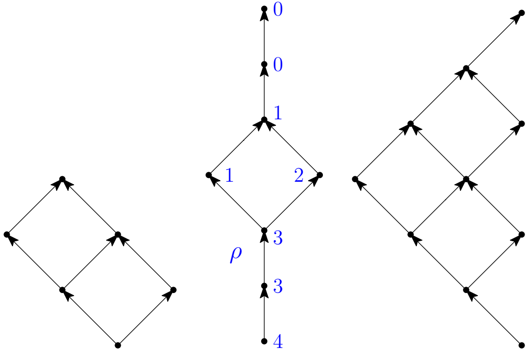
Reverse plane partitions via representations of quivers

Al Garver
(joint with Rebecca Patrias and Hugh Thomas)
arXiv: 1812.08345

University of Michigan

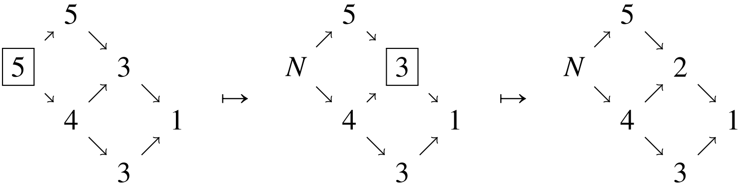
August 26, 2019



$[3] \times [2]$

A **reverse plane partition** on a poset P is an order-reversing map $\rho : \mathsf{P} \to \{0, 1, \ldots, N\}$ for some N.

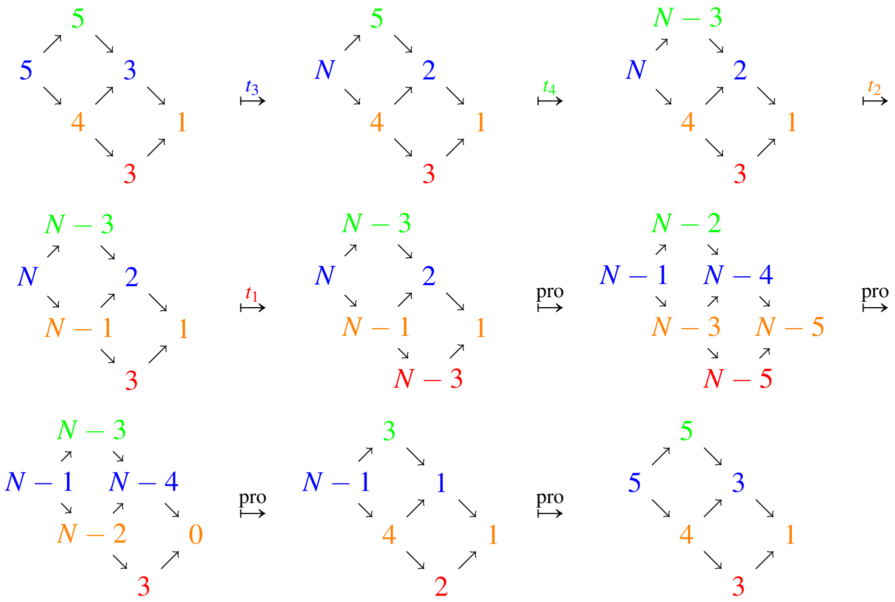
Toggling on reverse plane partitions



Given $\mathsf{x} \in \mathsf{P}$, one can **toggle** a reverse plane partition ρ as follows to produce a new reverse plane partition $t_{\mathsf{x}}\rho$.

$$t_{\mathsf{x}}\rho(\mathsf{y}) = \begin{cases} \max\limits_{\mathsf{y}<\mathsf{y}_1} \rho(\mathsf{y}_1) + \min\limits_{\mathsf{y}_2<\mathsf{y}} \rho(\mathsf{y}_2) - \rho(\mathsf{y}) & : \text{ if } \mathsf{y} = \mathsf{x} \\ \rho(\mathsf{y}) & : \text{ if } \mathsf{y} \neq \mathsf{x}, \end{cases}$$

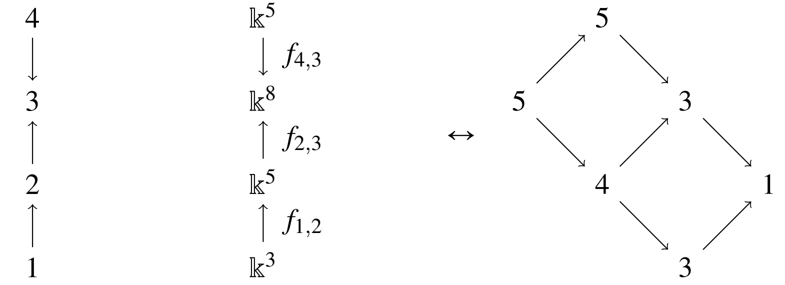
Periodicity for promotion pro $= t_1 t_2 t_4 t_3$



Theorem (Grinberg–Roby ‘15, Musiker–Roby ‘18)

Promotion on reverse plane partitions of $\mathsf{P} = [a] \times [b]$ has order $a + b$.

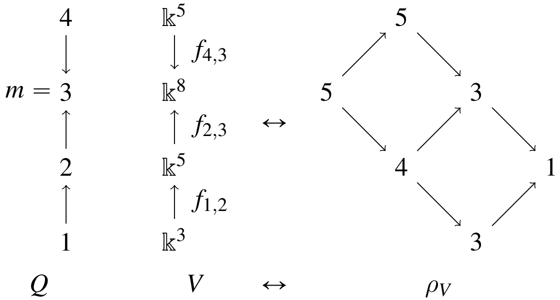
- Proved using a birational version of the promotion operator and then “tropicalizing”.
- We have a new proof of periodicity by showing that reverse plane partitions are equivalent to certain representations of quivers.



Q **quiver** | V **quiver representation** with dimension vector $\mathbf{dim}(V) = 3585$ | $\leftrightarrow$ | ρ_V

Theorem (G.–Patrias–Thomas ‘18)

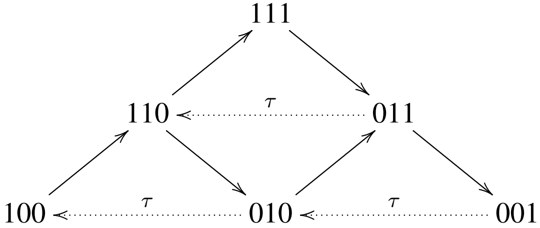
Let Q be a type A Dynkin quiver, m a vertex of Q, and $\mathsf{P}_{Q,m}$ the associated (minuscule) poset. There is a bijection between representations of Q all of whose indecomposable summands are supported at m and reverse plane partitions of $\mathsf{P}_{Q,m}$.



- To obtain ρ_V from V, one calculates the Jordan blocks of a generic nilpotent endomorphism of V.
- In the example, the Jordan blocks are ((3), (4,1), (5,3), (5)).

$$\underset{\text{quiver}}{Q} \quad \rightsquigarrow \quad \underset{\textbf{Auslander–Reiten quiver}}{\Gamma(Q)}$$

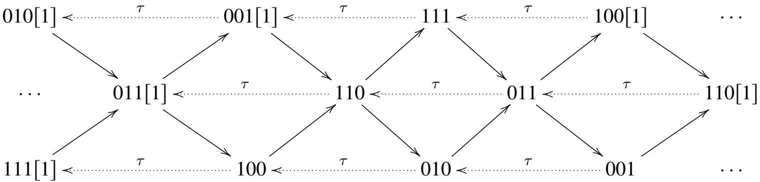
$$\{\text{vertices of } \Gamma(Q)\} \quad \leftrightarrow \quad \{\text{indecomposable representations of } Q\}$$
$$\{\text{arrows of } \Gamma(Q)\} \quad \leftrightarrow \quad \{\text{irreducible morphisms up to rescaling}\}$$



$\Gamma(Q)$ where $Q = 1 \leftarrow 2 \leftarrow 3$

The functor τ is known as the **Auslander–Reiten translation**.

The category of representations of Q sits inside the root category $\mathcal{R}_Q$.



$\Gamma(\mathcal{R}_Q)$where $Q = 1 \leftarrow 2 \leftarrow 3$

Lemma

Let Q be a type A Dynkin quiver. For any $X \in \mathcal{R}_Q$,

$$\tau^{|\{\textit{vertices of } Q\}|+1}(X) = X.$$

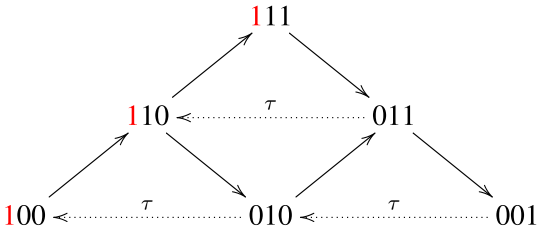
Promotion is compatible with τ

Lemma

Let Q be a type A Dynkin quiver. For any $X \in \mathcal{R}_Q$,

$$\tau^{|\{\text{vertices of } Q\}|+1}(X) = X.$$

For $Q = 1 \leftarrow 2 \leftarrow 3$, the possible associated posets are $[3]$ (when $m = 1, 3$) and $[2] \times [2]$ (when $m = 2$).



$\Gamma(Q)$ where $Q = 1 \leftarrow 2 \leftarrow 3$

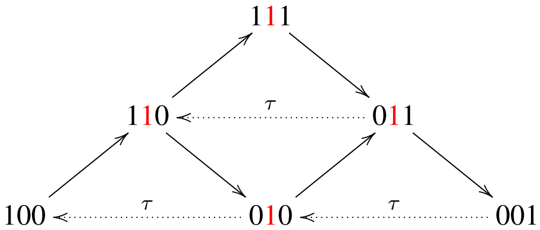
Promotion is compatible with τ

Lemma

Let Q be a type A Dynkin quiver. For any $X \in \mathcal{R}_Q$,

$$\tau^{|\{\text{vertices of } Q\}|+1}(X) = X.$$

For $Q = 1 \leftarrow 2 \leftarrow 3$, the possible associated posets are $[3]$ (when $m = 1, 3$) and $[2] \times [2]$ (when $m = 2$).



$\Gamma(Q)$ where $Q = 1 \leftarrow 2 \leftarrow 3$

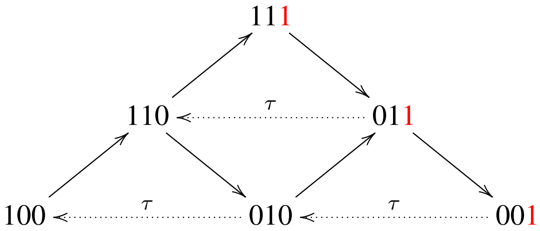
Promotion is compatible with τ

Lemma

Let Q be a type A Dynkin quiver. For any $X \in \mathcal{R}_Q$,

$$\tau^{|\{\text{vertices of } Q\}|+1}(X) = X.$$

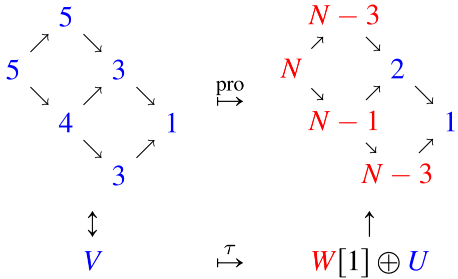
For $Q = 1 \leftarrow 2 \leftarrow 3$, the possible associated posets are $[3]$ (when $m = 1, 3$) and $[2] \times [2]$ (when $m = 2$).



$\Gamma(Q)$ where $Q = 1 \leftarrow 2 \leftarrow 3$

Theorem (G.–Patrias–Thomas '18)

Let Q be a type A Dynkin quiver and m a vertex of Q. Let V be a representation of Q where ρ_V is its corresponding reverse plane partition. Then $\rho_{\tau V} = pro\rho_V$ so $pro^{a+b}\rho_V = \rho_{\tau^{a+b}V} = \rho_V$.



We obtain the periodicity result.