

## I. C(N) LRM

- A.  $y = X\beta + \varepsilon$
- B.  $\varepsilon \sim (\underline{0}, \sigma^2 I)$
- C.  $E(\varepsilon | X) = 0$
- D.  $X$  of full column-rank
- E.  $X$  nonstochastic
- F.  $\varepsilon \sim N(0, \sigma^2 I)$



II. Least-squares (if  $N$ , also M) estimates:

$$\hat{b} = (X'X)^{-1}X'y \equiv A_y$$

$$A. E(\hat{b}) = E(A_y) = E(AX\beta + AE) = \beta$$

$$B. V(\hat{b}) = V(A_y) = AV(y)A' \\ = (X'X)^{-1}X'\sigma^2 I X(X'X)^{-1} \\ = \sigma^2 (X'X)^{-1}$$

= Cramer-Rao lower bound

= Show Consistency

$$C. \text{Estimate} \\ V(\hat{b}) = \sigma^2 (X'X)^{-1} \text{ by} \\ \widehat{V}(\hat{b}) = \frac{\mathbf{e}'\mathbf{e}}{n-k} (X'X)^{-1} = S^2 (X'X)^{-1} \\ V(b_k) = f(S^2, \bar{V}(X_k), R_{X_k, X_{-k}}^2) \\ \widehat{V}(b_k) = f(S^2, \bar{V}(X_k), R_{X_k, X_{-k}}^2)$$

## A. Confidence Intervals:

$$\text{pr} (b - \text{T.S.e.}(b) < \beta < b + \text{T.S.e.}(b))$$

$\uparrow \text{A}_y \quad \downarrow S^2 (X'X)^{-1} \quad \text{critical stat from } T_{n-k}$

## B. Wald Tests (Logic):

$$1. (\text{Estimate} - \text{Hypothesis}) / \text{s.e.}(\text{estimate}) \sim Z_{n-k}$$

$$2. [(\text{Estimate} - \text{Hypothesis})^2 / \text{Var}(\text{Estimate})] / J \sim F_{J, n-k}$$

$$\dagger (Rb - q)'[V(b)]^{-1}(Rb - q)$$

$$3. \text{Joint Confidence Regions: } \dagger [b - \beta]'[V(b)]^{-1}[b - \beta] \leq F_{J, n-k}$$

## C. Degradation of Fit Tests (Logic):

$$1. \frac{\Delta R^2 / \Delta k}{(1 - R_{\text{big model}}^2) / \Delta k} \sim F_{\Delta k, n-k}_{\text{big model}}$$

$$3. 2 \times (\ln L_U - \ln L_R) \sim \chi^2_{\epsilon}$$

$$2. \frac{(\Delta \mathbf{e}'\mathbf{e} / \Delta k)}{[\mathbf{e}'\mathbf{e}]_{\text{big model}}} \sim F_{\Delta k, n-k}_{\text{big model}}$$

## Post-Break Retester & Catch-up

(2)

### III. D. Lagrange-Multiplier Tests

1. Logics: If null true, then no constraints on opt., not binding:

a. If null true, then impose it as constraint on optimization, &  $\nabla_{\theta} \ln L = 0$  still

b. ... & Lagrangian multipliers,  $\lambda = 0$

2. Version 1a  $\Rightarrow b^* = b - \text{some positive-definite matrix}$

& test statistic:  $[R Q^{-1} R']^{-1} (R b - g) \sim \chi^2_k$

Version 1b  $\Rightarrow$  same, but also, often, in auxiliary post-estimation regression, coefficients are  $\lambda$ , so contrast them.

### E. Linear Combination & Joint Hypotheses by linear-algebra expressions

$$1. r' b = g \Rightarrow \frac{(r' b - g)}{\sqrt{r' V(b) r}} = T \sim t_{n-k}$$

$$2. R b = g \Rightarrow \frac{1}{J} (R b - g)' V(b)^{-1} (R b - g) \sim F_{J, n-k}$$

### F. Non-linear functions of parameter estimates:

1. "Delta Method": use asymptotic variance of linear-approx to function  
 $V(f(\hat{\theta})) \approx [\nabla_{\theta} f(\hat{\theta})]' V(\hat{\theta}) [\nabla_{\theta} f(\hat{\theta})]$

2. Simulation (a.k.a., parametric bootstrap):

$\hat{\theta} \sim N(\hat{\theta}, -\hat{H}')$ , so draw  $\hat{\theta}_i$  from MVN with that mean vector & that variance-covariance matrix, & calculate

a)  $\hat{f}(\hat{\theta}) = \sum_{i=1}^Z f(\hat{\theta}_i) / Z$

b)  $\hat{V}(f(\hat{\theta})) = \sum (f(\hat{\theta}_i) - \hat{f}(\hat{\theta}))^2 / Z$  (large) times.

(CLARIFY: ~~visit~~ <http://gking.harvard.edu/clarify/docs>)

#### IV. Structural Change:

- A. Chow Tests -- degradation of fit strategy...
- B. Dummy Variables, Dummy-variable Interactions -- Wald strategy
- C. Brief Intro to recursive estimation uses & test strategies.

#### V. Non-nested hypothesis Testing

- A. In all our tests so far, the null hyp.  $H_0$  could be expressed as a restriction on the alternative (more general). If not this overlap, e.g.

$$\begin{aligned} y &= X\beta + \varepsilon, & H_0 \\ y &= Z\gamma + \varepsilon, & H_A \end{aligned} \quad \left. \begin{array}{l} \text{with } X \cap Z^c \neq \emptyset \\ X^c \cap Z \neq \emptyset \\ \text{both non-empty} \end{array} \right\}$$

- B. Option 1: artificially nest the models --

$$y = [X \cap Z^c] \beta^* + [Z \cap X^c] \gamma^* + [X \cap Z] \omega + \varepsilon,$$

then test  $\beta^*$ ,  $\gamma^*$ , but... what of  $\omega$ ?

... neither model  $H_0$  nor  $H_A$  properly reflected in these tests of  $\beta^*$  or  $\gamma^*$

#### C. Davidson & McKinnon's J-test:

1. Consider:  $y = (1-\alpha)X\beta + \alpha Z\gamma + \varepsilon$ , & test

$$\alpha = 1 \text{ or } \alpha = 0 \dots$$

... but  $\alpha \gamma$  &  $(1-\alpha)\beta$   $\Rightarrow$  infinite set of  $\alpha, \gamma, \beta$  satisfy; and any perfect collinearity if any  $X \cap Z$ .

2. D&M suggest: ① Reg  $y$  on  $X$  & save  $\hat{y}_x$

② Reg  $y$  on  $Z$  and  $\hat{y}_x$

③ Test coefficient on  $\hat{y}_x = 0$  tests  $X$  model against  $Z$  model. Reject implies  $X$  model rejects  $Z$  model.

④ Reverse to test  $Z$  model v.  $X$  model.

(a) "Problem": all four possibilities exist

(b) "Encompassingness" interpretation.

3. See Greene<sup>7.3</sup> on Cox-Naor tests (rest more on ML). Clarke 2001 PA on "distribution free" nonnested tests.