B_{it}$ is the household's proceeds from selling existing houses.

Taken together, equation 2 says that the household's consumption in period t equals its wage income plus its capital income and proceeds from selling its stock of existing houses, minus its investment, its spending on newly built houses, and its spending on existing houses.

Equations 3 and 4 are the accumulation equations for capital in the consumption and housing sectors. They feature quadratic costs of adjustment to the level of the capital stock, the severity of which is parameterized by χ^C and χ^H.

The household also faces equations of motion for its numbers of buyers, houses for sale, and satisfied homeowners. Because the equations of motion are specific to the process for trade I assume in the housing market, I defer these equations to the subsection describing the Housing Market. Therefore, I will also defer discussion of the first order conditions (FOCs) of the household's problem.

2.2 Firms

There are two perfectly competitive firms in the economy, one of which produces the consumption good and one of which produces houses. Both firms rent capital and hire labor from the households, and the construction firm also rents land from the households. The consumption firm produces output according to the production function:

$$Y_t^C = Z_t^C (K_t^C)^{\nu_C} (L_t^C)^{1-\nu_C} \tag{5}$$

where Y_t^C is the production of the consumption good in period t, Z_t^C is total factor productivity in the consumption sector in period t, K_t^C is the capital stock employed in the consumption sector in period t, and L_t^C is the amount of labor employed in the consumption sector in period t. The price of output in the consumption sector is normalized to one. Therefore the consumption firm's problem is

$$\max_{K^C,L^C} Z_t^C (K_t^C)^{\nu_C} (L_t^C)^{1-\nu_C} - R_t^C K_t^C - W_t L_t^C \tag{6}$$

The FOCs of this problem are

$$R_t^C = \nu_C Z_t^C (K_t^C)^{\nu_C - 1} (L_t^C)^{1-\nu_C} \tag{7}$$

$$W_t = (1-\nu_C) Z_t^C (K_t^C)^{\nu_C} (L_t^C)^{-\nu_C} \tag{8}$$

The housing firm produces houses according to the production function:

$$Y_t^H = Z_t^H (K_t^H)^{\nu_H} (L_t^H)^{\rho_H} \tag{9}$$

where Y_t^H is the production of houses in period t, Z_t^H is total factor productivity in the housing sector in period t, K_t^H is the capital stock employed in the housing sector in period t, and L_t^H is the amount of labor employed in the housing sector in period t. I assume labor is freely mobile, so that the wage is the same across sectors. The housing firm's problem is

$$\max_{K^H,L^H} P_t^W Z_t^H (K_t^H)^{\nu_H} (L_t^H)^{\rho_H} - R_t^H K_t^H - W_t L_t^H \tag{10}$$

where, as noted above, P^W is the 'wholesale' price at which the housing firm sells newly built houses to the households as unmatched units. The FOCs of the housing firm's problem are

$$R_t^H = \nu_H P_t^W Z_t^H (K_t^H)^{\nu_H - 1} (L_t^H)^{\rho_H} \tag{11}$$

$$W_t = \rho_H P_t^W Z_t^H (K_t^H)^{\nu_H} (L_t^H)^{\rho_H - 1} \tag{12}$$

$$\tag{13}$$

In my calibration, I assume decreasing returns to scale in the housing sector. This assumption is meant to capture the idea that land acts as a fixed factor in the production of houses without explicitly modeling the details of land supply and demand. With no trend growth in productivity in the housing sector, the assumption of decreasing returns would lead to house prices steadily increasing with population. Therefore, I assume that Z_t^H grows at rate $1 - \nu_H - \rho_H$ over time. This assumption implies that in the absence of shocks, the price of housing will be constant over time.

2.3 The Housing Market

The housing market of this model exhibits search frictions. Buyers and sellers in the market for well-matched houses cannot transact on a frictionless spot market but instead form pairs according to a matching function that relates the numbers of active buyers and sellers into the number of successful matches. I assume that when buyers and sellers form a match, the seller's house is always a good match for the buyer. The implicit concept is that housing units and household preferences are heterogeneous, so that many houses a buyer visits will be ill-suited to their taste. A "match" between a buyer and a seller occurs when a buyer finds an appropriate house. The matching function is a reduced form way to capture the time consuming nature of search and matching in the housing market without specifying the microeconomic process by which matches are formed.

I consider two different matching functions in the model. The first is the Cobb-Douglas matching function that is standard in much of the literature (e.g., Wheaton 1990):

$$M(E_t, \gamma_t B_t, S_t) = A(E_t \gamma_t B_t)^{\phi} S_t^{1-\phi} \tag{14}$$

The second is the generalized urn-ball matching function of Head et al. (2010):

$$M(E_t, \gamma_t B_t, S_t) = AS_t(1 - e^{-\zeta \frac{E_t \gamma_t B_t}{S_t}}) \tag{15}$$

where M is the number of matches in a period. As is standard, I denote the 'tightness' of the housing market as

$$\theta_t = \frac{\gamma_t B_t}{S_t} \tag{16}$$

For convenience, I define the probability that a buyer exerting effort E_{it} meets a seller in a given period as

$$f(\theta, E_t, E_{it}) = \frac{E_{it} M(E_t, B_t, S_t)}{E_t \gamma_t B_t} = \begin{cases} AE_{it}(E_t\theta_t)^{\phi-1} & \text{w/Cobb-Douglas matching} \\ \frac{E_{it}A}{E_t\theta_t}(1 - e^{-E_t\theta_t\zeta}) & \text{w/ urn-ball matching} \end{cases} \tag{17}$$

At the cost of anticipating the equilibrium concept, in a symmetric equilibrium, in which all households choose the same search effort, $E_{it} = E_t$, the probability that a buyer will meet a seller will be

$$f(\theta_t, E_t) = f(\theta_t, E_t, E_t) = \frac{E_t M(E_t, B_t, S_t, \gamma_t)}{E_t \gamma_t B_t} = \begin{cases} AE_t^{\phi}\theta_t^{\phi-1} & \text{w/Cobb-Douglas matching} \\ \frac{A}{\theta_t}(1 - e^{-E_t\theta_t\zeta}) & \text{w/ urn-ball matching} \end{cases} \tag{18}$$

Similarly, I define the probability that a seller meets a buyer as

$$q(\theta_t, E_t) = \frac{M(E_t, B_t, S_t)}{S} = \begin{cases} A(E_t\theta_t)^{\phi} & \text{w/Cobb-Douglas matching} \\ A(1 - e^{-E_t\theta_t\zeta}) & \text{w/ urn-ball matching} \end{cases} \tag{19}$$

In the model, individuals remain in their current house so long as it remains 'well-matched', but each period well-matched homeowners face probability α of becoming mismatched. Once an individual becomes mismatched, the parent household no longer receives any utility from owning the house. Therefore, the parent household immediately puts the house for sale, and the individual begins looking for a new house to purchase. An individual who becomes mismatched is therefore simultaneously a seller and a potential buyer of housing. I assume that individuals who do not own a well-matched house live with other members of their household. Furthermore, I assume that the population of each household grows each period at rate g, with the new members being born as poorly-matched households. Finally, I assume that only a fraction γ_t of poorly matched individuals are able to search for a home each period. Implicitly, one could imagine financing or other constraints that prevent some households from searching in a given period.

The timing of each period is as follows:

1. Period starts.
2. Household receives flow utility from satisfied homeowners.
3. Housing market occurs
   Matches formed
   Bargaining and sales occur
4. Relocation shock hits
5. Production of goods and new houses
6. Housing company sells homeowners 'unmatched' new houses
7. Capital depreciates
8. Factors are paid and consumption occurs
9. New agents born
10. Period ends

Because matching is costly and time consuming, there is a surplus value associated with each match, which I will define in the section on Price Determination. I assume that buyers and sellers always exploit potential gains from trade, so that every match results in the sale of a house.

These assumptions give rise to the equations of motion for buyers (B_t), houses for sale (S_t), and non-traders (N_t):

$$\underbrace{B_{it+1}}_{\substack{\text{buyers}\\\text{next}\\\text{period}}} = \underbrace{B_{it}}_{\substack{\text{buyers}\\\text{this}\\\text{period}}} - \underbrace{(1-\alpha)}_{\substack{\text{no move}\\\text{shock}}} \underbrace{f(\theta_t, E_t, E_{it})}_{\substack{\text{probability}\\\text{buyer forms}\\\text{match}}} \underbrace{\gamma_t B_{it}}_{\substack{\text{active}\\\text{buy-}\\\text{ers}}} + \underbrace{\alpha N_{it}}_{\substack{\text{non-traders}\\\text{hit by move}\\\text{shock}}} + \underbrace{g(B_{it} + N_{it})}_{\substack{\text{population}\\\text{growth}}} \tag{20}$$

$$\underbrace{S_{it+1}}_{\substack{\text{sellers}\\\text{next}\\\text{period}}} = \underbrace{(1 - q(\theta_t, E_t))}_{\substack{\text{probability}\\\text{seller doesn't}\\\text{form match}}} \underbrace{S_{it}}_{\substack{\text{sellers}\\\text{this}\\\text{period}}} + \underbrace{\alpha}_{\substack{\text{move}\\\text{shock}}} (\underbrace{N_{it}}_{\substack{\text{non-}\\\text{traders}}} + \underbrace{f(\theta_t, E_t, E_{it})}_{\substack{\text{probability}\\\text{buyer forms}\\\text{match}}} \underbrace{\gamma_t B_{it}}_{\substack{\text{active}\\\text{buy-}\\\text{ers}}}) + \underbrace{Y^H_{it-2}}_{\substack{\text{new}\\\text{houses}}} \tag{21}$$

$$\underbrace{N_{it+1}}_{\substack{\text{non-}\\\text{traders}\\\text{next}\\\text{period}}} = \underbrace{(1-\alpha)}_{\substack{\text{no move}\\\text{shock}}} (\underbrace{N_{it}}_{\substack{\text{non-}\\\text{traders}\\\text{this}\\\text{period}}} + \underbrace{f(\theta_t, E_t, E_{it})}_{\substack{\text{probability}\\\text{buyer forms}\\\text{match}}} \underbrace{\gamma_t B_{it}}_{\substack{\text{active}\\\text{buy-}\\\text{ers}}}) \tag{22}$$

The first equation says that the number of buyers next period equals the number this period, less the buyers who purchase a house and do not receive a relocation shock, plus non-traders who receive a relocation shock and new agents. The second equation says that the number of homes for sale next period equals the number this period minus those sold, plus the homes for sale posted by newly mis-matched homeowners and new construction from two periods ago. The lag of two periods represents a time to build in the construction sector of approximately three quarters. The third equation says that the number of satisfied homeowners equals the number from last period, less those hit by a relocation shock, plus the number of buyers who successfully purchase a well-matched home. Because some agents are simultaneously buyers and sellers, it is not the case that $N_t + B_t + S_t$ equals the total population, which is $N_t + B_t$. The total housing stock in the economy is $N_t + S_t$.

2.4 Recursive Formulation of the Household's Problem

Given this structure for trade in the housing market, the household's problem can be re-formulated as a recursive problem with the following Bellman equation, where I have detrended the budget constraint and equations of motion to adjust for population growth:

$$\begin{aligned}
V(\tilde{B}_{it}, \tilde{S}_{it}, \tilde{N}_{it}, \tilde{K^C}_{it}, \tilde{K^H}_{it}) = \max_{\tilde{L}_{it}, E_{it}, \tilde{K}^C_{it+1}, \tilde{K}^H_{it+1}, \tilde{Y}^H_{it}} &\Big\{ (x\tilde{C}^{\lambda}_{it} + (1-x)\tilde{N}^{\lambda}_{it})^{\frac{1}{\lambda}} - \frac{\sigma}{2} E^2_{it}\gamma_t \tilde{B}_{it} \\
&- \rho \frac{\tilde{L}_{it}^{1+\frac{1}{\mu}}}{1+\frac{1}{\mu}} + \beta \mathbb{E}_t \big[V(\tilde{B}_{it+1}, \tilde{S}_{it+1}, \tilde{N}_{it+1}, \tilde{K^C}_{it+1}, \tilde{K^H}_{it+1}) \big] \Big\}
\end{aligned}$$

s.t.

$$\begin{aligned}
\tilde{C}_{it} &= W_t \tilde{L}_{it} + R^C_t \tilde{K^C}_{it} + R^H_t \tilde{K^H}_{it} + q(\theta_t, E_t) P^R_t \tilde{S}_{it} \\
&\quad - P^W_t \tilde{Y^H}_{it} - \tilde{I^C}_{it} - \tilde{I^H}_{it} - f(\theta_t, E_t, E_{it}) P^R_t \gamma_t \tilde{B}_{it} \\
\tilde{K^C}_{it+1} &= (1-\delta)\tilde{K^C}_{it} + \tilde{I^C}_{it} - \frac{\chi^C}{2} \Big(\frac{\tilde{K^C}_{it+1} - \tilde{K^C}_{it}}{\tilde{K^C}_{it}} \Big)^2 \tilde{K^C}_{it} \\
\tilde{K^H}_{it+1} &= (1-\delta)\tilde{K^H}_{it} + \tilde{I^H}_{it} - \frac{\chi^H}{2} \Big(\frac{\tilde{K^H}_{it+1} - \tilde{K^H}_{it}}{\tilde{K^H}_{it}} \Big)^2 \tilde{K^H}_{it} \\
(1+g)\tilde{B}_{it+1} &= \tilde{B}_{it} - (1-\alpha) f(\theta_t, E_t, E_{it}) \gamma_t \tilde{B}_{it} + \alpha \tilde{N}_{it} + g \\
(1+g)\tilde{S}_{it+1} &= (1 - q(\theta_t, E_t))\tilde{S}_{it} + \alpha(\tilde{N}_{it} f(\theta_t, E_t, E_{it}) \tilde{B}_{it}) + \tilde{Y^H}_{it-2} \\
(1+g)\tilde{N}_{it+1} &= (1-\alpha)(\tilde{N}_{it} + f(\theta_t, E_t, E_{it})\tilde{B}_{it})
\end{aligned}$$

For convenience, I define the marginal values to the household of having an additional buyer (V^B), seller (V^S), and non-trader (V^N):

$$
\begin{aligned}
\underbrace{V^B_{it}}_{\substack{\text{marginal}\\ \text{value}\\ \text{of}\\ \text{buyer}}} = & -\underbrace{MUC_{it}P^R_t}_{\substack{\text{house price}\\ \text{in utility}\\ \text{terms}}}\underbrace{f(\theta_t, E_t, E_{it})\gamma_t}_{\substack{\text{probability of}\\ \text{purchase}}} - \underbrace{\frac{\sigma}{2}E^2_{it}\gamma_t}_{\substack{\text{disutility}\\ \text{of}\\ \text{search}\\ \text{effort}}} + \beta\frac{1}{1+g}\mathbb{E}_t\Big[\underbrace{\big(1-(1-\alpha)f(\theta, E_t, E_{it})\gamma_t\big)V^B_{it+1}}_{\substack{\text{probability remain buyer times}\\ \text{marginal value of buyer next period}}} \\
& + \underbrace{\alpha f(\theta, E_t, E_{it})\gamma_t V^S_{it+1}}_{\substack{\text{probability become}\\ \text{seller times marginal}\\ \text{value of seller next}\\ \text{period}}} + \underbrace{(1-\alpha)f(\theta_t, E_t, E_{it})\gamma_t V^N_{it+1}}_{\substack{\text{probability become non-trader}\\ \text{times marginal value of}\\ \text{non-trader next period}}}\Big]
\end{aligned}
\tag{23}
$$

$$
\underbrace{V^S_{it}}_{\substack{\text{marginal}\\ \text{value}\\ \text{of}\\ \text{seller}}} = \underbrace{MUC_{it}P^R_t}_{\substack{\text{house price}\\ \text{in utlity}\\ \text{terms}}}\underbrace{q(\theta_t, E_t)}_{\substack{\text{probability}\\ \text{of sale}}} + \underbrace{(1-q(\theta_t, E_t))}_{\substack{\text{probability}\\ \text{seller next}\\ \text{period}}}\beta\frac{1}{1+g}\underbrace{\mathbb{E}_t\big[V^S_{it}\big]}_{\substack{\text{expected}\\ \text{marginal}\\ \text{value of}\\ \text{seller next}\\ \text{period}}}
\tag{24}
$$

$$
\underbrace{V^N_{it}}_{\substack{\text{marginal}\\ \text{value}\\ \text{of non-}\\ \text{trader}}} = \underbrace{MUN_{it}}_{\substack{\text{marginal flow}\\ \text{utility from}\\ \text{well-matched}\\ \text{home}}} + \beta\frac{1}{1+g}\mathbb{E}_t\big[\underbrace{\alpha(V^B_{it+1} + V^B_{it+1})}_{\substack{\text{probability of}\\ \text{move shock times}\\ \text{marginal values of}\\ \text{buyer and seller}\\ \text{next period}}} + \underbrace{(1-\alpha)V^N_{it+1}}_{\substack{\text{probability no}\\ \text{move shock times}\\ \text{marginal value of}\\ \text{non-trader next}\\ \text{period}}}\big]
\tag{25}
$$

where MUC_{it} denotes the marginal utility of consumption and MUN_{it} denotes the marginal flow utility of a well-matched house in epriod t. The household's Bellman equation features five control variables: K^C_{it+1}, K^H_{it+1}, Y^H_{it}, E_{it}, and $\tilde{L}_{it}$. The FOCs for these variables are:

$$\Big(1+\chi^C\frac{\tilde{K^C}_{it+1}-\tilde{K^C}_{it}}{\tilde{K^C}_{it}}\Big)M\tilde{U}C_{it}=\beta\mathbb{E}_t\Big[M\tilde{U}C_{it+1}\Big((1+R^C_{t+1}-\delta)+\frac{\chi^C}{2}\Big(\frac{\tilde{K^C}_{it+2}-\tilde{K^C}_{it+1}}{\tilde{K^C}_{it+1}}\Big)^2 + \chi^C\frac{\tilde{K^C}_{it+2}-\tilde{K^C}_{it+1}}{\tilde{K^C}_{it+1}}\Big)\Big] \tag{26}$$

$$\Big(1+\chi^H\frac{\tilde{K^H}_{it+1}-\tilde{K^H}_{it}}{\tilde{K^H}_{it}}\Big)M\tilde{U}C_{it}=\beta\mathbb{E}_t\Big[M\tilde{U}C_{it+1}\Big((1+R^H_{t+1}-\delta)+\frac{\chi^H}{2}\Big(\frac{\tilde{K^H}_{it+2}-\tilde{K^H}_{it+1}}{\tilde{K^H}_{it+1}}\Big)^2 + \chi^H\frac{\tilde{K^H}_{it+2}-\tilde{K^H}_{it+1}}{\tilde{K^H}_{it+1}}\Big)\Big] \tag{27}$$

$$M\tilde{U}C_{it}P^W_t=\beta^3\mathbb{E}_t[V^S_{it+3}] \tag{28}$$

$$\sigma E_{it}=\frac{\partial f(\theta,E_t,E_{it})}{\partial E_{it}}\Big(-M\tilde{U}C_{it}P^R_t+\beta\frac{1}{1+g}\times \mathbb{E}_t\big[(1-\alpha)(V^N_{it+1}-V^B_{it+1})\alpha V^S_{t+1}\big]\Big) \tag{29}$$

$$\tilde{L}_{it}=\big(M\tilde{U}C_{it}\frac{W_{it}}{\rho}\big)^{\mu} \tag{30}$$

Equations (26) and (27) are the household's Euler equations for consumption and housing capital, and reflect the quadratic costs of adjustment to both capital stocks. Equation (28) states that the wholesale price at which the household purchases unmatched houses from the construction firm, times the marginal utility of consumption, equals the discounted value of having an additional house for sale in three quarters. Equation (29) specifies the household's optimal search effort, at which the marginal disutility of additional search equals the marginal improvement in the probability of forming a match times the buyer's surplus from forming a match.

In a symmetric equilibrium in which all households choose the same search effort, equation (29), simplifies to

$$E_t^{2-\phi}=\frac{A(\theta_t)^{\phi-1}}{\sigma}\Big(-M\tilde{U}C_{it}P^R_t+\beta\frac{1}{1+g}\mathbb{E}_t\big[(1-\alpha)(V^N_{t+1}-V^B_{t+1})+\alpha V^S_{t+1}\big]\Big) \tag{31}$$

in the case of Cobb-Douglas matching, and to

$$E_t^2 = \frac{A(1-e^{-E_t\theta_t\zeta})}{\sigma\theta_t}\Big(-M\tilde{U}C_{it}P_t^R + \beta\frac{1}{1+g}\mathbb{E}_t\big[(1-\alpha)(V_{t+1}^N - V_{t+1}^B) + \alpha V_{t+1}^S\big]\Big) \quad (32)$$

in the case of urn-ball matching.

2.5 Price Determination

The buyer's surplus from purchasing a house at price P_t^R is the difference in utility from buying the house versus continuing to search, less the price of the house in utility terms:

$$\text{Buyer's Surplus} = \beta\frac{1}{1+g}\mathbb{E}_t\big[\underbrace{(1-\alpha)V_{t+1}^N}_{\substack{\text{no move}\\\text{shock}}} + \underbrace{\alpha V_{t+1}^S}_{\substack{\text{move}\\\text{shock}}} - \underbrace{V_{t+1}^B}_{\substack{\text{outside}\\\text{option}}}\big] - \underbrace{M\tilde{U}C_{it}P_t^R}_{\substack{\text{price in}\\\text{utility}\\\text{terms}}}$$

For the seller, selling a house at price P_t^R gives a payoff in utility terms of $-x\tilde{C}_t^{-\lambda}P_t^R$, while the payoff from not selling is the continuation value of keeping the house on the market, $\beta\frac{1}{1+g}\mathbb{E}_t\big[V_{t+1}^S\big]$. The seller's surplus is the difference:

$$\text{Seller's Surplus} = \underbrace{M\tilde{U}C_{it}P_t^R}_{\substack{\text{price in}\\\text{utility}\\\text{terms}}} - \beta\frac{1}{1+g}\mathbb{E}_t\big[\underbrace{V_{t+1}^S}_{\substack{\text{outside}\\\text{option}}}\big] \quad (33)$$

The match surplus is the total surplus to both parties from completing the transaction rather than parting ways, so it is the sum of the buyer's surplus and the seller's surplus:

$$\text{Match Surplus} = \beta\frac{1}{1+g}\mathbb{E}_t\big[(1-\alpha)(V_{t+1}^N - V_{t+1}^B) + \alpha(V_{t+1}^S - V_{t+1}^B) - V_{t+1}^S\big] \quad (34)$$

The purchase price of the house serves to divide the match surplus between the buyer and the seller. I will denote the share of the match surplus accruing to the buyer as η_t, postponing briefly a discussion of how this share is determined. Setting the buyer's surplus equal to η_t times the total match surplus gives the following rule for price determination:

$$P_t^R = \frac{1}{M\tilde{U}C_{it}}\beta\frac{1}{1+g}\mathbb{E}_t\Big[(1-\eta_t)\Big((1-\alpha)V_{t+1}^N - V_{t+1}^B + \alpha V_{t+1}^S\Big) + \eta_t V_{t+1}^S\Big] \tag{35}$$

Traditionally in the housing search and matching literature, it is assumed that the buyer receives a fixed share of the match surplus: $\eta_t = \eta$. This sharing rule for the surplus is motivated as the result of an asymmetric Nash bargain between the buyer and seller. I will call the model with this sharing rule the "random search" model.

More recently, Diaz and Jerez (2009) and Head et al. (2010) have explored search and matching models of the housing market under the competitive search framework introduced by Moen (1997) in the context of labor search. In the competitive search environment, sellers post list prices for their houses, and can credibly commit not to bargain with buyers over the price after a match forms. Moen shows that in a competitive search equilibrium, the share of the match surplus going to each party equals the elasticity of the matching function with respect to that party's side of the market. In such an environment, when the housing market "heats up", so that there are more buyers relative to sellers, the share of the match surplus going to the sellers will rise, while when the market "cools down", the share of the match surplus going to buyers will rise. Therefore, the competitive search framework has the potential to add volatility to house prices relative to the random search framework, in which the share of the match surplus accruing to the buyer remains constant.

Diaz and Jerez and Head et al. show that in the housing market, a competitive search equilibrium can be implemented similarly to a standard search equilibrium by adding an equation to endogenize η, the share of the match surplus going to the buyer, to be equal to the elasticity of the matching function with respect to buyers (in the case of my model, this will be effective buyers, or effort times buyers):

$$\eta_t = \frac{\partial M(E_t B_t, S_t)}{\partial E_t B_t} \frac{E_t B_t}{M(E_t B_t, S_t)} \tag{36}$$

With a Cobb-Douglas matching function, the elasticity of the matching function with respect to both sides of the market is constant. In this case, the competitive search model is equivalent to the random search model if the buyer's share of the match surplus is equal to the elasticity of the matching function with respect to buyers, i.e. if $\eta = \phi$. This equality, known as the Hosios condition after Hosios (1990), is a necessary condition for efficiency in the random search model. Therefore, as in much of the literature, I impose this condition in my calibration, implying that the two methods of determining the sharing rule are equivalent.

To allow the competitive search model to generate different results than the random search model, I follow Diaz and Jerez and Head et al. in using the urn-ball matching function described previously. In this case, which I will call the "competitive search" model, the buyer's share of the match surplus equals the elasticity of the matching function with respect to search effort times active buyers:

$$\eta_t = \frac{\partial M(E_t \gamma_t B_t, S_t)}{\partial E_t \gamma_t B_t} \frac{E_t \gamma_t B_t}{M(E_t \gamma_t B_t, S_t)} = \frac{E_t \theta_t \zeta}{e^{E_t \theta_t \zeta} - 1} \tag{37}$$

2.6 Sources of Stochastic Variation

I include five sources of stochastic variation in the model: a shock to the level of technology in the consumption sector, a shock to the level of technology in the housing sector, a shock to expectations of the future level of technology in the housing sector, a shock to the fraction of buyers who are active, and a shock to preferences for housing. I assume

Z_t^C, Z_t^H, γ_t, and y_t follow the $AR(1)$ processes:

$$\ln(Z_t^C) = \psi_C \ln(Z_{t-1}^C) + (1-\psi_C)\ln(\overline{Z^C}) + \epsilon_t^C + \epsilon_{t-20}^A \tag{38}$$

$$\epsilon_t^C \sim N(0, \sigma_C^2) \tag{39}$$

$$\epsilon_t^A \sim N(0, \sigma_A^2) \tag{40}$$

$$\ln(Z_t^H) = \psi_H \ln(Z_{t-1}^H) + (1-\psi_H)\ln(\overline{Z^H}) + \epsilon_t^H \tag{41}$$

$$\epsilon_t^H \sim N(0, \sigma_H^2) \tag{42}$$

$$\ln(\gamma_t) = \psi_\gamma \ln(\gamma_{t-1}) + (1-\psi_\gamma)\ln(\overline{\gamma}) + \epsilon_t^\gamma \tag{43}$$

$$\epsilon_t^\gamma \sim N(0, \sigma_\gamma^2) \tag{44}$$

where bars above variables represent their steady state values. I assume the shocks are independently distributed.

I will call the shock ϵ_t^A an anticipation shock, as it represents an anticipated movement in the future level of consumption productivity. Because I take the model period to be one quarter, anticipation shocks concern productivity changes 5 years in the future. These anticipation shocks are on average "correct" in the sense that the actual technology level in the housing sector equals the anticipated level in expectation. However, the presence of a contemporaneous or unanticipated shock to housing technology allows me to study the case of an unrealized expectation of a change in future productivity, which would correspond to ϵ_t^C being exactly equal to $-\epsilon_{t-20}^A$. This scenario is in the spirit of Beaudry and Portier's (2004) "Pigou Cycles".

2.7 Equilibrium

I define an equilibrium of the random search model as follows:

Definition A *recursive symmetric Nash equilibrium with random search and bargaining* of this model is a set of policy functions for the households and firms, equations of motion for the stocks of capital, buyers, sellers, and non-traders, and prices for factors of production, new houses, and existing houses, such that:

1. Households maximize their utility taking factor prices and the price of new houses as given;
2. Firms maximize their profits taking all prices as given;
3. The consumption good, new housing, and factor markets clear;
4. Every household chooses the same search effort;
5. The number of sales in of existing houses is consistent with the matching function; and
6. The price of existing houses is determined through asymmetric Nash bargaining with buyers receiving share η of the match surplus.

For the model with competitive search, my equilibrium definition is slightly different:

Definition A *recursive symmetric competitive search equilibrium* of this model is a set of policy functions for the households and firms, equations of motion for the stocks of capital, buyers, sellers, and non-traders, and prices for factors of production, new houses, and existing houses, such that:

1. Equilibrium conditions 1-5 above hold; and
2. The price of existing houses is such that the share of the match surplus accruing to the buyer equals the elasticity of the matching function with respect to effective buyers (search effort times active buyers).

Please see Appendix A for a complete list of equations characterizing both equilibria.

3 Empirics

3.1 State Space Representation

The model above can be linearized around its steady state equilibrium to give the following state space representation:

$$Z_t = \overline{Z}(\Theta) + \mathbf{B}(\Theta)Z_{t-1} + \mathbf{G}(\Theta)e_t \tag{45}$$

$$Y_t = \mathbf{H}Z_t \tag{46}$$

$$E[e_t e_t'] = \mathbf{V}(\Theta) \tag{47}$$

In the transition equation, (45), Θ is a vector of the structural parameters of the model, $\overline{Z}(\Theta)$ is a vector of the steady state values of the model variables (which are functions of Θ), and Z_t is a vector of the deviations of the model variables from their steady state values. $\mathbf{B}(\Theta)$ is a system matrix that relates this period's deviations from steady state to last period's. Finally, $\mathbf{G}(\Theta)$ is a policy function matrix and e_t is the vector of structural shocks to the economy.

In the observation equation, (46), Y_t is a vector of the observable variables I will use to estimate the model. $\mathbf{H}$ is a matrix of ones and zeros that selects the variables to be observed. In equation (47), $\mathbf{V}(\Theta)$ is the variance-covariance matrix of the shock process; I impose that the shocks are i.i.d., so $\mathbf{V}(\Theta)$ is a diagonal matrix.

Together, equations (45) and (46) form a system of Kalman filter equations. We can use the Kalman filter recursions to evaluate the log-likelihood of the model conditional on the structural parameters Θ and the observed data series Y_t.

3.2 Observed Data Series

I use GDP, the price of existing homes, sales volumes, and starts as my observable data series. Because I have four shocks in the model, using four observable data series allows me not to include any observation errors in the observation equation (46). The model makes different predictions for how these series will react to each of the four shocks in the model, so using these series in the estimation should allow for successful identification of the parameters of the shock process and the historical shocks.

I use the following procedure to match the data series I am using as closely as possible to the conceptual variables in my model. For all variables that are available on a monthly basis, I take simple averages to construct quarterly values. Next I convert all nominal variables to 2010 dollars using the CPI-U. Because the unit of analysis in my model is the household, I construct the GDP, Sales, and Starts series on a per household basis. To calculate the number of households, I divide the quarterly total population, which I construct as the average of the monthly population over the quarter, by the average household size provided by the Census. Unfortunately, average household size is only provided annually; I use a cubic spline to interpolate quarterly values. I then take logs of each series and regress the log values on a linear time trend, and for the not seasonally adjusted series, a set of quarter dummies. Finally I add the deviations from the linear time trend to the average value for each series over the sample period, 1982q3-2010q4.

The house price series I use is the CoreLogic Single Family Detached House Price Index (HPI). Because the CoreLogic HPI is not expressed in dollar terms, I normalize it to $194,592, the value of the FHFA U.S. single family detached HPI, in 2000q1. For my sales volume series, I take the total of new single family houses sold from the Census Bureau and existing single family houses sold from the National Association of Realtors (NAR). Using the total of new and existing home sales is conceptually appropriate because in the model, all new houses are immediately sold on a frictionless spot market to the households, who then market them for sale on the frictional housing market along with previously built homes. For my months' supply series, I add the inventory of existing single family homes published by the NAR to the number of newly constructed homes for sale published by the Census, and divide by the sum of existing single family home sales reported by the NAR and new single family houses sold reported by the Census. For my starts series, I use single family starts from the Census. The resulting series are shown in Figures 1 and 2.

3.3 Calibrated Parameters and Steady State

I calibrate the parameters that affect the steady state but not the shock process of the model, which is equivalent to imposing a degenerate prior distribution for their values in the estimation procedure. Table 2 shows the calibrated parameters. I take the model period to be one quarter. The only parameters that differ between the random and competitive search models concern the matching function and disutility of search effort. For these parameters, I calibrate the competitive search model to have the same steady state values of the buyer's share of the match surplus, η, search effort, E, and months' supply of housing as the random search model.

I also impose unit roots in the technology shocks in the model. I have experimented with estimating the persistence of these shocks; the results do not change appreciably. Finally, I calibrate the level of capital adjustment costs in both sectors, χ_C and χ_H, to be zero[3].

Some key steady state values implied by these parameters are described in Table 3. The steady state is the same in the random and competitive search models. Some steady state values bear discussion because they deviate from values in the data. The steady state proportion of the labor force, $\overline{L^H}$, is too low at 1.2%. This is chiefly because I calibrate my model using only single family home construction, whereas the set of all construction workers includes those working in multi-family and non-residential construction. Similarly, the number of sales per household is 1.2% per quarter. This implies an unrealistically long period between moves. This discrepancy results from the exclusion of multi-family dwellings from the sales figures I use to calibrate the model, as

[3]I may re-introduce capital adjustment costs into the estimation procedure in the future

well as the absence of renters from the model.

3.4 Estimation

I use the random walk Metropolis Hastings Algorithm to estimate the standard deviations of the shocks and the persistence of the housing preference shock. Table 4 displays the prior distributions I specify for these parameters.

I run the sampler for 40,000 iterations and drop the first 45% before conducting posterior simulations. Figure 3 illustrates the prior (in gray) and posterior (in black) distributions for each of the parameters to be estimated. The posterior means and standard deviations of the estimated parameters are listed in Table 5, and are illustrated in Figure 3. The posterior distributions for most parameters are similar for the random and competitive search models. In both models, most parameters are tightly identified by the estimation procedure.

The Laplace approximation of the log marginal density is 745.7 for the competitive search model and 715.5 for the random search model. According to DeJong and Dave, the implied poseterior odds ratio constitutes "decisive evidence" in favor of the competitive search model.

4 Analysis

4.1 Impulse Responses

I linearize the system of equations around the steady state to find impulse responses to the shocks in the model. Figures 4 through **??** show the impulse response functions for each of the shocks, which are normalized to be one standard deviation in size. The plotted values in all impulse responses are proportional deviations from the variable's steady state (i.e. a value of 0.01 is a 1% deviation from the steady state value). The time period covered is 240 quarters.

In response to an anticipated increase in consumption productivity, GDP is essentially unchanged on impact while construction and prices rise and sales rise a bit. When the shock is realized, construction falls sharply and GDP rises[4]. Prices remain at their new, higher level after the productivity shock is realized.

In response to an unanticipated consumption productivity shock, GDP, prices, and sales all rise on impact. Construction exhibits a hump-shaped response, while months' supply initally falls before rising above its steady state level.

The random and competitive search models respond very differently to the eligible buyers shock. In both cases sales rise sharply on impact and the market becomes much tighter, as represented by lower months' supply. In the random search model the fixed sharing rule for dividing the match surplus mutes the effect on prices, and therefore on construction. In the competitive search model, the tighter market gives the sellers greater bargaining power and a greater share of the match surplus, so the rise in prices is much more pronounced. The higher prices cause construction to rise sharply as well. Again, GDP is essentially flat in response to the shock.

In response to a housing productivity shock, GDP rises by a small amount, construction rises and house prices fall. Sales and months' supply both rise. Again, the impulse responses are quite similar between the random and competitive search models.

The differing reactions of GDP, prices, construction, and sales in response to the different shocks allow the estimation procedure to identify the parameters of the shock process and the historical shocks that have hit the housing market over the past thirty years.

4.2 Estimated Shocks

Figure 8 illustrates the smoothed shocks from the estimation procedure for the competitive model. Several patterns in particular emerge from the figure. First, there were

[4]The technology shocks in the model have a unit root, so the shocks will generally have permanent effects.

a series of positive shocks to anticipated consumption productivity in the early 2000s. From 2001q1 to 2004q4, the anticipated productivity shock was positive in all quarters but one, for a cumulative increase of 0.41 log points. In the enusing years this pattern reverses sharply, with overwhelmingly negative shocks to anticipated consumption productivity. Furthermore, in the period 2006q1-2009q4, a series of negative shocks to unanticipated productivity almost perfectly counteracts the earlier positive shocks to anticipated productivity (in fact, the cumulative size of these shocks at -0.48 log points is larger than the positive productivity shocks). There is also a large series of positive shocks to the fraction of eligible buyers beginning in the mid-1990s and intensifying in the 2000s, which reverses sharply in 2007. Finally, a number of negative shocks to housing productivity in the 2000s contributed to rising prices during the housing boom. Figure 9 illustrates the historical decomposition of the change in house prices over the sample period into the changes due to each shock.

4.3 Simulations

Because the model features unit root shocks, I use Monte Carlo simulations to assess the model's dynamic behavior. Tables 6 and 7 show the results of simulating the competitive and random search models using the mean estimated parameter values. For each model, I simulate 114 quarters of data (the same number as in my observed sample) 500 times. The tables show the same statistics for the simulated data that Table 1 shows for the actual data. Therefore, comparison with Table 1 should help in evaluating the model's empirical performance.

Qualitatively, the model is able to generate most of the observed correlations between key housing market variables. Prices, sales, and starts are positively correlated, and prices are comove positively with GDP. Starts are negatively correlated with months' supply and the vacancy rate. However, there is a slight negative correlation between starts and GDP, contrary to what is observed in the data. The competitive search model does a better job matching the observed correlations than the random search model, although neither model is able to match the strength of the observed correlations between prices, sales, and starts quantitatively.

5 Conclusion

This paper presents a search and matching model of the housing market embedded in a DSGE framework. Conditional on the observed data, a model with competitive search is more likely than a model with random search and bargaining. The model reproduces many of the stylized facts of the housing market, most notably the positive co-movement of prices, sales, and starts, and the negative co-movement of starts and months' supply. The estimation results imply that the recent housing boom was driven by a large increase in the fraction of eligible buyers and anticipated increases in future productivity in the consumption sector, while the ensuing bust was caused by a sharp reversal of these trends in conjunction with a series of unanticipated negative shocks to consumption productivity. I interpret this pattern as suggesting that the housing boom and bust were driven in part by expectations of above trend productivity growth that later turned out to be unfounded.

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Appendix

A Equilibrium Equations

The following system of equations characterizes a recursive Nash equilibrium of this model (the i subscripts have been dropped because households are identical):

$$(1+g)\tilde{K}^C_{t+1} = (1-\delta)\tilde{K}^C_t + \tilde{I}^C_t - \frac{\chi^C}{2}\Big(\frac{\tilde{K}^C_{t+1}-\tilde{K}^C_t}{\tilde{K}^C_t}\Big)^2\tilde{K}^C_t \tag{A.1}$$

$$(1+g)\tilde{K}^H_{t+1} = (1-\delta)\tilde{K}^H_t + \tilde{I}^H_t - \frac{\chi^H}{2}\Big(\frac{\tilde{K}^H_{t+1}-\tilde{K}^H_t}{\tilde{K}^H_t}\Big)^2\tilde{K}^H_t \tag{A.2}$$

$$(1+g)\tilde{B}_{t+1} = B_t - (1-\alpha)f(\theta_t,E_t)\gamma_t\tilde{B}_t + \alpha\tilde{N}_t \tag{A.3}$$

$$(1+g)\tilde{S}_{t+1} = (1-q(\theta_t,E_t))\tilde{S}_t + \alpha(\tilde{N}_t + f(\theta_t,E_t)\gamma_t\tilde{B}_t) + \tilde{Y}^H_{t-2} \tag{A.4}$$

$$(1+g)\tilde{N}_{t+1} = (1-\alpha)(\tilde{N}_t + f(\theta_t,E_t)\gamma_t\tilde{B}_t) \tag{A.5}$$

$$\begin{aligned} M\tilde{U}C_t &= \beta\mathbb{E}_t\Big[M\tilde{U}C_{t+1}\Big((1+R^C_{t+1}-\delta) + \frac{\chi^C}{2}(\frac{K^C_{t+2}-K^C_{t+1}}{K^C_{t+1}})^2 \\ &\qquad + \chi^C\frac{K^C_{t+2}-K^C_{t+1}}{K^C_{t+1}}\Big)\Big]\Big(1+\chi^C\frac{K^C_{t+1}-K^C_t}{K^C_t}\Big)^{-1} \end{aligned} \tag{A.6}$$

$$\begin{aligned} M\tilde{U}C_t &= \beta\mathbb{E}_t\Big[M\tilde{U}C_{t+1}\Big((1+R^H_{t+1}-\delta) + \frac{\chi^H}{2}(\frac{K^H_{t+2}-K^H_{t+1}}{K^H_{t+1}})^2 \\ &\qquad + \chi^H\frac{K^H_{t+2}-K^H_{t+1}}{K^H_{t+1}}\Big)\Big]\Big(1+\chi^H\frac{K^H_{t+1}-K^H_t}{K^H_t}\Big)^{-1} \end{aligned} \tag{A.7}$$

$$M\tilde{U}C_tP^W_t = \beta^3\mathbb{E}_t[V^S_{t+3}] \tag{A.8}$$

$$E_t^{2-\phi} = \frac{A(\theta_t)^{\phi-1}}{\sigma}\Big(-M\tilde{U}C_tP^R_t + \beta\frac{1}{1+g}\mathbb{E}_t\big[(1-\alpha)(V^N_{t+1}-V^B_{t+1}) + \alpha V^S_{t+1}\big]\Big) \tag{A.9}$$

$$\begin{aligned} V^B_t &= -M\tilde{U}C_tP^R_tf(\theta,E_t) - \frac{\sigma}{2}E_t^2 + \beta\frac{1}{1+g}\mathbb{E}_t\big[\big(1-(1-\alpha)f(\theta,E_t)\gamma_t\big)V^B_{t+1} \\ &\qquad + \alpha f(\theta,E_t)\gamma_tV^S_{t+1} + (1-\alpha)f(\theta,E_t)\gamma_tV^N_{t+1}\big] \end{aligned} \tag{A.10}$$

$$V^S_t = M\tilde{U}C_tP^R_tq(\theta_t,E_t) + (1-q(\theta_t,E_t))\beta\frac{1}{1+g}\mathbb{E}_t\big[V^S_{t+1}\big] \tag{A.11}$$

$$V^N_t = M\tilde{U}N_t + \beta\frac{1}{1+g}\mathbb{E}_t\big[(\alpha V^B_{it} + (1-\alpha)V^N_{t+1}\big] \tag{A.12}$$

$$\tilde{Y}^C_t = Z^C_t(\tilde{K}^C_t)^{\nu_C}(\tilde{L}^C_t)^{1-\nu_C} \tag{A.13}$$

$$\tilde{Y}^H_t = \tilde{Z}^H_t(\tilde{K}^H_t)^{\nu_H}(\tilde{L}^H_t)^{\rho_H} \tag{A.14}$$

$$R^C_t = \nu_CZ^C_t(\tilde{K}^C_t)^{\nu_C-1}(\tilde{L}^C_t)^{1-\nu_C} \tag{A.15}$$

$$W_t = (1-\nu_C)Z^C_t(\tilde{K}^C_t)^{\nu_C}(\tilde{L}^C_t)^{-\nu_C} \tag{A.16}$$

$$R_t^H = \nu_C \tilde{Z}_t^H (\tilde{K}_t^C)^{\nu_H - 1} (\tilde{L}_t^C)^{\rho_H} \tag{A.17}$$

$$W_t = \rho_H P_t^W \tilde{Z}_t^H (\tilde{K}_t^H)^{\nu_H} (\tilde{L}_t^H)^{\rho_H - 1} \tag{A.18}$$

$$P_t^R = \frac{1}{M\tilde{U}C_t} \beta \frac{1}{1+g} \mathbb{E}_t \Big[(1-\eta_t) \Big((1-\alpha) V_{t+1}^N - V_{t+1}^B + \alpha V_{t+1}^S \Big) + \eta_t V_{t+1}^S \Big] \tag{A.19}$$

$$M(E_t, \gamma_t \tilde{B}_t, \tilde{S}_t) = A(E_t \gamma_t \tilde{B}_t)^{\phi} \tilde{S}_t^{1-\phi} \tag{A.20}$$

$$\theta_t = \frac{\gamma_t \tilde{B}_t}{\tilde{S}_t} \tag{A.21}$$

$$f(\theta, E_t, E_{it}) = A E_{it} (E_t \theta_t)^{\phi - 1} \tag{A.22}$$

$$q(\theta_t, E_t) = A(E_t \theta_t)^{\phi} \tag{A.23}$$

$$\tilde{Y}_t^C = \tilde{C}_t + \tilde{I}_t^C + \tilde{I}_t^H \tag{A.24}$$

$$\tilde{L}_t = \tilde{L}_t^C + \tilde{L}_t^H \tag{A.25}$$

$$\tilde{L}_t = \big(M\tilde{U}C_t \frac{W_t}{\rho} \big)^{\mu} \tag{A.26}$$

$$\ln(\tilde{Z}_t^H) = \psi_H \ln(\tilde{Z}_{t-1}^H) + (1-\psi_H) \ln(\overline{Z^H}) + \epsilon_t^H \tag{A.27}$$

$$\ln(Z_t^C) = \psi_C \ln(Z_{t-1}^C) + (1-\psi_C) \ln(\overline{Z^C}) + \epsilon_t^C + \epsilon_{t-20}^A \tag{A.28}$$

$$\ln(\gamma_t) = \psi_\gamma \ln(\gamma_{t-1}) + (1-\psi_\gamma) \ln(\overline{\gamma}) + \epsilon_t^\gamma \tag{A.29}$$

$$G\tilde{D}P_t = \tilde{Y}_t^C + \overline{P^W} \tilde{Y}_t^H \tag{A.30}$$

$$TOM_t = \frac{1}{q(\theta_t, E_t)} \tag{A.31}$$

$$GDP_{obs,t} = \ln(GDP) \tag{A.32}$$

$$TOM_{obs,t} = \ln(3TOM_t) \tag{A.33}$$

$$PR_{obs,t} = \ln(PR_t) \tag{A.34}$$

$$Sales_{obs,t} = \ln(Sales_t) \tag{A.35}$$

$$Starts_{obs,t} = \ln(Y_t^H) \tag{A.36}$$

$$S_{obs,t} = \ln(S) \tag{A.37}$$

$$M\tilde{U}C_t = x\tilde{C}_t^{\lambda-1} (x\tilde{C}_t^{\lambda} + (1-x)\tilde{N}_t^{\lambda})^{\frac{1}{\lambda}-1} \tag{A.38}$$

$$M\tilde{U}N_t = (1-x)\tilde{N}_t^{\lambda-1} (x\tilde{C}_t^{\lambda} + (1-x)\tilde{N}_t^{\lambda})^{\frac{1}{\lambda}-1} \tag{A.39}$$

As in Iacoviello and Neri (2010) and Lambertini et al. (2010), I calculate GDP using the steady state price of housing. In the last six equations, I calculate the simulated data series in logs, which is how they are expressed in my estimation procedure, Figures 1 and 2, and Tables 1, 7, and 6. Note that I equate time on the market with months' supply in the data; I multiply the model's time on the market by 3 because the model period is quarterly, not monthly. Finally, I equate S_t, the number of sellers, with the homeowner vacancy rate from the Census Housing Vacancy Survey, whereas in reality many homes for sale remain occupied.

In the competitive search model, I replace equations A.20, A.22, A.23, and A.9 with the following equations, respectively (the only change in the price setting equation is that the buyer's share of the match surplus, η, is time varying):

$$M(E_t, \gamma_t \tilde{B}_t, \tilde{S}_t) = AS_t(1 - e^{-\zeta \frac{E_t \gamma_t \tilde{B}_t}{\tilde{S}_t}}) \tag{A.40}$$

$$f(\theta_t, E_t) = \frac{E_{it} A}{E_t \theta_t}(1 - e^{-E_t \theta_t \zeta}) \tag{A.41}$$

$$q(\theta_t, E_t) = A(1 - e^{-E_t \theta_t \zeta}) \tag{A.42}$$

$$E_t^2 = \frac{A}{\sigma \theta_t}(1 - e^{-E_t \theta_t \zeta})\Big(- M\tilde{U}C_t P_t^R + \tag{A.43}$$

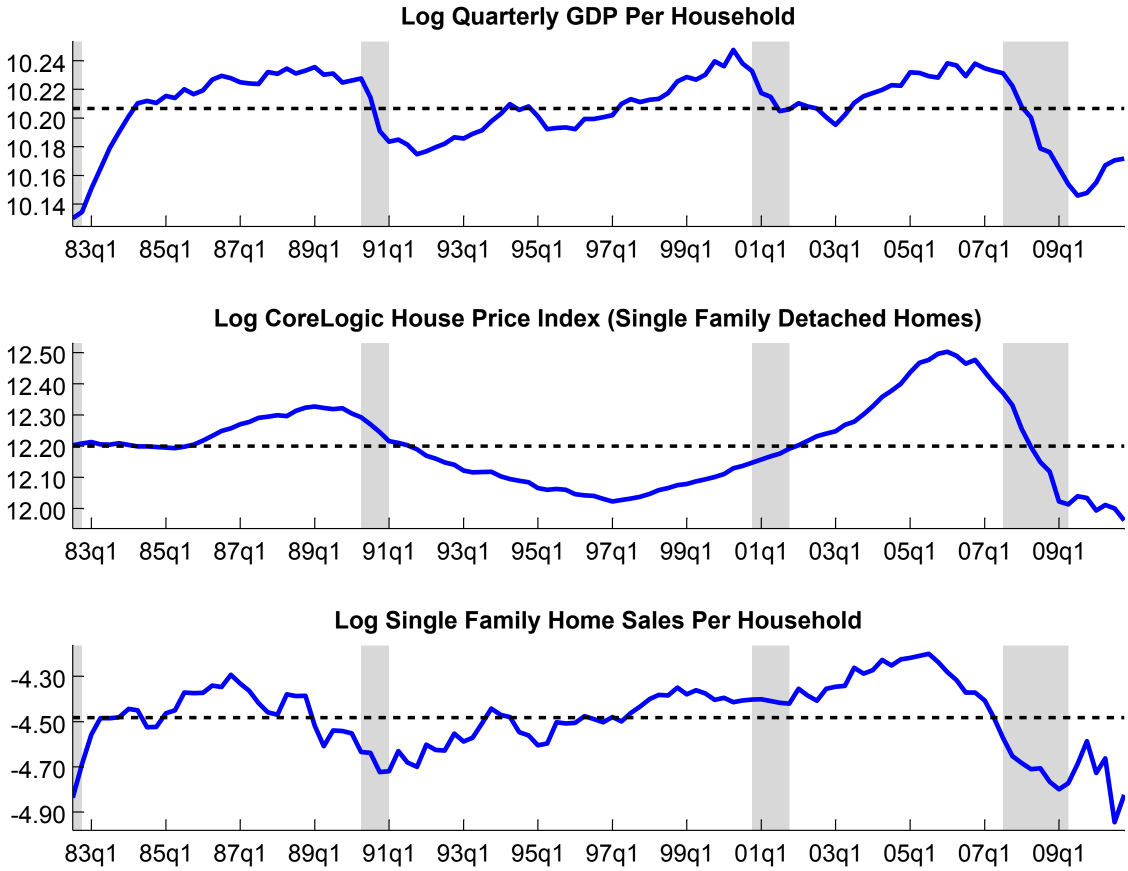
$$\beta \frac{1}{1+g} \mathbb{E}_t \big[(1-\alpha)(V_{t+1}^N - V_{t+1}^B) + \alpha V_{t+1}^S\big]\Big) \tag{A.44}$$

I also add an equation for the determination of the buyer's share of the match surplus:

$$\eta_t = \frac{E_t \theta_t \zeta}{e^{E_t \theta_t \zeta} - 1} \tag{A.45}$$

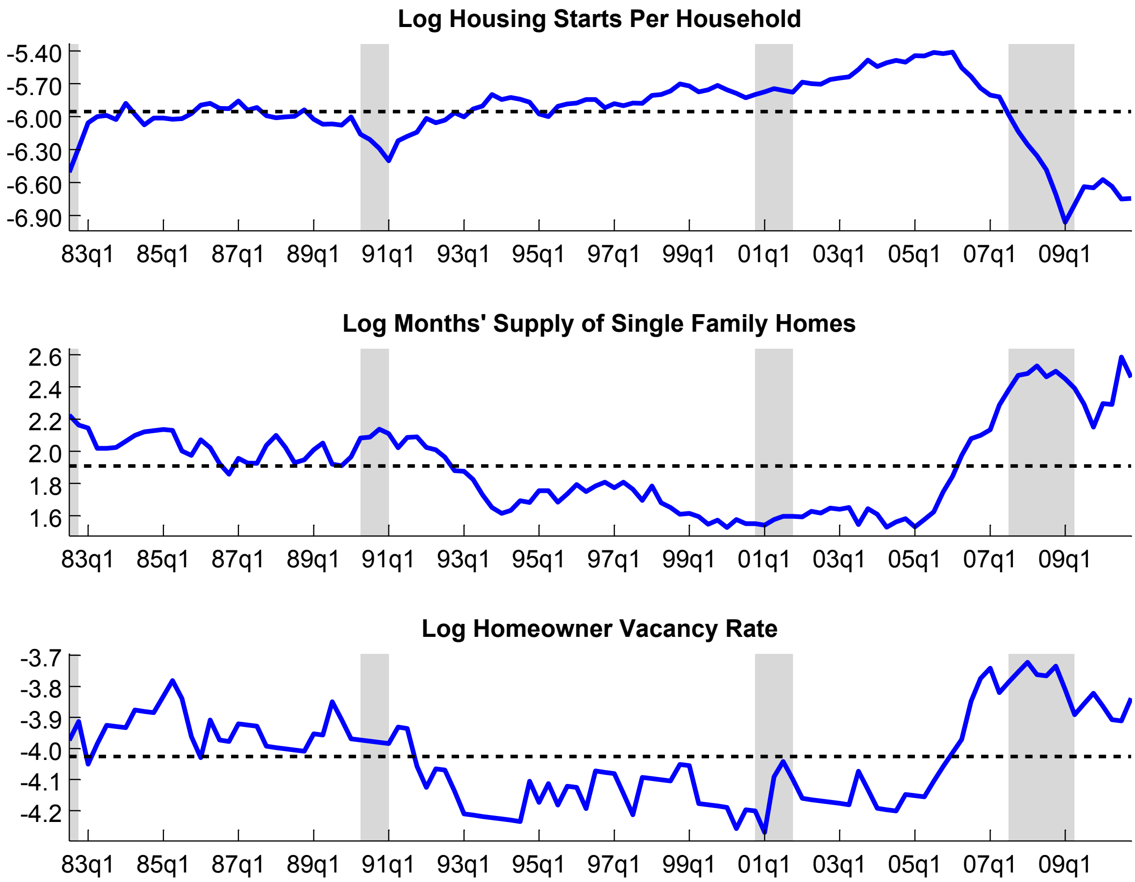
B Tables and Figures

Figure 1: Housing Market Time Series 1983q1-2010q4



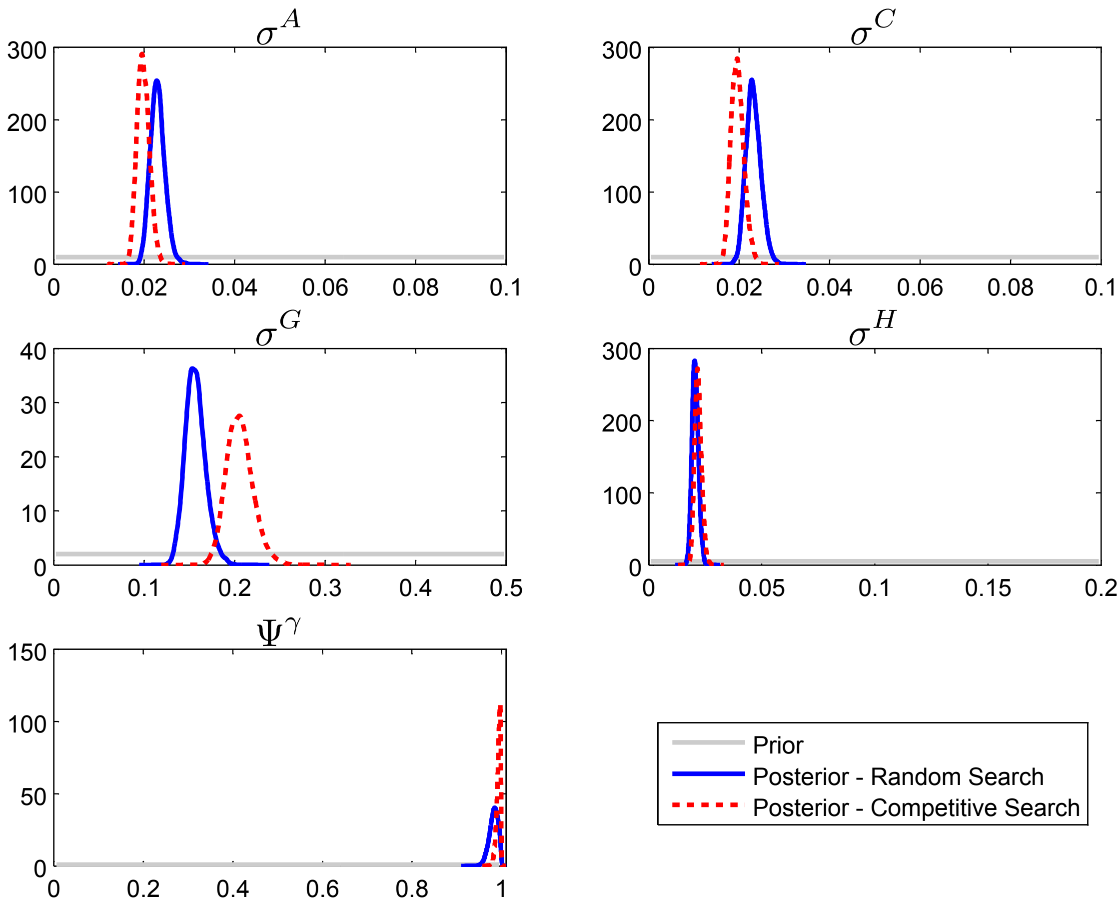
Series are seasonally adjusted log values from 1982q3-2010q4, expressed as deviations from linear trend plus average value over sample period. Shaded areas are NBER recession dates. Please see Observed Data Series section for details on series construction.

Figure 2: Housing Market Time Series 1983q1-2010q4 (cont'd.)



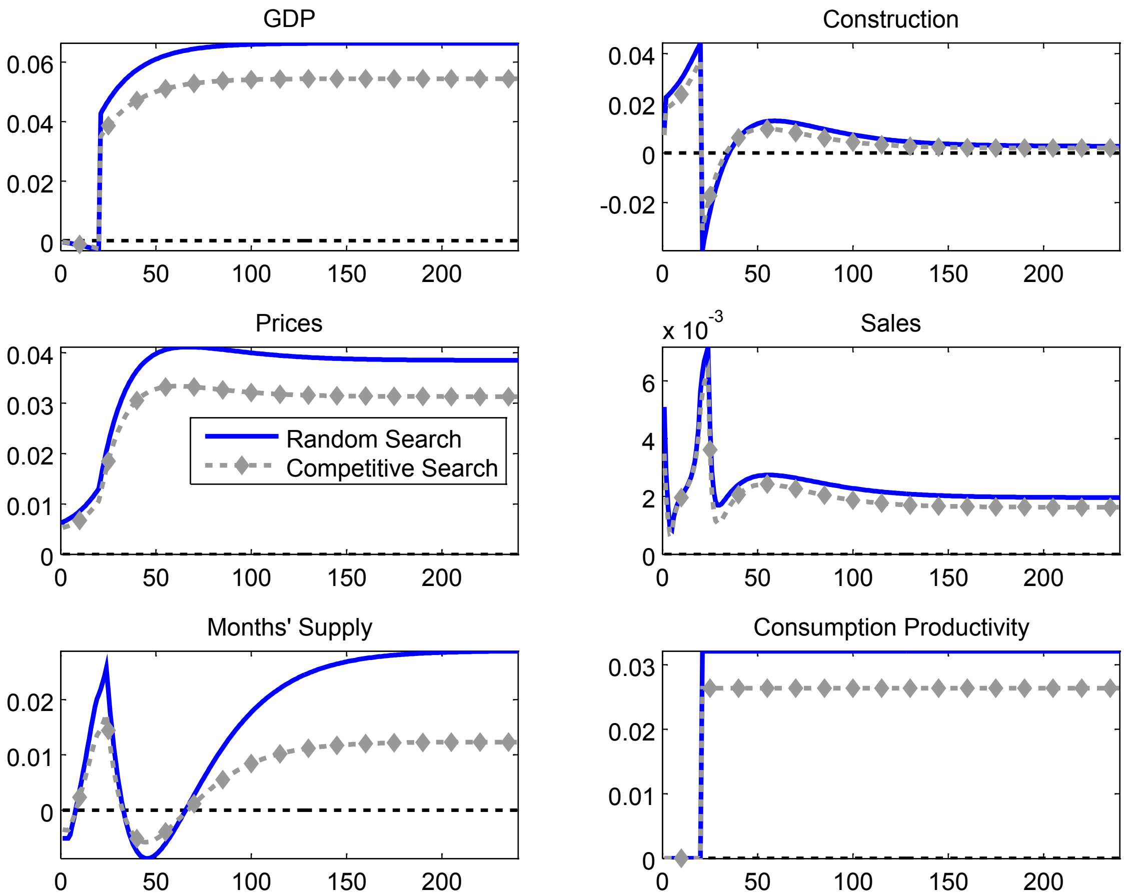
Series are seasonally adjusted log values from 1982q3-2010q4, expressed as deviations from linear trend plus average value over sample period. Shaded areas are NBER recession dates. Please see Observed Data Series section for details on series construction.

Figure 3: Priors and Posteriors for Estimated Parameters



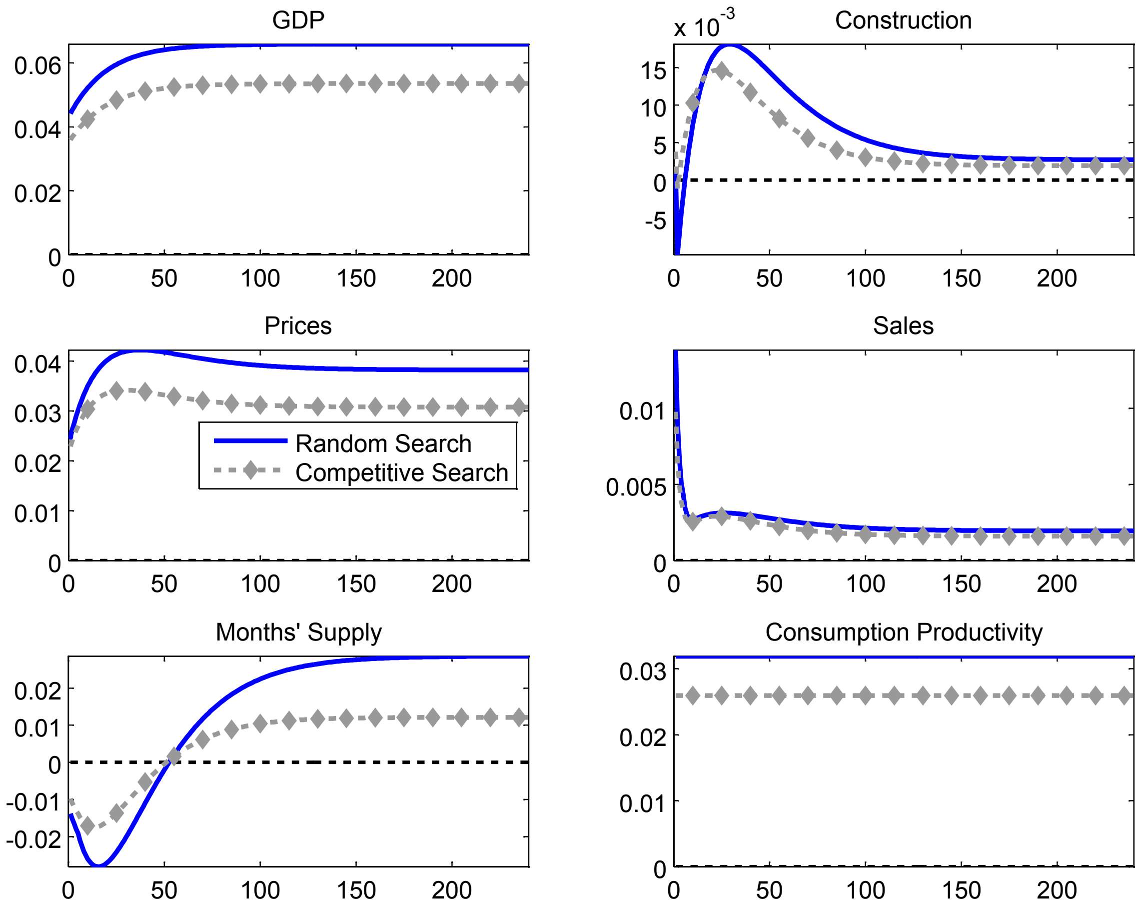
Posteriors obtained from random walk Metropolis Hastings Algorithm.

Figure 4: Impulse Responses to an Anticipated Consumption Productivity Shock



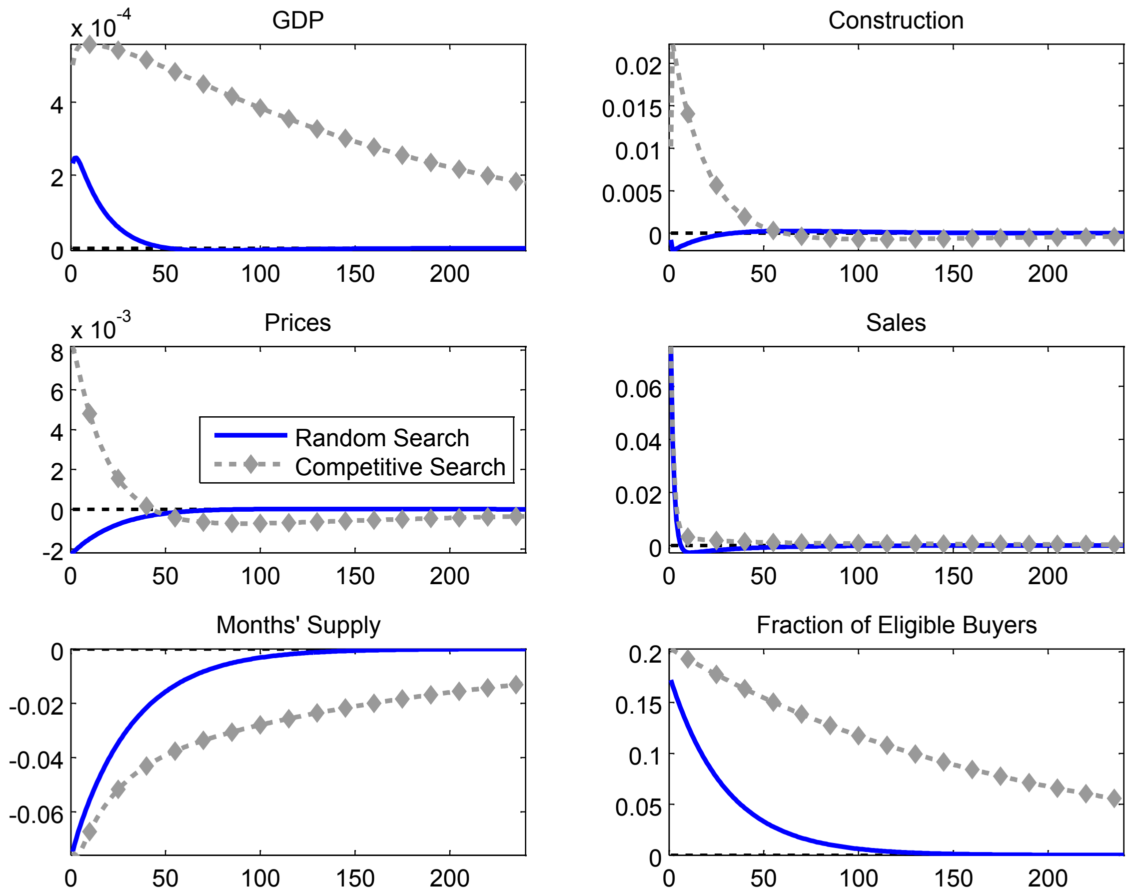
Time period is quarterly. Values shown are log deviations from steady state values.

Figure 5: Impulse Responses to an Unanticipated Consumption Productivity Shock



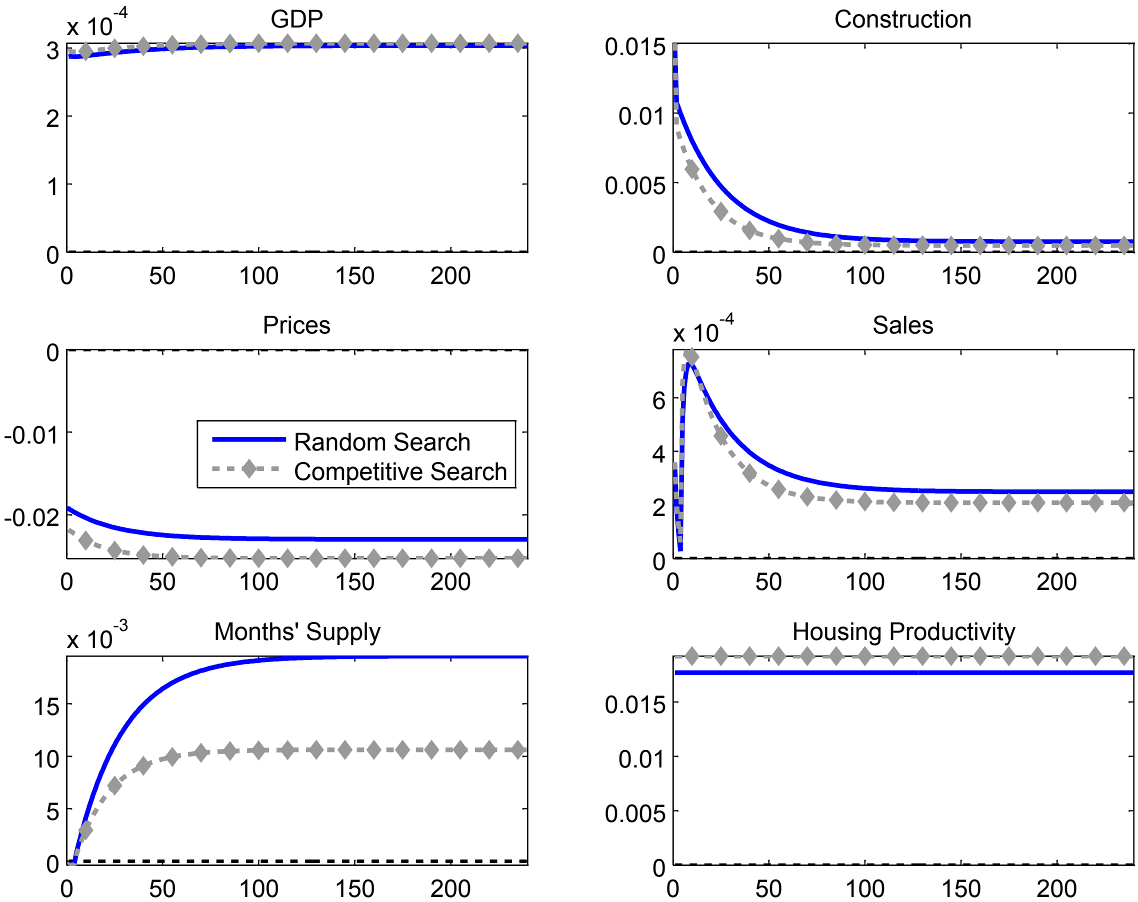
Time period is quarterly. Values shown are log deviations from steady state values.

Figure 6: Impulse Responses to an Eligible Buyers Shock



Time period is quarterly. Values shown are log deviations from steady state values.

Figure 7: Impulse Responses to an Unanticipated Housing Productivity Shock



Time period is quarterly. Values shown are log deviations from steady state values.

Figure 8: Smoothed Shocks - Competitive Search Model

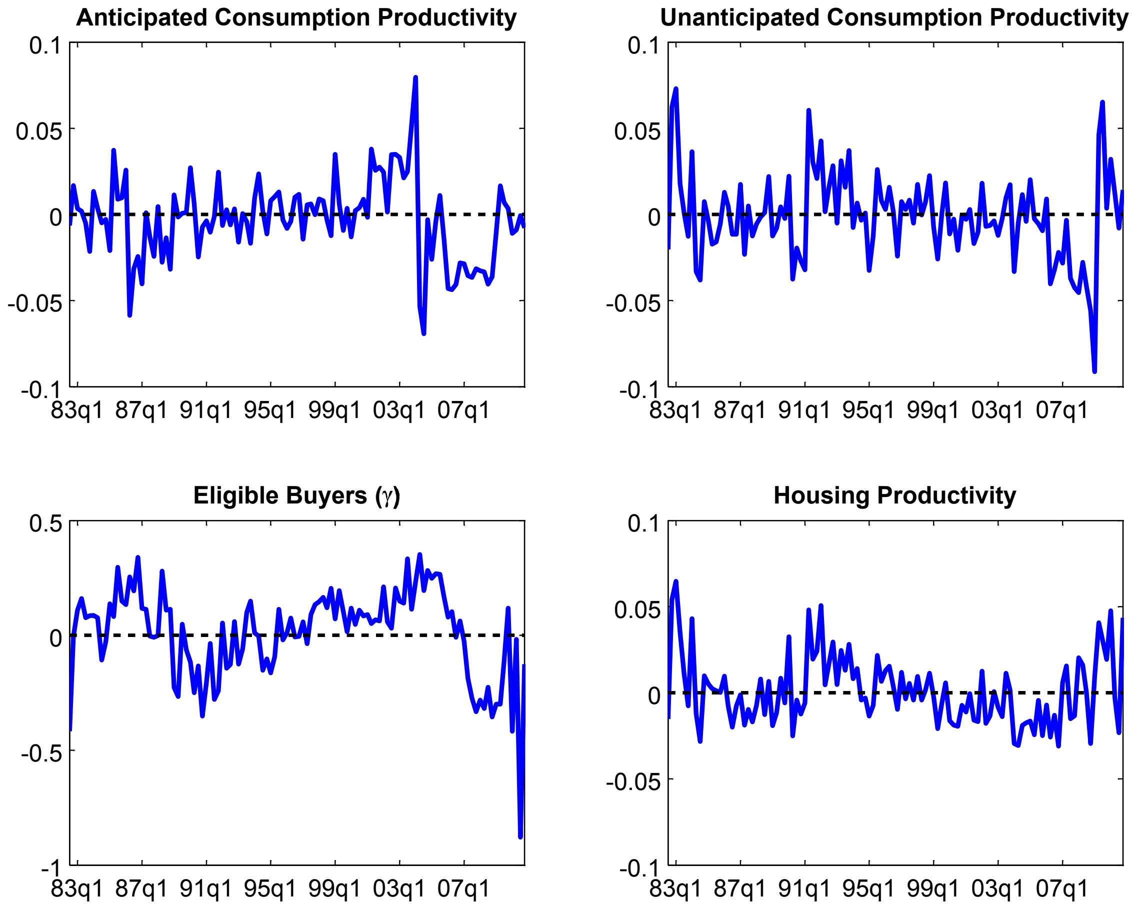


Figure 9: Historical Decomposition - Competitive Search Model

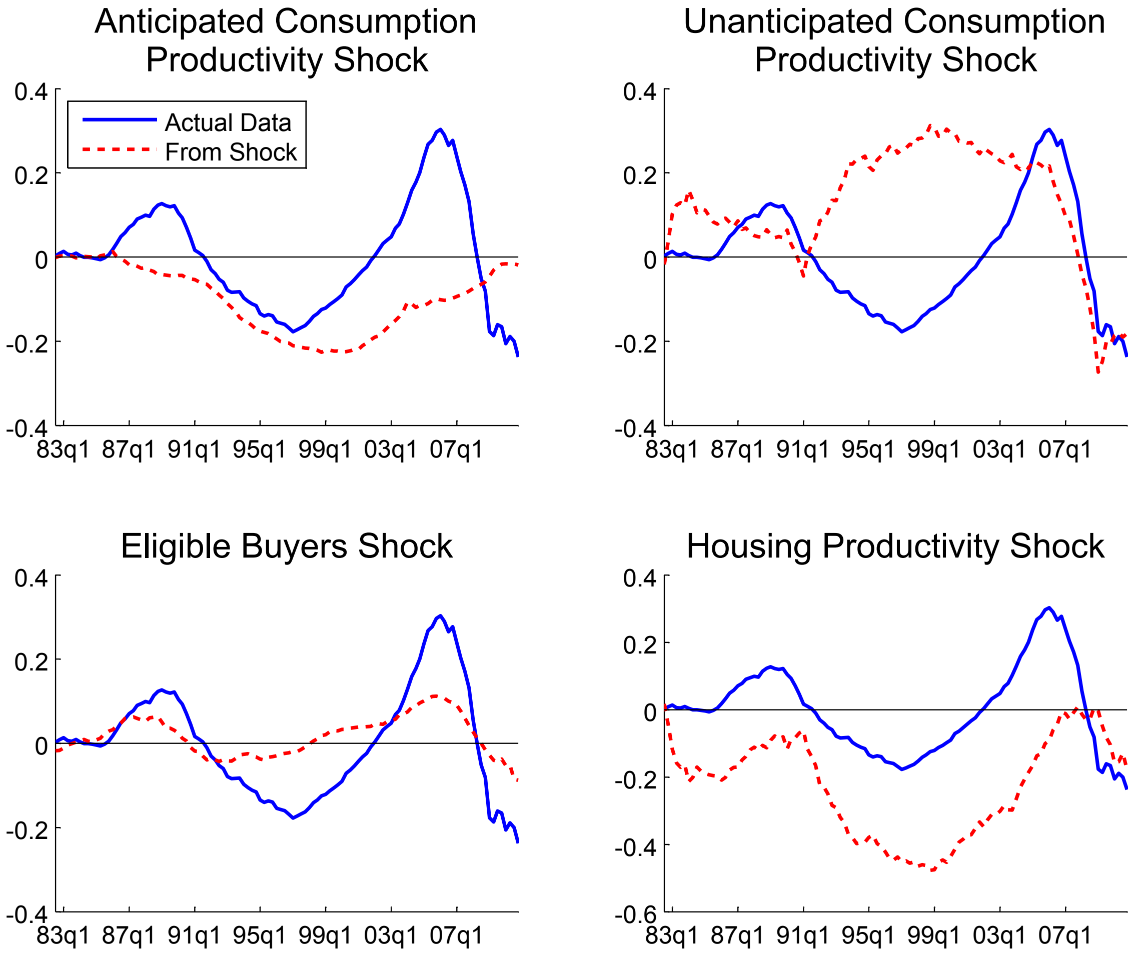


Table 1: Housing Market Time Series

	Standard Deviation	Correlation Coefficient with GDP	Prices	Sales	Starts	Months' Supply	Vacancy Rate
GDP	0.024	1.000					
		(1.000)					
Prices	0.130	0.510	1.000				
		(0.000)	(1.000)				
Sales	0.152	0.668	0.474	1.000			
		(0.000)	(0.000)	(1.000)			
Starts	0.314	0.618	0.467	0.894	1.000		
		(0.000)	(0.000)	(0.000)	(1.000)		
Months' Supply	0.277	-0.378	-0.010	-0.724	-0.800	1.000	
		(0.000)	(0.915)	(0.000)	(0.000)	(1.000)	
Vacancy Rate	0.143	-0.100	0.204	-0.422	-0.582	0.860	1.000
		(0.288)	(0.030)	(0.000)	(0.000)	(0.000)	(1.000)

Series are seasonally adjusted log values from 1982Q3-2010Q4, expressed as deviations from linear trend plus average value over sample period. p-values for correlation coefficients are in parentheses. GDP, sales, and starts expressed on a per household basis. Prices are from CoreLogic House Price Index for single family detached homes. Sales, sales, starts, and months' supply are for single family homes only. Vacancy rate is for home owners only. Please see Estimation section for details on series construction.

Table 2: Calibrated Parameters

Parameter	Symbol	Value	Reason
Competitive Search Model			
Discount Factor	β	0.995	0.98 annually
Depreciation Rate of Productive Capital	δ	0.026	0.1 annually
Relocation Probability	α	.009	Home sales per household $\approx$ .0114
Population Growth Rate	g	.0026	Starts per household $\approx$ .0027
SS Consumption Productivity	$\overline{Z^C}$	324.9	Quarterly GDP $\approx$ \$27,092
SS Housing Productivity	$\overline{Z^H}$	0.0059	House Price $\approx$ \$200,750
CES Utility Parameter	λ	-1	Elasticity of Substitution $= \frac{1}{2}$
Preference for housing vs. consumption	x	0.99998	Quarterly Consumption $\approx$17,950
Frisch Elasticity of Labor Supply	μ	1	Standard Value
Disutility of Labor	ρ	1005	Labor Supply $= 1$
SS fraction of eligible buyers	$\overline{\gamma}$	0.7	$\theta \approx 0.7$
Efficiency of Matching Function	A	1.31	Months' Supply $\approx$ 7 months
Disutility of Search Effort	σ	24608	Finding Rate $\approx$ .6
Capital's Share in Consumption Production Function	ν_C	.35	Standard Value
Capital's Share in Housing Production Function	ν_H	.19	Albouy and Ehrlich 2011
Labor's Share in Housing Production Function	ρ_H	.56	Albouy and Ehrlich 2011
Persistence of Consumption Productivity Shock	ψ_C	1	Unit Root Technology Shock
Persistence of Housing Productivity Shock	ψ_C	1	Unit Root Technology Shock
Consumption Capital Adjustment Costs	χ_C	0	Might drop this parameter
Housing Capital Adjustment Costs	χ_H	0	Might drop this parameter
Urn-ball Generalization Parameter	ζ	1.08	Steady State $\eta = 0.81$
Random Search Model (where different)			
Efficiency of Matching Function	A	1	Months' Supply $\approx$ 7
Buyers' Exponent in Matching Function	ϕ	.79	Genesove and Han 2010
Disutility of Search Effort	σ	24610	Finding Rate $\approx$ .6
Buyer's Share of Match Surplus	η	.81	Hosios Condition

Table 3: Steady State Values

Variable	Symbol	Value
Total Labor Supply	L	1
Proportion of Labor Force in Housing	L^H	0.016
Wholesale Housing Price	P^W	$194,323
Retail Housing Price	P^R	$200,750
Number of Eligible Buyers	γB	0.019
Houses for Sale	S	0.026
Market Tightness	θ	0.74
Probability of Sale	$q(E, \theta)$	0.443
Probability of Purchase	$f(E, \theta)$	0.598
Search Effort	E	0.517
Months' Supply	TOM	6.77
Sales	$M(E, \theta, \gamma)$	0.011

Table 4: Prior Distributions for Estimated Parameters

Variable	Symbol	Distribution
Persistence of Eligible Buyers Shock	ψ_γ	Uniform(0.001,0.999)
Standard Error of Anticipated Consumption Productivity Shock	σ_A	Uniform(0,0.1)
Standard Error of Unanticipated Consumption Productivity Shock	σ_C	Uniform(0, 0.1)
Standard Error of Eligible Buyers Shock	σ_γ	Uniform(0, 0.5)
Standard Error of Housing Productivity Shock	σ_H	Uniform(0, 0.2)

Table 5: Posterior Values for Estimated Parameters

Variable	Symbol	Posterior Mean (S.D.) Competitive Search	Posterior Mean (S.D.) Random Search
Persistence of Eligible Buyers Shock	ψ_γ	0.9945 (0.0032)	0.9669 (0.0098)
Standard Error of Anticipated Consumption Productivity Shock	σ_A	0.0264 (0.0019)	0.0321 (0.0022)
Standard Error of Unanticipated Consumption Productivity Shock	σ_C	0.0259 (0.0019)	0.0319 (0.0021)
Standard Error of Eligible Buyers Shock	σ_γ	0.2027 (0.0137)	0.1721 (0.0108)
Standard Error of Housing Productivity Shock	σ_H	0.0192 (0.0013)	0.0177 (0.0013)

Table 6: Simulations of Competitive Search Model

	Standard Deviation of First Difference	Correlation Coefficient with GDP	Prices	Sales	Starts	Months' Supply	Vacancy Rate
GDP	0.050	1.000 (1.000)					
Prices	0.033	0.771 (0.014)	1.000 (1.000)				
Sales	0.085	0.123 (0.240)	0.181 (0.212)	1.000 (1.000)			
Starts	0.069	-0.017 (0.166)	0.168 (0.155)	0.253 (0.117)	1.000 (1.000)		
Months' Supply	0.076	-0.120 (0.106)	-0.252 (0.083)	-0.450 (0.010)	-0.397 (0.053)	1.000 (1.000)	
Vacancy Rate	0.041	-0.101 (0.099)	-0.229 (0.090)	-0.261 (0.092)	-0.346 (0.084)	0.975 (0.000)	1.000 (1.000)

Average values from 500 model simulations of 114 quarters each. Numbers in parentheses are average p-values of correlation coefficients over each simulation.

Table 7: Simulations of Random Search Model

	Standard Deviation of First Difference	Correlation Coefficient with GDP	Prices	Sales	Starts	Months' Supply	Vacancy Rate
GDP	0.061	1.000					
		(1.000)					
Prices	0.032	0.843	1.000				
		(0.007)	(1.000)				
Sales	0.088	0.144	0.119	1.000			
		(0.258)	(0.263)	(1.000)			
Starts	0.081	-0.063	0.127	0.033	1.000		
		(0.158)	(0.145)	(0.317)	(1.000)		
Months' Supply	0.078	-0.176	-0.228	-0.339	-0.090	1.000	
		(0.094)	(0.084)	(0.040)	(0.195)	(1.000)	
Vacancy Rate	0.042	-0.153	-0.205	-0.129	-0.046	0.971	1.000
		(0.102)	(0.090)	(0.258)	(0.207)	(0.000)	(1.000)

Average values from 500 model simulations of 114 quarters each. Numbers in parentheses are average p-values of correlation coefficients over each simulation.