Polychotomous (Nominal, Categorical, $M>2$) ①
Dependent Variables

- m Categories
- Model $P_j/p_m = \dfrac{F(XB_j)}{1-F(XB_j)} = G(XB_j)$ n.b. $\dfrac{p_m}{p_m} = \dfrac{F(XB_m)}{1-F(XB_m)} = 1$

$\Rightarrow m-1$ models of prob. category j v. base (m).

- (n.b., for 2 categories $\Rightarrow$ odds)

$$\Rightarrow p_m = \frac{1}{1+\sum_{j=1}^{m-1} G(XB_j)} \quad \text{b/c} \quad \sum_{j=1}^{m-1} \frac{P_j}{P_m} = \frac{1-P_m}{P_m} = \frac{1}{P_m} - 1$$

$$\Rightarrow 1 + \sum_{j=1}^{m-1} \frac{P_j}{P_m} = \frac{1}{P_m}$$

$$\Rightarrow p_m = 1 \Big/ 1 + \sum_{j=1}^{m-1} G(XB_j)$$

$$\Rightarrow P_j = \frac{G(XB_j)}{1+\sum_{j=1}^{m-1} G(XB_j)}$$

let $G(XB_j) = e^{XB_j}$ $\Rightarrow$ $P_j = \dfrac{e^{XB_j}}{1+\sum_{j=1}^{m-1} e^{XB_j}}$ & $P_m = \dfrac{1}{1+\sum_{j=1}^{m-1} e^{XB_j}}$

Multinomial Logit:

$$p_i = \frac{e^{xB_i}}{1+\sum_{j=1}^{m-1} e^{xB_j}} \quad ; \quad p_m = \frac{1}{1+\sum_{j=1}^{m-1} e^{xB_j}}$$

$\Rightarrow$ interp. as set of logits, cat. by cat. against base cat.

Likelihood Function:

$y_{ij} = 1$ if obs i in category j
$\quad = 0$ otherwise

$$L = \prod_{i=1}^{n} P_{i1}^{y_{i1}} P_{i2}^{y_{i2}} \cdots P_{im}^{y_{im}} \quad \left(= P_{ik}^{y_{ik}} \text{ where } k = \text{true category obs } i \right)$$

$$\ln L = \sum_{i=1}^{n} \sum_{j=1}^{m} y_{ij} P_{ij} \qquad (p_{ij} \text{ given above})$$

$$\frac{\partial \ln L}{\partial \beta_k} \begin{cases} = \sum_{i=1}^{n} \left[\frac{y_{ik}}{P_{ik}} P_{ik}(1-P_{ik}) + \sum_{\substack{j=1 \\ j\neq k}}^{m} \frac{y_{ij}}{P_{ij}} (-P_{ij}P_{ik}) \right] X_i \\ = \sum_{i=1}^{n} \left[y_{ik} - P_{ik} \left(\sum_{j=1}^{m} y_{ij} \right) \right] X_i \end{cases}$$

$$= \sum_{i=1}^{n} (y_{ik} - P_{ik}) X_i$$

$\frac{\partial \ln L}{\partial \beta_k} \Rightarrow$ normal equations again