Ex: A projectile is fired with an initial speed of 200 m/s and angle of elevation 60°. Find
(a) the range of the projectile,
(b) the maximum height reached, and
(c) the speed at impact.

(a) range = \( \frac{1}{2} V_0^2 \sin(2\alpha) \) = \( \frac{(200)^2 \sin(\frac{2\pi}{3})}{9.8} \) \approx 3534.80 m

(b) Maximize y. \( \frac{dy}{dt} = |V_0| \sin(\alpha) - g \cdot t \)

Solve
\[ 0 = |V_0| \sin(\alpha) - g \cdot t \]

\[ t = \frac{|V_0| \sin(\alpha)}{g} \]

\[ \Rightarrow t = \frac{200 \sin(\frac{\pi}{3})}{9.8} \]

Then \( y = 200 \sin(\frac{\pi}{3}) \left( \frac{200 \sin(\frac{\pi}{3})}{9.8} \right) - \frac{9.8}{2} \left( \frac{200 \sin(\frac{\pi}{3})}{9.8} \right)^2 \)

\[ \approx 1530.61 \text{ m} \]

(c) Impact at \( t_i = \frac{2(200) \sin(\frac{\pi}{3})}{9.8} \)

Find \( |V(t_i)| \).

\[ |V(t_i)| = \sqrt{200^2 \cos^2(\frac{\pi}{3}) + (200 \sin(\frac{\pi}{3}) - 9.8(400) \left( \frac{\sin(\frac{\pi}{3})}{9.8} \right))^2} \]

\[ = 200 \text{ m/s} \]
14.1 Functions with multiple variables

A function \( f \) is a rule that assigns to each ordered pair \((x, y)\) a real number \( f(x, y) \) whenever \((x, y)\) is in the domain \( D \) of \( f \).

Sketch the domain of

(a) \( f(x, y) = \sqrt{x - 2} + \sqrt{y - 1} \)

(b) \( g(x, y) = \frac{x - y}{x + y} \)

(a) \( D = \{(x, y) \mid x \geq 2, y \geq 1\} \)

(b) \( D = \{(x, y) \mid x \neq -y\} \)

\[ = \mathbb{R}^2 \setminus \{(x, y) \mid x = -y\} \]

The graph of \( f(x, y) \) is the set \( \{(x, y, z) \mid (x, y) \in D \text{ and } z = f(x, y)\} \subset \mathbb{R}^3 \)

Ex: \( f(x, y) = \sqrt{9 - x^2 - y^2} \), sketch its graph

\[ D = \{(x, y) \mid x^2 + y^2 \leq 9\} \]

Top half of sphere of radius 3
Contour Maps

Understand or graph a function by studying the equations \( f(x,y) = k \).

Level curves

Plotting several level curves gives us a contour map for \( f(x,y) \).

Ex: Draw a contour map for \( f(x,y) = x^2 - y^2 \)

\[
\begin{align*}
  k &= 0 \quad & x^2 &= y^2 \\
  k &= 1 \Rightarrow 1 &= x^2 - y^2 \\
  y &= \pm \sqrt{x^2 - 1} \\
  k &= -1 \Rightarrow x &= \pm \sqrt{y^2 + 1}
\end{align*}
\]

Level curves appear more close together when \( f(x,y) \) is changing more quickly, further apart when it's changing more slowly.

Ex: (p. 940) Match the functions with their graphs

(a) \( f(x,y) = \frac{1}{1 + x^2 + y^2} \)  
(b) \( f(x,y) = \frac{1}{1 + x^2 y^2} \)

(c) \( f(x,y) = \ln(x^2 + y^2) \)  
(d) \( f(x,y) = \cos(\sqrt{x^2 + y^2}) \)

(e) \( f(x,y) = |xy| \)  
(f) \( f(x,y) = \cos(xy) \)
I: \((b)\), set \(x = 0\) or \(y = 0\); then \(f(x, y) = 1\)

II: \((c)\), set \(x = 0\) or \(y = 0\); then \(f(x, y) = 1\)
set \(y = 1\); then \(f(x, 1) = \cos(x)\)

III: \((a)\)

IV: \((c)\) set \(x = 1\); then \(f(1, y) = \ln(1 + y^2)\)

V: \((c)\) \(z = \sqrt{x^2 + y^2}\) is a cone opening around the positive \(z\)-axis

VI: \((e)\) set \(x = 1\); then \(f(1, y) = |y|\).

If \(f(x, y, z)\) is a function of 3 variables, we can attempt to understand its graph by studying the level surfaces \(k = f(x, y, z)\).

Ex: What are level surfaces of
(a) \(f(x, y, z) = x^2 + y^2 + z^2\)?
(b) \(f(x, y, z) = x + 3y + 5z\)?

(a) \(k = x^2 + y^2 + z^2\); the level surfaces are all spheres of radius \(\sqrt{k}\) centered at \((0, 0, 0)\)
(b) \(k = x + 3y + 5z\); the level surfaces are all planes with normal vector \(<1, 3, 5>\).