Modeling and Interpreting Interactions

ps699: Winter 2012

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Overview

• Interactions in Pol-Sci: ubiquitous, but should be more
  – In present context, concern re: coefficient heterogeneity is a call for interactions—i.e., models of conditional, i.e., context-variant, coefficients.

• From theory to empirical-model specification: Arguments that imply interactions (& some that don’t), & how to write.

• Interpretation:
  – Effects = derivatives & differences, not coefficients!
  – Std Errs (etc.): effects vary, so do std errs (etc.)!

• Presentation: Tables & Graphs, & Choosing between equivalent Specifications

• Use & abuse of some common-practice “rules”

• Extensions:
  – Split-sample v. dummy-interaction
  – Common 2nd-moment implications of interactions
  – Interactions with uncertainty = random coefficients = hierarchical...
Interactions in Pol-Sci Research

• Common. ‘96–‘01 *AJPS, APSR, JoP*:
  – 54% some stat meth (=s.e.’s), of which 24% = interax (so interax ≈ 12.5% or 1/8th total; more if exclude *CP*).
  – (N.b., most rest QualDep & frml thry, not counted, & “thry” in denom) so understate tech nature discipline)

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>American Political Science Review</td>
<td>279</td>
<td>274 77%</td>
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<td>70 40%</td>
<td>10 6% 14%</td>
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<td>226 80%</td>
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<td>2446</td>
<td>1323 54%</td>
<td>311 13% 24%</td>
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Interactions in Pol-Sci Theory

- Ubiquitous, but our Theories/Substance say should be even more; core classes of argument inherently interactive:
  - **INSTITUTIONAL**: institutions are inherently interactive variables:
    - Institutions funnel, moderate, shape, condition, constrain, refract, magnify, augment, dampen, mitigate political processes that...
      - ...translate societal interest-structures into effective political pressures,
      - ...&/or pressures into public-policy responses,
      - ...&/or policies to outcomes.
  - I.e., they affect (*modify, condition, moderate...*) effects ≡ *interaction.*
Interactions in Pol-Sci Theory

- Views from across institutionalist perspectives:
  - Hall: “institutionalist model→policy more than sum countervailing pressure from soc grps; that press mediated by organizational dynamic.”
  - Ikenberry: “[Political struggles] mediated by inst’l setting where [occur]”
  - Steinmo & Thelen: “inst’s...constrain & refract politics... [effects of] macro-structures magnify or mitigated by intermediate-level inst’s... help us...explain the contingent nature of pol-econ development...”
  - Shepsle: “SIE clearly a move [to] incorporating inst’l features into R-C. Structure & procedure combine w/ preferences to produce outcomes.”
Interactions in Pol-Sci Theory

- Ubiquitous, but our Theories/Substance say should be even more; core classes of argument inherently interactive:
  - **INSTITUTIONAL:** …
  - **STRATEGIC:** actors’ choices (outcomes) conditional upon institutional/structural environ., opportunity set, & (interdep.) other actors’ choices.
  - **CONTEXTUAL:** actors’ choices (outcomes) conditional upon environment, opportunity set, & (interdep.) aggregates of other actors’ choices.
Interactions in Pol-Sci Theory

• Across subfields:
  – Comparative Politics *examples*:
    • Electoral system & societal structure interact to produce party system.
    • Divided government & polarization retards legislative productivity, ⇒ conditional dynamics.
    • Corruption interactive product of institutional & societal structures.
  – International Relations *examples*:
    • System polarity & offense-defense balance ⇒ war propensity.
    • Terrorist targeting & counterterrorism responses depend “grievance” & resources
  – American Politics …
Interactions in Pol-Sci Theory

- Political Economy:
  - Electoral & partisan cycles (i.e., effects) depend on pol & econ inst’l, struct’l, contextual conditions

- Political Behavior:
  - Gov’t inst’s shape voter behavior: balancing (Kedar, Alesina); economic voting (Powell & Whitten); etc.

- Legislative Studies:
  - Effects divided government different in (depend on) presidential v. parliamentary.

- Political Development:
  - Effect inequality on democratization depends cleavage structure.
Theory & Substance: Everyone’s Favorite “Model”

Economics Affects Politics and Society
Politics Affects Economics and Society
Society Affects Politics and Economics

Picture & text seem relate more directly to the “ubiquitous endogeneity” of Society, Economy, & Polity. For “ubiquitous context-conditionality”, think “affects the effects of {Politics, Economics, Society} on {Society, Polity, Economy}.”
Theory & Substance:
An Old (& still) Favorite “Model” of Mine

The Cycle of Political Economy

Examples of the Elements at Each Stage:

(A) Interests:
   - Sectoral Structure of Economy
   - Income Distribution
   - Age Distribution
   - Trade Openness

Elections:
   - Electoral Law
   - Voter Participation

Government Formation:
   - Fractionalization
   - Polarization

(B) Representation:
   - Partisanship

Policy:
   - Fiscal Policy
   - Monetary Policy
   - Institutional Adjustment

Government Termination:
   - Replacement Risk

(C) Outcomes:
   - Unemployment
   - Inflation
   - Growth
   - Sectoral Shift
   - Debt
   - Institutional Change

Again, the ubiquitous context-conditionality perhaps not fully overtly shown, but implication here that effect factors at each stage tend depend on others at that and other stages.
Theory & Substance:  
An Newer Favorite “Model” of Mine  

• **Complex Context-Conditionality:**  
  – Effect of (almost) everything depends on (almost) everything else.  
  – E.g., Principal-Agent Situations  

• If fully principal, \( y_1 = f(X) \); if fully agent, \( y_2 = g(Z) \); institutions: \( 0 \leq h(I) \leq 1 \).

\[
y = h(I) f(X) + \{1 - h(I)\} g(Z)
\]

\[
\Rightarrow \frac{\partial y}{\partial x} = h(I) \frac{\partial f(X)}{\partial x} \quad ; \quad \frac{\partial y}{\partial z} = -h(I) \frac{\partial g(Z)}{\partial z};
\]

\[
\frac{\partial y}{\partial i} = \frac{\partial h(I)}{\partial i} \left[ f(X) - g(Z) \right]
\]
(Complex) Context-Conditionality: (Hallmark of Modern Pol-Sci Theory?)

- Principal-Agent (Shared Control) Situations, for example:
  - If fully principal: \( y_1 = f(X) \);
  - If fully agent: \( y_2 = g(Z) \);
  - Institutions\(=>\)Monitoring & Enforcement costs principal must pay to induce agent behave as principal would: \( 0 \leq h(I) \leq 1 \).
  - RESULT:

\[
y = h(I)f(X) + \{1 - h(I)\}g(Z)
\]

\[
\Rightarrow \quad \frac{\partial y}{\partial x} = h(I) \frac{\partial f(X)}{\partial x} ;
\]

\[
\frac{\partial y}{\partial z} = - h(I) \frac{\partial g(Z)}{\partial z} ;
\]

\[
\frac{\partial y}{\partial i} = \frac{\partial h(I)}{\partial i} \left[ f(X) - g(Z) \right]
\]

- In words...
  ... 
  ...
  ...i.e., effect of anything depends on everything else!
Not Every Argument Is an Interactive Argument

- **Not Interactive:**
  - $X$ affects $Y$ through its effect on $Z$: $X \Rightarrow Z \Rightarrow Y$
    - In (political) psychology / behavior, this called *mediation*. Interaction is called *moderation* in this literature.
  - $X$ and $Z$ affect each other: $X \Leftrightarrow Z$.
    - I.e., $X$ and $Z$ endogenous to each other. Note: irrelevant to Gauss-Markov (OLS is BLUE); merely implies care to what partials (coefficients) mean.
  - $Y$ depends on $X$ controlling for $Z$, or $Y$ depends on $X$ & $Z$: $E(Y|X,Z)=f(Z), \ E(Y|X)=f(Z), \ Y=f(X,Z)$
    - I.e., e.g., showing outcomes differ across $2 \times 2$ of $X$ & $Z$ insufficient; issues is difference of differences across rows or down columns.

- **Interactive**: *Effect of $X$ on $Y$ depends on $Z$* ($\Rightarrow$ converse: *Effect of $Z$ on $Y$ depends on $X$)*:

  \[
  \frac{\partial Y}{\partial X} = f(Z) \iff \frac{\partial Y}{\partial Z} = f(X)
  \]
From Theory/Substance to Empirical-Model Specification

- Classic Comparative-Politics Example:
  - Societal Fragmentation, $S\text{Frag}$, &
  - Electoral-System Proportionality, $D\text{Mag}$,
  - $\Rightarrow$ Effective # Parliamentary Parties: $E\text{NPP}$

- “Theory”: $E\text{NPP} = f(S\text{Frag}, D\text{Mag}, \cdot, \varepsilon)$

- Hypotheses:
  \[
  \frac{\partial E\text{NPP}}{\partial S\text{Frag}} \geq 0 \quad \frac{\partial E\text{NPP}}{\partial D\text{Mag}} \geq 0
  \]

- Empirical Specification: Lots ways get there...
A Typical Linear-Interactive Specification

- Want linear \( f(\cdot) \) w/ these properties; many ways to get there:

\[
\begin{align*}
\text{ENPP} &= \beta_0 + \beta_1 \text{SFrag} + \beta_2 \text{DMag} + \varepsilon \\
\frac{\partial \text{ENPP}}{\partial \text{SFrag}} &= \beta_1 \Rightarrow f(\text{DMag}) = \alpha_0 + \alpha_1 \text{DMag} \\
\frac{\partial \text{ENPP}}{\partial \text{DMag}} &= \beta_2 \Rightarrow f(\text{SFrag}) = \gamma_0 + \gamma_1 \text{SFrag} \\
\Rightarrow \text{ENPP} &= \beta_0 + (\alpha_0 + \alpha_1 \text{DMag}) \text{SFrag} + (\gamma_0 + \gamma_1 \text{SFrag}) \text{DMag} + \varepsilon \\
&= \beta_0 + \alpha_0 \text{SFrag} + \alpha_1 \text{DMag} \text{SFrag} + \gamma_0 \text{DMag} + \gamma_1 \text{SFrag} \text{DMag} + \varepsilon \\
&= \beta_0 + \alpha_0 \text{SFrag} + \gamma_0 \text{DMag} + (\alpha_1 + \gamma_1) \text{SFrag} \text{DMag} + \varepsilon \\
&= \beta_0 + \beta_{SF} \text{SFrag} + \beta_{DM} \text{DMag} + \beta_{SFDM} \text{SFrag} \text{DMag} + \varepsilon \\
\Rightarrow \frac{\partial \text{ENPP}}{\partial \text{SFrag}} &= \beta_{SF} + \beta_{SFDM} \text{ DMag} \\
\frac{\partial \text{ENPP}}{\partial \text{DMag}} &= \beta_{DM} + \beta_{SFDM} \text{SFrag}
\end{align*}
\]
Interpretation of *Effects*:
Derivatives & Differences, *Not* Coefficients

- **Standard Linear Interactive Model:**
  \[ EN = \beta_0 + \beta_{SF} SF + \beta_{DM} DM + \beta_{SFDM} SF \times DM + \ldots + \varepsilon \]

- **Effect of *SF*rag on *ENPP* (is a function of *DMag*):**
  \[ \text{Effect}(SF) \equiv \frac{\partial EN}{\partial SF} = \beta_{SF} + \beta_{SFDM} DM \]
  \[ \Delta EN = \beta_{SF} \Delta SF + \beta_{SFDM} DM \cdot \Delta SF \]
  \[ \equiv \frac{\Delta EN}{\Delta SF} = \beta_{SF} + \beta_{SFDM} DM \]

- **Effect of *DMag* on *ENPP* (is f of *SF*rag):**
  \[ \text{Effect}(DMag) \equiv \frac{\partial ENPP}{\partial DMag} = \beta_{DM} + \beta_{SFDM} SFrag \]
  \[ \equiv \frac{\Delta ENPP}{\Delta DM} = \beta_{DM} \Delta DM + \beta_{SFDM} SFrag \cdot \Delta DM \]
  \[ \equiv \frac{\Delta ENPP}{\Delta DM} = \beta_{DM} + \beta_{SFDM} SFrag \]}
Interpretation of *Effects*: NOTES

- “Main Effect” & “Interactive Effect”:
  - For example, $\beta_{SF} = \text{“main effect of SFrang”}$
  - ....but $\beta_{SF}$ is merely the effect of SFrang at other variable(s) involved in interaction with it=0, so:
    - *Other-var*(s)=0 may be extreme in the sample, or beyond sample range, or even logically impossible.
    - *Other-var*(s)=0 substantive meaning of 0 altered by rescaling
      - E.g., by “centering” (centering changes nothing, btw...)
    - *Other-var*(s)=0 may not have anything substantively main about it
    - Is no Main Effect or separately & Interactive Effect; is just the effect, which conditional, varies:

\[
\text{Effect}(SF) \equiv \frac{\partial EN}{\partial SF} = \beta_{SF} + \beta_{SFDM} \, DM \quad ; \quad \text{Effect}(DM) \equiv \frac{\partial EN}{\partial DM} = \beta_{DM} + \beta_{SFDM} \, SF
\]
Interpretation of Effects: NOTES²

\[ EN = \beta_0 + \beta_{SF}SF + \beta_{DM}DM + \beta_{SFDM}SF \times DM + ... + \varepsilon \]

- COEFFICIENTS ARE NOT EFFECTS. EFFECTS ARE DERIVATIVES &/OR DIFFERENCES.

  - Only in **purely** linear-additive-separable model are they equal because only there do derivatives simply equal coefficients.

  - \( \beta_{SF} \) is **not** “effect of S\text{Frag} ‘independent of’…” & definitely not its “effect ‘controlling for’…other variable(s) in the interaction”

- Cannot substitute linguistic invention for understanding model’s logic (its simple math)
Interpretation of *Effects*: NOTES\textsuperscript{3}

- Interactions are logically symmetric:
  - For any function, not just lin-add.\[ \frac{\partial \left\{ \frac{\partial y}{\partial x} \right\}}{\partial z} \equiv \frac{\partial \left\{ \frac{\partial y}{\partial z} \right\}}{\partial x} \equiv \frac{\partial^2 y}{\partial x \partial z} \equiv \frac{\partial^2 y}{\partial z \partial x} \]
  - If argue effect \( x \) depends \( z \), must also believe effect \( z \) depends \( x \).

- Interactions often have 2\textsuperscript{nd}-moment (variance, i.e., heteroskedacidity) implications too:
  - Larger district magnitudes, \( DMag \), are “permissive” elect sys: *allow* more parties…
  - Fewer *Veto Actors* *allow* greater policy-change… (both need additional assumpts)

- All of this holds for any type of variable:
  - Measurement: binary, continuous…
  - *Level*: micro or macro; \( i, j, k, \ldots \)
Frequent 2nd-Moment Implications Interactions

- **DMag** permissive ele sys: **allows** more parties...
  \[ NP = \beta_0 + \beta_1 DM + \varepsilon \ ; \ V(\varepsilon) = f(DM) \ , \text{ e.g., } \sigma_0 + \sigma_1 DM \]
  - Note: unmodeled interactions look like heteroskedasticity; that’s general, actually. Anything unmodeled gets into \( e^2 \)...

- Few **Veto Actors allows** greater policy-change...

\[ y = \beta_0 + \beta_1 VP + \varepsilon \ ; \ V(\varepsilon) = f(VP) \ , \text{ e.g., } \sigma_0 + \sigma_1 VP \]

- I.e., these are Rndm-Coeff &/or Het-sked Props...
Interpretation of Effects: Standard Errors for Effects

\[ ENPP = \beta_0 + \beta_{SF} \text{SFrag} + \beta_{DM} \text{DMag} + \beta_{SFDM} \text{SFragDMag} + \ldots + \varepsilon \]

- Std Errs reported with regression output are for coefficients, not for effects.
  - The s.e. \((t\text{-stat}, p\text{-level})\) for \(\hat{\beta}_{SF}\) regards the estimated effect of SFrag on ENPP at DMag=0 (...which is logically impossible).
- Effect of \(x\) depends on \(z\) & v.v. (i.e., which was the point, remember?), so does the s.e.:

\[
\text{Effect}(x) = \frac{\partial y}{\partial x} = \beta_x + \beta_{xz}z \Rightarrow \text{Est.Eff.}(x) = E\left(\frac{\partial y}{\partial x}\right) = \hat{\beta}_x + \hat{\beta}_{xz}z
\]

\[
\text{Est.Var.}\{\text{Est.Eff.}(x)\} = E\left[\text{Var}\left(E\left(\frac{\partial y}{\partial x}\right)\right)\right] = E\left[\text{Var}\{\hat{\beta}_x + \hat{\beta}_{xz}z\}\right]
\]

\[
= V\left\{\hat{\beta}_x + \hat{\beta}_{xz}z\right\} = V\left\{\hat{\beta}_x\right\} + V\left\{\hat{\beta}_{xz}\right\} \cdot z^2 + 2 \cdot C\left(\hat{\beta}_x, \hat{\beta}_{xz}\right)z
\]

- In words… More Generally:

\[
V(x'\hat{\beta}) = x'\left[V(\hat{\beta})\right]x
\]
From Hypotheses to Hypotheses Tests:

Does \( Y \) Depend on \( X \) or \( Z \)?

\[ ENPP = \beta_0 + \beta_{SF} S\text{Frag} + \beta_{DM} D\text{Mag} + \beta_{SFDM} S\text{FragDMag} + \ldots + \varepsilon \]

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Mathematical Expression (^{91})</th>
<th>Statistical test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x ) affects ( y ), or ( y ) is a function of (depends on) ( x )</td>
<td>( y = f(x) ) ( \frac{\partial y}{\partial x} = \beta_x + \beta_{xz} z \neq 0 )</td>
<td>( F )- test: ( H_0: \beta_x = \beta_{xz} = 0 )</td>
</tr>
<tr>
<td>( x ) increases ( y )</td>
<td>( \frac{\partial y}{\partial x} = \beta_x + \beta_{xz} z &gt; 0 )</td>
<td>( \text{Multiple } t)-tests: ( H_0: \beta_x + \beta_{xz} z \leq 0 )</td>
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<td>( x ) decreases ( y )</td>
<td>( \frac{\partial y}{\partial x} = \beta_x + \beta_{xz} z &lt; 0 )</td>
<td>( \text{Multiple } t)-tests: ( \beta_x + \beta_{xz} z \geq 0 )</td>
</tr>
<tr>
<td>( z ) affects ( y ), or ( y ) is a function of (depends on) ( z )</td>
<td>( y = g(z) ) ( \frac{\partial y}{\partial z} = \beta_z + \beta_{xz} x \neq 0 )</td>
<td>( F )- test: ( H_0: \beta_z = \beta_{xz} = 0 )</td>
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### From Hypotheses to Hypotheses Tests:

Is $Y$’s Dependence on $X$ Conditional on $Z$ & v.v.? How?

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<th>Hypothesis</th>
<th>Mathematical Expression $^92$</th>
<th>Statistical test</th>
</tr>
</thead>
</table>
| The effect of $x$ on $y$ depends on $z$ | $y=f(xz, \bullet)$  
\[ \frac{\partial y}{\partial x} = \beta_x + \beta_{xz} z = g(z) \]  
\[ \frac{\partial (\frac{\partial y}{\partial x})}{\partial z} = \frac{\partial^2 y}{\partial x \partial z} = \beta_{xz} = 0 \] | $t$-test: $H_0: \beta_{xz} = 0$ |
| The effect of $x$ on $y$ increases in $z$ | $\frac{\partial (\frac{\partial y}{\partial x})}{\partial z} = \frac{\partial^2 y}{\partial x \partial z} = \beta_{xz} > 0$ | $t$-test: $H_0: \beta_{xz} \leq 0$ |
| The effect of $x$ on $y$ decreases in $z$ | $\frac{\partial (\frac{\partial y}{\partial x})}{\partial z} = \frac{\partial^2 y}{\partial x \partial z} = \beta_{xz} < 0$ | $t$-test: $H_0: \beta_{xz} \geq 0$ |
| The effect of $z$ on $y$ depends on $x$ | $y=f(xz, \bullet)$  
\[ \frac{\partial y}{\partial z} = \beta_z + \beta_{xz} x = h(x) \]  
\[ \frac{\partial (\frac{\partial y}{\partial z})}{\partial x} = \frac{\partial^2 y}{\partial z \partial x} = \beta_{xz} = 0 \] | $t$-test: $H_0: \beta_{xz} = 0$ |
| The effect of $z$ on $y$ increases in $x$ | $\frac{\partial (\frac{\partial y}{\partial z})}{\partial x} = \frac{\partial^2 y}{\partial z \partial x} = \beta_{xz} > 0$ | $t$-test: $H_0: \beta_{xz} \leq 0$ |
| The effect of $z$ on $y$ decreases in $x$ | $\frac{\partial (\frac{\partial y}{\partial z})}{\partial x} = \frac{\partial^2 y}{\partial z \partial x} = \beta_{xz} < 0$ | $t$-test: $H_0: \beta_{xz} \geq 0$ |

**Discussion:** Alternative views how explore this...

### Does $Y$ Depend on $X$, $Z$, or $XZ$?

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<tr>
<th>Hypothesis</th>
<th>Mathematical Expression $^93$</th>
<th>Statistical Test</th>
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<tbody>
<tr>
<td>$y$ is a function of (depends on) $z$, $z$, and/or their interaction</td>
<td>$y=f(x,z,xz)$</td>
<td>$F$-test: $H_0: \beta_x = \beta_z = \beta_{xz} = 0$</td>
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</table>
Discussion re: determining if $Z$ conditions effect of $X$, etc.

• Some suggest need find some $Z_0$ & $Z_1$ (in sample?) for which $\frac{dY}{dX}$ differ significantly:

  - $\frac{dY}{dX}|_{Z_0}=b_x+b_{xz}Z_0$, $\frac{dY}{dX}|_{Z_1}=b_x+b_{xz}Z_1$

  - So: $\frac{dY}{dX}|_{Z_0}-\frac{dY}{dX}|_{Z_1}$

    - $=b_x+b_{xz}Z_0-(b_x+b_{xz}Z_1)=b_{xz}(Z_0-Z_1)$

    - Wald test this signif diff zero =>

      - $=\frac{b_{xz}(Z_0-Z_1)}{[\text{var}\{b_{xz}(Z_0-Z_1)\}]^{\frac{1}{2}}}$
      - $=\frac{b_{xz}(Z_0-Z_1)}{[\text{var}\{b_{xz}\}(Z_0-Z_1)^2]^{\frac{1}{2}}}$
      - $=\frac{b_{xz}(Z_0-Z_1)}{[\text{s.e.}\{b_{xz}\}(Z_0-Z_1)]=b_{xz}/\text{s.e.}\{b_{xz}\},}$

    - I.e., just the standard (Wald) t-test on interactive coeff, and that’s because that coefficient is cross-derivative about which the hypothesis asks!
Use & Abuse of Some Common ‘Rules’

- **Centering to Redress Colinearity Concerns:**
  - Adds no info, so changes *nothing*; no help with colinearity or anything else; only moves substantive content of $x=0, z=0$.
  - Specifically, makes coeff. on $x$ ($z$), effect when $z$ ($x$) at sample-mean, the new 0. Do only if aids presentation.

- **Must Include All Components (if $x \cdot z$, then $x & z$):**
  - Application of Occam’s Razor &/or scientific caution (e.g., greater flexibility to allow linear w/in lin-interax model), but
  - *Not* a logical or statistical requirement.
  - Safer rule than opposite & to check almost always, but
  - *Not* override *theory & evidence*, esp. if (insofar as strongly) agree to exclude…

- **Pet-Peeve: Linguistic Gymnastics to Dodge the Math**
  - “Main effect, Interactive effect”: *the* effect in model is $dy/dx$.
  - Discussion of [coefficients & s.e.’s] as if [effects & s.e.’s].
Presentation: Marginal-Effects / Differences Tables & Graphs

- Plot/Tabulate Effects, $dy/dx$, over Meaningful &/or Illuminating Ranges of $z$, with Conf. Int.’s
  
  
  $\frac{dy}{dx} \pm t_{df,p} \sqrt{Var(dy/dx)} = \hat{\beta}_x + \hat{\beta}_{xz} z \pm t_{df,p} \sqrt{V(\hat{\beta}_x) + V(\hat{\beta}_{xz}) z^2} + 2C(\hat{\beta}_x, \hat{\beta}_{xz}) z$

- Explain axes
- Explain shape
- Linear-interax:
  - *Will cross 0 & be insig @ 0.*
- Rescaling &
  - “main effect”
  - “centering”
  - Max(Asterisks)

![Figure 5. Marginal Effect of Runoff, Extending the Range of Groups](image-url)
• Predictions, \( E(y|x,z) \): notice how helpful in matrix form:

\[
\hat{y} \pm t_{df,p} \sqrt{Var(\hat{y})} = \beta_0 + \beta_x x + \beta_z z + \beta_{xz} xz \pm t_{df,p} \sqrt{V(\hat{\beta}_0) + V(\hat{\beta}_x)x^2 + V(\hat{\beta}_z)z^2 + V(\hat{\beta}_{xz})(xz)^2 + 2C(\hat{\beta}_0, \hat{\beta}_x)x + 2C(\hat{\beta}_0, \hat{\beta}_z)z + 2C(\hat{\beta}_0, \hat{\beta}_{xz})xz + 2C(\hat{\beta}_x, \hat{\beta}_z)xz + 2C(\hat{\beta}_x, \hat{\beta}_{xz})x^2z + 2C(\hat{\beta}_z, \hat{\beta}_{xz})xz^2}
\]

– Here’s one place a little matrix algebra would help:

\[
\hat{y} \pm t_{df,p} \sqrt{Var(\hat{y})} = \beta_0 + \beta_x x + \beta_z z + \beta_{xz} xz \pm t_{df,p} \sqrt{x'V(\hat{\beta})x}
\]

\[
= \beta_0 + \beta_x x + \beta_z z + \beta_{xz} xz \pm t_{df,p} \sqrt{1 \ x \ z \ xz}
\]

\[
\begin{bmatrix}
\hat{V}(\hat{\beta}_0) & \hat{C}(\hat{\beta}_0, \hat{\beta}_x) & \hat{C}(\hat{\beta}_0, \hat{\beta}_z) & \hat{C}(\hat{\beta}_0, \hat{\beta}_{xz}) \\
\hat{C}(\hat{\beta}_0, \hat{\beta}_x) & \hat{V}(\hat{\beta}_x) & \hat{C}(\hat{\beta}_x, \hat{\beta}_z) & \hat{C}(\hat{\beta}_x, \hat{\beta}_{xz}) \\
\hat{C}(\hat{\beta}_0, \hat{\beta}_z) & \hat{C}(\hat{\beta}_x, \hat{\beta}_z) & \hat{V}(\hat{\beta}_z) & \hat{C}(\hat{\beta}_z, \hat{\beta}_{xz}) \\
\hat{C}(\hat{\beta}_0, \hat{\beta}_{xz}) & \hat{C}(\hat{\beta}_x, \hat{\beta}_{xz}) & \hat{C}(\hat{\beta}_z, \hat{\beta}_{xz}) & \hat{V}(\hat{\beta}_{xz})
\end{bmatrix}
\]

– Use spreadsheet or stat-graph software (…list coming...)
Presentation¹:
Choose Illuminating Graphics & Base Cases

• Interpretation same regardless of “type” of interax: effect always \( \equiv dy/dx \), but present appropriately:
  – All combos Dummy, Discrete, or Continuous:
    • Dummy-Dummy\( \Rightarrow \)\( >4 \) (or 2\#interacting variables) points estimated, so box & whisker or histograms effective
    • Dummy-Continuous or Discrete\( (\text{few}) \)-Continuous\( \Rightarrow \)\( >2 \) (or \# categories) slopes, so \( E(y|x,z) \) as line or \( dy/dx \) as box & whisker or histograms effective
    • Continuous – Continuous (or Disc. Many)\( \Rightarrow \)Effect-lines best or (slices from) contour plot (i.e., slices from 3D)
  – Powers (e.g., \( X \) & \( X^2 \Rightarrow \)parabola) viewable as interax w/ self; certain slope shifts too (e.g., \( dy/dx=a \) for \( x<x^0 \) & \( b \) for \( x>x^0 \) is \( x \) interact w/ dummy for condition)
Choose Illuminating Graphics & Base Cases

- Interpretation same regardless of “type” of interax: \( \textit{effect} \) always \( \equiv \frac{dy}{dx} \), but present appropriately...
  - Always plot over substantively revealing ranges.
  - Especially with sets of dummies, have several (identical) specification options:
    - (full-set or set-less-1): choose which (& what base if use set-less-1) to abet presentation & discussion
    - (overlapping or disjoint): choose to facilitate presentation & discussion.
  - Scale Effectively: e.g., center only if & to extent that aids presentation & discussion (b/c centering does nothing else)
Presentation: Choose Illuminating Graphics & Base Cases. Examples.

Dummy-Continuous Interaction: could also plot two $E(\text{Cands}|\text{Groups})$ lines, with c.i.’s, effectively.
Presentation $^4$: Choose Illuminating Graphics & Base Cases. Examples.

**Dummy-Dummy Interaction:** could also plot four $E(Supp|gender,party)$ box-whiskers effectively.
Presentation: Choose Illuminating Graphics & Base Cases. Examples.

Presentation: Choose Illuminating Graphics & Base Cases. Examples.

Figure 14. Marginal Effect of Parliamentary Support for Government, Pairwise-Interaction Model, with 90% Confidence Intervals

\[ \text{GovDur} = \beta_0 + \beta_{np} NP + \beta_{ps} PS + \beta_{pd} PD + \beta_{npps} NP \times PS + \beta_{nppd} NP \times PD + \beta_{pdps} PD \times PS + \varepsilon \] [25]
Elaborations, Complications, & Extensions: Sample-Splitting v. (Dummy-)Interacting

- Split-sample (e.g., unit-by-unit) \( \approx \) Full-Dum Interax:
  - Subsample by binary (or multinomial, e.g., by-unit in TSCS) category to estimate separately \( \approx \) Include dummy for each category (or set-less-1) & interact each dummy with each \( x \) (and include \( x \) by itself also if set-less-1)
    - Coeff’s same (or equal substantive content if using set-1 dummies).
    - S.E.’s same except \( s^2 \) part of OLS’s \( s^2(X'X)^{-1} \) versus \( s_i^2 \) for splitting
    - Can make essentially exact by allow \( s_i^2 \) (FWLS)
  - Subsample by hi/lo values of some non-nominal regressor is equiv to nominalizing the info in that var & dummy-interact
    - I.e., wasting information, when usually have too little (non-parametric or extreme-measurement-error arguments might justify)
    - So usually a bad idea… (could discuss arg’s for it, under rare circ.)
Elaborations, Complications, & Extensions: Sample-Splitting v. (Dummy-)Interacting²

- Split-sample abets eyeballing, obfuscates statistical analysis, of the main point: the different effects by category.
  - What’s s.e/signif. of $b_{1i} - b_{1j}$? Need:
    \[
    \text{s.e.}(b_{1i} - b_{1j}) = \sqrt{V(b_{1i}) + V(b_{1j}) - 2C(b_{1i}, b_{1j})} = \sqrt{V(b_{1i}) + V(b_{1j})}
    \]
  - Luckily, cov=0, but, still, squaring 2 terms, sum, & root in head?
- Can choose full dummy set to mirror the split-sample estimates directly (& report that way, if wish) or the set-less-one to get significance of differences b/w samples directly (in the standard reported t-test)
  - Same thing, so choose form to optimize presentational efficacy.
- One advantage of hierarchical models (random coeff.) is how it affords, naturally, various positions b/w these extremes.
  - E.g., can “borrow strength” across units.
Typical 2\textsuperscript{nd}-Moment Implications of Interactions

- \textit{DMag} permissive ele sys: \textbf{allow} more parties…
  \[ NP = \beta_0 + \beta_1 DM + \varepsilon \, ; \quad V(\varepsilon) = f(DM) \, , \text{e.g.,} \, \sigma_0 + \sigma_1 DM \]

- Few \textbf{Veto Actors allow} greater policy-change…

- I.e., these are Rndm-Coeff &/or Het-sked Props…
  \[ NP = \beta_0 + \beta_1 DM + \beta_2 SF + \beta_3 DM \times SF + \varepsilon \, ; \quad V(\varepsilon) = f(DM) \, , \text{e.g.,} \, \sigma_0 + \sigma_1 DM \]
Sandwich Estimators

\[ EN = \beta_0 + \beta_1 SF + \beta_2 DM + \varepsilon \]

\[ \frac{\partial EN}{\partial SF} = \beta_1 = \alpha_0 + \alpha_1 DM + \omega_1 \]

\[ \frac{\partial EN}{\partial DM} = \beta_2 = \gamma_0 + \gamma_1 SF + \omega_2 \]

\[ \Rightarrow EN = \beta_0 + (\alpha_0 + \alpha_1 DM + \omega_1) SF + (\gamma_0 + \gamma_1 SF + \omega_2) DM + \varepsilon \]

\[ = \beta_0 + \alpha_0 SF + (\alpha_1 + \gamma_1) DM \times SF + \gamma_0 DM + \{\varepsilon + \omega_1 SF + \omega_2 DM\} \]

\[ = b_0 + b_1 SF + b_2 DM \times SF + b_3 DM + \varepsilon^* \]

- Notice the compound error term:
  - \( V(\varepsilon^*) \) will not be \( \sigma^2 I \) even if \( \varepsilon \) is, so \( V(b) \) doesn’t reduce to \( \sigma^2(X'X)^{-1} \), so OLS s.e.’s wrong.
  - Be OK on average (unbiased) & in limit (consistent) if element of \( V(\varepsilon^*) \) “orthogonal to \( xx' \)”
  - But def’ly not because \( \varepsilon^* \) includes \( x \) & \( z \), which part of \( X \)!
Sandwich Estimators

\[ V(b_{LS}) = (X'X)^{-1}X'[V(\varepsilon + \omega_1SF + \omega_2DM)]X(X'X)^{-1} \]

- Brilliant insight of ‘robust’ (i.e., consistent) “sandwich” estimators:
  - Only need formula that accounts relation \( V(\varepsilon^*) \) to “\( X'X \)”
    i.e., regressors, squares, & cross-prod’s involved in \( X'[\cdot]X \)
- \( \Rightarrow \) “, robust” (or, in HM: “, cluster”) can work:
  \[ V(\varepsilon^*_i)_{RE} = \sigma^2_\varepsilon + \sigma^2_\omega_1 x^2_i + \sigma^2_\omega_2 z^2_i \]
  so track \( e^2 \) rel \( xx' \) & \( zz' \)
  - White’s heteroskedasticity-consistent s.e.’s:

\[
[\cdot] = \frac{1}{n} \sum_{i=1}^{n} e^2_i x_i x_i'
\]
Cross-Level Interactions

• Nothing much different if interactions between variables that vary at different levels (note, e.g., not many subscripts used above):
  – If CLRM assumptions apply, then unbiased, consistent, and efficient.

  • Two main issues of concern, though:
    – *Un- or insufficiently modeled* parameter heterogeneity (incl. intercept): can cause bias, if pattern un/insuff. het. relates to $X$, 
    – Non-spherical error-covariance matrix:
      » An efficiency & proper s.e.’s issue, not bias/consistency.
      » As just seen, surely will arise, & likely in different specific forms depending assumed error-components structure.
  
  – As before: Effects, their variances, symmetry of interactive propositions, that neither *micro-* nor *macro-level* coefficients=effects...all that applies...
Cross-Level Interactions:

\[
\text{reg spend L.spend unem left growthpc depratio cdem trade lowwage fdi skand skand_unem}
\]

\[
\text{spend}_{it} = \beta_i^0 + \beta_i^l \text{left}_{it} + ... + \varepsilon_{it}
\]

\[
\beta_i^0 = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0
\]

\[
\beta_i^l = \gamma_0 + \gamma_1 \text{skand}_i + u_i^1
\]

\[
\Rightarrow \text{spend}_{it} = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0 + \gamma_0 \text{left}_{it}
\]

\[
+ \gamma_1 \text{left}_{it} \times \text{skand}_i + \text{left}_{it} u_i^1 + ... + \varepsilon_{it}
\]

gathering terms:

\[
\text{spend}_{it} = \alpha_0 + ... + \alpha_1 \text{skand}_i + \gamma_0 \text{left}_{it}
\]

\[
+ \gamma_1 \text{left}_{it} \times \text{skand}_i + \left( u_i^0 + \text{left}_{it} u_i^1 + \varepsilon_{it} \right)
\]

\[
\Rightarrow \frac{\partial \text{spend}}{\partial \text{left}} = b_{left} + b_{lftsk} \text{skand} + u_i^1 \quad \& \quad \frac{\partial \text{spend}}{\partial \text{skand}} = b_{skand} + b_{lftsk} \text{left}
\]

\[
V(\varepsilon_i^*)_{HM} = \sigma_0^2 + \sigma_1^2 x_i x_i' + \sigma_2^2 z_i z_i' \quad \Rightarrow \text{cluster:} \quad [\cdot] = \sum_{j=1}^J \left\{ \sum_{i=1}^n \frac{(\Sigma e_{ij} x_{ij})' (\Sigma e_{ij} x_{ij})}{n_i} \right\}
\]
From CLRM to Multilevel Model

\[
\text{spend}_{it} = \beta_i^0 + \beta_i^l \text{left}_{it} + \ldots + \varepsilon_{it}
\]

\[
\beta_i^0 = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0
\]

\[
\beta_i^l = \gamma_0 + \gamma_1 \text{skand}_i + u_i^1
\]

● If CLRM assumptions apply, then OLS unbiased, consistent, and efficient.

  - Two main issues of concern:
    - Parameter heterogeneity: (see pictures on next slide)
      - systematic &/or stochastic (fixed v. rndm intrcpt/coeff)
      - can cause bias if pattern unmodeled hetero relates to \( X \),
    - Non-spherical error cov-mat: an efficiency & proper s.e.’s issue, not a bias/consistency one
      - But “mere inefficiency” can be serious.
      - And accurate std err’s very important.
From the CLRM to HLM

- Examples of parameter heterogeneity that covaries w/ x values, so bias:

  Note: FE v. RE both theoretically could cause bias if cov w/ x, but latter identified by orthogonality assumption, as we’ll see.
From the CLRM to RE Model

- Arbitrary R.E. Model: Odd that std. lin-interax model:
  - Assumes know $y=f(X)+\text{error}$: $y_i = \beta_0 + \beta_x x_i + \beta_z z_i + \beta_{xz} x_i z_i + \varepsilon_i$
  - But $dy/dx=f(z)$ w/o error!: $\frac{dy}{dx} = \beta_x + \beta_{xz} x z$
  - So, try:

\[
 y = \beta_0 + \beta_1 x + \beta_2 z + \varepsilon^0 \\
 \frac{dy}{dx} \equiv \beta_1 = \alpha_0 + \alpha_1 z + \varepsilon^1 \\
 \frac{dy}{dz} \equiv \beta_2 = \gamma_0 + \gamma_1 x + \varepsilon^2 \\
 \Rightarrow y = \beta_0 + \left(\alpha_0 + \alpha_1 z + \varepsilon^1\right) x + \left(\gamma_0 + \gamma_1 x + \varepsilon^2\right) z + \varepsilon^0 \\
 = \beta_0 + \alpha_0 x + \gamma_0 z + (\alpha_1 + \gamma_1) x z + (\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z)
\]
  - => std. lin-interact...except compound error-term...

- Hierarchical/Multilevel/Mixed-Effect Model: Same model, except $x_{ij}$ & $z_j$, & specifically: $\varepsilon^* = \varepsilon^0_{ij} + \varepsilon^1_{ij} x_{ij} + \varepsilon^2_{ij} z_j$
  - So also std lin-interact, but w/ diff compound-error structure

- These also called “error-component” models
From CLRM to Hierarchical Model

- Std. HLM: Same model, except $x_{ij}$ & $z_j$, $\varepsilon_i$
  - So a std. lin-interact too, but with different compound-error stochastic properties.

\[
\text{spend}_{it} = \beta_i^0 + \beta_i^l \text{left}_{it} + \ldots + \varepsilon_{it}
\]
\[
\beta_i^0 = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0
\]
\[
\beta_i^l = \gamma_0 + \gamma_1 \text{skand}_i + u_i^1
\]
\[
\Rightarrow \text{spend}_{it} = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0 + \gamma_0 \text{left}_{it}
\]
\[
\ldots + \gamma_1 \text{left}_{it} \times \text{skand}_i + \text{left}_{it} u_i^1 + \ldots + \varepsilon_{it}
\]

Gathering terms:
\[
\text{spend}_{it} = \alpha_0 + \ldots + \alpha_1 \text{skand}_i + \gamma_0 \text{left}_{it}
\]
\[
\ldots + \gamma_1 \text{left}_{it} \times \text{skand}_i + (u_i^0 + \text{left}_{it} u_i^1 + \varepsilon_{it})
\]
\[
\Rightarrow \frac{\partial \text{spend}}{\partial \text{left}} = b_{\text{left}} + b_{\text{lfsk}} \text{skand} + u_i^1 \quad \& \quad \frac{\partial \text{spend}}{\partial \text{skand}} = b_{\text{skand}} + b_{\text{lfsk}} \text{left}
\]
Properties of OLS under HLM Conditions

- Properties of OLS Estimates of Linear-Interaction Model if truly RE/HLM:

\[ y = \beta_0 + \beta_x x + \beta_z z + \beta_{xz} xz + (\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z) = X\beta + (\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z) \]

- So, OLS coeff. est’s still differ from truth by \( A\varepsilon^* \):

\[
\hat{\beta}_{LS} = (X'X)^{-1}X'y = (X'X)^{-1}X'[X\beta + (\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z)] = (X'X)^{-1}X'X\beta + (X'X)^{-1}X'(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z) = \beta + (X'X)^{-1}X'\varepsilon^* 
\]

- So, OLS coeff. est’s unbiased & consistent:

\[
E(\hat{\beta}_{LS}) = E[\beta + (X'X)^{-1}X'\varepsilon^*] = E[\beta + (X'X)^{-1}X'(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z)] = \beta + (X'X)^{-1}X'[E(\varepsilon^0) + E(\varepsilon^1)x + E(\varepsilon^2)z] = \beta + (X'X)^{-1}X'[0 + 0 + 0] = \beta. \quad Q.E.D.
\]

- Note: only works for models w/ additively separable stochastic component; not necessarily for others (e.g., logit/probit)
Properties of OLS under HLM Conditions

• But, OLS s.e.’s will be wrong; not \( s^2(X'X)^{-1} \), but:

\[
V(\hat{\beta}_{LS}) = V\left[\beta + (X'X)^{-1}X'e^*\right]
\]

\[
= V[\beta] + V\left[(X'X)^{-1}X'e^*\right] + 2C\left[\beta, (X'X)^{-1}X'e^*\right]
\]

\[
= 0 + V\left[(X'X)^{-1}X'e^*\right] + 0
\]

\[
= (X'X)^{-1}X'V(e^*)X(X'X)^{-1}
\]

\[
= (X'X)^{-1}X'\left[V\left(e^0 + e^1x + e^2z\right)\right]X(X'X)^{-1}
\]

\[
= (X'X)^{-1}X'\left[V\left(e^0\right) + V\left(e^1x\right) + V\left(e^2z\right)\right]X(X'X)^{-1}
\]

(the covariance terms are assumed zero)
Sandwich Estimators

\[ V(\hat{\beta}_{LS}) = (X'X)^{-1} X' [V(\epsilon^0) + V(\epsilon^I x) + V(\epsilon^2 z)] X(X'X)^{-1} \]

- Not \( \sigma^2 I \) (even if each \( \epsilon^* \) component is), so whole thing doesn’t reduce to \( \sigma^2 (X'X)^{-1} \), so OLS s.e.’s wrong.
- Be OK on avg (unbiased) & in limit (consistent) if that term varied in way “orthogonal to \( xx' \)”
  - But, as b4, def’ly not b/c \([\cdot]\) includes \( x \) & \( z \), which part of \( X \).
  - =brilliant insight of ‘robust’ (i.e., consistent) s.e. est’s:
    - Only need s.e. formula that accounts relation \( V(\epsilon^*) \) to “\( xx' \)”, i.e., to the regressors, their squares, & cross-prod’s involved in \( X' [\cdot] X \)”
- \( \Rightarrow \), robust” & “, cluster” can work (for RE & HLM, resp’ly)
  - \( \hat{V}(\hat{\beta})_{RE} = \sigma^2 (I + xx' + zz') \) so track \( e^2 \) rel \( xx' \) & \( zz' \) \( \Rightarrow \)
    - i.e., White’s het-consistent s.e.’s
  - \( \hat{V}(\hat{\beta})_{HM} = \sigma_0^2 I + \sigma_1^2 xx' + \sigma_2^2 zz' \) sim but grpng \( \Rightarrow \)
    - i.e., het-cluster consistent s.e.’s
From the CLRM to HLM

\[ y = \beta_0 + \alpha_0 x + \gamma_0 z + (\alpha_1 + \gamma_1)xz + (\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z) \]

\[ V(\hat{\beta}_{LS}) = (X'X)^{-1}X'[V(\varepsilon^0) + V(\varepsilon^1 x) + V(\varepsilon^2 z)]X(X'X)^{-1} \]

- \Rightarrow appropriate “, robust” & “, cluster” can work
  - I.e., *asymptotically* std errs right...
    - Note: generally need large \( n_j \), more than just large \( n \), for cluster.
  - I.e., *coefficients still inefficient*.
    - Want/need efficiency, or \( n_j \) low? HLM/RE or FGLS/FWLS.
    - Note: similarity RE and HLM, RE & FWLS. As suggests, RE only helps efficiency and only rightly does so if that’s all it does. (I.e., if the RE’s orthogonal to X.)
  - I.e., “work” thusly for models with additively-separable stochastic components
    - As w/ all such “sandwich” estimators, logical disconnect in applying them to models w/o such separability.
Additional Topics

• Contents:
  – Interactions in nonlinear/QualDep models
  – Nonlinear Least Squares and complex context-conditionality. Applications:
    • “Multiple Hands on the Wheel”
    • “Veto Actors Bargaining in Common Pools” (Multiple Effects of Multiple Policymakers)
Elaborations, Complications, & Extensions: Interax in QualDep (Inherently Interactive) Models

- Probit/Logit Models w/ Interactions
  - Probit: \( p(y = 1) = \Phi(x'\beta) \)  - Logit: \( p(y = 1) = \frac{\exp(x'\beta)}{1 + \exp(x'\beta)} = [1 + \exp(-x'\beta)]^{-1} \)

- Marginal Effects: (nonlinear, so must specify @ what \( x \))
  - Start w/ \( x'\beta \) pure lin-add, model inherently inter. b/c S-shaped:
    - Probit:
      \[
      \frac{\partial p}{\partial x_k} = \frac{\partial \Phi(x'\beta)}{\partial x_k} = \phi(x'\beta) \cdot \frac{\partial x'\beta}{\partial x_k} = \phi(x'\beta) \cdot \beta_k
      \]
    - Logit:
      \[
      \frac{\partial p}{\partial x_k} = \frac{\partial \{e^{x'\beta} [1+e^{x'\beta}]^{-1}\}}{\partial x} = \frac{e^{x'\beta}}{1+e^{x'\beta}} \cdot \beta_k - \frac{e^{x'\beta}}{(1+e^{x'\beta})^2} \cdot e^{x'\beta} \cdot \beta_x
      \]
      \[
      = \left[ \frac{e^{x'\beta}(1+e^{x'\beta})}{(1+e^{x'\beta})^2} - \frac{(e^{x'\beta})^2}{(1+e^{x'\beta})^2} \right] \cdot \beta_k = \frac{e^{x'\beta}}{(1+e^{x'\beta})^2} \cdot \beta_k
      \]
      \[
      = \frac{e^{x'\beta}}{1+e^{x'\beta}} \cdot \frac{1}{1+e^{x'\beta}} \cdot \beta_k = p \cdot (1 - p) \cdot \beta_k
      \]
  - If \( x'\beta = \ldots + \beta_x x + \beta_z z + \beta_{xz} xz \ldots \Rightarrow \) same except \( dx'\beta/dx = \beta_x x + \beta_{xz} z \); underlying propensity, i.e., movement along S-shape also interact explicitly \( x \& z \). [Discuss meaning inherent v. explicit interax...]

- Probit:
  \[
  \frac{\partial p}{\partial x} = \phi(x'\beta) \cdot (\beta_x + \beta_{xz} z)
  \]
- Logit:
  \[
  \frac{\partial p}{\partial x} = p \cdot (1 - p) \cdot (\beta_x + \beta_{xz} z)
  \]
Elaborations, Complications, & Extensions: Interax in Nonlin/Qual (Inherently Interax) Models

- **Standard Errors?**
  - **Delta Method:**

\[
\text{Asym.Var.}(f(\hat{\beta})) \\
\approx \left[ \nabla_{\beta} f(\hat{\beta}) \right]' V(\hat{\beta}) \left[ \nabla_{\beta} f(\hat{\beta}) \right]
\]

- **Probit Marginal-Effect s.e.:**

\[
\frac{\partial \{ \phi(x'\hat{\beta}) \frac{\partial x'\hat{\beta}}{\partial x_1} \}}{\partial \hat{\beta}_1} \ldots \frac{\partial \{ \phi(x'\hat{\beta}) \frac{\partial x'\hat{\beta}}{\partial x_k} \}}{\partial \hat{\beta}_k} \\
\frac{\partial \{ \phi(x'\hat{\beta}) \frac{\partial x'\hat{\beta}}{\partial x_1} \}}{\partial \hat{\beta}_1} \ldots \frac{\partial \{ \phi(x'\hat{\beta}) \frac{\partial x'\hat{\beta}}{\partial x_k} \}}{\partial \hat{\beta}_k}
\]

- Logit: same, except \( \hat{p}(1 - \hat{p}) \frac{\partial x'\hat{\beta}}{\partial x} \) replaces \( \phi(x'\hat{\beta}) \frac{\partial x'\hat{\beta}}{\partial x} \)

- For first-difference effects, similar, but need specify from what \( x \) to what \( x \), and not just at what \( x \).

- Or you could *CLARIFY...* or *mfx...*
Elaborations, Complications, & Extensions:
Interax in QualDep Models: Cross-Derivatives

- Recall Marginal Effects:
  \[
  \frac{\partial p}{\partial x} = \phi(x'\beta) \cdot (\beta_x + \beta_{xz}z)
  \]

\[
\frac{\partial p}{\partial x} = \frac{e^{x'\beta}}{1+e^{x'\beta}} \cdot \frac{1}{1+e^{x'\beta}} \cdot (\beta_x + \beta_{xz}z) = p \cdot (1-p) \cdot (\beta_x + \beta_{xz}z)
\]

- So, what about conditioning effect of \( z(x) \) on the effect of \( x(z) \) on the probability \( y=1 \)?

\[
\frac{\partial}{\partial z} \left( \frac{\partial p}{\partial x} \right) = \phi(x'\beta) \cdot \beta_{xz} + \frac{\partial \{ \phi(x'\beta) \}}{\partial (x'\beta)} (\beta_x + \beta_{xz}z)(\beta_z + \beta_{xz}x)
\]

\[
\frac{\partial}{\partial z} \left( e^{x'\beta} (1 + e^{x'\beta})^{-2} (\beta_x + \beta_{xz}z) \right) = e^{x'\beta} (1 + e^{x'\beta})^{-2} \beta_{xz}
\]

\[
+ e^{x'\beta} (\beta_z + \beta_{xz}x)(1 + e^{x'\beta})^{-2} (\beta_x + \beta_{xz}z) - 2 \cdot e^{x'\beta} (1 + e^{x'\beta})^{-3} e^{x'\beta} (\beta_z + \beta_{xz}x)(\beta_x + \beta_{xz}z)
\]

\[
= \hat{p}(1 - \hat{p}) \beta_{xz} + \hat{p}(1 - \hat{p})(\beta_z + \beta_{xz}x)(\beta_x + \beta_{xz}z) - 2\hat{p}(1 - \hat{p})(1 + e^{x'\beta})^{-1} e^{x'\beta} (\beta_z + \beta_{xz}x)(\beta_x + \beta_{xz}z)
\]

\[
= \hat{p}(1 - \hat{p}) \beta_{xz} + \hat{p}(1 - \hat{p})[1 - 2\hat{p}] (\beta_z + \beta_{xz}x)(\beta_x + \beta_{xz}z)
\]
**Complex Context-Conditionality and Nonlinear Least-Squares**

- **Complex Context Conditionality**: The effect of anything depends on most everything else. E.g.:
  - **Policymaking**:
    - Socioeconomic-structure of interests
    - Party-system and internal party-structures
    - Electoral system & Governmental system
    - Socio-economic realities linking policies to outcomes
  - **Voting**:
    - Voter preferences & informational environment
    - Party/candidate locations & informational environment
    - Electoral & governmental system
  - **Institutions**: Sets of institutions; effect each depends configuration others present (e.g., that core of VoC claim).
  - **Strategic Interdependence**: each actors’ action depends on everyone else’s; complex feedback (see Franzese & Hays....)
Complex Context-Conditionality

- **Empirically**: Multicollinear Nightmare: Options?
  - Ignore context conditionality (stay linear-additive):
    - Inefficient at best, biased more usually, and, anyway, context-conditionality is often our interest!
  - Isolate one or some very few interactions for close study; ignore rest (stay linear-interactive):
    - Same issues, but to degree lessened by amount interax allow, but demands on data rise rapidly w/ that amount.
  - “Structured Case Analysis”:
  - **EMTI**\textsuperscript{TM}: Lean harder on thry/subst to specify more precisely the nature interax: functional form, precise measures, etc.
    - Refines question put to the data (usually changes default tests also).
    - *GIVEN* thry/subst. specification into empirical model, can estimate complex interactivity. Side benefits. But must *give*. 
Nonlinear Least-Squares

**Estimate NLS:**

\[ y = f(X, \beta) + \varepsilon \quad \text{with} \quad \varepsilon \sim g(\varepsilon) \]

\[ \Rightarrow E(y) = f(X, \beta), \text{ so } y = f(X, \hat{\beta}) + \hat{\varepsilon} \]

\[ \Rightarrow \text{Min } \hat{\varepsilon}'\hat{\varepsilon} \Rightarrow \text{Min } [y - f(X, \tilde{\beta})]'[y - f(X, \tilde{\beta})] \]

\[ \Rightarrow \text{Min } \text{SSE} = y'y - y'(X, \tilde{\beta}) - f(X, \tilde{\beta})'y + f(X, \tilde{\beta})'f(X, \tilde{\beta}) \]

\[ \Rightarrow \text{FOC: } \nabla_{\beta}\text{SSE}=0 \Rightarrow -2\nabla_{\beta}f(X, \tilde{\beta})'y + 2\nabla_{\beta}f(X, \tilde{\beta})'f(X, \tilde{\beta}) = 0 \]

So, if, e.g., \( f(X, \beta) = X\beta, \) then \( \nabla_{\beta}f(X, \tilde{\beta})' = X, \) so FOC: \( X'y = X'X\tilde{\beta} \Rightarrow \hat{\beta}_{LS} = (X'X)^{-1}X'y, \) and if

\[ V(\varepsilon) = \Omega = \sigma^2I, \text{ then } \hat{V}(\hat{\varepsilon})_{LS} = \frac{1}{n-k}[y - f(X, \tilde{\beta}_{LS})]'[y - f(X, \tilde{\beta}_{LS})] \quad \text{(also, as always)}. \]

That is, intuitively, writing \( \nabla_{\beta}f(X, \tilde{\beta}_{LS}) \) as simply \( \nabla, \) we have:

\[ \hat{\beta}_{LS} = (\nabla'\nabla)^{-1}\nabla'y \]

\[ \hat{V}(\hat{\beta}_{LS})_{LS} = \hat{V}[\nabla'\nabla]^{-1}\nabla'y] = (\nabla'\nabla)^{-1}\nabla'\hat{V}(y)\nabla(\nabla'\nabla)^{-1} \quad \text{& if } \hat{V}(y) = \sigma^2I, \text{ reduces to } \sigma^2(\nabla'\nabla)^{-1}. \]

which if \( f(X, \tilde{\beta}_{LS}) = X\tilde{\beta}_{LS} \) meaning \( \nabla = X, \) & if \( \Omega = \sigma^2I, \) gives the familiar

\[ \hat{\beta}_{LS} = (X'X)^{-1}X'y \quad \& \quad \hat{V}(\hat{\beta}_{LS})_{LS} = \sigma^2(X'X)^{-1}, \text{ as always}. \]

- **NLS is BLUE under same conditions OLS, w/ \( \nabla \) for \( X. \)**
- **Interpreting NLS (already know how): Effects = deriv’s & 1st-diff’s; s.e.’s by Delta Method or simulation…**
Generalized Nonlinear Least-Squares

**GNLS:**

\[ y = f(X, \beta) + \varepsilon \quad \text{with} \quad V(\varepsilon) = \sigma^2 \Omega \neq \sigma^2 I \]

\[ \Rightarrow \hat{\beta}_{\text{GNLS}} = (\nabla' \Omega^{-1} \nabla)^{-1} \nabla' \Omega^{-1} y \]

\[ \Rightarrow V(\hat{\beta}_{\text{GNLS}}) = (\nabla' \Omega^{-1} \nabla)^{-1} \nabla' \Omega^{-1} V(y) \Omega^{-1} \nabla (\nabla' \Omega^{-1} \nabla)^{-1} \]

\[ = (\nabla' \Omega^{-1} \nabla)^{-1} \nabla' \Omega^{-1} \Omega \Omega^{-1} \nabla (\nabla' \Omega^{-1} \nabla)^{-1} \]

\[ = (\nabla' \Omega^{-1} \nabla)^{-1} \nabla' \Omega^{-1} \nabla (\nabla' \Omega^{-1} \nabla)^{-1} = (\nabla' \Omega^{-1} \nabla)^{-1} \]

- GNLS is BLUE in same cond’s NLS, but \( \Omega \) for \( I \).
- …don’t know \( \Omega \), so need consistent 1\(^{st} \) stage (e.g., NLS)

**FGNLS is asymptotically BLUE:**

\[ y = f(X, \beta) + \varepsilon \quad \text{with} \quad V(\varepsilon) = \sigma^2 \Omega \neq \sigma^2 I \]

\[ \Rightarrow \hat{\beta}_{\text{FGNLS}} = (\nabla' \hat{\Omega}^{-1} \nabla)^{-1} \nabla' \hat{\Omega}^{-1} y \]

\[ \Rightarrow V(\hat{\beta}_{\text{FGNLS}}) = (\nabla' \hat{\Omega}^{-1} \nabla)^{-1} \nabla' \hat{\Omega}^{-1} V(y) \hat{\Omega}^{-1} \nabla (\nabla' \hat{\Omega}^{-1} \nabla)^{-1} \]

\[ = (\nabla' \hat{\Omega}^{-1} \nabla)^{-1} \]

\[ = (\nabla' \hat{\Omega}^{-1} \nabla)^{-1} \nabla' \hat{\Omega}^{-1} \nabla (\nabla' \hat{\Omega}^{-1} \nabla)^{-1} = (\nabla' \hat{\Omega}^{-1} \nabla)^{-1} \]
Nonlinear Least-Squares & EMTI

- **EITM: Empirical Implications of Theoretical Models**
  - Vision: Theory ⇒ more, sharper predictions ⇒ better tests, which therefore inform theory more, which...

- **TMEI: Theory-specified Models for Empirical Inference**
  - Vision: Theory structures empirical models & relations b/w obs ⇒ specification & (causal) i.d. of empirical models

- **TIEM: Theoretical Implications of Empirical Measures**
  - Vision: Emp. regularities, findings, measures inform theory dev’p.

- **EMTI: Empirical Models of Theoretical Intuitions**
  - Vision: Intuitions derived from theoretical models specify empirical models. I.e., empirical specification to match intuitions, not model.

- Note: Strongly counter some alternative moves stats & econometrics, & related; there toward non-parametric, matching, & experimentation—there, “model-dependence” a 4-letter word. Alternative audiences & rhetorical purposes?
  - Convince skeptic some causal effect exists, vs.
  - For the convinced, give richer, portable model of how world works.
Nonlinear Least-Squares: “Multiple Hands on the Wheel” Model (Franzese, PA ‘03)

- Monetary Policy in Open & Institutionalized Econ
  - Key C&IPE Insts/Struct: CBI, ER-Regime, Mon. Open
    - CBI ≡ ° Govt Delegated Mon Pol to CB
    - Peg ≡ ° Domestic (CB&Gov) Delegate to Peg-Curr (CB&Gov)
    - FinOp ≡ ° Dom cannot delegate b/c effectively del’d to globe
  - Effect of ev’thing to which for. & dom. mon. pol-mkrs would respond diff’ly depends on combo insts-structs & v.v., & through intl inst-structs, for. on dom. & v.v.

\[
\pi = \begin{cases} 
    P \cdot E \cdot C \cdot \pi_1(X_1) + P \cdot E \cdot (1-C) \cdot \pi_2(X_2) \\
    + P \cdot (1-E) \cdot C \cdot \pi_3(X_3) + P \cdot (1-E) \cdot (1-C) \cdot \pi_4(X_4) \\
    (1-P) \cdot E \cdot C \cdot \pi_5(X_5) + (1-P) \cdot E \cdot (1-C) \cdot \pi_6(X_6) \\
    +(1-P) \cdot (1-E) \cdot C \cdot \pi_7(X_7) + (1-P) \cdot (1-E) \cdot (1-C) \cdot \pi_8(X_8)
\end{cases}
\]

- Multicolinear Nightmare:
  - \(2^3=8\) inst-struct conds, \(i\), times \(k\) factors per \(\pi_i(X_i)\) if lin-interact
  - Exponentially more if all polynomials; \(k!/2(k-2)!\) if all pairs.
  - Good thing can lean on some thry to specify more precisely!
Nonlinear Least-Squares:
“Multiple Hands on the Wheel” Model

- **CB & Govt Interaction** ([Franzese, *AJPS* ‘99]):
  \[E(\pi) = c \cdot \pi_c(x_c) + (1-c) \cdot \pi_g(x_g)\]

\[\pi_c = \pi_c, \quad \pi_g(x_g) = \pi_g(GP, UD, BC, TE, EY, FS, AW, \pi_a)\]

- **Full Monetary Exposure & Atomistic** \(\Rightarrow\) **zero domestic autonomy** \(\Rightarrow\)

\[\pi_1(x_1) = \pi_2(x_2) = \pi_5(x_5) = \pi_6(x_6) = \pi_a\]

\[\Rightarrow \begin{cases} E \cdot \pi_a + P \cdot (1-E) \cdot C \cdot \pi_3(x_3) + P \cdot (1-E) \cdot (1-C) \cdot \pi_4(x_4) \\ +(1-P) \cdot (1-E) \cdot C \cdot \pi_c + (1-P) \cdot (1-E) \cdot (1-C) \cdot \pi_g(x_8) \end{cases}\]

- **s.t. that, full e.r. fix** \(\Rightarrow\)**CB&Govt match peg** \(\Rightarrow\)

\[\pi_3(x_3) = \pi_4(x_4) = \pi_p\]

\[\Rightarrow \begin{cases} E \cdot \pi_a + P \cdot (1-E) \cdot \pi_p \\ +(1-P) \cdot (1-E) \cdot [C \cdot \pi_c + (1-C) \cdot \pi_g(x_8)] \end{cases}\]
Nonlinear Least-Squares: “Multiple Hands on the Wheel” Model

- Compact & intuitive, yet gives all theoretically expected interactions, in the form expected

\[
\pi = E \cdot \pi_a + (1 - E) \cdot \left\{ P \cdot \pi_p + (1 - P) \cdot \left[ C \cdot \pi_c + (1 - C) \cdot \pi_g(X_g) \right] \right\}
\]

\[
\Rightarrow
\]

\[
\frac{\partial \pi}{\partial E} = \pi_a(P^*, E^*, C^*, X^*, \pi_a^*) - \left\{ P \cdot \pi_p(P^*, E^*, C^*, X^*, \pi_p^*) + (1 - P) \cdot \left[ C \cdot \pi_c + (1 - C) \cdot \pi_g(X_g) \right] \right\}
\]

\[
\frac{\partial \pi}{\partial P} = (1 - E) \cdot \left\{ \pi_p(P^*, E^*, C^*, X^*, \pi_p^*) - \left[ C \cdot \pi_c + (1 - C) \cdot \pi_g(X_g) \right] \right\}
\]

\[
\frac{\partial \pi}{\partial C} = (1 - E) \cdot \left\{ (1 - P) \cdot \left[ \pi_c - \pi_g(X_g) \right] \right\}
\]

\[
\frac{\partial \pi}{\partial X} = (1 - E) \cdot \left\{ (1 - P) \cdot \left[ (1 - C) \cdot \frac{\partial \pi_g}{\partial X} \right] \right\}
\]

\[
\frac{\partial \pi}{\partial x^*} = E \cdot \frac{\partial \pi_a}{\partial x^*} + (1 - E) \cdot \left\{ P \cdot \frac{\partial \pi_p}{\partial x^*} + (1 - P) \cdot \left[ (1 - C) \cdot \frac{\partial \pi_g}{\partial \pi_a} \cdot \frac{\partial \pi_a}{\partial x^*} \right] \right\}
\]
Nonlinear Least-Squares: “Multiple Hands on the Wheel” Model

- Effectively Estimable, yet gives all theoretically expected interactions, in the form expected

\[
E(\pi) = B_0 + \beta_e F \cdot \beta_{\pi} \cdot \pi + \left(1 - \beta_e F\right) \cdot \left[\left(\beta_{gp} GP + \beta_{ey} EY + \beta_{up} UP + \beta_{bc} BC + \beta_{aw} AW + \beta_{fs} FS + \beta_{te} TE + \beta_{a} \pi_a\right) \cdot \left(1 - \beta_{c1} C\right) + \beta_{c1} C \cdot \beta_{c2} \cdot \left(1 - \beta_{sp} SP - \beta_{mp} MP\right) + \beta_{sp} SP \cdot \beta_{\pi} \cdot \pi_{sp} + \beta_{mp} MP \cdot \beta_{\pi} \cdot \pi_{mp}\right]
\]

- Just 14 parameters (plus intercepts & dynamics, assuming those constant), just 3 more than lin-add!

- Parameters substantive meaning, too:
  - Degree to which...constrains certain set of actors.
  - Yields est. of inflation-target hypothetical fully indep CB
    - \(\Rightarrow\) general strategy for estimating/measuring unobservables
      - If know role factor will play & explanators of factor well enough, can estimate unobservables conditional on both those theories, if both powerful enough & enough empirical variation.
Nonlinear Least-Squares: “Multiple Hands on the Wheel” Model

- Neat, but does it work? (Try it! Data online; stata: help nl). Estimated Equation, w/ Std.Errs.:

$E(\pi) \approx \left(1 - 0.44^{1.14} E\right) \left\{\begin{array}{l}
 0.53^{0.30} + 0.55^{0.05} \pi_{t-1} - 0.12^{0.04} \pi_{t-2} + 0.44^{0.14} E \pi_a + \\
 1.0^{0.05} SP \cdot 0.59^{0.07} \pi_{sp} + 0.22^{0.12} MP \cdot 0.59^{0.07} \pi_{mp} + \\
 (1 - 1.0^{-0.5} SP - 0.22 \cdot 12 MP) \left\{1.0^{1.11} C(-0.59^{-1.2}) + \\
 (1 - 1.0^{-1.11} C) \left\{-0.60^{0.30} GP + 2.6^{0.13} EY + 16^{0.46} UP - 11^{0.24} BC \\
 + 12^{0.49} AW - 1.3^{0.30} FS - 8.2^{0.49} TE + 0.64^{0.24} \pi_a \right\}\right\}\right\}$

- Estimated Effects (highly context-conditional):

$E\left(\frac{d\pi}{dC}\right) = (1 - 0.44 \cdot E) \cdot \left\{(1 - b_p P) \cdot \left[0.6GP - 2.6EY - 16UP + 11BC - 1.2AW + 1.1FS + 8.2TE - 0.64\pi_a\right] - 0.59\right\}$

$E\left(\frac{d\pi}{dx}\right) = (1 - 0.44E) \cdot \left\{(1 - SP - 22MP) \cdot \left[1 - C\right] b_x \right\}$

$E\left(\frac{d\pi}{dP}\right) = (1 - 0.44E) \cdot b_p \cdot \left[0.59\pi_p - \left[1 - C\right] \cdot (-0.6GP + 2.6EY + 16UP - 11BC + 1.2AW - 1.1FS - 8.2TE + 0.64\pi_a) - 0.59C\right]$
Nonlinear Least-Squares: “Multiple Hands on the Wheel” Model

Table 2: Estimated Effects of Domestic Political-Economic Conditions, $d\pi/x$, as Function of Central Bank Autonomy, CBA, International Monetary Exposure, $E$, and Exchange-Rate Regime, $P$

<table>
<thead>
<tr>
<th></th>
<th>Little Exposed ($E=0.40$)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Float</td>
<td>Basket Peg</td>
<td>Simple Peg</td>
<td>Float</td>
<td>Basket Peg</td>
<td>Simple Peg</td>
<td>Float</td>
<td>Basket Peg</td>
<td>Simple Peg</td>
<td>Float</td>
<td>Basket Peg</td>
<td>Simple Peg</td>
<td>Float</td>
</tr>
<tr>
<td><em>central</em></td>
<td>0.26</td>
<td>1.563.79</td>
<td>1.224.61</td>
<td>0.000.09</td>
<td>1.352.69</td>
<td>1.059.53</td>
<td>0.000.07</td>
<td>1.142.60</td>
<td>0.894.47</td>
<td>0.000.06</td>
<td>1.049.53</td>
<td>0.894.47</td>
<td>0.000.06</td>
</tr>
<tr>
<td>bank</td>
<td>0.46</td>
<td>1.120.57</td>
<td>0.877.44</td>
<td>0.000.06</td>
<td>0.970.50</td>
<td>0.759.39</td>
<td>0.000.05</td>
<td>0.819.44</td>
<td>0.641.34</td>
<td>0.000.05</td>
<td>0.759.39</td>
<td>0.641.34</td>
<td>0.000.05</td>
</tr>
<tr>
<td><em>auton.</em></td>
<td>0.66</td>
<td>0.678.37</td>
<td>0.531.29</td>
<td>0.000.04</td>
<td>0.587.32</td>
<td>0.459.25</td>
<td>0.000.03</td>
<td>0.495.28</td>
<td>0.388.22</td>
<td>0.000.03</td>
<td>0.459.25</td>
<td>0.388.22</td>
<td>0.000.03</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Estimated Impact of a Post-Election Year ($d\pi/dEY$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>central</em></td>
</tr>
<tr>
<td>bank</td>
</tr>
<tr>
<td><em>auton.</em></td>
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</tbody>
</table>

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<tr>
<th>Estimated Impact of 10% Increase in Union Density ($0.1\cdot d\pi/dUP$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>central</em></td>
</tr>
<tr>
<td>bank</td>
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<tr>
<td><em>auton.</em></td>
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<tr>
<th>Estimated Impact of 1% Increase in Financial-Sector Employment-Share ($d\pi/dFS$)</th>
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<tbody>
<tr>
<td><em>central</em></td>
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<tr>
<td>bank</td>
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<tr>
<td><em>auton.</em></td>
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<tr>
<th>Estimated Impact of 1% Increase in Average Inflation Abroad ($d\pi/d\pi_a$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>central</em></td>
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<tr>
<td>bank</td>
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<tr>
<td><em>auton.</em></td>
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</tbody>
</table>

**Notes:** These are first-year effects, meaning before the estimated dynamics unfold. Standard errors noted in superscripts.
Nonlinear Least-Squares: “Multiple Hands on the Wheel” Model

Figure 1: Estimated Partisan Cycles in the Linear & Theoretically Informed Models at High & Low CBA, E, & MP
Figure 2: Estimated Domestic-Inflation Effect of Actual or Counter-Factual $SP$ in 21 Countries, 1957-90. Estimates plotted for $d\text{INF}/d\text{SP}$ at the values of all other variables in the equation actually occurring in that country-year. For counter-factual pegs, peg country assumed to have OECD-average inflation that year. Shading separates countries and extends from 1955 to 1990 in each country, left to right.
Multiple Policymakers: Veto Actors Bargaining in Common Pools

- Multiple implications for policy outcomes dispersal of policymaking-authority across diverse actors:
  - Veto-Actor Theory (Tsebelis ‘02) emphasizes:
    - Privileges S.Q., & so retards policy adjustment, reduces change.
  - Collective-Action/Common-Pool Theories (WSJ ‘81):
    - Externalities & so overexploit/underinvest public goods.
  - Bargaining & Delegation Theories rather stress:
    - Bargaining Strengths/Positions, yielding Weighted Compromise.

- This project attempts a synthesis:
  - Disting. theoretically/conceptually many effects of # (fragment.) & diversity (polar., partisan) policymakers.
  - Empirical model of many effects distinctly & effectively.
  - Preliminary application to evolution fiscal policy (pub debt) in developed democracies, 1950s-90s.
Veto Actors: Deadlock, Delayed Stabilization, & Policy-Adjustment Retardation

- Tsebelis (‘95b, ‘99, ‘00, ‘02): Essential Argument:
  - ↑ # &/or ideological/interest polarization of pol-mkng actors whose approval required to ΔSQ, i.e., veto actors, ⇒, loosely, ↓ probability &/or magnitude policy Δ.
  - I.e., strictly, as size W(SQ) ↓, which generally does as # &/or polarization VA ↑, range possible policy Δ(SQ) ↓.
  - ⇒ following empirical prediction (Tsebelis 2002, Fig. 1.7):
    - Suggests both mean/expected policy-Δ & variance pol & pol-Δ ↑↓ as size of W(SQ) ↑↓ (aside: why only suggests)
    - No prediction of pol-level or of direction pol-Δ, only of E(|Δp|), V(Δp).
Veto-Actor Implications

↑ # (\text{\textasciitilde} \text{Frag}) & Polar of VA Privileges SQ ⇒

- Retards policy-adjustment rates/delays stabilization,
- ↓ range of possible policy-\Delta, & so, possibly,
- ↓ magnitude/variance policy- \Delta (1^{st} & 2^{nd}-order E(\Delta)).

- Results, e.g. in fiscal policy, deficits & debts; originally mixed, but tighter specify thry into empirical analysis:
  - (F ’00, ’02) \textbf{How model:} policy-adjustment-rate effect = conditional coefficient on LDV in dynamic model, not level.
  - (F ’00, ’02) \textbf{How measure:} frag & polar in VA theory =
    - raw #, not eff. # (size-wtd) VA;
    - max range pref’s, not \textit{V(pref’s)} or \textit{sd(pref’s)}, (size-wtd)

⇒ Model: \[ y_t = \ldots + \theta(\#VA, Range\{pref(VA)\}) \times y_{t-1} \ldots \]
& or \[ V(y_t) = f(\#, Range) \Rightarrow \text{empirical support.} \]
Common-Pool Theory (1)

- Weingast, Shepsle, Johnsen (1981): districting & distributive/pork-barrel spend \((\text{law of } 1/n)\)
  - Benefits concentrate district \(i\): \(B_i = f(C), \ f' > 0 \& f'' < 0\)
  - Costs disperse across \(n\) districts: \(C_i = C/n\)
  - \(\therefore\) optimal project-size from \(i\)'s view \(\uparrow\) in \# districts: \(f'(C^*) = 1/n\) \((\ldots \log\text{-linearly?})\)

- Alternative Decision Rules/Processes \(\ldots\) \(\Rightarrow\)
  - \(\ldots\) \(\text{Law of } 1/n\) is general, & stronger as legislative behavior more Universalistic & less Minimal-Winning, which tendency \(\uparrow\) as rational ignorance, winning-coalition uncertainty, or legislative-rule closure to amend or veto \(\uparrow\).

- E.g., PubRev = common pool for \(n\) reps, overused to distribute bens; this CA prob worsens “proportionally” by \(\text{law } 1/n\), i.e. at rate b/w those at which \((n+1)/2n\) (MWC) \& \(1/n\) (uni) \(\downarrow\) in \(n\).
Manifestations of Common Pools

- Velasco (‘98, ‘99, ‘00): inter-temporal totality pub rev is C-P to today’s policymakers ⇒ deficits & debts also law of 1/n
- Peterson & co’s, Treisman: federalism ⇒ multiple tax authorities ⇒ several common-pool problems:
  - Inter-jurisdiction competition (w/ high factor mobility) ⇒ C-P of investment resources ⇒ over-fishing: taxes too low.
  - National govt as lender last resort ⇒ subnational jurisdictions see fed guarantee & funds as common pool ⇒ excessive borrowing by subnat’l units. (e.g., EU, EMU & Euro ⇒ common pools…)
- Again, should be quite general:
  - Anything that gains set of pol-makers credit ⇒ underinvested as ↑n
  - Anything that gains set of pol-makers blame ⇒ overexploited as ↑n
- E.g., (thry 2nd-best), ELECTIONEERING:
  - Magnitude incentive electioneer fades w/ n (see, e.g., Goodhart)
  - Control over electioneering diminishes w/ n.
- Notice: CP not arise in Tsebelis’ VA Theory b/c # & pref’s of VA’s exog & predetermined, whereas in CP theory: prefs=f(#).
Modeling Common-Pool Effects

- CP Effects distinguishable from VA Effects:
  - C-P Effects on *levels*, not (as in VA) in dynamics.
  - Proportional to $1/n$ for *equal-sized* actors. Standard Olsonian encompassingness logic $\Rightarrow$ proper $n$ here is *size-weighted* (effective & s.d./var.)
  - Fractionalization (#) & esp. polarization (het.) relate to VA effects; CP, conversely, relate primarily to #, although het. can exacerbate some CA probs.

- Suggests Proper Model of Policy-Response to some public demand for, $x_1'\beta_1$, or against, $x_2'\beta_2$:
  - $\ldots+(x_1'\beta_1)(1-f(ln(Eff#)))+(x_2'\beta_2)(1+f(ln(Eff#)))+\ldots$
  - Same $f(ln(Eff#))$, b/c overexploit/underinvest same °
Bargaining, Delegation, & Compromise

- **Explicit extensive-form delegation & bargaining games**: huge theoretical & empirical literature
- **F (‘99, ‘02, ‘03): less context-specific empirical strategy…**
  - Because broad comparativist seek thry that *travels*, not that requires different model each context.
- **Offering is roughly equivalent Nash Bargaining.**
  - Most ext forms ⇒ eqbm bounded by actors’ ideal pts:
    - Convex set/hull, upper-contour set (=core of coop. game thry),
    - So like Tsebelis, but further, though short of explicit ext-form
  - Policy outside that range possible,
    - e.g., if uncertainty resolved unfavorably,
    - but that ⇒ highly unlikely that E(pol) outside this range
  - Thus, E(pol)=some convex-combo (wtd-avg) pol-mkrs’ ideals ⇒ convex-combo emp. models ≈ compromise
    - If Nash Bargain, e.g., (n.b. NB=coop. game-thry but equiv. sev. reasonable ext-form non-coop barg. games: Rubinstein ‘82), ⇒ (geometric) *wtd-influence pol-mkng*; i.e., simple wtd-avg.
Empirical Manifestations & Model of Compromise Policymaking

- Re: def’s & debt, Cusack (‘99, ‘01; cf., Clark ‘03)
  - Arg: left more Keynes-active counter-cyc; right less, even pro-cyc
  - Add Nash-Barg Model ⇒ wtd-avg pol-mkr partisanship
    conditions ⇔ Keynesian cntr-cyc fisc-pol response to macroecon.

Empirical Implementation:

- Ideally:
  - Describe barg power party $i$ as $f$(charact’s $i$ & barg envir, $j$, ⇒ $f(v_{ij})$
  - Desc. pol response to conditions $x_k$ if $i$ sole pol-mkng control: $q_i(x_k)$
  - Then embed Nash-Barg sol’n, $\Sigma_i f(v_{ij})q_i(x_k)$, in emp. model to est.

- Currently:
  - Assume wtd-avg compromise outcome pre-estimation.
  - I.e., simply assume by measure & specification that Policy responds to $WtdPartisanshipCurrGovt \times MacroeconomicConditions$. 
Empirical Model of the Theoretical Synthesis (1)

- Different aspects of policy-maker fragmentation, polarization, & partisanship:
  - V-A Effects: raw # (frag) and ideological ranges (polar)
  - C-P Effects: eff # (frag) & maybe, ideol. s.d./var (polar)
  - D-B Effects: power-wtd mean ideologies (partisanship)

- Different ways these distinct effects manifest in pol:
  - V-A (primarily) to slow pol-adjust (delay stabilization);
  - C-P induces over-draw from common resources (incl. from future as in debt); under-invest in common properties (less electioneering), log-proportionately
  - D-B induces convex-combinatorial (compromise) policies, incl. greater left-activist/right-conservative Keynesian-countercyclical/conservative pro-cyclical, in proportion to degree left/right controls policymaking
Empirical Model of the Theoretical Synthesis (2)

- …implies specification where:
  - Abs # VA & ideol range modify pol-adjust rates
  - (log) Eff # pol-mkrs & s.d. ideol (wtd measures) gauge C-P prob in electioneering (+debt-lvl effect?)
  - Some barg process among partisan pol-mkrs (e.g., Nash ⇒ wtd-influence) determines combo reflected in net policy responsiveness to macro (° K-activism)

⇒

\[
D_{it} = \alpha_i + \left(1 + \rho_n N\text{op}_{it} + \rho_{ar} A\text{RwiG}_{it}\right) \times \left(\rho_{1D_{i,t-1}} + \rho_{2D_{i,t-2}} + \rho_{3D_{i,t-3}}\right) \\
+ \left(\beta_{\Delta Y} \Delta Y_{i,t} + \beta_{\Delta U} \Delta U_{i,t} + \beta_{\Delta P} \Delta P_{i,t}\right) \times \left(1 + \beta_{cg} C\text{oG}_{it}\right) \\
+ \left(\gamma_{e1} E_{it} + \gamma_{e2} E_{i,t-1}\right) \times \left(1 + \gamma_{en} E\text{NoP}_{it} + \gamma_{sd} S\text{DwiG}_{it}\right) + x'_{it} \eta + z'_{it} \omega + \varepsilon_{it}
\]
Empirical Model Specification & Data

\[ D_{it} = \alpha_i + (1 + \rho_n NoP_{it} + \rho_{ar} ARwiG_{it}) \times (\rho_1 D_{it-1} + \rho_2 D_{it-2} + \rho_3 D_{it-3}) + x_i' \eta + z_i' \omega + \varepsilon_{it} \]

- \( D_{it} \) = Debt (\%GDP);
- \( NoP \& ARwiG = \text{raw Num of Prtys in Govt} \& \text{Abs Range w/i Govt} \):
  - VA conception, so modify dynamics. Expect \( \rho_n \) & \( \rho_{ar} > 0 \).
  - By thry \& for efficiency: modify all lag dynamics same.
- \( CoG \) (govt center, left to right, 0-10):
  - Modifies response to macroecon (equally, by thry \& for eff’cy): \( \beta_{cg} < 0 \).
  - Macroec: \( \Delta Y = \text{real GDP growth}; \Delta U = \Delta \text{unemp rate}; \Delta P = \text{infl rate} \).
- \( x' \eta \) = controls: set pol-econ cond’s response to which not partisan-differentiated or gov comm-pool: (e.g., E(real-int)-E(real-grow), ToT)
- \( ENoP \& SDwiG = \text{Effective Num of Prtys in govt} \& \text{Std Dev w/i Govt} \):
  - Frag \& Polar by \text{wtd-influence concept}. CP lvl-effects modify (at same rate) electioneering, \( E_t \), pre-elect-year, \& \( E_{t-1} \), post-elect-yr.: \( \gamma_{en} \) \& \( \gamma_{sd} < 0 \).
- \( z' \omega \) = set of constituent terms in the interactions:
- \( ENoP, SDwiG \text{ may have positive coeff’s by CP effect lvl debt, but issue is temporal fract more than curr. govt fract. Thry o/w says omit.} \)
|                          | Coeff. | Std. Err. | t-Stat. | Pr($T>|t|$) |
|--------------------------|--------|-----------|---------|-------------|
| **Dependent Variables**  |        |           |         |             |
| $D_{t-1}$                | 1.212  | 0.060     | 20.112  | 0.000       |
| $D_{t-2}$                | -0.153 | 0.085     | -1.792  | 0.074       |
| $D_{t-3}$                | -0.121 | 0.045     | -2.677  | 0.008       |
| **Macroeconomic Conditions** |        |           |         |             |
| $\Delta Y$              | -0.336 | 0.111     | -3.033  | 0.003       |
| $\Delta U$              | 0.992  | 0.308     | 3.219   | 0.001       |
| $\Delta P$              | -0.188 | 0.063     | -2.965  | 0.003       |
| **Controls**             |        |           |         |             |
| $x_1$ (open)            | 15.891 | 5.279     | 3.010   | 0.003       |
| $x_2$ (ToT)             | 0.388  | 1.744     | 0.222   | 0.824       |
| $x_3$ (open $\cdot$ ToT) | -10.681 | 5.156   | -2.072  | 0.039       |
| $x_4$ (dxrig)           | -0.036 | 0.066     | -0.544  | 0.587       |
| $x_5$ (oy)              | 2.064  | 1.094     | 1.886   | 0.060       |
| **Pre- and Post-Electoral Indicators** |        |           |         |             |
| $E_t$                    | 0.687  | 0.568     | 1.210   | 0.227       |
| $E_{t-1}$               | 1.490  | 0.645     | 2.310   | 0.021       |
| **Constituent Terms from the Interactions** |        |           |         |             |
| $z_1$ (CoG)             | 0.051  | 0.131     | 0.390   | 0.697       |
| $z_2$ (ENoP)            | 0.281  | 0.446     | 0.629   | 0.530       |
| $z_3$ (SDwiG)           | 0.542  | 0.437     | 1.242   | 0.215       |
| $z_4$ (NoP)             | 0.181  | 0.277     | 0.654   | 0.514       |
| $z_5$ (ARwiG)           | -0.312 | 0.259     | -1.205  | 0.228       |

**Summary Statistics**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$ (Deg. Free)</td>
<td>735</td>
<td>(691)</td>
<td>$s_e^2$</td>
<td>2.525</td>
</tr>
<tr>
<td>$R^2$ ($\bar{R}^2$)</td>
<td>0.991</td>
<td>(0.990)</td>
<td>DW-Stat.</td>
<td>2.101</td>
</tr>
</tbody>
</table>
Pace Brambor et al. (‘06), but joint-significance of multiple-policymaker conditioning effects \((\gamma_{en}, \gamma_{sd}, \rho_{n}, \rho_{ar}, \beta_{cg})\) overwhelmingly rejects excluding \((p \approx .001)\), whereas joint-sig coeff’s on constit. terms, \(z\), clearly fails reject \((p \approx .602)\) exclusion. (Almost) All theory says should be zero, so…

| Lagged | Coeff. | Std. Err. | t-Stat. | Pr\((T > |t|)\) |
|--------|--------|-----------|---------|----------------|
| \(D_{t-1}\) | 1.207  | 0.060     | 20.290  | 0.000          |
| Dependent Variables |        |           |         |                |
| \(D_{t-2}\) | -0.158 | 0.085     | -1.851  | 0.065          |
| \(D_{t-3}\) | -0.117 | 0.045     | -2.577  | 0.010          |
| \(\rho_{n}\) (veto-actor effect: fractionalization) | 0.011  | 0.005     | 2.369   | 0.018          |
| \(\rho_{ar}\) (veto-actor effect: polarization) | -0.002 | 0.004     | -0.437  | 0.662          |

Macroeconomic Conditions

| \(\Delta Y\) | -0.375 | 0.087 | -4.332 | 0.000 |
| \(\Delta U\) | 1.095  | 0.286 | 3.829  | 0.000 |
| \(\Delta P\) | -0.207 | 0.053 | -3.889 | 0.000 |
| \(\beta_{cg}\) (partisan-compromise bargaining) | -0.051 | 0.020 | -2.484 | 0.013 |

Controls

| \(x_1\) (open) | 16.128 | 5.314 | 3.035 | 0.002 |
| \(x_2\) (ToT) | 0.414  | 1.728 | 0.239 | 0.811 |
| \(x_3\) (open·ToT) | -10.780 | 5.194 | -2.076 | 0.038 |
| \(x_4\) (dxrig) | -0.038 | 0.066 | -0.578 | 0.563 |
| \(x_5\) (oy) | 1.898  | 1.100 | 1.724  | 0.085 |

Pre- and Post-Electoral Indicators

| \(E_t\) | 0.475 | 0.420 | 1.133 | 0.258 |
| \(E_{t-1}\) | 1.146 | 0.562 | 2.040 | 0.042 |

| \(\gamma_{en}\) (common-pool effect: fractionalization) | -0.570 | 0.209 | -2.727 | 0.007 |
| \(\gamma_{sd}\) (common-pool effect: polarization) | 0.881  | 0.586 | 1.503  | 0.133 |

Summary Statistics

| N (Deg. Free) | 735 (696) | \(s^2_e\) | 2.522 |
| \(R^2 (R^2)\) | 0.991 (0.990) | DW-Stat. | 2.099 |
### Veto-Actor Effects: Estimates of Policy-Adjustment Rate

<table>
<thead>
<tr>
<th>Adjustment Rates</th>
<th>NoP=1</th>
<th>NoP=2</th>
<th>NoP=3</th>
<th>NoP=4</th>
<th>NoP=5</th>
<th>NoP=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag Coefficient(a)</td>
<td>0.943</td>
<td>0.952</td>
<td>0.960</td>
<td>0.969</td>
<td>0.978</td>
<td>0.986</td>
</tr>
<tr>
<td>Policy-Adjust/Yr(b)</td>
<td>0.057</td>
<td>0.048</td>
<td>0.040</td>
<td>0.031</td>
<td>0.022</td>
<td>0.014</td>
</tr>
<tr>
<td>Long-Run Mult.(c)</td>
<td>17.498</td>
<td>20.639</td>
<td>25.154</td>
<td>32.200</td>
<td>44.727</td>
<td>73.208</td>
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<tr>
<td>½-Life(d)</td>
<td>11.778</td>
<td>13.956</td>
<td>17.087</td>
<td>21.971</td>
<td>30.654</td>
<td>50.397</td>
</tr>
<tr>
<td>90%-Life(e)</td>
<td>39.127</td>
<td>46.362</td>
<td>56.761</td>
<td>72.985</td>
<td>101.832</td>
<td>167.415</td>
</tr>
</tbody>
</table>

### Bargaining Effects: Estimates of Keynesian Fiscal Responsiveness

<table>
<thead>
<tr>
<th>Mean Econ. Performance</th>
<th>Mean Econ. Performance</th>
<th>Mean Economic Performance</th>
<th>Mean Econ. Performance</th>
<th>Mean Econ. Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2 std. dev.</td>
<td>-1 std. dev.</td>
<td>+1 std. dev.</td>
<td>+2 std. dev.</td>
<td></td>
</tr>
<tr>
<td><strong>Growth</strong></td>
<td><strong>d(UE)</strong></td>
<td><strong>Infl</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2.354</td>
<td>0.454</td>
<td>3.261</td>
<td>6.069</td>
<td></td>
</tr>
<tr>
<td>1.915</td>
<td>1.034</td>
<td>0.153</td>
<td>-0.728</td>
<td></td>
</tr>
<tr>
<td>-3.593</td>
<td>1.230</td>
<td>6.054</td>
<td>10.877</td>
<td></td>
</tr>
</tbody>
</table>

### Collective-Action/Common-Pool Effects: Estimates of Electoral Debt-Cycle Magnitude

| CoG | E(D|Econ)\(f\) | E(D|Econ) | E(D|Econ) | E(D|Econ) | E(D|Econ) |
|-----|----------------|-----------|-----------|-----------|-----------|
| 3.0 | 3.157          | 0.599     | -1.959    | -4.516    | -7.074    |
| 4.2 | 2.930          | 0.556     | -1.818    | -4.192    | -6.566    |
| 5.4 | 2.703          | 0.513     | -1.677    | -3.867    | -6.058    |
| 6.6 | 2.476          | 0.470     | -1.536    | -3.543    | -5.549    |
| 7.8 | 2.250          | 0.427     | -1.396    | -3.218    | -5.041    |
| 9.0 | 2.023          | 0.384     | -1.255    | -2.894    | -4.533    |

### Fiscal-Cycle Magnitude\(g\)


### Collective-Action/Common-Pool Effects: Estimates of Electoral Debt-Cycle Magnitude

<table>
<thead>
<tr>
<th>Electoral-Cycle Magnitude(h)</th>
<th>NoP=1</th>
<th>NoP=2</th>
<th>NoP=3</th>
<th>NoP=4</th>
<th>NoP=5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Electoral-Cycle Magnitude</strong></td>
<td>1.07410</td>
<td>0.86454</td>
<td>0.65497</td>
<td>0.44541</td>
<td>0.23585</td>
</tr>
</tbody>
</table>
Extension & Refinement

\[ E(y_t) = \delta^0 + x_t^0 b^0 + \left( \rho_0 + \rho_1 \ln(NoP_t) + \rho_2 \ln(1 + ARwiG_t) \right) y_{t-1} \]

\[ + \left( \begin{array}{c}
    x_t^1 b^1 + \sum_{i=1}^{l} p(c_{it}) \times q_i(x_t^2)
  \end{array} \right) \times \left[ 1 + \alpha_1 \ln(NoP_t) + \alpha_2 \ln(1 + ARwiG_t) \right] \times \left[ 1 + \gamma_1 \ln(ENoP_t) + \gamma_2 \ln(1 + SDwiG_t) \right] \]

- \( x^0 \) = factors that affect policy-outcomes unless pol-mkrs act to change status quo, i.e., that have effect on pol-out directly.
- \( x^1 \) = factors affecting policy-outcomes if policymakers act to change status quo, without partisan-differentiated response
- \( x^2 \) = factors affecting policy-outcomes if policymakers act to change status quo, with partisan-differentiated response
- \( \{ NoP, ARwiG \} \) = sources of veto-actor effects; as before
- \( \{ ENoP, SDwiG \} \) = sources of common-pool effects; as before
- \( \{ p(c_{it}), q_j(x_t) \} \) = sources of bargaining & delegation effects:
  - \( p(c_{it}) \): Effective policy-influence of party \( i \) in context \( t \). (E.g., as now: cabinet seat-shares, but could become richer model.)
  - \( q_j(x_t) \): Model of response of party \( i \) to pol-econ conditions \( x_t \). (E.g., as now: \( CoG_t \times Macrocon_t \), but could become richer model.)
## Results of Fuller Model

| Temporal Dynamics | Coeff. | Std. Err. | t-Stat. | Pr(\(T>|t|\)) |
|-------------------|--------|-----------|---------|----------------|
| D(t-1)            | 1.197  | 0.059     | 20.144  | 0.000          |
| D(t-2)            | -0.139 | 0.085     | -1.629  | 0.104          |
| D(t-3)            | -0.121 | 0.045     | -2.698  | 0.007          |

### Veto-Actor Effect on Outcome-Adjustment Rate

|                   | Coeff. | Std. Err. | t-Stat. | Pr(\(T>|t|\)) |
|-------------------|--------|-----------|---------|----------------|
| **NoP**           | 0.008  | 0.004     | 1.883   | 0.060          |
| **Open**          | 16.624 | 3.758     | 4.423   | 0.000          |
| **Open*ToT**      | -11.190| 3.135     | -3.569  | 0.000          |
| **Ele(t)**        | 0.315  | 0.363     | 0.867   | 0.386          |
| **Ele(t-1)**      | 0.873  | 0.399     | 2.186   | 0.029          |
| **OY**            | 2.833  | 1.295     | 2.187   | 0.029          |
| **DXRIG3**        | -0.073 | 0.072     | -1.009  | 0.314          |

### Common-Pool Effect on Policy Response

|                   | Coeff. | Std. Err. | t-Stat. | Pr(\(T>|t|\)) |
|-------------------|--------|-----------|---------|----------------|
| **ln(ENoP)**      | -0.277 | 0.071     | -3.903  | 0.000          |

### **x2**: Variables with Effects via Partisan-Differentiated Policy Response

|                   | Coeff. | Std. Err. | t-Stat. | Pr(\(T>|t|\)) |
|-------------------|--------|-----------|---------|----------------|
| **Growth**        | -0.238 | 0.084     | -2.815  | 0.005          |
| **d(UE)**         | 0.749  | 0.228     | 3.289   | 0.001          |
| **Inflation**     | -0.137 | 0.047     | -2.947  | 0.003          |

### Bargaining-Compromise Effects on Partisan Policy-Responses

|                   | Coeff. | Std. Err. | t-Stat. | Pr(\(T>|t|\)) |
|-------------------|--------|-----------|---------|----------------|
| **CoG**           | -0.049 | 0.026     | -1.893  | 0.059          |

### Veto-Actor Effect on Policy-Adjustment Rate

|                   | Coeff. | Std. Err. | t-Stat. | Pr(\(T>|t|\)) |
|-------------------|--------|-----------|---------|----------------|
| **NoP**           | 0.215  | 0.121     | 1.773   | 0.077          |

### Common-Pool Effect on Debt Level

|                   | Coeff. | Std. Err. | t-Stat. | Pr(\(T>|t|\)) |
|-------------------|--------|-----------|---------|----------------|
| **ln(ENoP)**      | 1.128  | 0.486     | 2.320   | 0.021          |

### Summary Statistics

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<td><strong>s_e^2</strong></td>
<td>2.510</td>
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<tr>
<td><strong>R^2 (R^2)</strong></td>
<td>0.991 (0.990)</td>
<td><strong>DW-Stat.</strong></td>
<td>2.090</td>
<td></td>
</tr>
</tbody>
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