

Simplified Exposition Bias if LDV w/ Corr Errors

$$Y_t = X_t \beta + \delta Y_{t-1} + \varepsilon_t$$

$$\varepsilon_t = \rho \varepsilon_{t-1} + \omega_t$$

$$\begin{aligned} \text{Cov}(Y_{t-1}, \varepsilon_t) &= \text{Cov}(Y_{t-1}, \rho \varepsilon_{t-1} + \omega_t) \\ &= \text{Cov}(X_{t-1} \beta + \delta Y_{t-2} + \varepsilon_{t-1}, \rho \varepsilon_{t-1} + \omega_t) \\ &= \rho \text{Cov}(\varepsilon_{t-1}, \varepsilon_{t-1}) + \rho \delta \text{Cov}(Y_{t-2}, \varepsilon_{t-1}) \end{aligned}$$

$$\text{Cov}(Y_{t-1}, \varepsilon_t) = \rho \frac{\sigma_\omega^2}{1-\rho^2} + \rho \delta \text{Cov}(Y_{t-1}, \varepsilon_t)$$

$$\Rightarrow (1 - \rho\delta) \text{Cov}(Y_{t-1}, \varepsilon_t) = \rho \frac{\sigma_\omega^2}{1-\rho^2}$$

$$\Rightarrow \text{Cov}(Y_{t-1}, \varepsilon_t) = \frac{\rho \sigma_\omega^2}{(1-\rho^2)(1-\rho\delta)} \neq 0$$

UNLESS $\rho = 0$