True or False? (p. 1022)

2. False. $f_{xy}(x,y) = 2y$ and $f_{yx}(x,y) = 1$
   This contradicts Clairaut's Theorem.

3. False. $f_{xy} = \frac{\partial^2 f}{\partial y \partial x}$

4. True. $D_k f(x,y,z) = \lim_{h \to 0} \frac{f(x,y,z+h) - f(x,y,z)}{h} = f_z(x,y,z)$

11. True. $y = \langle a, b \rangle$
    $D_z f(x,y) = f_x(x,y)a + f_y(x,y)b$
    $= \cos(x)a + \cos(y)b$
    $-1 \leq \cos(x) \leq 1$
    $-a \leq \cos(x)a \leq a$
    $-6 \leq \cos(x)b \leq b$
    $-a - 6 \leq D_z f(x,y) \leq a + b$
    The biggest $a + b$ can be is $\sqrt{2}$.
    The smallest $-a - 6$ can be is $-\sqrt{2}$.

Contour maps plot level curves $f(x,y) = k$.
where $k = 0, \pm 1, \pm 2, \ldots$
or $k = 0, \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \ldots$
or $k = 0, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm 1, \ldots$

etc.

* White space indicates the function is not changing
  or changing only small amounts.

* Higher concentration of level curves indicates that the
  function is changing more quickly.
4. [10 points] Match each of the nine sets of level curves below with the appropriate function.

- $\sin(xy)$ _______ ix _______
- $\sin(x)\sin(y)$ _______ viii _______
- $\sin(x) + \sin(y)$ _______ vii _______
- $\cos(x)/(x^2 + y^2 + 1)$ _______ i _______
- $\sin(x)/(x^2 + y^2 + 1)$ _______ vi _______
- $(x^2 + y^2)^{1/2}$ _______ v _______
- $e^x\cos(y)$ _______ ii _______
- $(1 - x^2)(1 - y^2)$ _______ iii _______
- $\sin(x + y)$ _______ iv _______

(i) ![Curve Image]
(ii) ![Curve Image]
(iii) ![Curve Image]
(iv) ![Curve Image]
(v) ![Curve Image]
(vi) ![Curve Image]
(vii) ![Curve Image]
(viii) ![Curve Image]
(ix) ![Curve Image]
\[ \sin(xy): \] 
- If \( x = 0 \) or \( y = 0 \) \( \Rightarrow \) \( \sin(xy) = 0 \)
- As \( x \) and \( y \) get further from the origin, \( \sin(xy) \) oscillates faster and faster.

\[ \sin(x)\sin(y); \] 
- \( x = 0 \) or \( y = 0 \) \( \Rightarrow \) \( \sin(x)\sin(y) = 0 \)
- When \( x = y \), \( \sin(x)\sin(y) = \sin^2(x) \)
- When \( x = -y \), \( \sin(x)\sin(y) = -\sin^2(x) \)

\[ \cos(x)/\sqrt{x^2+y^2+1}; \] 
- \( x = -y \) \( \Rightarrow \) \( \cos(x)/\sqrt{x^2+y^2+1} = 0 \)
- \( x = y \) \( \Rightarrow \) \( \cos(x)/\sqrt{x^2+y^2+1} = 2\sin(x) \)
- If \( x \) is close to \( \pm \frac{\pi}{2} \) and so is \( y \), then \( \cos(x)/\sqrt{x^2+y^2+1} \) is close to \( 0 \)

\[ \sin(x)/(x^2+y^2+1); \] 
- \( x = 0 \) \( \Rightarrow \) \( \sin(x)/(x^2+y^2+1) = 0 \)

\[ (x^2+y^2)^{1/2}; \] 
- This is the top half of an infinite cone with vertex at the origin.

\[ e^x\cos(y); \] 
- \( y = \pm \frac{\pi}{2} \) \( \Rightarrow \) \( e^x\cos(y) = 0 \)

\[ (1-x^2)(1-y^2); \] 
- \( x = \pm 1 \) or \( y = \pm 1 \) \( \Rightarrow \) \( (1-x^2)(1-y^2) = 0 \)
\sin(x+y): along any diagonal \( y = x + k \) we have \( \sin(x+y) = \sin(2x+k) \).

Ex: (8 p. 1022)

(a) Estimate \( f(3,2) \)

(b) Is \( f_x(3,2) \) positive or negative?

(c) Which is greater, \( f_y(3,1) \) or \( f_y(3,2) \)?

(a) Point \((3,2)\) is closer to \( F(x,y) = 50 \) than \( f(x,y) = 50 \). So \( f(3,2) \approx 57 \).

(b) As one moves to the right of \((2,3)\), the level curves have smaller heights. Moving to the left of \((2,3)\), one encounters level curves with increasing heights. Thus \( f_x(2,3) < 0 \).

(c) \( f_y(2,1) \approx \frac{f(2,1) - f(2,0.8)}{0.2} = \frac{50 - 40}{0.2} = \frac{10}{0.2} = 50 \)

\( f_y(2,2) \approx \frac{f(2,2.4) - f(2,1.5)}{0.9} = \frac{80 - 70}{0.9} = \frac{10}{0.9} = \frac{100}{9} \)

\( f_y(2,1) > f_y(2,2) \)

Ex: (p. 1022 11(c))

(c) Estimate \( T_{xy}(6,4) \).

\( T_x(6,4) \approx \frac{1}{2} \left( \frac{8}{2} + \frac{6}{2} \right) = \frac{14}{4} = 3.5 \)

\( T_y(6,2) \approx \frac{1}{2} \left( \frac{13+3}{2} \right) = \frac{16}{4} = 4 \)
\[ T_x(6,6) \approx \frac{1}{2} \left( \frac{7 + 5}{2} \right) = \frac{12}{4} = 3 \]

\[
\begin{array}{ccc}
T_x(6,2) & T_x(6,4) & T_x(6,6) \\
4 & 3.5 & 3
\end{array}
\]

\[ \Rightarrow T_y(6,4) \approx -\frac{1}{4} \]

Ex: (p. 922 13) Find the curvature of \( y = x^4 \) at \((1,1)\).

\[ \mathbf{r}(t) = \langle t, t^4, 0 \rangle \]

\[ \mathbf{r}'(t) = \langle 1, 4t^3, 0 \rangle \quad |\mathbf{r}'(t)| = (1 + 16t^6)^{3/2} \]

\[ \mathbf{r}''(t) = \langle 0, 12t^2, 0 \rangle \]

\[ \mathbf{r}'(t) \times \mathbf{r}''(t) = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
1 & 4t^3 & 0 \\
0 & 12t^2 & 0
\end{vmatrix} = \langle 0, 0, 12t^2 \rangle \]

\[ \kappa(t) = \frac{12t^2}{(1 + 16t^6)^{2/3}} \]

\[ \kappa(1) = \frac{12}{17^{2/3}} \]