Modeling and Interpreting Interactions

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Overview

- Interactions in Pol-Sci: ubiquitous, but should be more
  - In present context, concern re: coefficient heterogeneity is a call for interactions—i.e., models of conditional, i.e., context-variant, coefficients.
- From theory to empirical-model specification: Arguments that imply interactions (& some that don't), & how to write.
- Interpretation:
  - Effects = derivatives & differences, not coefficients!
  - Std Errs (etc.): effects vary, so do std errs (etc.)!
- Presentation: Tables & Graphs, & Choosing between equivalent Specifications
- Use & abuse of some common-practice "rules"
- Extensions:
  - Split-sample v. dummy-interaction
  - Common 2nd-moment implications of interactions
  - Interactions with uncertainty = random coefficients = hierarchical...

Interactions in Pol-Sci Research

- Common. '96-'01 *AJPS*, *APSR*, *JoP*:
  - 54% some stat meth (=s.e.'s), of which 24% = interax (so interax ≈ 12.5% or 1/8th total; more if exclude *CP*).
  - (N.b., most rest QualDep & frml thry, not counted, & "thry" in denom) so understate tech nature discipline)

Journal (1996-2001)	***Total Articles***	***Statistical Analysis***		***Interaction-Term Usage***		
		Count	*% of Tot*	*Count*	*% of Tot*	*% of Stat*
American Political Science Review	279	274	77%	69	19%	25%
American Journal Political Science	355	155	55%	47	17%	30%
Comparative Politics	130	12	9%	1	1%	8%
Comparative Political Studies	189	92	49%	23	12%	25%
International Organization	170	43	25%	9	5%	21%
International Studies Quarterly	173	70	40%	10	6%	14%
Journal of Politics	284	226	80%	55	19%	24%
Legislative Studies Quarterly	157	104	66%	19	12%	18%
World Politics	116	28	24%	6	5%	25%
TOTALS	**2446**	**1323**	**54%**	**311**	**13%**	**24%**

Interactions in Pol-Sci Theory[1]

- Ubiquitous, but our Theories/Substance say should be even more; core classes of argument inherently interactive:
  - **INSTITUTIONAL**: institutions are inherently interactive variables:
    - Institutions funnel, moderate, shape, condition, constrain, refract, magnify, augment, dampen, mitigate political processes that...
      - ...translate societal interest-structures into effective political pressures,
      - ...&/or pressures into public-policy responses,
      - ...&/or policies to outcomes.
    - I.e., they *affect (modify, condition, moderate...) effects* ≡ ***interaction.***

Interactions in Pol-Sci Theory^2

- Views from across institutionalist perspectives:
  - Hall: "institutionalist model=>policy more than sum countervailing pressure from soc grps; that press **mediated** by organizational dynamic."
  - Ikenberry: "[Political struggles] **mediated** by inst'l setting where [occur]"
  - Steinmo & Thelen: "inst's…**constrain & refract** politics… [effects of] macro-structures magnify or mitigated by intermediate-level inst's… help us…explain the contingent nature of pol-econ development…"
  - Shepsle: "SIE clearly a move [to] incorporating inst'l features into R-C. Structure & procedure combine w/ preferences to produce outcomes."

Interactions in Pol-Sci Theory[3]

- Ubiquitous, but our Theories/Substance say should be even more; core classes of argument inherently interactive:
  - **INSTITUTIONAL**: ...
  - **STRATEGIC**: actors' choices (outcomes) conditional upon institutional/structural environ., opportunity set, & (interdep.) other actors' choices.
  - **CONTEXTUAL**: actors' choices (outcomes) conditional upon environment, opportunity set, & (interdep.) aggregates of other actors' choices.

Interactions in Pol-Sci Theory[4]

- Across subfields:
  - Comparative Politics *examples:*
    - Electoral system & societal structure interact to produce party system.
    - Divided government & polarization retards legislative productivity, $\Rightarrow$ conditional dynamics.
    - Corruption interactive product of institutional & societal structures.
  - International Relations *examples:*
    - System polarity & offense-defense balance $\Rightarrow$ war propensity.
    - Terrorist targeting & counterterrorism responses depend "grievance" & resources
  - American Politics ...

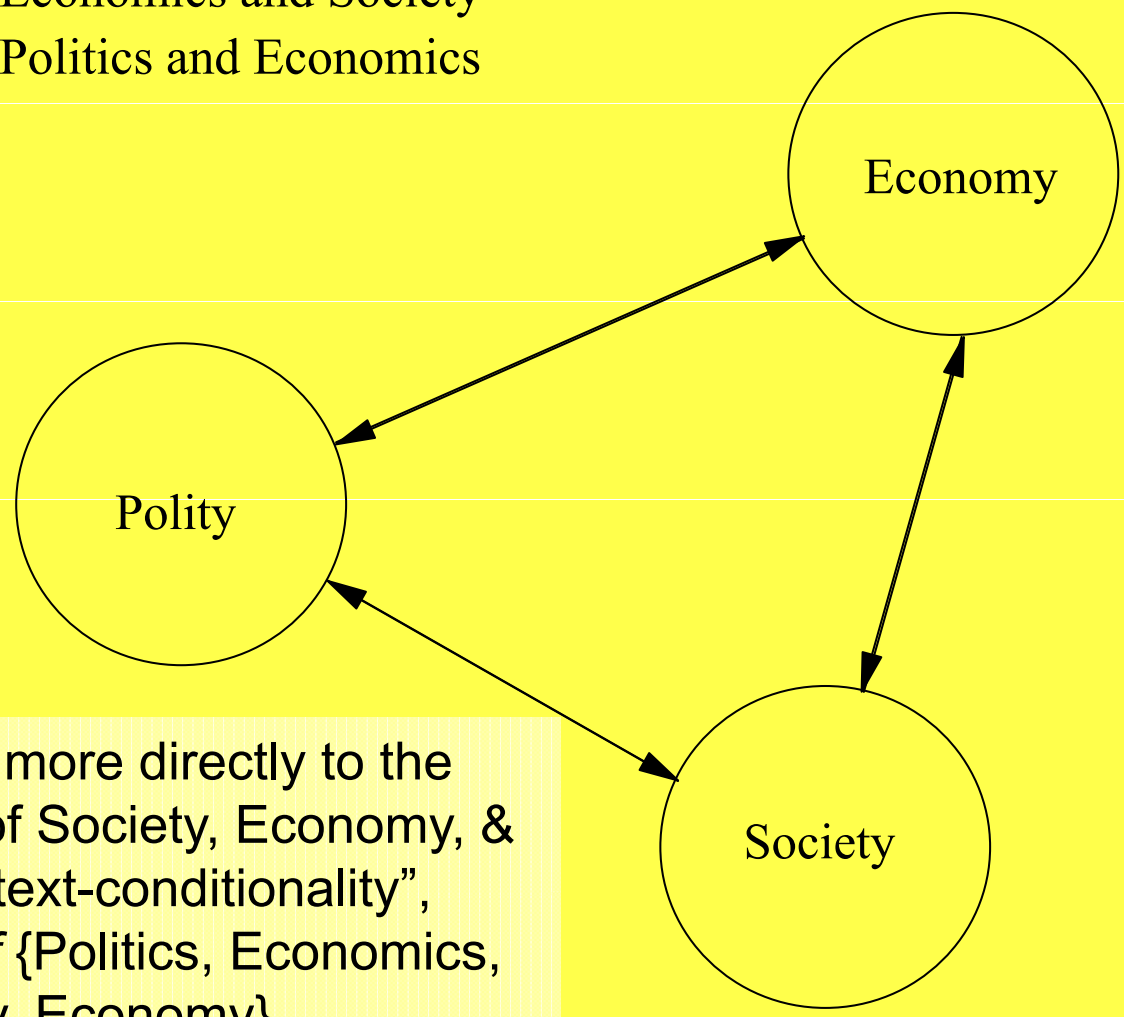
Interactions in Pol-Sci Theory[5]

- Political Economy:
  - Electoral & partisan cycles (i.e., effects) depend on pol & econ inst'l, struct'l, contextual conditions
- Political Behavior:
  - Gov't inst's shape voter behavior: balancing (Kedar, Alesina); economic voting (Powell & Whitten); etc.
- Legislative Studies:
  - Effects divided government different in (depend on) presidential v. parliamentary.
- Political Development:
  - Effect inequality on democratization depends cleavage structure.

Theory & Substance:

Everyone's Favorite "Model"

Economics Affects Politics and Society
Politics Affects Economics and Society
Society Affects Politics and Economics



Picture & text seem relate more directly to the "ubiquitous endogeneity" of Society, Economy, & Polity. For "ubiquitous context-conditionality", think "affects *the effects of* {Politics, Economics, Society} *on* {Society, Polity, Economy}.

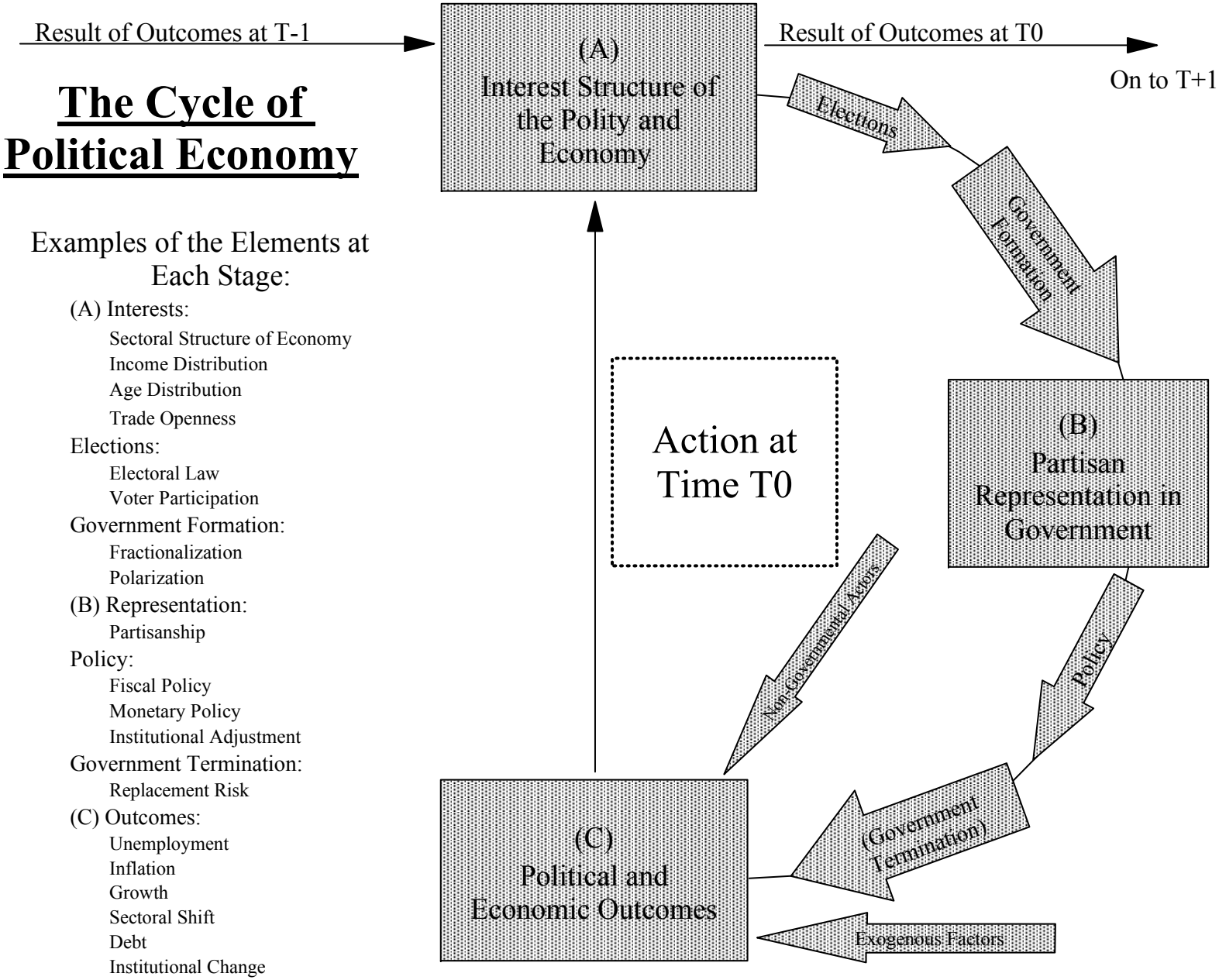
Theory & Substance:

An Old (& still) Favorite "Model" of Mine

The Cycle of Political Economy

Examples of the Elements at Each Stage:

- (A) Interests:
  - Sectoral Structure of Economy
  - Income Distribution
  - Age Distribution
  - Trade Openness
- Elections:
  - Electoral Law
  - Voter Participation
- Government Formation:
  - Fractionalization
  - Polarization
- (B) Representation:
  - Partisanship
- Policy:
  - Fiscal Policy
  - Monetary Policy
  - Institutional Adjustment
- Government Termination:
  - Replacement Risk
- (C) Outcomes:
  - Unemployment
  - Inflation
  - Growth
  - Sectoral Shift
  - Debt
  - Institutional Change



Again, the ubiquitous context-conditionality perhaps not fully overtly shown, but implication here that effect factors at each stage tend depend on others at that and other stages.

Theory & Substance:

An Newer Favorite "Model" of Mine

- ***Complex Context-Conditionality*:**
  - Effect of (almost) everything depends on (almost) everything else.
  - E.g., Principal-Agent Situations
    - If fully principal, $y_1=f(\boldsymbol{X})$; if fully agent, $y_2=g(\boldsymbol{Z})$; institutions: $0\leq h(\boldsymbol{I})\leq 1$.

$$y = h(\mathbf{I})f(\mathbf{X}) + \{1 - h(\mathbf{I})\}\, g(\mathbf{Z})$$

$$\Rightarrow \frac{\partial y}{\partial x} = h(\mathbf{I})\frac{\partial f(\mathbf{X})}{\partial x} \quad ; \quad \frac{\partial y}{\partial z} = -h(\mathbf{I})\frac{\partial g(\mathbf{Z})}{\partial z};$$

$$\frac{\partial y}{\partial i} = \frac{\partial h(\mathbf{I})}{\partial i}\left[f(\mathbf{X}) - g(\mathbf{Z})\right]$$

(Complex) Context-Conditionality: (Hallmark of Modern Pol-Sci Theory?)

- **Principal-Agent (Shared Control) Situations, for example:**
  - **If fully principal:** $y_1 = f(\mathbf{X})$;
  - **If fully agent:** $y_2 = g(\mathbf{Z})$;
  - **Institutions=>Monitoring & Enforcement costs principal must pay to induce agent behave as principal would:** $0 \leq h(\mathbf{I}) \leq 1$.
  - **RESULT:**

$$y = h(\mathbf{I}) f(\mathbf{X}) + \{1 - h(\mathbf{I})\} g(\mathbf{Z})$$

$$\Rightarrow \frac{\partial y}{\partial x} = h(\mathbf{I}) \frac{\partial f(\mathbf{X})}{\partial x} \quad ;$$

$$\frac{\partial y}{\partial z} = -h(\mathbf{I}) \frac{\partial g(\mathbf{Z})}{\partial z} \quad ;$$

$$\frac{\partial y}{\partial i} = \frac{\partial h(\mathbf{I})}{\partial i} \left[f(\mathbf{X}) - g(\mathbf{Z}) \right]$$

- **In words…**

…

…

…i.e., effect of anything depends on everything else!

Not Every Argument Is an Interactive Argument

- Not Interactive:
  - **X** affects **Y** through its effect on **Z**: **X**⇒**Z**⇒**Y**
    - In (political) psychology / behavior, this called *mediation*. Interaction is called *moderation* in this literature.
  - **X** and **Z** affect each other: **X**⇔**Z**.
    - I.e., **X** and **Z** endogenous to each other. Note: irrelevant to Gauss-Markov (OLS is BLUE); merely implies care to what partials (coefficients) mean.
  - **Y** depends on **X** controlling for **Z**, or **Y** depends on **X** & **Z**: $E(\mathbf{Y}|\mathbf{X},\mathbf{Z})=f(\mathbf{Z})$, $E(\mathbf{Y}|\mathbf{X})=f(\mathbf{Z})$, $\mathbf{Y}=f(\mathbf{X},\mathbf{Z})$
    - I.e., e.g., showing outcomes differ across 2×2 of **X** & **Z** insufficient; issues is difference of differences across rows or down columns.
- <u>**Interactive**</u>: ***<u>Effect</u> of X on Y* depends on Z** (⇒ converse: Effect of **Z** on **Y** depends on **X**):

$$\frac{\partial Y}{\partial X} = f(Z) \Leftrightarrow \frac{\partial Y}{\partial Z} = f(X)$$

From Theory/Substance to Empirical-Model Specification

- **Classic Comparative-Politics Example:**
  - Societal Fragmentation, ***SFrag***, &
  - Electoral-System Proportionality, ***DMag***,
  - $\Rightarrow$Effective # Parliamentary Parties: ***ENPP***
- *"Theory"*: $\boldsymbol{ENPP = f(SFrag\ ,\ DMag\ ,\ \cdot\ ,\ \varepsilon)}$
- Hypotheses: $\dfrac{\partial ENPP}{\partial SFrag} \geq 0$ $\dfrac{\partial ENPP}{\partial DMag} \geq 0$

$$\frac{\partial\left\{\frac{\partial ENPP}{\partial SFrag}\right\}}{\partial DMag} \equiv \frac{\partial\left\{\frac{\partial ENPP}{\partial DMag}\right\}}{\partial SFrag} \equiv \frac{\partial^2 ENPP}{\partial SFrag \partial DMag} \equiv \frac{\partial^2 ENPP}{\partial DMag \partial SFrag} \geq 0$$

- Empirical Specification: Lots ways get there...

A Typical Linear-Interactive Specification

- Want linear $f(\cdot)$ w/ these properties; many ways to get there:

$$ENPP = \beta_0 + \beta_1 SFrag + \beta_2 DMag + \varepsilon$$

$$\frac{\partial ENPP}{\partial SFrag} = \beta_1 \overset{?}{\rightarrow} f(DMag) = \alpha_0 + \alpha_1 DMag$$

$$\frac{\partial ENPP}{\partial DMag} = \beta_2 \overset{?}{\rightarrow} f(SFrag) = \gamma_0 + \gamma_1 SFrag$$

$$\Rightarrow ENPP = \beta_0 + (\alpha_0 + \alpha_1 DMag)SFrag + (\gamma_0 + \gamma_1 SFrag)DMag + \varepsilon$$

$$= \beta_0 + \alpha_0 SFrag + \alpha_1 DMagSFrag + \gamma_0 DMag + \gamma_1 SFragDMag + \varepsilon$$

$$= \beta_0 + \alpha_0 SFrag + \gamma_0 DMag + (\alpha_1 + \gamma_1)SFragDMag + \varepsilon$$

$$= \beta_0 + \beta_{SF} SFrag + \beta_{DM} DMag + \beta_{SFDM} SFragDMag + \varepsilon$$

$$\Rightarrow \frac{\partial ENPP}{\partial SFrag} = \beta_{SF} + \beta_{SFDM} DMag$$

$$\frac{\partial ENPP}{\partial DMag} = \beta_{DM} + \beta_{SFDM} SFrag$$

Interpretation of *Effects*:
Derivatives & Differences, *Not* Coefficients

- Standard Linear Interactive Model:

$$EN = \beta_0 + \beta_{SF} SF + \beta_{DM} DM + \beta_{SFDM} SF \times DM + \ldots + \varepsilon$$

- Effect of ***SFrag*** on ***ENPP*** (is a *function* of ***DMag***):

$$Effect\left(SF\right) \equiv \frac{\partial EN}{\partial SF} = \beta_{SF} + \beta_{SFDM} DM$$

$$\Delta EN = \beta_{SF} \Delta SF + \beta_{SFDM} DM \cdot \Delta SF$$

$$\equiv \frac{\Delta EN}{\Delta SF} = \beta_{SF} + \beta_{SFDM} DM$$

- Effect of ***DMag*** on ***ENPP*** (is *f* of ***SFrag***):

$$Effect\left(DMag\right) \equiv \frac{\partial ENPP}{\partial DMag} = \beta_{DM} + \beta_{SFDM} SFrag$$

$$\equiv \Delta ENPP = \beta_{DM} \Delta DM + \beta_{SFDM} SFrag \cdot \Delta DM$$

$$\equiv \frac{\Delta ENPP}{\Delta DM} = \beta_{DM} + \beta_{SFDM} SFrag$$

Interpretation of *Effects*: NOTES[1]

- "Main Effect" & "Interactive Effect":
  - For example, β_{SF} = "*main effect* of **SFrag**"
  - **....but** β_{SF} is merely the effect of **SFrag** at other variable(s) involved in interaction with it=0, so:
    - *Other-var(s)=0* may be extreme in the sample, or beyond sample range, or even logically impossible.
    - *Other-var(s)=0* substantive meaning of 0 altered by rescaling
      - E.g., by "centering" (centering changes nothing, btw...)
    - *Other-var(s)=0* may not have anything substantively *main* about it
  - Is no Main Effect or separately & Interactive Effect; is just the effect, which conditional, varies:

$$\boldsymbol{Effect}\left(\boldsymbol{SF}\right) \equiv \frac{\partial \boldsymbol{EN}}{\partial \boldsymbol{SF}} = \boldsymbol{\beta}_{\boldsymbol{SF}} + \boldsymbol{\beta}_{\boldsymbol{SFDM}}\boldsymbol{DM} \quad ; \quad \boldsymbol{Effect}\left(\boldsymbol{DM}\right) \equiv \frac{\partial \boldsymbol{EN}}{\partial \boldsymbol{DM}} = \boldsymbol{\beta}_{\boldsymbol{DM}} + \boldsymbol{\beta}_{\boldsymbol{SFDM}}\boldsymbol{SF}$$

Interpretation of *Effects*: NOTES^2

$$EN = \beta_0 + \beta_{SF} SF + \beta_{DM} DM + \beta_{SFDM} SF \times DM + \ldots + \varepsilon$$

- **COEFFICIENTS ARE NOT EFFECTS. EFFECTS ARE DERIVATIVES &/OR DIFFERENCES.**
  - Only in ***purely*** linear-additive-separable model are they equal because only there do derivatives simply = coefficients.
  - β_{SF} is ***not*** "effect of **SFrag** 'independent of'…" & definitely not its "effect 'controlling for'…other variable(s) in the interaction"
- Cannot substitute linguistic invention for under-standing model's logic (its simple math)

Interpretation of *Effects*: NOTES^3

- Interactions are logically symmetric:
  - For any function, not just lin-add.
  - If argue effect x depends z, must also believe effect z depends x.

$$\frac{\partial\left\{\frac{\partial y}{\partial x}\right\}}{\partial z} \equiv \frac{\partial\left\{\frac{\partial y}{\partial z}\right\}}{\partial x} \equiv \frac{\partial^2 y}{\partial x \partial z} \equiv \frac{\partial^2 y}{\partial z \partial x}$$

- Interactions often have 2nd-moment (variance, i.e., heteroskedacity) implications too:
  - **Larger district magnitudes, *DMag***, are "permissive" elect sys: <u>*allow*</u> more parties...
  - Fewer ***Veto Actors*** <u>*allow*</u> greater policy-change... (both need additional assumpts)
- All of this holds for any type of variable:
  - Measurement: binary, continuous...
  - <u>***Level***</u>: micro or macro; i, j, k, ...

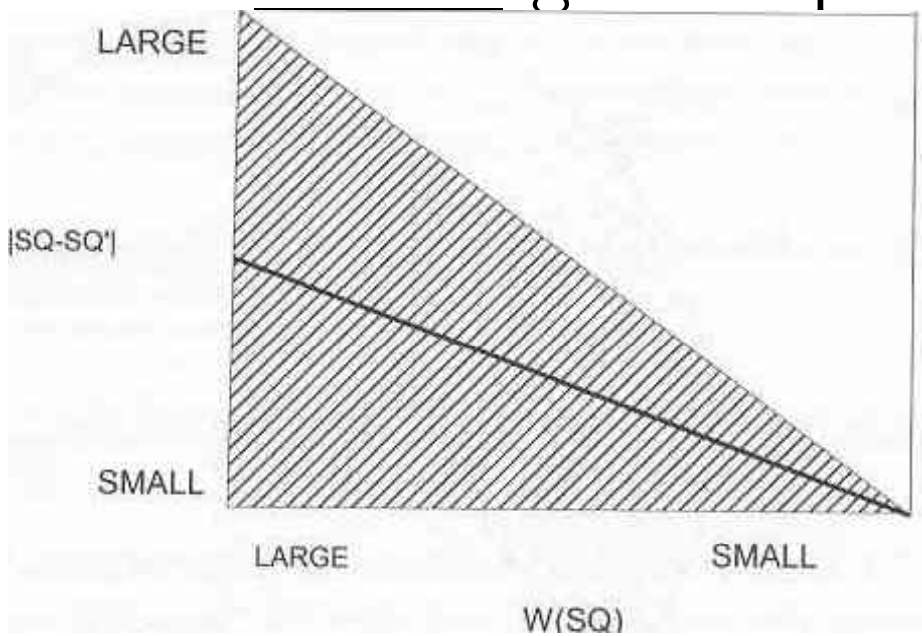
Frequent 2nd-Moment Implications Interactions

- ***DMag*** permissive ele sys: ***allows*** more parties…

$$NP = \beta_0 + \beta_1 DM + \varepsilon \;\; ; \;\; V(\varepsilon) = f(DM) \;\; \text{, e.g., } \sigma_0 + \sigma_1 DM$$

  – Note: unmodeled interactions look like heteroskedasticity; that's general, actually. Anything unmodeled gets into $\mathbf{e}^2$…

- Few ***Veto Actors*** ***allows*** greater policy-change…



$$y = \beta_0 + \beta_1 VP + \varepsilon \;\; ; \;\; V(\varepsilon) = f(VP) \;\; \text{, e.g., } \sigma_0 + \sigma_1 VP$$

- I.e., these are Rndm-Coeff &/or Het-sked Props…

Interpretation of *Effects*:
Standard Errors for *Effects*

$$\boldsymbol{ENPP = \beta_0 + \beta_{SF} SFrag + \beta_{DM} DMag + \beta_{SFDM} SFragDMag + \ldots + \varepsilon}$$

- Std Errs reported with regression output are for coefficients, not for effects.
  - The s.e. (t-stat, p-level) for $\hat{\beta}_{SF}$ regards the estimated effect of ***SFrag*** on ***ENPP*** at ***DMag***=0 (…which is logically impossible).
- Effect of x depends on z & v.v. (i.e., which was the point, remember?), so does the s.e.:

$$\boldsymbol{Effect}(\boldsymbol{x}) \equiv \frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}} = \boldsymbol{\beta}_x + \boldsymbol{\beta}_{xz}\boldsymbol{z} \;\Rightarrow\; \boldsymbol{Est.Eff.}(\boldsymbol{x}) \equiv E\left(\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}}\right) = \hat{\boldsymbol{\beta}}_x + \hat{\boldsymbol{\beta}}_{xz}\boldsymbol{z}$$

$$\boldsymbol{Est.Var.}\left\{\boldsymbol{Est.Eff.}(\boldsymbol{x})\right\} \equiv E\left[Var\left\{E\left(\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}}\right)\right\}\right] = E\left[Var\left\{\hat{\boldsymbol{\beta}}_x + \hat{\boldsymbol{\beta}}_{xz}\boldsymbol{z}\right\}\right]$$

$$= \widehat{V\left\{\hat{\boldsymbol{\beta}}_x + \hat{\boldsymbol{\beta}}_{xz}\boldsymbol{z}\right\}} = \widehat{V\left\{\hat{\boldsymbol{\beta}}_x\right\}} + \widehat{V\left\{\hat{\boldsymbol{\beta}}_{xz}\right\}} \cdot \boldsymbol{z}^2 + 2 \cdot \widehat{C\left(\hat{\boldsymbol{\beta}}_x, \hat{\boldsymbol{\beta}}_{xz}\right)}\boldsymbol{z}$$

- In words… More Generally: $\widehat{V(\mathbf{x}'\hat{\boldsymbol{\beta}})} = \mathbf{x}'\left[\widehat{V(\hat{\boldsymbol{\beta}})}\right]\mathbf{x}$

From Hypotheses to Hypotheses Tests:

Does Y Depend on X or Z?

$$ENPP = \beta_0 + \beta_{SF} SFrag + \beta_{DM} DMag + \beta_{SFDM} SFragDMag + \ldots + \varepsilon$$

Hypothesis	*Mathematical Expression*[91]	*Statistical test*
x affects y, or *y is a function of (depends on) x*	*y=f(x)* $\partial y/\partial x = \beta_x + \beta_{xz} z \neq 0$	*F- test:* H_0: $\beta_x = \beta_{xz} = 0$
x increases y	$\partial y/\partial x = \beta_x + \beta_{xz} z > 0$	*Multiple t-tests:* H_0: $\beta_x + \beta_{xz} z \leq 0$
x decreases y	$\partial y/\partial x = \beta_x + \beta_{xz} z < 0$	*Multiple t- tests:* $\beta_x + \beta_{xz} z \geq 0$
z affects y, or *y is a function of (depends on) z*	*y=g(z)* $\partial y/\partial z = \beta_z + \beta_{xz} x \neq 0$	*F- test:* H_0: $\beta_z = \beta_{xz} = 0$
z increases y	$\partial y/\partial z = \beta_z + \beta_{xz} x > 0$	*Multiple t-tests:* H_0: $\beta_z + \beta_{xz} x \leq 0$
z decreases y	$\partial y/\partial z = \beta_z + \beta_{xz} x < 0$	*Multiple t- tests:* H_0: $\beta_z + \beta_{xz} x \geq 0$

From Hypotheses to Hypotheses Tests:

Is Y's Dependence on X Conditional on Z & v.v.? How?

Hypothesis	*Mathematical Expression*[92]	*Statistical test*
The effect of x on y depends on z	$y=f(xz,\bullet)$ $\partial y/\partial x = \beta_x + \beta_{xz} z = g(z)$ $\partial(\partial y/\partial x)/\partial z = \partial^2 y/\partial x \partial z = \beta_{xz} = 0$	*t-test:* H_0: $\beta_{xz} = 0$
The effect of x on y increases in z	$\partial(\partial y/\partial x)/\partial z = \partial^2 y/\partial x \partial z = \beta_{xz} > 0$	*t-test:* H_0: $\beta_{xz} \leq 0$
The effect of x on y decreases in z	$\partial(\partial y/\partial x)/\partial z = \partial^2 y/\partial x \partial z = \beta_{xz} < 0$	*t-test:* H_0: $\beta_{xz} \geq 0$
The effect of z on y depends on x	$y=f(xz,\bullet)$ $\partial y/\partial z = \beta_z + \beta_{xz} x = h(x)$ $\partial(\partial y/\partial z)/\partial x = \partial^2 y/\partial z \partial x = \beta_{xz} = 0$	*t-test:* H_0: $\beta_{xz} = 0$
The effect of z on y increases in x	$\partial(\partial y/\partial z)/\partial x = \partial^2 y/\partial z \partial x = \beta_{xz} > 0$	*t-test:* H_0: $\beta_{xz} \leq 0$
The effect of z on y decreases in x	$\partial(\partial y/\partial z)/\partial x = \partial^2 y/\partial z \partial x = \beta_{xz} < 0$	*t-test:* H_0: $\beta_{xz} \geq 0$

Discussion: Alternative views how explore this…

Does Y Depend on X, Z, or XZ?

Hypothesis	*Mathematical Expression*[93]	*Statistical Test*
y is a function of (depends on) z, z, and/or their interaction	$y=f(x,z,xz)$	*F-test:* H_0: $\beta_x = \beta_z = \beta_{xz} = 0$

Discussion re: determining if Z conditions effect of X, etc.

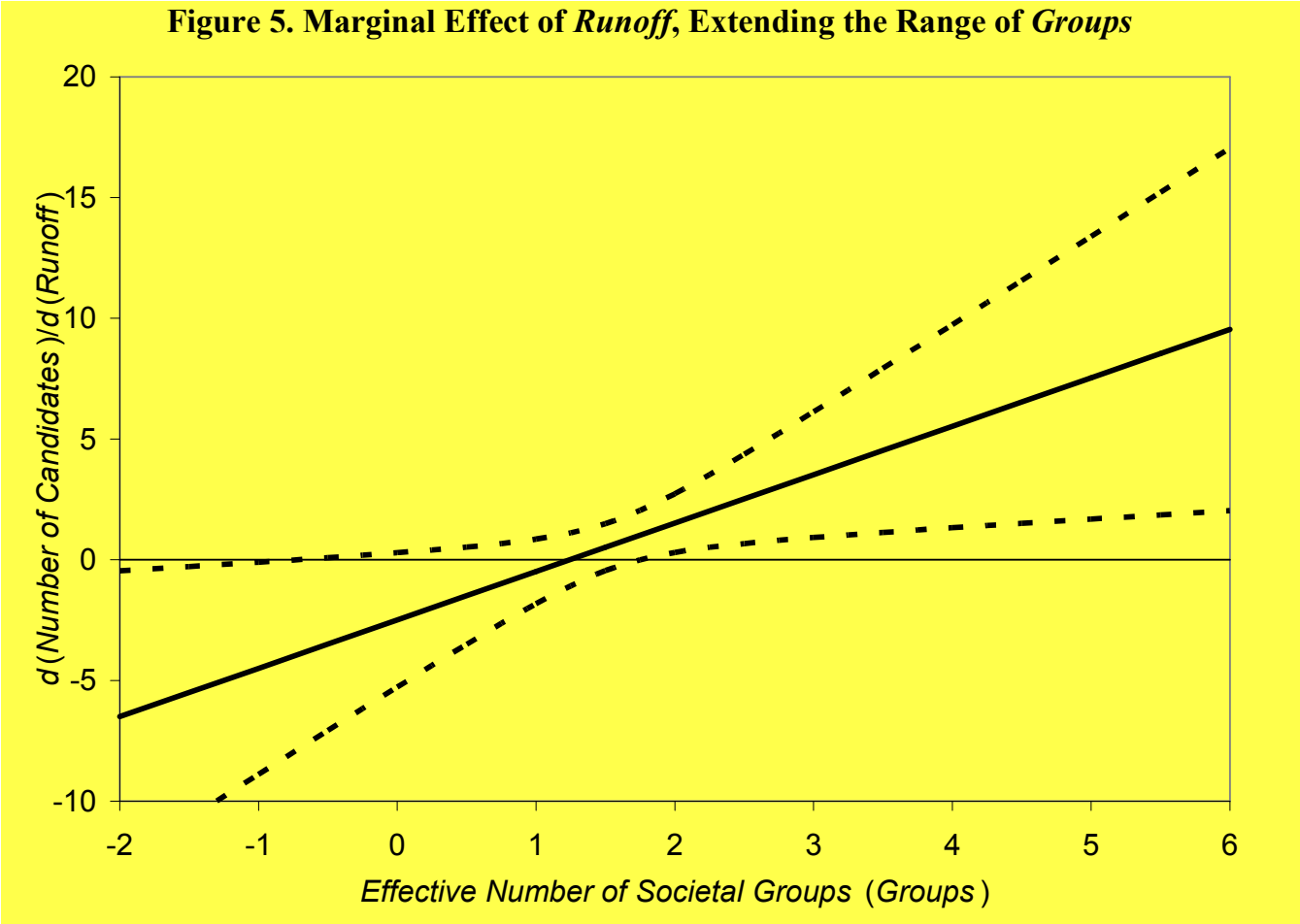
- Some suggest need find some Z0 & Z1 (in sample?) for which dY/dX differ significantly:
- $dY/dX|Z_0=b_x+b_{xz}Z_0$, $dY/dX|Z_1=b_x+b_{xz}Z_1$
  - So: $dY/dX|Z_0-dY/dX|Z_1$
    - $=b_x+b_{xz}Z_0-(b_x+b_{xz}Z_1)=b_{xz}(Z_0-Z_1)$
    - Wald test this signif diff zero =>
      - $=b_{xz}(Z_0-Z_1)/[var\{b_{xz}(Z_0-Z_1)\}]^{.5}$
      - $=b_{xz}(Z_0-Z_1)/[var\{b_{xz}\}(Z_0-Z_1)^2]^{.5}$
      - $=b_{xz}(Z_0-Z_1)/[s.e.\{b_{xz}\}(Z_0-Z_1)]=b_{xz}/s.e.\{b_{xz}\}$,
        - » I.e., just the standard (Wald) t-test on interactive coeff, and that's because that coefficient is cross-derivative about which the hypothesis asks!

Use & Abuse of Some Common 'Rules'

- *Centering to Redress Colinearity Concerns*:
  - Adds no info, so changes ***nothing***; no help with colinearity or anything else; only moves substantive content of x=0,z=0.
  - Specifically, makes coeff. on x (z), effect when z (x) at sample-mean, the new 0. Do only if aids presentation.
- *Must Include All Components* (if $x{\cdot}z$, then $x\&z$):
  - Application of Occam's Razor &/or scientific caution (e.g., greater flexibility to allow linear w/in lin-interax model), but
  - ***Not*** a logical or statistical requirement.
  - Safer rule than opposite & to check almost always, but
  - ***Not*** override *theory & evidence, esp. if (inosfar as strongly) agree to exclude...*
- *Pet-Peeve: Linguistic Gymnastics to Dodge the Math*
  - "Main effect, Interactive effect": ***the*** effect in model is dy/dx.
  - Discussion of [coefficients & s.e.'s] as if [effects & s.e.'s].

<u>Presentation</u>: Marginal-Effects / Differences Tables & Graphs

- Plot/Tabulate **Effects**, dy/dx, over Meaningful &/or Illuminating Ranges of z, with Conf. Int.'s
  - $d\hat{y}/dx \pm t_{df,p}\sqrt{Var(d\hat{y}/dx)} = \hat{\beta}_x + \hat{\beta}_{xz}z \pm t_{df,p}\sqrt{V(\hat{\beta}_x) + V(\hat{\beta}_{xz})z^2 + 2C(\hat{\beta}_x, \hat{\beta}_{xz})z}$
  - Explain axes
  - Explain shape
  - Linear-interax:
    - *Will* cross 0 & be insig @ 0.
  - Rescaling &
    - "main effect"
    - "centering"
    - Max(Asterisks)



Presentation: Expected-Value/Predictions Tables & Graphs

- ***Predictions***, $E(y|x,z)$: notice how helpful in matrix form:

$$\hat{y} \pm t_{df,p}\sqrt{Var(\hat{y})} =$$

$$\hat{\beta}_0 + \hat{\beta}_x x + \hat{\beta}_z z + \hat{\beta}_{xz} xz \pm t_{df,p}\sqrt{\begin{array}{l} V(\hat{\beta}_0) + V(\hat{\beta}_x)x^2 + V(\hat{\beta}_z)z^2 + V(\hat{\beta}_{xz})(xz)^2 \\ + 2C(\hat{\beta}_0,\hat{\beta}_x)x + 2C(\hat{\beta}_0,\hat{\beta}_z)z + 2C(\hat{\beta}_0,\hat{\beta}_{xz})xz \\ + 2C(\hat{\beta}_x,\hat{\beta}_z)xz + 2C(\hat{\beta}_x,\hat{\beta}_{xz})x^2 z + 2C(\hat{\beta}_z,\hat{\beta}_{xz})xz^2 \end{array}}$$

  – Here's one place a little matrix algebra would help:

$$\hat{y} \pm t_{df,p}\sqrt{Var(\hat{y})} = \hat{\beta}_0 + \hat{\beta}_x x + \hat{\beta}_z z + \hat{\beta}_{xz} xz \pm t_{df,p}\sqrt{\mathbf{x}'\hat{V}(\hat{\boldsymbol{\beta}})\mathbf{x}}$$

$$= \hat{\beta}_0 + \hat{\beta}_x x + \hat{\beta}_z z + \hat{\beta}_{xz} xz \pm t_{df,p}\sqrt{\begin{bmatrix} 1 & x & z & xz \end{bmatrix}\begin{bmatrix} \hat{V}(\hat{\beta}_0) & \hat{C}(\hat{\beta}_0,\hat{\beta}_x) & \hat{C}(\hat{\beta}_0,\hat{\beta}_z) & \hat{C}(\hat{\beta}_0,\hat{\beta}_{xz}) \\ \hat{C}(\hat{\beta}_0,\hat{\beta}_x) & \hat{V}(\hat{\beta}_x) & \hat{C}(\hat{\beta}_x,\hat{\beta}_z) & \hat{C}(\hat{\beta}_x,\hat{\beta}_{xz}) \\ \hat{C}(\hat{\beta}_0,\hat{\beta}_z) & \hat{C}(\hat{\beta}_x,\hat{\beta}_z) & \hat{V}(\hat{\beta}_z) & \hat{C}(\hat{\beta}_z,\hat{\beta}_{xz}) \\ \hat{C}(\hat{\beta}_0,\hat{\beta}_{xz}) & \hat{C}(\hat{\beta}_x,\hat{\beta}_{xz}) & \hat{C}(\hat{\beta}_z,\hat{\beta}_{xz}) & \hat{V}(\hat{\beta}_{xz}) \end{bmatrix}\begin{bmatrix} 1 \\ x \\ z \\ xz \end{bmatrix}}$$

  – Use spreadsheet or stat-graph software (…list coming...)

Presentation[1]:
Choose Illuminating Graphics & Base Cases

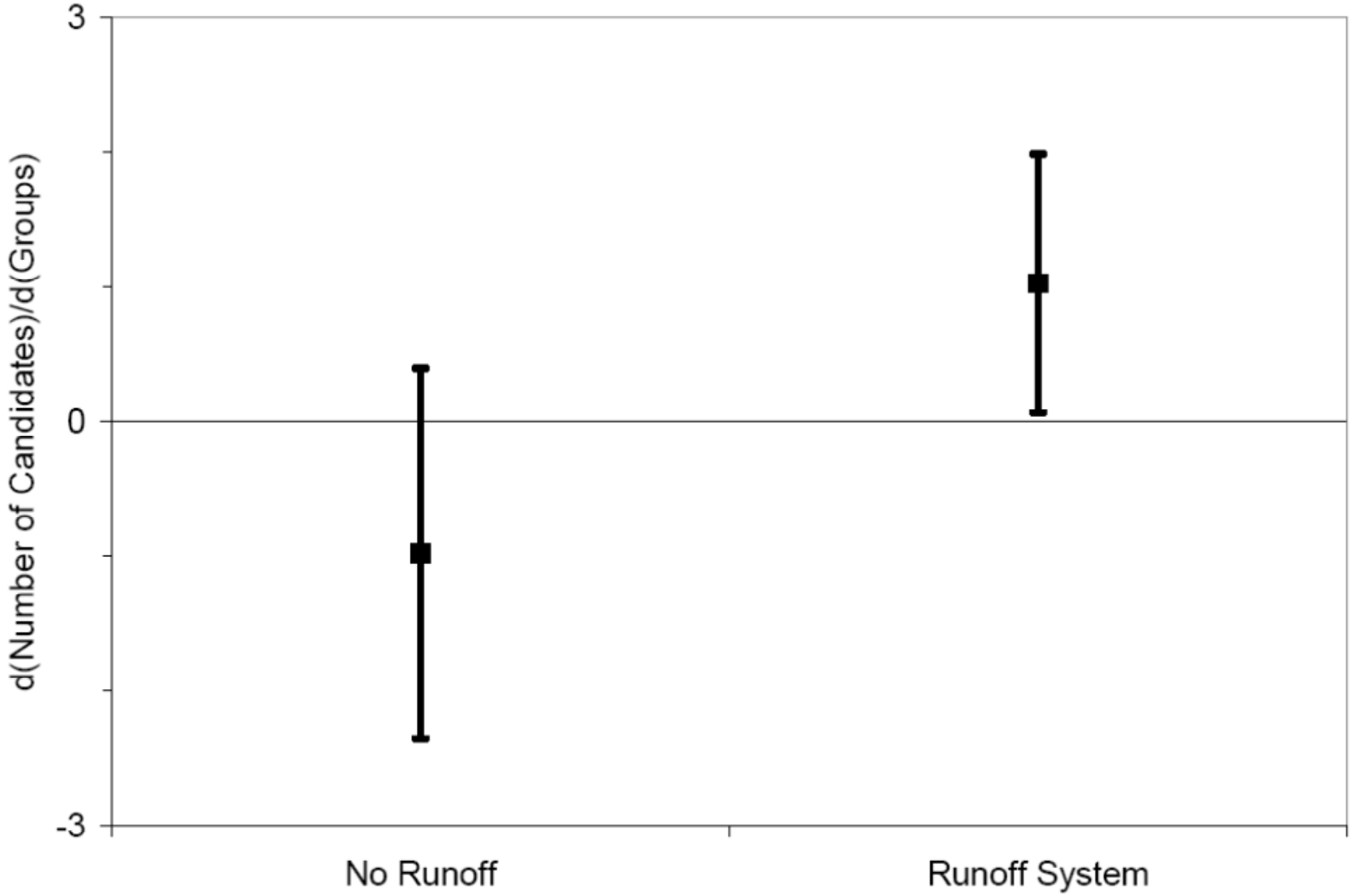
- Interpretation same regardless of "type" of interax: ***effect*** always ≡ ***dy/dx***, but present appropriately:
  - All combos Dummy, Discrete, or Continuous:
    - Dummy-Dummy=>4 (or $2^{\#\text{interacting variables}}$) points estimated, so box & whisker or histograms effective
    - Dummy-Continuous or Discrete(*few*)-Continuous=>2 (or # categories) slopes, so E(y|x,z) as line or dy/dx as box & whisker or histograms effective
    - Continuous – Continuous (or Disc. Many)=>Effect-lines best or (slices from) contour plot (i.e., slices from 3D)
  - Powers (e.g., X & X^2=>parabola) viewable as interax w/ self; certain slope shifts too (e.g., $dy/dx=a$ for $x<x^0$ & b for $x>x^0$ is x interact w/ dummy for condition)

Presentation^2:

Choose Illuminating Graphics & Base Cases

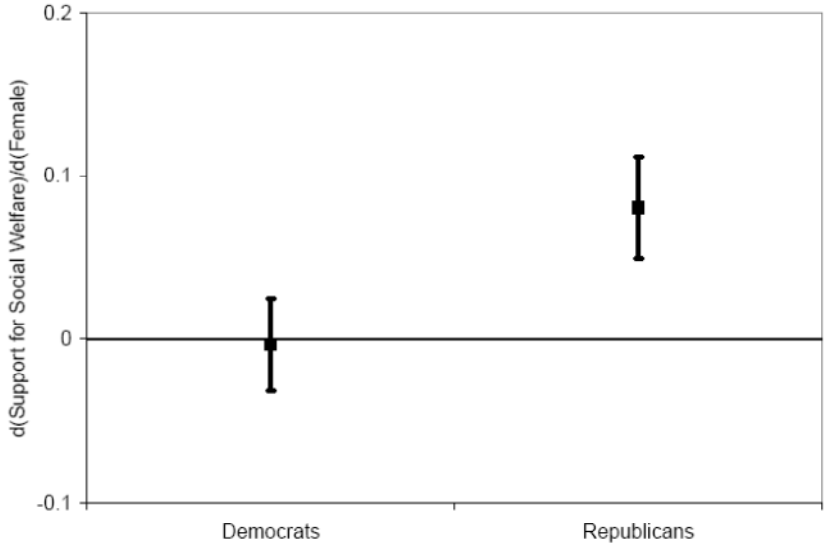
- Interpretation same regardless of "type" of interax: ***effect*** always ≡ ***dy/dx***, but present appropriately...
  - Always plot over substantively revealing ranges.
  - Especially with sets of dummies, have several (identical) specification options:
    - (full-set or set-less-1): choose which (& what base if use set-less-1) to abet presentation & discussion
    - (overlapping or disjoint): choose to facilitate presentation & discussion.
  - Scale Effectively: e.g., center only if & to extent that aids presentation & discussion (b/c centering does nothing else)

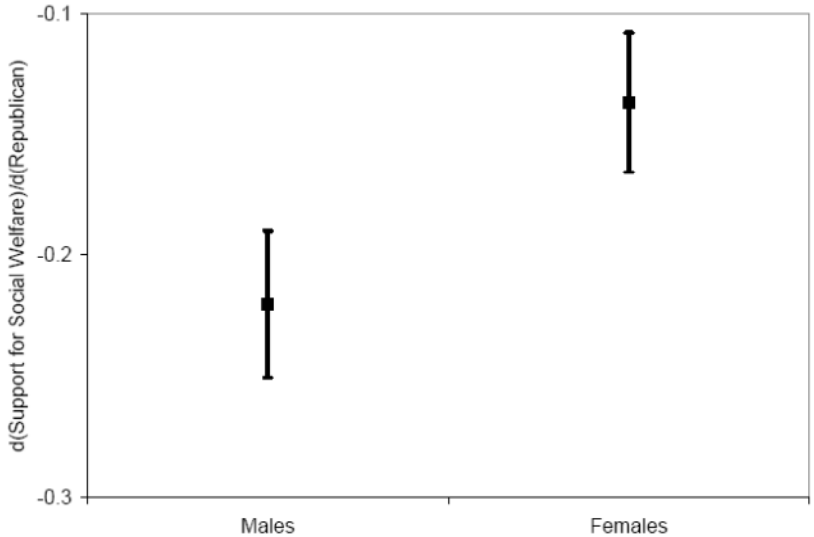
Presentation[3]: Choose Illuminating Graphics & Base Cases. Examples.



Dummy-Continuous Interaction: could also plot two E(Cands|Groups) lines, with c.i.'s, effectively.

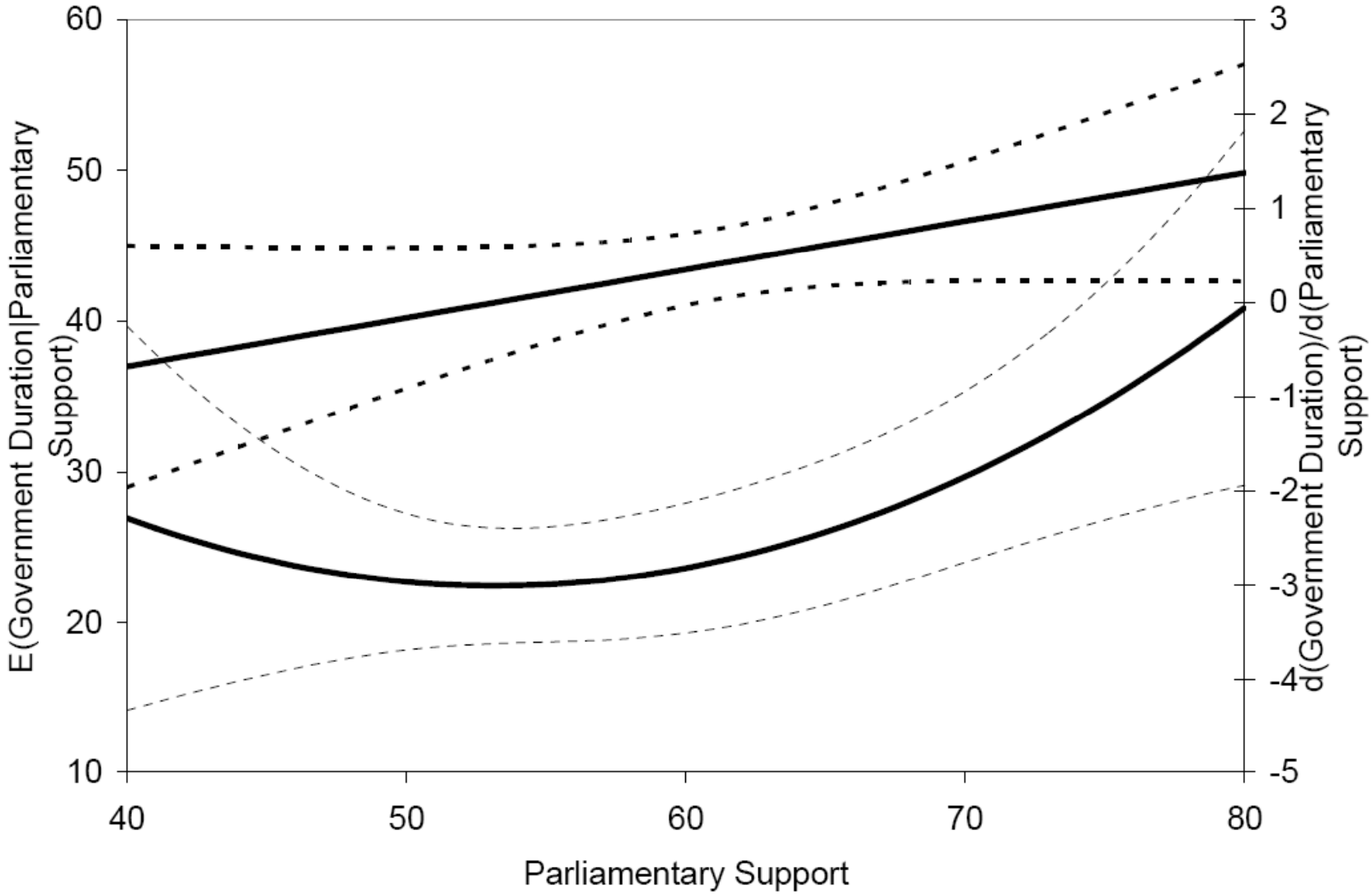
Presentation[4]: Choose Illuminating Graphics & Base Cases. Examples.





Dummy-Dummy Interaction: could also plot four E(Supp|gender,party) box-whiskers effectively.

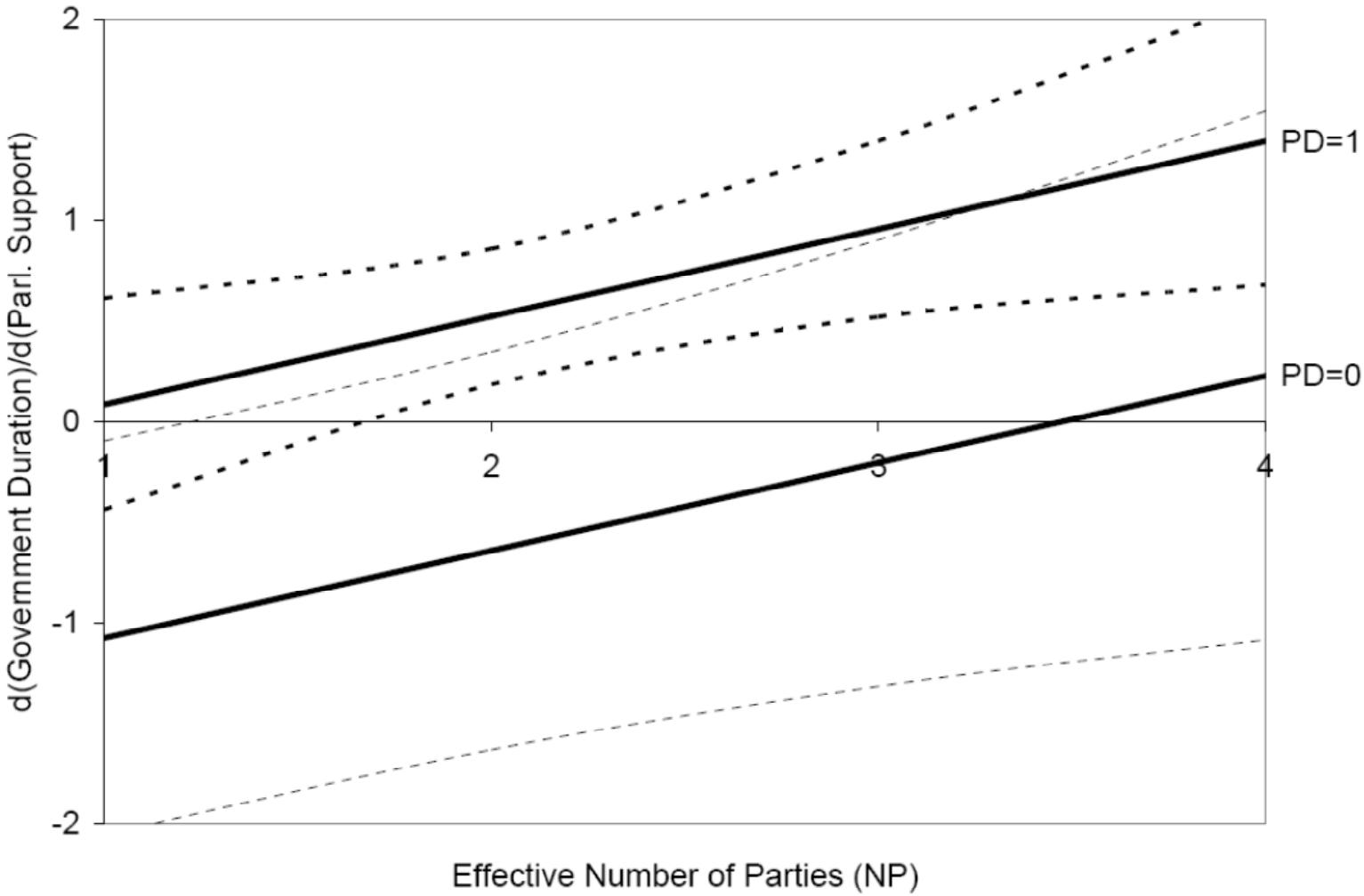
Presentation[5]: Choose Illuminating Graphics & Base Cases. Examples.



Quadratic Model of Government Duration as Function of Parliamentary Support of Governing Parties.

Presentation[6]: Choose Illuminating Graphics & Base Cases. Examples.

Figure 14. Marginal Effect of *Parliamentary Support for Government*, Pairwise-Interaction Model, with 90% Confidence Intervals



$$GovDur = \beta_0 + \beta_{np} NP + \beta_{ps} PS + \beta_{pd} PD + \beta_{npps} NP \times PS + \beta_{nppd} NP \times PD + \beta_{pdps} PD \times PS + \varepsilon \quad [25]$$

Elaborations, Complications, & Extensions: Sample-Splitting v. (Dummy-)Interacting[1]

- Split-sample (e.g., **unit-by-unit**) ≈ Full-Dum Interax:
  - Subsample by binary (or multinomial, e.g., by-unit in TSCS) category to estimate separately ≈ Include dummy for each category (or set-less-1) & interact each dummy with each x *(*and include x by itself also if set-less-1*)*
    - Coeff's same (or equal substantive content if using *set-1* dummies).
    - S.E.'s same except s^2 part of OLS's $s^2(\mathbf{X}'\mathbf{X})^{-1}$ versus s_i^2 for splitting
    - Can make essentially exact by allow s_i^2 (FWLS)
  - Subsample by hi/lo values of some non-nominal regressor is equiv to *nominalizing* the info in that var & dummy-interact
    - I.e., wasting information, when usually have too little (non-parametric or extreme-measurement-error arguments might justify)
    - So usually a bad idea… *(could discuss arg's for it, under rare circ.)*

Elaborations, Complications, & Extensions: Sample-Splitting v. (Dummy-)Interacting[2]

- Split-sample abets eyeballing, obfuscates statistical analysis, of the main point: the different effects by category.
  - What's s.e/signif. of b_{1i}-b_{1j}? Need:

$$s.e.\left(b_{1i}-b_{1j}\right)=\sqrt{V\left(b_{1i}\right)+V\left(b_{1j}\right)-2C\left(b_{1i},b_{1j}\right)}=\sqrt{V\left(b_{1i}\right)+V\left(b_{1j}\right)}$$

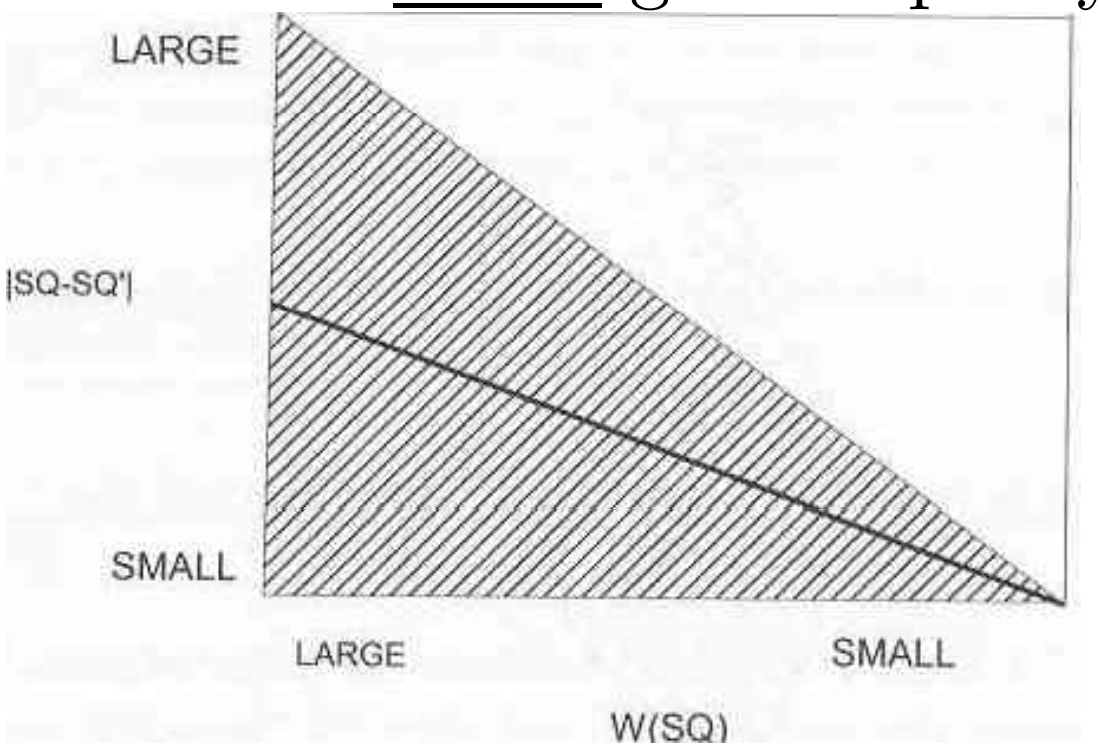
  - Luckily, cov=0, but, still, squaring 2 terms, sum, & root in head?
- Can choose *full dummy set* to mirror the split-sample estimates directly (& report that way, if wish) or the *set-less-one* to get significance of differences b/w samples directly (in the standard reported *t*-test)
  - Same thing, so choose form to optimize presentational efficacy.
- One advantage of hierarchical models (random coeff.) is how it affords, naturally, various positions b/w these extremes.
  - E.g., can "borrow strength" across units.

Typical 2nd-Moment Implications of Interactions

- ***DMag*** permissive ele sys: ***allow*** more parties…

$$NP = \beta_0 + \beta_1 DM + \varepsilon \;\; ; \;\; V(\varepsilon) = f(DM) \;\; \text{, e.g., } \sigma_0 + \sigma_1 DM$$

- Few ***Veto Actors*** ***allow*** greater policy-change…



$$y = \beta_0 + \beta_1 VP + \varepsilon \;\; ; \;\; V(\varepsilon) = f(VP) \;\; \text{, e.g., } \sigma_0 + \sigma_1 VP$$

- I.e., these are Rndm-Coeff &/or Het-sked Props…

$$NP = \beta_0 + \beta_1 DM + \beta_2 SF + \beta_3 DM \times SF + \varepsilon \;\; ; \;\; V(\varepsilon) = f(DM) \;\; \text{, e.g., } \sigma_0 + \sigma_1 DM$$

Sandwich Estimators[1]

$$EN = \beta_0 + \beta_1 SF + \beta_2 DM + \varepsilon$$

$$\partial EN / \partial SF = \beta_1 = \alpha_0 + \alpha_1 DM + \omega_1$$

$$\partial EN / \partial DM = \beta_2 = \gamma_0 + \gamma_1 SF + \omega_2$$

$$\Rightarrow EN = \beta_0 + \left(\alpha_0 + \alpha_1 DM + \omega_1\right) SF + \left(\gamma_0 + \gamma_1 SF + \omega_2\right) DM + \varepsilon$$

$$= \beta_0 + \alpha_0 SF + \left(\alpha_1 + \gamma_1\right) DM \times SF + \gamma_0 DM + \left\{\varepsilon + \omega_1 SF + \omega_2 DM\right\}$$

$$= b_0 + b_1 SF + b_2 DM \times SF + b_3 DM + \varepsilon^*$$

- Notice the compound error term:
  - V($\boldsymbol{\varepsilon}^*$) will not be $\sigma^2 \boldsymbol{I}$ even if $\boldsymbol{\varepsilon}$ is, so V($\mathbf{b}$) doesn't reduce to $\sigma^2(\mathbf{X'X})^{-1}$, so OLS s.e.'s wrong.
  - Be OK on average (unbiased) & in limit (consistent) if element of V($\boldsymbol{\varepsilon}^*$) "orthogonal to $\mathbf{xx'}$"
  - But def'ly not because $\boldsymbol{\varepsilon}^*$ includes $\boldsymbol{x}$ & $\boldsymbol{z}$, which part of $\mathbf{X}$!

Sandwich Estimators[2]

$$V(\mathbf{b}_{LS}) = (\mathbf{X'X})^{-1}\mathbf{X'}\left[V(\boldsymbol{\varepsilon} + \omega_1\mathbf{SF} + \omega_2\mathbf{DM}\right]\mathbf{X}(\mathbf{X'X})^{-1}$$

- Brilliant insight of 'robust' (i.e., consistent) "sandwich" estimators:
  - Only need formula that accounts relation V($\boldsymbol{\varepsilon}^*$) to "$\mathbf{X'X}$", i.e., regressors, squares, & cross-prod's involved in $\mathbf{X'}[\cdot]\mathbf{X}$"
- $\Rightarrow$ ", robust" (or, in HM: ", cluster") can work:

$V(\varepsilon_i^*)_{RE} = \sigma_\varepsilon^2 + \sigma_{\omega_1}^2 x_i^2 + \sigma_{\omega_2}^2 z_i^2$ so track e^2 rel $\boldsymbol{xx'}$ & $\mathbf{zz'}$ $\Rightarrow$ White's *heteroskedasticity-consistent s.e.'s*:

$$[\cdot] = \frac{1}{n}\sum_{i=1}^{n} e_i^2 \boldsymbol{x}_i \boldsymbol{x}_i'$$

Cross-Level Interactions

- Nothing much different if interactions between variables that vary at different levels (note, e.g., not many subscripts used above):
  - If CLRM assumptions apply, then unbiased, consistent, and efficient.
    - Two main issues of concern, though:
      - ***Un- or insufficiently modeled*** parameter heterogeneity (incl. intercept): can cause bias, if pattern un/insuff. het. relates to $\mathbf{X}$,
      - Non-spherical error-covariance matrix:
        - » An efficiency & proper s.e.'s issue, not bias/consistency.
        - » As just seen, surely will arise, & likely in different specific forms depending assumed error-components structure.
  - As before: Effects, their variances, symmetry of interactive propositions, that neither *micro-* nor *macro-level* coefficients=effects…all that applies…

Cross-Level Interactions:

```
reg spend L.spend unem left growthpc depratio
    cdem trade lowwage fdi skand skand_unem
```

$$\text{spend}_{it} = \beta_i^0 + \beta_i^l \text{left}_{it} + \ldots + \varepsilon_{it}$$

$$\beta_i^0 = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0$$

$$\beta_i^l = \gamma_0 + \gamma_1 \text{skand}_i + u_i^1$$

$$\Rightarrow \text{spend}_{it} = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0 + \gamma_0 \text{left}_{it}$$

$$+ \gamma_1 \text{left}_{it} \times \text{skand}_i + \text{left}_{it} u_i^1 + \ldots + \varepsilon_{it}$$

gathering terms:

$$\text{spend}_{it} = \alpha_0 + \ldots + \alpha_1 \text{skand}_i + \gamma_0 \text{left}_{it}$$

$$+ \gamma_1 \text{left}_{it} \times \text{skand}_i + \left(u_i^0 + \text{left}_{it} u_i^1 + \varepsilon_{it} \right)$$

$$\Rightarrow \frac{\partial \text{spend}}{\partial \text{left}} = b_{left} + b_{lftsk} \text{skand} + \text{u}_i^1 \quad \& \quad \frac{\partial \text{spend}}{\partial \text{skand}} = b_{skand} + b_{lftsk} \text{left}$$

$$V(\varepsilon_i^*)_{HM} = \sigma_0^2 + \sigma_1^2 \boldsymbol{x}_i \boldsymbol{x}_i' + \sigma_2^2 \boldsymbol{z}_i \boldsymbol{z}_i' \Rightarrow \textit{cluster:} \quad [\cdot] = \sum_{j=1}^{J} \left\{ \left(\sum_{i=1}^{n_j} e_{ij} \boldsymbol{x}_{ij}\right)' \left(\sum_{i=1}^{n_j} e_{ij} \boldsymbol{x}_{ij}\right) \right\}$$

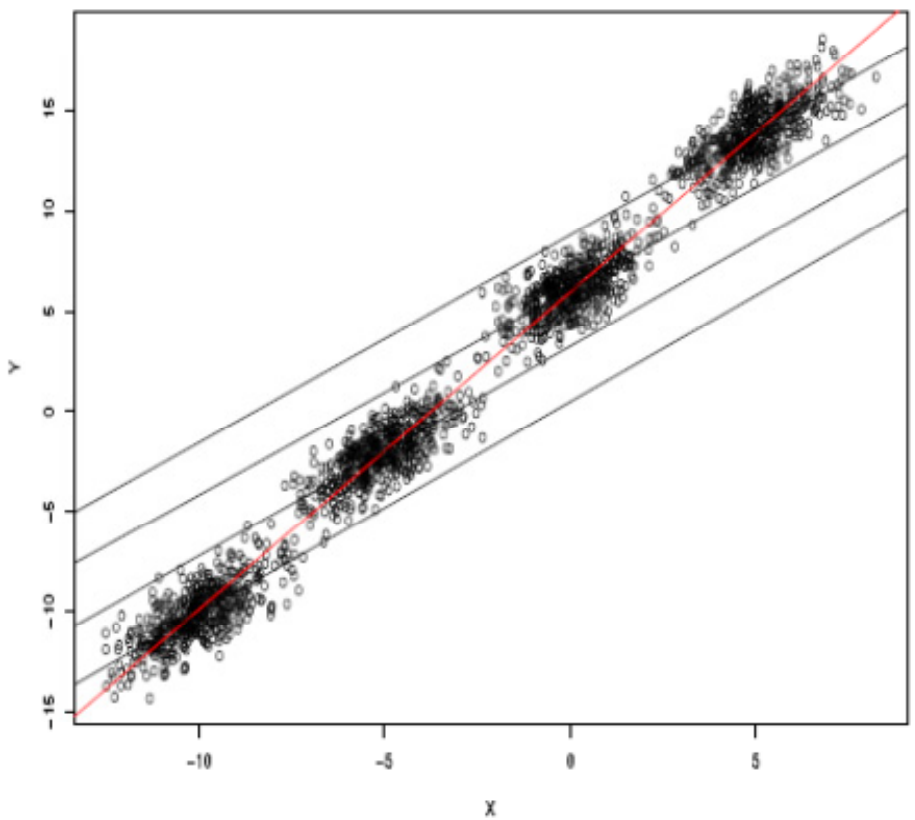
From CLRM to Multilevel Model

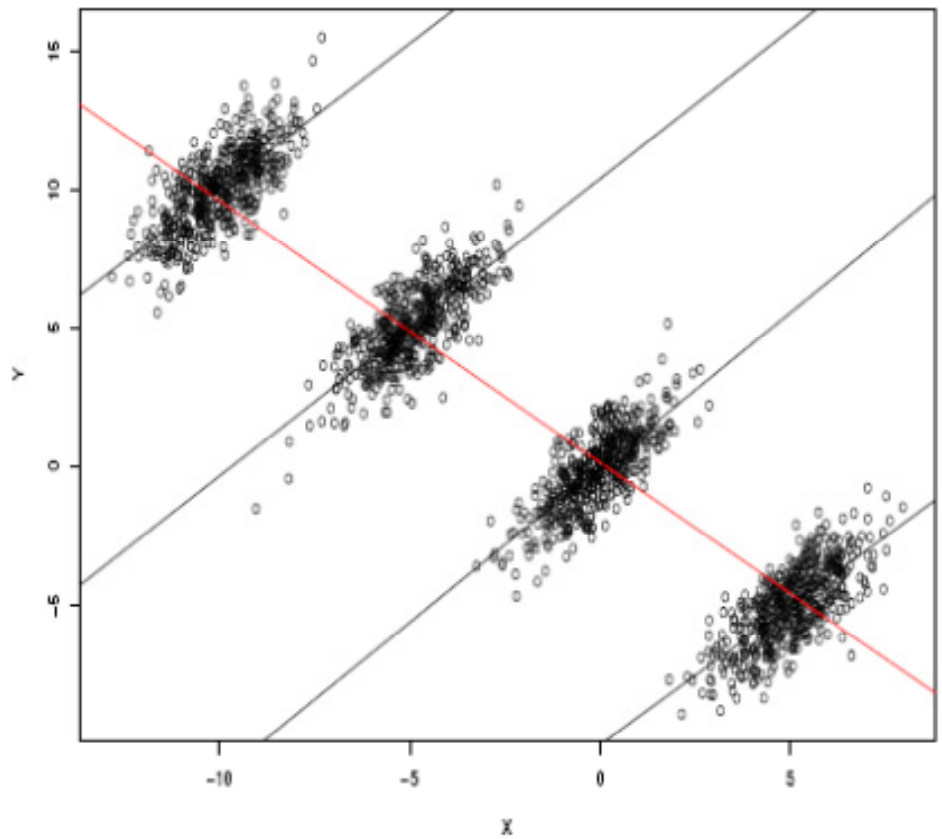
$$\text{spend}_{it} = \beta_i^0 + \beta_i^l \text{left}_{it} + \ldots + \varepsilon_{it}$$
$$\beta_i^0 = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0$$
$$\beta_i^l = \gamma_0 + \gamma_1 \text{skand}_i + u_i^1$$

- If CLRM assumptions apply, then OLS unbiased, consistent, and efficient.
  - Two main issues of concern:
    - Parameter heterogeneity: (see pictures on next slide)
      - systematic &/or stochastic (fixed v. rndm intrcpt/coeff)
      - can cause bias if pattern unmodeled hetero relates to **X**,
    - Non-spherical error cov-mat: an efficiency & proper s.e.'s issue, not a bias/consistency one
      - But "mere inefficiency" can be serious.
      - And accurate std err's very important.

From the CLRM to HLM

- Examples of parameter hetereogeneity that covaries w/ **x** values, so bias:





  - Note: FE v. RE both theoretically could cause bias if cov w/ **x**, but latter identified by orthogonality assumption, as we'll see.

From the CLRM to RE Model

- Arbitrary R.E. Model: Odd that std. lin-interax model:
  - Assumes know *y=f(X)+error*: $y_i = \beta_0 + \beta_x x_i + \beta_z z_i + \beta_{xz} x_i z_i + \varepsilon_i$
  - But *dy/dx=f(z)* w/o error!: $\frac{\partial y}{\partial x} = \beta_x + \beta_{xz} xz$
  - So, try:

$$
\begin{aligned}
y &= \beta_0 + \beta_1 x + \beta_2 z + \varepsilon^0 \\
\frac{\partial y}{\partial x} &\equiv \beta_1 = \alpha_0 + \alpha_1 z + \varepsilon^1 \\
\frac{\partial y}{\partial z} &\equiv \beta_2 = \gamma_0 + \gamma_1 x + \varepsilon^2 \\
\Rightarrow y &= \beta_0 + \left(\alpha_0 + \alpha_1 z + \varepsilon^1\right) x + \left(\gamma_0 + \gamma_1 x + \varepsilon^2\right) z + \varepsilon^0 \\
&= \beta_0 + \alpha_0 x + \gamma_0 z + \left(\alpha_1 + \gamma_1\right) xz + \left(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z\right)
\end{aligned}
$$

  - => std. lin-interact...except compound error-term…
- Hierarchical/Multilevel/Mixed-Effect Model: Same model, except x_{ij} & z_j, & specifically: $\varepsilon^* = \varepsilon_{ij}^0 + \varepsilon_j^1 x_{ij} + \varepsilon_{ij}^2 z_j$
  - So also std lin-interact, but w/ diff compound-error structure
- These also called "error-component" models

From CLRM to Hierarchical Model

- Std. HLM: Same model, except x_{ij} & z_j, &
  - So a std. lin-interact too, but with different compound-error stochastic properties.

$$\text{spend}_{it} = \beta_i^0 + \beta_i^l \text{left}_{it} + \ldots + \varepsilon_{it}$$

$$\beta_i^0 = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0$$

$$\beta_i^l = \gamma_0 + \gamma_1 \text{skand}_i + u_i^1$$

$$\Rightarrow \text{spend}_{it} = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0 + \gamma_0 \text{left}_{it}$$

$$+\gamma_1 \text{left}_{it} \times \text{skand}_i + \text{left}_{it} u_i^1 + \ldots + \varepsilon_{it}$$

gathering terms:

$$\text{spend}_{it} = \alpha_0 + \ldots + \alpha_1 \text{skand}_i + \gamma_0 \text{left}_{it}$$

$$+\gamma_1 \text{left}_{it} \times \text{skand}_i + \left(u_i^0 + \text{left}_{it} u_i^1 + \varepsilon_{it}\right)$$

$$\Rightarrow \frac{\partial \text{spend}}{\partial \text{left}} = b_{left} + b_{lftsk} \text{skand} + \text{u}_i^1 \quad \& \quad \frac{\partial \text{spend}}{\partial \text{skand}} = b_{skand} + b_{lftsk} \text{left}$$

Properties of OLS under HLM Conditions

- Properties of OLS Estimates of Linear-Interaction Model if truly RE/HLM:

$$y = \beta_0 + \beta_x x + \beta_z z + \beta_{xz} xz + \left(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z\right) = \mathbf{X}\boldsymbol{\beta} + \left(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z\right)$$

- So, OLS coeff. est's still differ from truth by $\mathbf{A}\boldsymbol{\varepsilon}^*$:

$$\begin{aligned}\hat{\boldsymbol{\beta}}_{\mathbf{LS}} &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\left[\mathbf{X}\boldsymbol{\beta} + \left(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z\right)\right] \\ &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X}\boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\left(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z\right) = \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\varepsilon}^*\end{aligned}$$

- So, OLS coeff. est's unbiased & consistent:

$$\begin{aligned}E\left(\hat{\boldsymbol{\beta}}_{\mathbf{LS}}\right) &= E\left[\boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\varepsilon}^*\right] = E\left[\boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\left(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z\right)\right] \\ &= \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'E\left(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z\right) = \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\left[E\left(\varepsilon^0\right) + E\left(\varepsilon^1\right)x + E\left(\varepsilon^2\right)z\right] \\ &= \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\left[0 + E\left(\varepsilon^1\right)x + E\left(\varepsilon^2\right)z\right] = \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'[0+0+0] = \boldsymbol{\beta}. \quad Q.E.D.\end{aligned}$$

  - Note: only works for models w/ additively separable stochastic component; not necessarily for others (e.g., logit/probit)

Properties of OLS under HLM Conditions

- But, OLS s.e.'s will be wrong; not $s^2(\mathbf{X'X})^{-1}$, but:

$$
\begin{aligned}
V\left(\hat{\boldsymbol{\beta}}_{\mathbf{LS}}\right) &= V\left[\boldsymbol{\beta} + \left(\mathbf{X'X}\right)^{-1}\mathbf{X'}\boldsymbol{\varepsilon}^*\right] \\
&= V\left[\boldsymbol{\beta}\right] + V\left[\left(\mathbf{X'X}\right)^{-1}\mathbf{X'}\boldsymbol{\varepsilon}^*\right] + 2C\left[\boldsymbol{\beta}, \left(\mathbf{X'X}\right)^{-1}\mathbf{X'}\boldsymbol{\varepsilon}^*\right] \\
&= 0 + V\left[\left(\mathbf{X'X}\right)^{-1}\mathbf{X'}\boldsymbol{\varepsilon}^*\right] + 0 \\
&= \left(\mathbf{X'X}\right)^{-1}\mathbf{X'}V\left(\boldsymbol{\varepsilon}^*\right)\mathbf{X}\left(\mathbf{X'X}\right)^{-1} \\
&= \left(\mathbf{X'X}\right)^{-1}\mathbf{X'}\left[V\left(\boldsymbol{\varepsilon}^0 + \boldsymbol{\varepsilon}^1\boldsymbol{x} + \boldsymbol{\varepsilon}^2\boldsymbol{z}\right)\right]\mathbf{X}\left(\mathbf{X'X}\right)^{-1} \\
&= \left(\mathbf{X'X}\right)^{-1}\mathbf{X'}\left[V\left(\boldsymbol{\varepsilon}^0\right) + V\left(\boldsymbol{\varepsilon}^1\boldsymbol{x}\right) + V\left(\boldsymbol{\varepsilon}^2\boldsymbol{z}\right)\right]\mathbf{X}\left(\mathbf{X'X}\right)^{-1}
\end{aligned}
$$

(the covariance terms are assumed zero)

Sandwich Estimators

$$V\left(\hat{\boldsymbol{\beta}}_{\mathbf{LS}}\right) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\left[V\left(\boldsymbol{\varepsilon}^{0}\right) + V\left(\boldsymbol{\varepsilon}^{1}\boldsymbol{x}\right) + V\left(\boldsymbol{\varepsilon}^{2}\boldsymbol{z}\right)\right]\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}$$

- Not $\boldsymbol{\sigma}^2\boldsymbol{I}$ (even if each $\boldsymbol{\varepsilon}^*$ component is), so whole thing doesn't reduce to $\boldsymbol{\sigma}^2(\mathbf{X}'\mathbf{X})^{-1}$, so OLS s.e.'s wrong.
- Be OK on avg (unbiased) & in limit (consistent) if that term varied in way "orthogonal to $\mathbf{xx}'$"
  - But, as b4, def'ly not b/c [·] includes $\boldsymbol{x}$ & $\boldsymbol{z}$, which part of $\mathbf{X}$.
  - =brilliant insight of 'robust' (i.e., consistent) s.e. est's:
    - Only need s.e. formula that accounts relation V($\boldsymbol{\varepsilon}^*$) to "$\mathbf{xx}'$", i.e., to the regressors, their squares, & cross-prod's involved in $\mathbf{X}'[\cdot]\mathbf{X}$"
- ⇒", robust" & ", cluster" can work (for RE & HLM, resp'ly)
  - $\hat{V}\left(\hat{\boldsymbol{\beta}}\right)_{RE} = \sigma^2\left(\mathbf{I} + \boldsymbol{xx}' + \boldsymbol{zz}'\right)$ so track e^2 rel $\boldsymbol{xx}'$ & $\mathbf{zz}'$ ⇒ $[\cdot] = \frac{1}{n}\sum_{i=1}^{n} e_i^2 \boldsymbol{X_i X_i}'$
    - i.e., White's het-consistent s.e.'s
  - $\hat{V}\left(\hat{\boldsymbol{\beta}}\right)_{HM} = \sigma_0^2\mathbf{I} + \sigma_1^2\boldsymbol{xx}' + \sigma_2^2\boldsymbol{zz}'$ sim but grpng ⇒ $[\cdot] = \sum_{j=1}^{J}\left\{\left(\sum_{i=1}^{n_j} e_{ij}\boldsymbol{X_{ij}}\right)'\left(\sum_{i=1}^{n_j} e_{ij}\boldsymbol{X_{ij}}\right)\right\}$
    - i.e., het-cluster consistent s.e.'s

From the CLRM to HLM

$$\boldsymbol{y} = \boldsymbol{\beta_0} + \alpha_0 \boldsymbol{x} + \gamma_0 \boldsymbol{z} + \left(\alpha_1 + \gamma_1\right) \boldsymbol{xz} + \left(\boldsymbol{\varepsilon}^0 + \boldsymbol{\varepsilon}^1 \boldsymbol{x} + \boldsymbol{\varepsilon}^2 \boldsymbol{z}\right)$$

$$V\left(\hat{\boldsymbol{\beta}}_{\mathbf{LS}}\right) = \left(\mathbf{X}'\mathbf{X}\right)^{-1} \mathbf{X}' \left[V\left(\boldsymbol{\varepsilon}^0\right) + V\left(\boldsymbol{\varepsilon}^1 \boldsymbol{x}\right) + V\left(\boldsymbol{\varepsilon}^2 \boldsymbol{z}\right)\right] \mathbf{X} \left(\mathbf{X}'\mathbf{X}\right)^{-1}$$

- $\Rightarrow$appropriate “, robust” & “, cluster” can work
  - I.e., <u>*asymptotically*</u> std errs right…
    - Note: generally need large n_j, more than just large n, for cluster.
  - I.e., <u>*coefficients still inefficient*</u>.
    - Want/need efficiency, or n_j low? HLM/RE or FGLS/FWLS.
    - Note: similarity RE and HLM, RE & FWLS. As suggests, RE only helps efficiency and only rightly does so if that’s all it does. (I.e., if the RE’s orthogonal to X.)
  - I.e., “work” thusly for models with additively-separable stochastic components
    - As w/ all such “sandwich” estimators, logical disconnect in applying them to models w/o such separability.

Additional Topics

- Contents:
  - Interactions in nonlinear/QualDep models
  - Nonlinear Least Squares and complex context-conditionality. Applications:
    - “Multiple Hands on the Wheel”
    - “Veto Actors Bargaining in Common Pools” (Multiple Effects of Multiple Policymakers)

Elaborations, Complications, & Extensions:

Interax in QualDep (Inherently Interactive) Models

- Probit/Logit Models w/ Interactions
  - Probit: $p(y=1)=\Phi\left(\mathbf{x}'\boldsymbol{\beta}\right)$ – Logit: $p(y=1)=\dfrac{\exp(\mathbf{x}'\boldsymbol{\beta})}{1+\exp(\mathbf{x}'\boldsymbol{\beta})}=\left[1+\exp(-\mathbf{x}'\boldsymbol{\beta})\right]^{-1}$
- Marginal Effects: (nonlinear, so must specify @ what $\mathbf{x}$)
  - Start w/ $\mathbf{x}'\boldsymbol{\beta}$ pure lin-add, model inherently inter. b/c S-shaped:
    - Probit: $\dfrac{\partial p}{\partial x_k}=\dfrac{\partial\Phi\left(\mathbf{x}'\boldsymbol{\beta}\right)}{\partial x_k}=\phi\left(\mathbf{x}'\boldsymbol{\beta}\right)\cdot\dfrac{\partial\mathbf{x}'\boldsymbol{\beta}}{\partial x_k}=\phi\left(\mathbf{x}'\boldsymbol{\beta}\right)\cdot\beta_k$
    - Logit:

$$\frac{\partial p}{\partial x_k}=\frac{\partial\left\{e^{\mathbf{x}'\boldsymbol{\beta}}[1+e^{\mathbf{x}'\boldsymbol{\beta}}]^{-1}\right\}}{\partial x}=\frac{e^{\mathbf{x}'\boldsymbol{\beta}}}{1+e^{\mathbf{x}'\boldsymbol{\beta}}}\cdot\beta_k-\frac{e^{\mathbf{x}'\boldsymbol{\beta}}}{(1+e^{\mathbf{x}'\boldsymbol{\beta}})^2}\cdot e^{\mathbf{x}'\boldsymbol{\beta}}\cdot\beta_x$$

$$=\left[\frac{e^{\mathbf{x}'\boldsymbol{\beta}}(1+e^{\mathbf{x}'\boldsymbol{\beta}})}{(1+e^{\mathbf{x}'\boldsymbol{\beta}})^2}-\frac{(e^{\mathbf{x}'\boldsymbol{\beta}})^2}{(1+e^{\mathbf{x}'\boldsymbol{\beta}})^2}\right]\cdot\beta_k=\frac{e^{\mathbf{x}'\boldsymbol{\beta}}}{(1+e^{\mathbf{x}'\boldsymbol{\beta}})^2}\cdot\beta_k$$

$$=\frac{e^{\mathbf{x}'\boldsymbol{\beta}}}{1+e^{\mathbf{x}'\boldsymbol{\beta}}}\cdot\frac{1}{1+e^{\mathbf{x}'\boldsymbol{\beta}}}\cdot\beta_k=p\cdot\left(1-p\right)\cdot\beta_k$$

  - If $\mathbf{x}'\boldsymbol{\beta}=\ldots+\beta_x x+\beta_z z+\beta_{xz}xz\ldots\Rightarrow$ same except $d\mathbf{x}'\boldsymbol{\beta}/dx=\beta_x x+\beta_{xz}z$; underlying propensity, i.e., movement along S-shape also interact explicitly x & z. [Discuss meaning inherent v. explicit interax…]
    - Probit: $\dfrac{\partial p}{\partial x}=\phi\left(\mathbf{x}'\boldsymbol{\beta}\right)\cdot\left(\beta_x+\beta_{xz}z\right)$ • Logit: $\dfrac{\partial p}{\partial x}=p\cdot\left(1-p\right)\cdot\left(\beta_x+\beta_{xz}z\right)$

Elaborations, Complications, & Extensions: Interax in Nonlin/Qual (Inherently Interax) Models

- Standard Errors?
  - ***Delta Method:***

$$Asym.Var.\left(f(\hat{\boldsymbol{\beta}})\right) \approx \left[\nabla_{\boldsymbol{\beta}} f\left(\hat{\boldsymbol{\beta}}\right)\right]' V\left(\hat{\boldsymbol{\beta}}\right)\left[\nabla_{\boldsymbol{\beta}} f\left(\hat{\boldsymbol{\beta}}\right)\right]$$

  - Probit Marginal-Effect s.e.:

$$\begin{bmatrix} \dfrac{\partial\left\{\phi\left(\mathbf{x}'\hat{\boldsymbol{\beta}}\right)\frac{\partial \mathbf{x}'\hat{\boldsymbol{\beta}}}{\partial x_1}\right\}}{\partial \hat{\beta}_1} \\ \vdots \\ \dfrac{\partial\left\{\phi\left(\mathbf{x}'\hat{\boldsymbol{\beta}}\right)\frac{\partial \mathbf{x}'\hat{\boldsymbol{\beta}}}{\partial x_k}\right\}}{\partial \beta_k} \end{bmatrix}' \begin{bmatrix} \hat{V}\left(\hat{\beta}_1\right) & \cdots & \hat{C}\left(\hat{\beta}_1, \hat{\beta}_k\right) \\ \vdots & \ddots & \vdots \\ \hat{C}\left(\hat{\beta}_1, \hat{\beta}_k\right) & \cdots & \hat{V}\left(\hat{\beta}_k\right) \end{bmatrix} \begin{bmatrix} \dfrac{\partial\left\{\phi\left(\mathbf{x}'\hat{\boldsymbol{\beta}}\right)\frac{\partial \mathbf{x}'\hat{\boldsymbol{\beta}}}{\partial x_1}\right\}}{\partial \hat{\beta}_1} \\ \vdots \\ \dfrac{\partial\left\{\phi\left(\mathbf{x}'\hat{\boldsymbol{\beta}}\right)\frac{\partial \mathbf{x}'\hat{\boldsymbol{\beta}}}{\partial x_k}\right\}}{\partial \beta_k} \end{bmatrix}$$

  - Logit: same, except $\hat{p}(1-\hat{p})\frac{\partial \mathbf{x}'\hat{\boldsymbol{\beta}}}{\partial x}$ replaces $\phi(\mathbf{x}'\hat{\boldsymbol{\beta}})\frac{\partial \mathbf{x}'\hat{\boldsymbol{\beta}}}{\partial x}$
  - For first-difference effects, similar, but need specify from what **x** to what **x**, and not just at what **x**.
- Or you could *CLARIFY*... or *mfx*...

Elaborations, Complications, & Extensions:

Interax in QualDep Models: Cross-Derivatives

- Recall Marginal Effects: $\frac{\partial p}{\partial x} = \phi\left(\mathbf{x}'\boldsymbol{\beta}\right)\cdot\left(\beta_x + \beta_{xz}z\right)$

$$\frac{\partial p}{\partial x} == \frac{e^{\mathbf{x}'\boldsymbol{\beta}}}{1+e^{\mathbf{x}'\boldsymbol{\beta}}}\cdot\frac{1}{1+e^{\mathbf{x}'\boldsymbol{\beta}}}\cdot\left(\beta_x + \beta_{xz}z\right) = p\cdot\left(1-p\right)\cdot\left(\beta_x + \beta_{xz}z\right)$$

- So, what about conditioning effect of z (x) on the effect of x (z) on the probability y=1?

$$\frac{\partial\left(\frac{\partial p}{\partial x}\right)}{\partial z} = \phi\left(\mathbf{x}'\boldsymbol{\beta}\right)\cdot\beta_{xz} + \frac{\partial\left\{\phi\left(\mathbf{x}'\boldsymbol{\beta}\right)\right\}}{\partial\left(\mathbf{x}'\boldsymbol{\beta}\right)}\left(\beta_x + \beta_{xz}z\right)\left(\beta_z + \beta_{xz}x\right)$$

$$\partial\left\{e^{\mathbf{x}'\boldsymbol{\beta}}\left(1+e^{\mathbf{x}'\boldsymbol{\beta}}\right)^{-2}\left(\beta_x + \beta_{xz}z\right)\right\}\Big/\partial z = e^{\mathbf{x}'\boldsymbol{\beta}}\left(1+e^{\mathbf{x}'\boldsymbol{\beta}}\right)^{-2}\beta_{xz}$$

$$+ e^{\mathbf{x}'\boldsymbol{\beta}}\left(\beta_z + \beta_{xz}x\right)\left(1+e^{\mathbf{x}'\boldsymbol{\beta}}\right)^{-2}\left(\beta_x + \beta_{xz}z\right) - 2\cdot e^{\mathbf{x}'\boldsymbol{\beta}}\left(1+e^{\mathbf{x}'\boldsymbol{\beta}}\right)^{-3}e^{\mathbf{x}'\boldsymbol{\beta}}\left(\beta_z + \beta_{xz}x\right)\left(\beta_x + \beta_{xz}z\right)$$

$$= \hat{p}(1-\hat{p})\beta_{xz} + \hat{p}(1-\hat{p})\left(\beta_z + \beta_{xz}x\right)\left(\beta_x + \beta_{xz}z\right) - 2\hat{p}(1-\hat{p})\left(1+e^{\mathbf{x}'\boldsymbol{\beta}}\right)^{-1}e^{\mathbf{x}'\boldsymbol{\beta}}\left(\beta_z + \beta_{xz}x\right)\left(\beta_x + \beta_{xz}z\right)$$

$$= \hat{p}(1-\hat{p})\beta_{xz} + \hat{p}(1-\hat{p})[1-2\hat{p}]\left(\beta_z + \beta_{xz}x\right)\left(\beta_x + \beta_{xz}z\right)$$

Complex Context-Conditionality and Nonlinear Least-Squares

- ***Complex Context Conditionality***: The effect of anything depends on most everything else. E.g.:
  - *Policymaking*:
    - Socioeconomic-structure of interests
    - Party-system and internal party-structures
    - Electoral system & Governmental system
    - Socio-economic realities linking policies to outcomes
  - *Voting:*
    - Voter preferences & informational environment
    - Party/candidate locations & informational environment
    - Electoral & governmental system
  - *Institutions*: Sets of institutions; effect each depends configuration others present (e.g., that core of *VoC* claim).
  - *Strategic Interdependence*: each actors' action depends on everyone else's; complex feedback (see Franzese & Hays....)

Complex Context-Conditionality

- Empirically⇒Multicolinear Nightmare: Options?
  - Ignore context conditionality (stay linear-additive):
    - Inefficient at best, biased more usually, and, anyway, context-conditionality is often our interest!
  - Isolate one or some very few interactions for close study; ignore rest (stay linear-interactive):
    - Same issues, but to degree lessened by amount interax allow, but demands on data rise rapidly w/ that amount.
  - "Structured Case Analysis":
    - May help 'theory generation', but, for empirical evaluation, doesn't help; worsens problem! (See Franzese *OxfHndbk CP* 2007).
  - **EMTI**^{TM}: Lean harder on thry/subst to specify more precisely the nature interax: functional form, precise measures, etc.
    - Refines question put to the data (usually changes default tests also).
    - *GIVEN* thry/subst. specification into empirical model, can estimate complex interactivity. Side benefits. But must *give*.

Nonlinear Least-Squares

- Estimate NLS:

$$\mathbf{y} = f(\mathbf{X},\boldsymbol{\beta})+\boldsymbol{\varepsilon} \quad \text{with} \quad \boldsymbol{\varepsilon} \sim g(\boldsymbol{\varepsilon})$$

$$\Rightarrow E\left(\mathbf{y}\right) = f(\mathbf{X},\boldsymbol{\beta}), \text{ so } \mathbf{y}=f(\mathbf{X},\hat{\boldsymbol{\beta}}) + \hat{\boldsymbol{\varepsilon}}$$

$$\Rightarrow \underset{\tilde{\boldsymbol{\beta}}}{Min} \ \hat{\boldsymbol{\varepsilon}}'\hat{\boldsymbol{\varepsilon}} \Rightarrow \underset{\tilde{\boldsymbol{\beta}}}{Min} \ [\mathbf{y} - f(\mathbf{X},\tilde{\boldsymbol{\beta}})]'[\mathbf{y} - f(\mathbf{X},\tilde{\boldsymbol{\beta}})]$$

$$\Rightarrow \underset{\tilde{\boldsymbol{\beta}}}{Min} \ \text{SSE}=\mathbf{y}'\mathbf{y} - \mathbf{y}'f(\mathbf{X},\tilde{\boldsymbol{\beta}}) - f(\mathbf{X},\tilde{\boldsymbol{\beta}})'\mathbf{y} + f(\mathbf{X},\tilde{\boldsymbol{\beta}})'f(\mathbf{X},\tilde{\boldsymbol{\beta}})$$

$$\Rightarrow \text{FOC: } \nabla_{\tilde{\boldsymbol{\beta}}}\text{SSE}=0 \Rightarrow -2\nabla_{\tilde{\boldsymbol{\beta}}}f(\mathbf{X},\tilde{\boldsymbol{\beta}})'\mathbf{y} + 2\nabla_{\tilde{\boldsymbol{\beta}}}f(\mathbf{X},\tilde{\boldsymbol{\beta}})'f(\mathbf{X},\tilde{\boldsymbol{\beta}}) = 0$$

So, if, e.g., $f(\mathbf{X},\boldsymbol{\beta}) = \mathbf{X}\boldsymbol{\beta}$, then $\nabla_{\tilde{\boldsymbol{\beta}}}f(\mathbf{X},\tilde{\boldsymbol{\beta}})'=\mathbf{X}$, so FOC: $\mathbf{X}'\mathbf{y} = \mathbf{X}'\mathbf{X}\tilde{\boldsymbol{\beta}} \Rightarrow \hat{\boldsymbol{\beta}}_{LS} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y}$, and if $\mathbf{V}(\boldsymbol{\varepsilon}) \equiv \boldsymbol{\Omega} = \sigma^2\mathbf{I}$, then $\widehat{\mathbf{V}(\hat{\boldsymbol{\varepsilon}})}_{LS} = \dfrac{1}{n-k}\left[\mathbf{y} - f\left(\mathbf{X},\hat{\boldsymbol{\beta}}_{LS}\right)\right]'\left[\mathbf{y} - f\left(\mathbf{X},\hat{\boldsymbol{\beta}}_{LS}\right)\right]$ (also, as always).

That is, intuitively, writing $\nabla_{\hat{\boldsymbol{\beta}}}f(\mathbf{X},\hat{\boldsymbol{\beta}}_{LS})$ as simply ∇, we have:

$$\hat{\boldsymbol{\beta}}_{LS} = (\nabla'\nabla)^{-1}\nabla'\mathbf{y}$$

$$\widehat{\mathbf{V}\left(\hat{\boldsymbol{\beta}}_{LS}\right)}_{LS} = \widehat{\mathbf{V}\left[(\nabla'\nabla)^{-1}\nabla'\mathbf{y}\right]} = (\nabla'\nabla)^{-1}\nabla'\widehat{\mathbf{V}(\mathbf{y})}\nabla(\nabla'\nabla)^{-1}; \text{ \& if } \widehat{\mathbf{V}(\mathbf{y})} = \sigma^2\mathbf{I}, \text{ reduces to } \sigma^2(\nabla'\nabla)^{-1}.$$

which if $f(\mathbf{X},\hat{\boldsymbol{\beta}}_{LS}) = \mathbf{X}\hat{\boldsymbol{\beta}}_{LS}$ meaning $\nabla = \mathbf{X}$, &, if $\boldsymbol{\Omega}=\sigma^2\mathbf{I}$, gives the famiar

$\hat{\boldsymbol{\beta}}_{LS} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ & $\widehat{\mathbf{V}(\hat{\boldsymbol{\beta}}_{LS})}_{LS} = \sigma^2(\mathbf{X}'\mathbf{X})^{-1}$, as always.

- NLS is BLUE under same conditions OLS, w/ ∇ for $\mathbf{X}$.
- Interpreting NLS (already know how): Effects = deriv's & 1st-diff's; s.e.'s by Delta Method or simulation…

Generalized Nonlinear Least-Squares

- GNLS:

$$\mathbf{y} = f(\mathbf{X},\boldsymbol{\beta}) + \boldsymbol{\varepsilon} \quad \text{with} \quad V(\boldsymbol{\varepsilon}) = \sigma^2\boldsymbol{\Omega} \neq \sigma^2\mathbf{I}$$

$$\Rightarrow \hat{\boldsymbol{\beta}}_{GNLS} = (\nabla'\boldsymbol{\Omega}^{-1}\nabla)^{-1}\nabla'\boldsymbol{\Omega}^{-1}\mathbf{y}$$

$$\Rightarrow V(\hat{\boldsymbol{\beta}}_{GNLS}) = (\nabla'\boldsymbol{\Omega}^{-1}\nabla)^{-1}\nabla'\boldsymbol{\Omega}^{-1}V(\mathbf{y})\boldsymbol{\Omega}^{-1}\nabla(\nabla'\boldsymbol{\Omega}^{-1}\nabla)^{-1}$$

$$= (\nabla'\boldsymbol{\Omega}^{-1}\nabla)^{-1}\nabla'\boldsymbol{\Omega}^{-1}\boldsymbol{\Omega}\boldsymbol{\Omega}^{-1}\nabla(\nabla'\boldsymbol{\Omega}^{-1}\nabla)^{-1}$$

$$= (\nabla'\boldsymbol{\Omega}^{-1}\nabla)^{-1}\nabla'\boldsymbol{\Omega}^{-1}\nabla(\nabla'\boldsymbol{\Omega}^{-1}\nabla)^{-1} = (\nabla'\boldsymbol{\Omega}^{-1}\nabla)^{-1}$$

  - GNLS is BLUE in same cond's NLS, but $\boldsymbol{\Omega}$ for $\mathbf{I}$.
  - …don't know $\boldsymbol{\Omega}$, so need consistent 1st stage (e.g., NLS)
- FGNLS is asymptotically BLUE:

$$\mathbf{y} = f(\mathbf{X},\boldsymbol{\beta}) + \boldsymbol{\varepsilon} \quad \text{with} \quad V(\boldsymbol{\varepsilon}) = \sigma^2\boldsymbol{\Omega} \neq \sigma^2\mathbf{I}$$

$$\Rightarrow \hat{\boldsymbol{\beta}}_{FGNLS} = (\nabla'\hat{\boldsymbol{\Omega}}^{-1}\nabla)^{-1}\nabla'\hat{\boldsymbol{\Omega}}^{-1}\mathbf{y}$$

$$\Rightarrow V(\hat{\boldsymbol{\beta}}_{FGNLS}) = (\nabla'\hat{\boldsymbol{\Omega}}^{-1}\nabla)^{-1}\nabla'\hat{\boldsymbol{\Omega}}^{-1}V(\mathbf{y})\hat{\boldsymbol{\Omega}}^{-1}\nabla(\nabla'\hat{\boldsymbol{\Omega}}^{-1}\nabla)^{-1}$$

$$= (\nabla'\hat{\boldsymbol{\Omega}}^{-1}\nabla)^{-1}$$

Nonlinear Least-Squares & EMTI

- EITM: Empirical Implications of Theoretical Models
  - *Vision:* Theory ⇒ more, sharper predictions ⇒ better tests, which therefore inform theory more, which…
- TMEI: Theory-specified Models for Empirical Inference
  - *Vision:* Theory structures empirical models & relations b/w obs ⇒ specification & (causal) i.d. of empirical models
- TIEM: Theoretical Implications of Empirical Measures
  - *Vision:* Emp. regularities, findings, measures inform theory dev'p.
- EMTI: Empirical Models of Theoretical Intuitions
  - *Vision:* Intuitions derived from theoretical models specify empirical models. I.e., empirical specification to match intuitions, not model.
- Note: Strongly counter some alternative moves stats & econometrics, & related; there toward non-parametric, matching, & experimentation—there, "model-dependence" a 4-letter word. Alternative audiences & rhetorical purposes?
  - Convince skeptic some causal effect exists, vs.
  - For the convinced, give richer, portable model of how world works.

Nonlinear Least-Squares:

"Multiple Hands on the Wheel" Model (Franzese, *PA* '03)

- Monetary Policy in Open & Institutionalized Econ
  - Key C&IPE Insts/Struct: CBI, ER-Regime, Mon. Open
    - ° CBI ≡ ° Govt Delegated Mon Pol to CB
    - ° Peg ≡ ° Domestic (CB&Gov) Delegate to Peg-Curr (CB&Gov)
    - ° FinOp ≡ ° Dom cannot delegate b/c effectively del'd to globe
  - Effect of ev'thing to which for. & dom. mon. pol-mkrs would respond diff'ly depends on combo insts-structs & v.v., &, through intl inst-structs, for. on dom. & v.v.

$$\pi = \begin{cases} P\cdot E\cdot C\cdot \pi_1(X_1) + P\cdot E\cdot (1-C)\cdot \pi_2(X_2) \\ +P\cdot (1-E)\cdot C\cdot \pi_3(X_3) + P\cdot (1-E)\cdot (1-C)\cdot \pi_4(X_4) \\ (1-P)\cdot E\cdot C\cdot \pi_5(X_5) + (1-P)\cdot E\cdot (1-C)\cdot \pi_6(X_6) \\ +(1-P)\cdot (1-E)\cdot C\cdot \pi_7(X_7) + (1-P)\cdot (1-E)\cdot (1-C)\cdot \pi_8(X_8) \end{cases}$$

  - Multicolinear Nightmare:
    - $2^3=8$ inst-struct conds, *i*, times *k* factors per $\pi_i(\mathbf{X}_i)$ if lin-interact
    - Exponentially more if all polynominials; k!/2(k-2)! if all pairs.
    - Good thing can lean on some thry to specify more precisely!

Nonlinear Least-Squares: "Multiple Hands on the Wheel" Model

- CB & Govt Interaction (Franzese, *AJPS* '99):

$$E(\pi) = c \cdot \pi_c(\mathbf{x}_c) + (1-c) \cdot \pi_g(\mathbf{x}_g)$$

$$\pi_c = \bar{\pi}_c \qquad \pi_g(\mathbf{x}_g) = \pi_g(GP, UD, BC, TE, EY, FS, AW, \pi_a)$$

- Full Monetary Exposure & Atomistic ⇒ zero domestic autonomy ⇒

$$\widehat{\pi_1(\mathbf{x}_1)} = \widehat{\pi_2(\mathbf{x}_2)} = \widehat{\pi_5(\mathbf{x}_5)} = \widehat{\pi_6(\mathbf{x}_6)} = \pi_a$$

$$\Rightarrow \begin{cases} E \cdot \pi_a + P \cdot (1-E) \cdot C \cdot \pi_3(\mathbf{x}_3) + P \cdot (1-E) \cdot (1-C) \cdot \pi_4(\mathbf{x}_4) \\ +(1-P) \cdot (1-E) \cdot C \cdot \bar{\pi}_c + (1-P) \cdot (1-E) \cdot (1-C) \cdot \pi_g(\mathbf{x}_8) \end{cases}$$

- s.t. that, full e.r.fix⇒CB&Gov match peg⇒

$$\widehat{\pi_3(\mathbf{x}_3)} = \widehat{\pi_4(\mathbf{x}_4)} = \pi_p$$

$$\Rightarrow \begin{cases} E \cdot \pi_a + P \cdot (1-E) \cdot \pi_p \\ +(1-P) \cdot (1-E) \cdot \left[C \cdot \bar{\pi}_c + (1-C) \cdot \pi_g(\mathbf{x}_8) \right] \end{cases}$$

Nonlinear Least-Squares: "Multiple Hands on the Wheel" Model

- Compact & intuitive, yet gives all theoretically expected interactions, in the form expected

$$\pi = E \cdot \pi_a + (1-E) \cdot \left\{ P \cdot \pi_p + (1-P) \cdot \left[C \cdot \overline{\pi_c} + (1-C) \cdot \pi_g(X_g) \right] \right\}$$

$$\Rightarrow$$

$$\frac{\partial \pi}{\partial E} = \pi_a\left(P^*, E^*, C^*, X^*, \pi_a^*\right) - \left\{ P \cdot \pi_p\left(P^*, E^*, C^*, X^*, \pi_p^*\right) + (1-P) \cdot \left[C \cdot \overline{\pi_c} + (1-C) \cdot \pi_g(X_g) \right] \right\}$$

$$\frac{\partial \pi}{\partial P} = (1-E) \cdot \left\{ \pi_p\left(P^*, E^*, C^*, X^*, \pi_p^*\right) - \left[C \cdot \overline{\pi_c} + (1-C) \cdot \pi_g(X_g) \right] \right\}$$

$$\frac{\partial \pi}{\partial C} = (1-E) \cdot \left\{ (1-P) \cdot \left[\overline{\pi_c} - \pi_g(X_g) \right] \right\}$$

$$\frac{\partial \pi}{\partial x} = (1-E) \cdot \left\{ (1-P) \cdot \left[(1-C) \cdot \frac{\partial \pi_g}{\partial x} \right] \right\}$$

$$\frac{\partial \pi}{\partial z^*} = E \cdot \frac{\partial \pi_a}{\partial z^*} + (1-E) \cdot \left\{ P \cdot \frac{\partial \pi_p}{\partial z^*} + (1-P) \cdot \left[(1-C) \cdot \frac{\partial \pi_g}{\partial \pi_a} \cdot \frac{\partial \pi_a}{\partial z^*} \right] \right\}$$

Nonlinear Least-Squares: "Multiple Hands on the Wheel" Model

- Effectively Estimable, yet gives all theoretically expected interactions, in the form expected

$$E(\pi) = \boldsymbol{B_0} + \beta_e E \cdot \beta_{\pi^*}\pi_a + \left(1 - \beta_e E\right) \cdot \left\{ \begin{array}{l} \left[\begin{array}{l} \left(\beta_{gp} GP + \beta_{ey} EY + \beta_{up} UP + \beta_{bc} BC + \beta_{aw} AW + \beta_{fs} FS + \beta_{te} TE + \beta_a \pi_a\right) \\ \cdot \left(1 - \beta_{c1} C\right) + \beta_{c1} C \cdot \beta_{c2} \end{array} \right] \\ \cdot (1 - \beta_{sp} SP - \beta_{mp} MP) + \beta_{sp} SP \cdot \beta_{\pi^*}\pi_{sp} + \beta_{mp} MP \cdot \beta_{\pi^*}\pi_{mp} \end{array} \right\}$$

- Just 14 parameters (plus intercepts & dynamics, assuming those constant), just 3 more than lin-add!
- Parameters substantive meaning, too:
  - Degree to which…constrains certain set of actors.
  - Yields est. of inflation-target hypothetical fully indep CB
    - ⇒ general strategy for estimating/measuring unobservables
      - If know role factor will play & explanators of factor well enough, can estimate unobservables conditional on both those theories, if both powerful enough & enough empirical variation.

Nonlinear Least-Squares: "Multiple Hands on the Wheel" Model

- Neat, but does it work? (**Try it!** Data online; **stata**: **`help nl`**). Estimated Equation, w/ Std. Errs.:

$$E(\pi)\approx\left\langle\begin{array}{l} .53^{.30}+.55^{.05}\pi_{t-1}-.12^{.04}\pi_{t-2}+.44^{.14}E\cdot\pi_a+ \\ (1-.44^{.14}E)\cdot\left\{\begin{array}{l} 1.0^{.05}SP\cdot.59^{.07}\pi_{sp}+.22^{.12}MP\cdot.59^{.07}\pi_{mp}+ \\ (1-1.0^{.05}SP-.22^{.12}MP)\cdot\left[\begin{array}{l} 1.0^{.11}C\cdot(-.59^{1.2})+ \\ (1-1.0^{.11}C)\cdot\left(\begin{array}{l} -.60^{.30}GP+2.6^{1.3}EY+16^{4.6}UP-11^{2.4}BC \\ +1.2^{.49}AW-1.1^{.30}FS-8.2^{4.9}TE+.64^{.24}\pi_a \end{array}\right) \end{array}\right] \end{array}\right\} \end{array}\right.$$

- Estimated Effects (highly context-conditional):

$$E\left(\frac{d\pi}{dC}\right)=(1-.44\cdot E)\cdot\left\{(1-b_pP)\cdot\left[(.6GP-2.6EY-16UP+11BC-1.2AW+1.1FS+8.2TE-.64\pi_a)-.59\right]\right\}$$

$$E\left(\frac{d\pi}{dx}\right)=(1-.44E)\cdot\left\{(1-SP-.22MP)\cdot\left[(1-C)\cdot b_x\right]\right\}$$

$$E\left(\frac{d\pi}{dP}\right)=(1-.44E)\cdot b_p\cdot\left\{.59\pi_p-\left[(1-C)\cdot(-.6GP+2.6EY+16UP-11BC+1.2AW-1.1FS-8.2TE+.64\pi_a)-.59C\right]\right\}$$

$$E\left(\frac{d\pi}{dE}\right)=.44\cdot\left(\pi_a-\left\{b_pP\cdot.59\pi_p+(1-b_pP)\cdot\left[(1-C)\cdot(-.6GP+2.6EY+16UP-11BC+1.2AW-1.1FS-8.2TE+.64\pi_a)-.59C\right]\right\}\right)$$

Nonlinear Least-Squares: "Multiple Hands on the Wheel" Model

Table 2: Estimated Effects of Domestic Political-Economic Conditions, $d\pi/x$, as Function of Central Bank Autonomy, *CBA*, International Monetary Exposure, *E*, and Exchange-Rate Regime, *P*

		Little Exposed (E=0.40)			*Moderately Exposed (E=0.65)*			*Highly Exposed (E=0.90)*		
		Float	*Basket Peg*	*Simple Peg*	*Float*	*Basket Peg*	*Simple Peg*	*Float*	*Basket Peg*	*Simple Peg*
		Estimated Impact of a Post-Election Year ($d\pi/dEY$)								
central	*0.26*	$+1.563^{.79}$	$+1.224^{.61}$	$+0.000^{.09}$	$+1.352^{.69}$	$+1.059^{.53}$	$+0.000^{.07}$	$+1.142^{.60}$	$+0.894^{.47}$	$+0.000^{.06}$
bank	*0.46*	$+1.120^{.57}$	$+0.877^{.44}$	$+0.000^{.06}$	$+0.970^{.50}$	$+0.759^{.39}$	$+0.000^{.05}$	$+0.819^{.44}$	$+0.641^{.34}$	$+0.000^{.05}$
auton.	*0.66*	$+0.678^{.37}$	$+0.531^{.29}$	$+0.000^{.04}$	$+0.587^{.32}$	$+0.459^{.25}$	$+0.000^{.03}$	$+0.495^{.28}$	$+0.388^{.22}$	$+0.000^{.03}$
		Estimated Impact of 10% Increase in Union Density ($0.1 \cdot d\pi/dUP$)								
central	*0.26*	$+0.98^{.25}$	$+0.76^{.18}$	$+0.00^{.05}$	$+0.84^{.21}$	$+0.66^{.16}$	$+0.00^{.04}$	$+0.71^{.19}$	$+0.56^{.14}$	$+0.00^{.04}$
bank	*0.46*	$+0.70^{.18}$	$+0.55^{.13}$	$+0.00^{.04}$	$+0.61^{.15}$	$+0.47^{.11}$	$+0.00^{.03}$	$+0.51^{.14}$	$+0.40^{.10}$	$+0.00^{.03}$
auton.	*0.66*	$+0.42^{.13}$	$+0.33^{.10}$	$+0.00^{.02}$	$+0.37^{.11}$	$+0.29^{.08}$	$+0.00^{.02}$	$+0.31^{.10}$	$+0.24^{.08}$	$+0.00^{.02}$
		Estimated Impact of 1% Increase in Financial-Sector Employment-Share ($d\pi/dFS$)								
central	*0.26*	$-0.66^{.18}$	$-0.52^{.12}$	$-0.00^{.03}$	$-0.57^{.16}$	$-0.45^{.11}$	$-0.00^{.03}$	$-0.48^{.15}$	$-0.38^{.11}$	$-0.00^{.03}$
bank	*0.46*	$-0.47^{.13}$	$-0.37^{.09}$	$-0.00^{.02}$	$-0.41^{.12}$	$-0.32^{.08}$	$-0.00^{.02}$	$-0.35^{.11}$	$-0.27^{.08}$	$-0.00^{.02}$
auton.	*0.66*	$-0.29^{.10}$	$-0.22^{.07}$	$-0.00^{.01}$	$-0.25^{.09}$	$-0.19^{.06}$	$-0.00^{.01}$	$-0.21^{.08}$	$-0.16^{.06}$	$-0.00^{.01}$
		Estimated Impact of 1% Increase in Average Inflation Abroad ($d\pi/d\pi_a$)								
central	*0.26*	$+0.49^{.14}$	$+0.41^{.13}$	$+0.11^{.05}$	$+0.50^{.12}$	$+0.43^{.11}$	$+0.17^{.07}$	$+0.52^{.10}$	$+0.46^{.10}$	$+0.24^{.09}$
bank	*0.46*	$+0.38^{.10}$	$+0.32^{.09}$	$+0.11^{.04}$	$+0.41^{.08}$	$+0.36^{.08}$	$+0.17^{.06}$	$+0.44^{.08}$	$+0.39^{.08}$	$+0.24^{.08}$
auton.	*0.66*	$+0.27^{.06}$	$+0.24^{.06}$	$+0.11^{.04}$	$+0.32^{.06}$	$+0.28^{.06}$	$+0.17^{.06}$	$+0.36^{.06}$	$+0.33^{.06}$	$+0.24^{.08}$

NOTES: These are *first-year effects*, meaning before the estimated dynamics unfold. Standard errors noted in superscripts.

Nonlinear Least-Squares: "Multiple Hands on the Wheel" Model

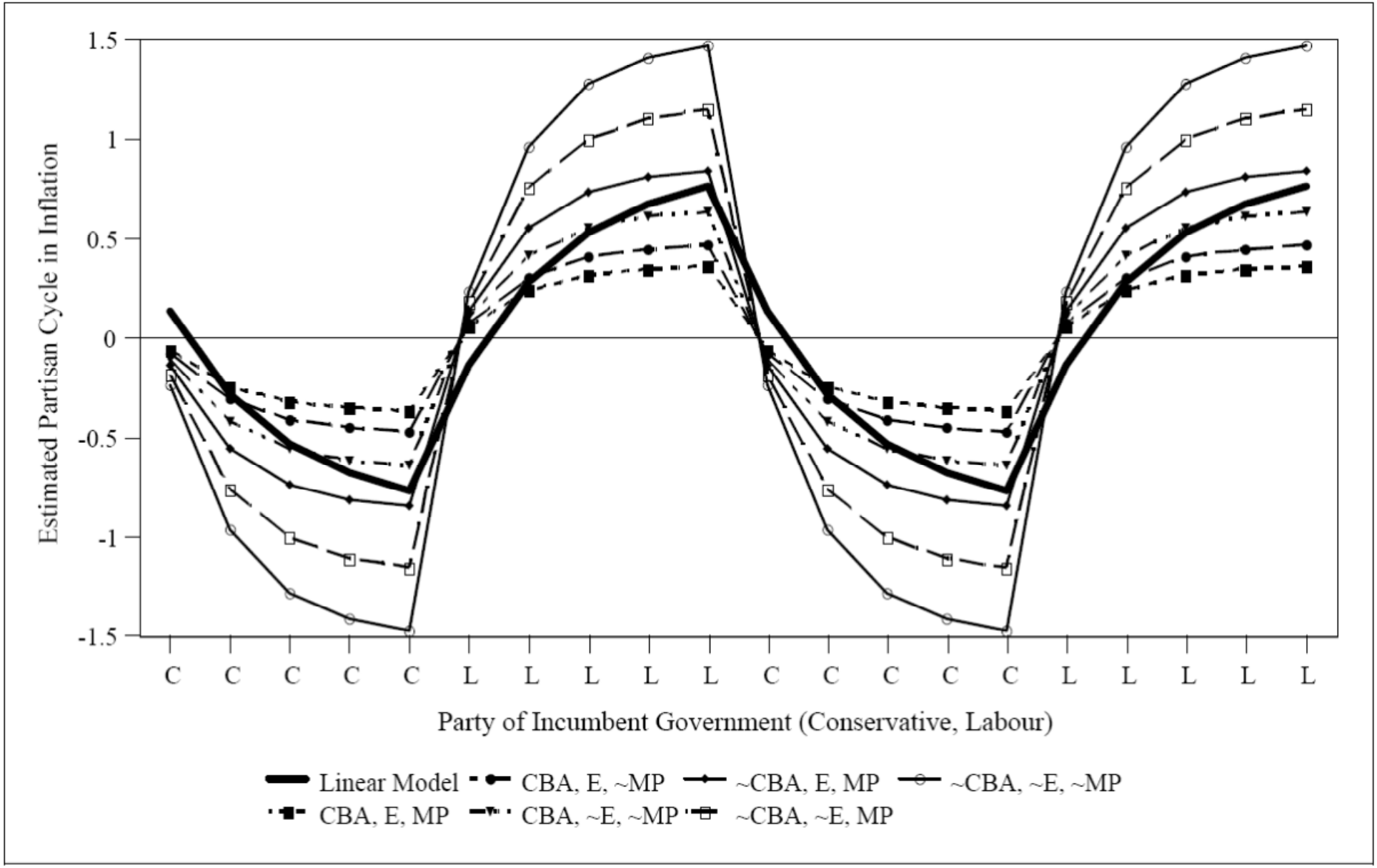


Figure 1: Estimated Partisan Cycles in the Linear & Theoretically Informed Models at High & Low *CBA*, *E*, & *MP*

Nonlinear Least-Squares: "Multiple Hands on the Wheel" Model

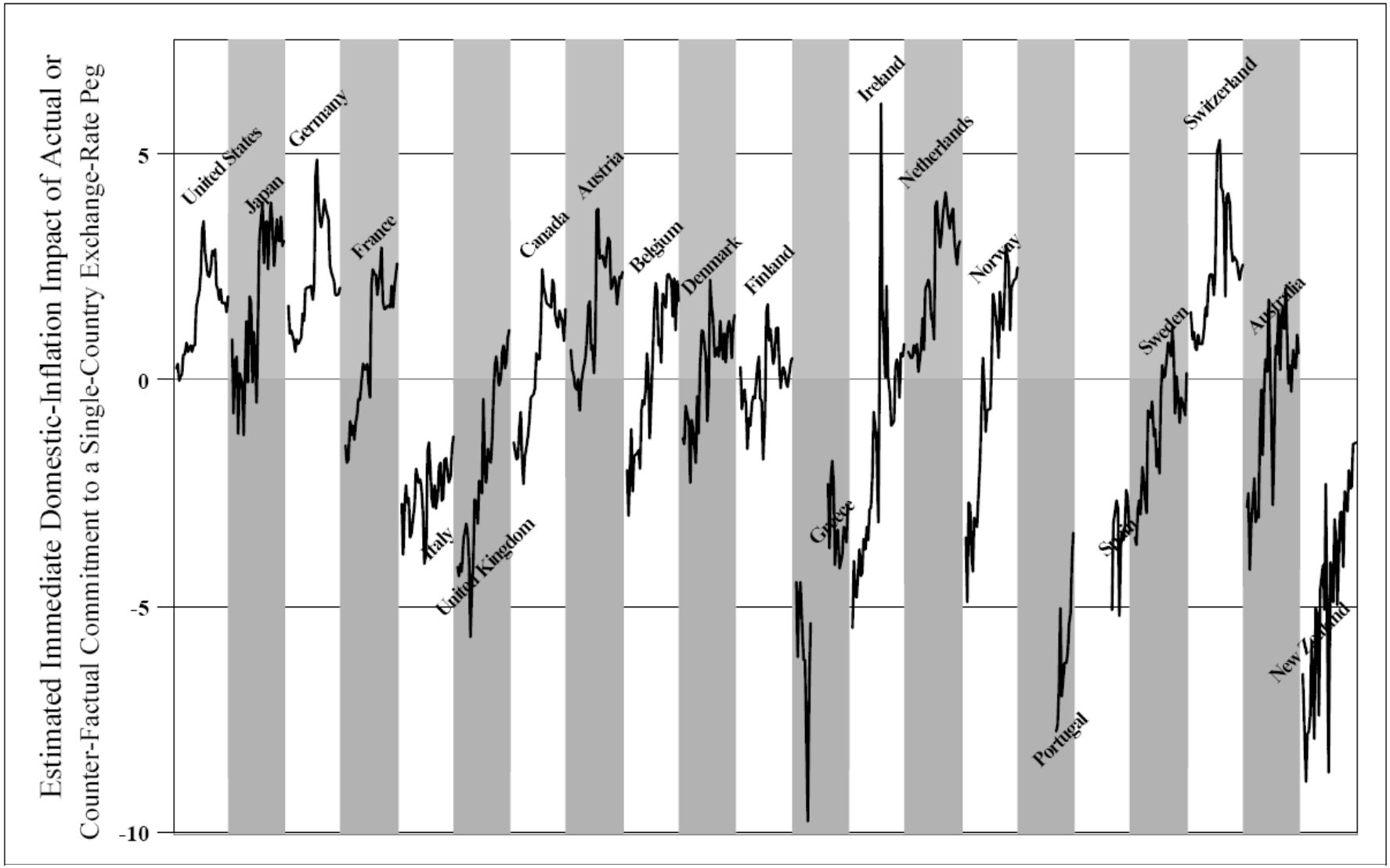


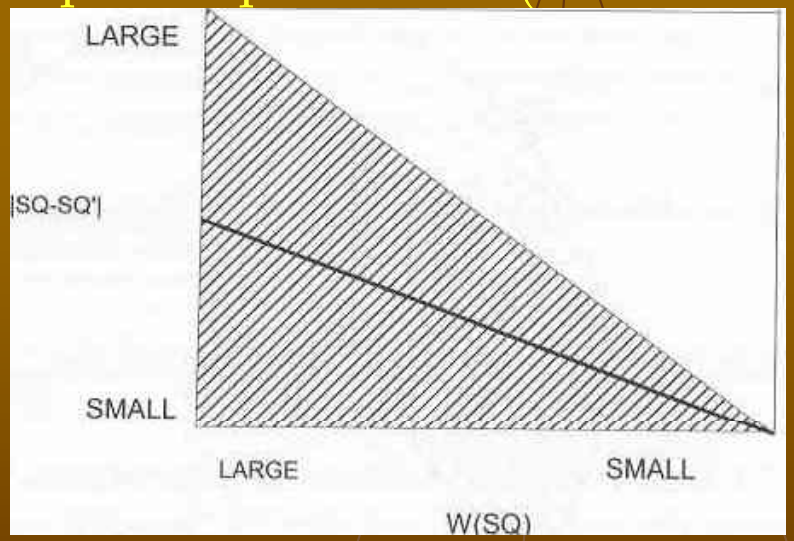
Figure 2: Estimated Domestic-Inflation Effect of Actual or Counter-Factual *SP* in 21 Countries, 1957-90. Estimates plotted for dINF/dSP at the values of all other variables in the equation actually occurring in that country-year. For counter-factual pegs, peg country assumed to have OECD-average inflation that year. Shading separates countries and extends from 1955 to 1990 in each country, left to right.

Multiple Policymakers: Veto Actors Bargaining in Common Pools

- Multiple implications for policy outcomes dispersal of policymaking-authority across diverse actors:
  - Veto-Actor Theory (Tsebelis '02) emphasizes:
    - Privileges *S.Q.*, & so retards policy adjustment, reduces change.
  - Collective-Action/Common-Pool Theories (WSJ '81):
    - Externalities & so overexploit/underinvest public goods.
  - Bargaining & Delegation Theories rather stress:
    - Bargaining Strengths/Positions, yielding Weighted Compromise.
- This project attempts a synthesis:
  - Disting. theoretically/conceptually many effects of # (fragment.) & diversity (polar., partisan) policymakers.
  - Empirical model of many effects distinctly & effectively.
  - Preliminary application to evolution fiscal policy (pub debt) in developed democracies, 1950s-90s.

Veto Actors: Deadlock, Delayed Stabilization, & Policy-Adjustment Retardation

- Tsebelis ('95b, '99, '00, '02): Essential Argument:
  - ↑ # &/or ideological/interest polarization of pol-mkng actors whose approval required to ΔSQ, i.e., *veto actors*, ⇒, loosely, ↓ probability &/or magnitude policy Δ.
  - I.e., strictly, as size W(SQ) ↓, which generally does as # &/or polarization VA ↑, range *possible* policy Δ(SQ) ↓.
  - ⇒ following empirical prediction (Tsebelis 2002, Fig. 1.7):



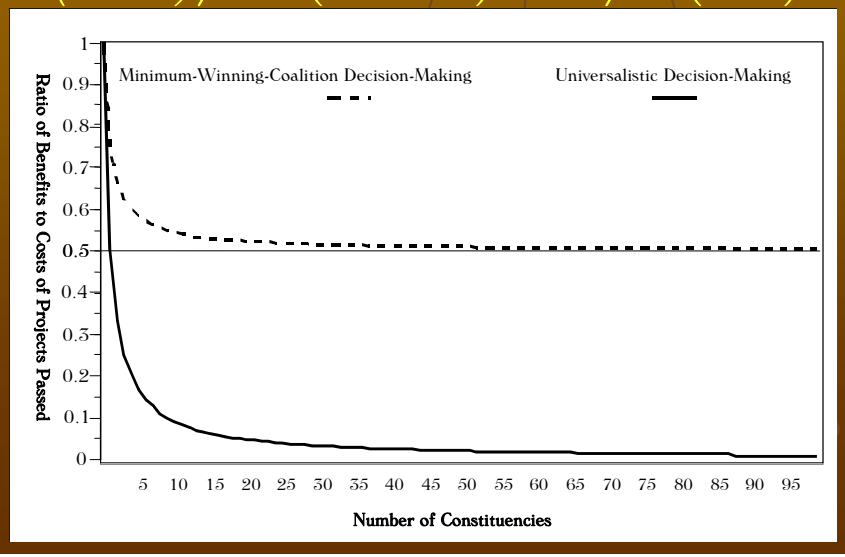
  - Suggests both mean/expected policy-Δ & variance pol & pol-Δ ↑↓ as size of W(SQ) ↑↓ (aside: why only suggests)
  - No prediction of pol-level or of direction pol-Δ, only of E(|Δp|), V(Δp).

Veto-Actor Implications

- ↑ # (=Frag) & Polar of VA Privileges SQ ⇒
  - Retards policy-adjustment rates/delays stabilization,
  - ↓ range of possible policy-Δ, & so, possibly,
  - ↓ magnitude/variance policy- Δ (1st- & 2nd-order E(Δ)).
- Results, e.g. in fiscal policy, deficits & debts; originally mixed, but tighter specify thry into empirical analysis:
  - (F '00, '02) **How model:** policy-adjustment-rate effect = conditional coefficient on LDV in dynamic model, not level.
  - (F '00, '02) **How measure:** frag & polar in VA theory =
    - raw #, not eff. # (size-wtd) VA;
    - max range pref's, not *V*(*pref's*) or *sd*(*pref's*), (size-wtd)
- ⇒ Model: $y_t = \ldots + \theta(\#VA, Range\{pref(VA)\}) \times y_{t-1} \ldots$ &/or $V(y_t) = f(\#, Range)$ ⇒ empirical support.

Common-Pool Theory (1)

- Weingast, Shepsle, Johnsen (1981): districting & distributive/pork-barrel spend (***law of 1/n***)
  - Benefits concentrate district i: $B_i = f(C)$, $f' > 0$ & $f'' < 0$
  - Costs disperse across n districts: $C_i = C/n$
  - ∴ optimal project-size from i's view ↑ in # districts: $f'(C^*) = 1/n$ (…log-linearly?)
- Alternative Decision Rules/Processes […] ⇒
  - […] *Law of 1/n* is general, & stronger as legislative behavior more Universalistic & less Minimal-Winning, which tendency ↑ as rational ignorance, winning-coalition uncertainty, or legislative-rule closure to amend or veto ↑.
  - E.g., PubRev = common pool for n reps, overused to distribute bens; this CA prob worsens "proportionally" by *law 1/n*, i.e. at rate b/w those at which $(n+1)/2n$ (MWC) & $1/n$ (uni) ↓ in n



Manifestations of Common Pools

- Velasco ('98, '99, '00): inter-temporal totality pub rev is C-P to today's policymakers ⇒ deficits & debts also *law of 1/n*
- Peterson & co's, Treisman: federalism ⇒ multiple tax authorities ⇒ several common-pool problems:
  - Inter-jurisdiction competition (w/ high factor mobility) ⇒ C-P of investment resources ⇒ over-fishing: taxes too low.
  - National govt as lender last resort ⇒ subnational jurisdictions see fed guarantee & funds as common pool ⇒ excessive borrowing by subnat'l units. (e.g., EU, EMU & Euro ⇒ common pools…)
- Again, should be quite general:
  - **Anything that gains set of pol-makers credit ⇒ underinvested as ↑n**
  - **Anything that gains set of pol-makers blame ⇒ overexploited as ↑n**
- E.g., (thry 2nd-best), ELECTIONEERING:
  - Magnitude incentive electioneer fades w/ n (see, e.g., Goodhart)
  - Control over electioneering diminishes w/ n.
- Notice: CP not arise in Tsebelis' VA Theory b/c # & pref's of VA's exog & predetermined, whereas in CP theory: prefs=*f(#)*.

Modeling Common-Pool Effects

- CP Effects distinguishable from VA Effects:
  - C-P Effects on *levels*, not (as in VA) in dynamics.
  - Proportional to *1/n* for *equal-sized* actors. Standard Olsonian encompassingness logic ⇒ proper *n* here ***is*** *size-weighted* (effective & s.d./var.)
  - Fractionalization (#) & esp. polarization (het.) relate to VA effects; CP, conversely, relate primarily to #, although het. can exacerbate some CA probs.
- Suggests Proper Model of Policy-Response to some public demand for, $\mathbf{x_1}'\boldsymbol{\beta_1}$, or against, $\mathbf{x_2}'\boldsymbol{\beta_2}$:
  - $\ldots+(\mathbf{x_1}'\boldsymbol{\beta_1})(1-f(ln(Eff\#))+(\mathbf{x_2}'\boldsymbol{\beta_2})(1+f(ln(Eff\#))+\ldots$
  - Same $f(ln(Eff\#)$, b/c overexploit/underinvest same °

Bargaining, Delegation, & Compromise

- **Explicit extensive-form delegation & bargaining games:** huge theoretical & empirical literature
- F ('99,'02,'03): less context-specific empirical strategy…
  - Because broad comparativist seek thry that *travels*, not that requires different model each context.
- Offering is roughly equivalent Nash Bargaining.
  - Most ext forms ⇒ eqbm bounded by actors' ideal pts:
    - Convex set/hull, upper-contour set (=core of coop. game thry),
    - So like Tsebelis, but further, though short of explicit ext-form
  - Policy outside that range possible,
    - e.g., if uncertainty resolved unfavorably,
    - but that ⇒ highly unlikely that E(pol) outside this range
  - Thus, E(pol)=some convex-combo (wtd-avg) pol-mkrs' ideals ⇒ convex-combo emp. models ≈ compromise
    - If Nash Bargain, e.g., (n.b. NB=coop. game-thry but equiv. sev. reasonable ext-form non-coop barg. games: Rubinstein '82), ⇒ (geometric) ***wtd-influence pol-mkng***; i.e., simple wtd-avg.

Empirical Manifestations & Model of Compromise Policymaking

- Re: def's & debt, Cusack ('99, '01; cf., Clark '03)
  - *Arg:* left more Keynes-active counter-cyc; right less, even pro-cyc
  - *Add Nash-Barg Model* $\Rightarrow$ wtd-avg pol-mkr partisanship conditions ^{o} Keynesian cntr-cyc fisc-pol response to macroecon.
- Empirical Implementation:
  - Ideally:
    - Describe barg power party i as $f($charact's i & barg envir, j, $\Rightarrow f(\boldsymbol{v}_{ij})$
    - Desc. pol response to conditions $\mathbf{x}_{\mathrm{k}}$ if i sole pol-mkng control: $q_i(\mathbf{x}_{\mathrm{k}})$
    - Then embed Nash-Barg sol'n, $\Sigma_{\mathrm{i}} f(\boldsymbol{v}_{ij}) q_i(\mathbf{x}_{\mathrm{k}})$, in emp. model to est.
  - Currently:
    - Assume wtd-avg compromise outcome pre-estimation.
    - I.e., simply assume by measure & specification that Policy responds to *WtdPartisanshipCurrGovt* $\times$ *MacroeconomicConditions*.

Empirical Model of the Theoretical Synthesis (1)

- Different aspects of policy-maker fragmentation, polarization, & partisanship:
  - V-A Effects: raw # (frag) and ideological ranges (polar)
  - C-P Effects: eff # (frag) &, maybe, ideol. s.d./var (polar)
  - D-B Effects: power-wtd mean ideologies (partisanship)
- Different ways these distinct effects manifest in pol:
  - V-A (primarily) to slow pol-adjust (delay stabilization);
  - C-P induces over-draw from common resources (incl. from future as in debt); under-invest in common properties (less electioneering), log-proportionately
  - D-B induces convex-combinatorial (compromise) policies, incl. greater left-activist/right-conservative Keynesian-countercyclical/conservative pro-cyclical, in proportion to degree left/right controls policymaking

Empirical Model of the Theoretical Synthesis (2)

- …implies specification where:
  - Abs # VA & ideol range modify pol-adjust rates
  - (log) Eff # pol-mkrs & s.d. ideol (wtd measures) gauge C-P prob in *electioneering* (+debt-lvl effect?)
  - Some barg process among partisan pol-mkrs (e.g., Nash ⇒ wtd-influence) determines combo reflected in net policy responsiveness to macro (° K-activism)
- ⇒

$$
\begin{aligned}
D_{it} = {} & \alpha_i + \left(1 + \rho_n NoP_{it} + \rho_{ar} ARwiG_{it}\right) \times \left(\rho_1 D_{i,t-1} + \rho_2 D_{i,t-2} + \rho_3 D_{i,t-3}\right) \\
& + \left(\beta_{\Delta Y} \Delta Y_{i,t} + \beta_{\Delta U} \Delta U_{i,t} + \beta_{\Delta P} \Delta P_{i,t}\right) \times \left(1 + \beta_{cg} CoG_{it}\right) \\
& + \left(\gamma_{e1} E_{it} + \gamma_{e2} E_{i,t-1}\right) \times \left(1 + \gamma_{en} ENoP_{it} + \gamma_{sd} SDwiG_{it}\right) + \mathbf{x}'_{it}\boldsymbol{\eta} + \mathbf{z}'_{it}\boldsymbol{\omega} + \varepsilon_{it}
\end{aligned}
$$

Empirical Model Specification & Data

$$D_{it} = \alpha_i + \left(1 + \rho_n NoP_{it} + \rho_{ar} ARwiG_{it}\right) \times \left(\rho_1 D_{i,t-1} + \rho_2 D_{i,t-2} + \rho_3 D_{i,t-3}\right) + \mathbf{x}'_{it}\boldsymbol{\eta} + \mathbf{z}'_{it}\boldsymbol{\omega} + \varepsilon_{it}$$
$$+ \left(\beta_{\Delta Y}\Delta Y_{i,t} + \beta_{\Delta U}\Delta U_{i,t} + \beta_{\Delta P}\Delta P_{i,t}\right) \times \left(1 + \beta_{cg} CoG_{it}\right) + \left(\gamma_{e1} E_{it} + \gamma_{e2} E_{i,t-1}\right) \times \left(1 + \gamma_{en} ENoP_{it} + \gamma_{sd} SDwiG_{it}\right)$$

- D_{it} = Debt (%GDP);
- *NoP* & *ARwiG* = raw *N*um *o*f *P*rtys in *G*ovt & *A*bs *R*ange *w/i G*ovt:
  - VA conception, so modify dynamics. Expect ρ_n & ρ_{ar} >0.
  - By thry & for efficiency: modify all lag dynamics same.
- *CoG* (govt center, left to right, 0-10):
  - Modifies response to macroecon (equally, by thry & for eff'cy) : β_{cg}<0.
  - Macroec: ΔY = real GDP growth; ΔU = Δ unemp rate; ΔP = infl rate.
- ***x'η*** = controls: set pol-econ cond's response to which not partisan-differentiated or gov comm-pool: (e.g., E(real-int)-E(real-grow), *ToT*)
- *ENoP* & *SDwiG* = *E*ffective *N*um *o*f *P*rtys in govt & *S*td *D*ev *w/i G*ovt:
  - Frag & Polar by *wtd-influence* concept. CP lvl-effects modify (at same rate) electioneering, E_t, pre-elect-year, & E_{t-1}, post-elect-yr.: γ_{en} & γ_{sd}<0.
- ***z'ω*** = set of constituent terms in the interactions:
  - *ENoP*, *SDwiG* <u>*may*</u> have positive coeff's by CP effect lvl debt, but issue is *temporal fract* more than curr. govt *fract*. Thry o/w says omit.

		Coeff.	Std. Err.	*t*-Stat.	Pr(*T*>|*t*|)
Lagged Dependent Variables	D_{t-1}	1.212	0.060	20.112	0.000
	D_{t-2}	-0.153	0.085	-1.792	0.074
	D_{t-3}	-0.121	0.045	-2.677	0.008
$\boldsymbol{\rho_n}$ (*veto-actor effect: fractionalization*)		**0.007**	**0.006**	**1.089**	**0.277**
$\boldsymbol{\rho_{ar}}$ (*veto-actor effect: polarization*)		**-0.000**	**0.006**	**-0.013**	**0.990**
Macroeconomic Conditions	ΔY	-0.336	0.111	-3.033	0.003
	ΔU	0.992	0.308	3.219	0.001
	ΔP	-0.188	0.063	-2.965	0.003
$\boldsymbol{\beta_{cg}}$ (*partisan-compromise bargaining*)		**-0.037**	**0.037**	**-0.988**	**0.323**
Controls	x_1 (*open*)	15.891	5.279	3.010	0.003
	x_2 (*ToT*)	0.388	1.744	0.222	0.824
	x_3 (*open·ToT*)	-10.681	5.156	-2.072	0.039
	x_4 (*dxrig*)	-0.036	0.066	-0.544	0.587
	x_5 (*oy*)	2.064	1.094	1.886	0.060
Pre- and Post-Electoral Indicators	E_t	0.687	0.568	1.210	0.227
	E_{t-1}	1.490	0.645	2.310	0.021
$\boldsymbol{\gamma_{en}}$ (*common-pool effect: fractionalization*)		**-0.547**	**0.182**	**-3.001**	**0.003**
$\boldsymbol{\gamma_{sd}}$ (*common-pool effect: polarization*)		**0.573**	**0.486**	**1.179**	**0.239**
Constituent Terms from the Interactions	z_1 (*CoG*)	0.051	0.131	0.390	0.697
	z_2 (*ENoP*)	0.281	0.446	0.629	0.530
	z_3 (*SDwiG*)	0.542	0.437	1.242	0.215
	z_4 (*NoP*)	0.181	0.277	0.654	0.514
	z_5 (*ARwiG*)	-0.312	0.259	-1.205	0.228
Summary Statistics					
N (Deg. Free)		735 (691)		s_e^2	2.525
R^2 ($\overline{R}^2$)		0.991 (0.990)		**DW-Stat.**	2.101

- Pace Brambor et al. ('06), but joint-significance of multiple-policymaker conditioning effects (γ_{en}, γ_{sd}, ρ_n, ρ_{ar}, β_{cg}) overwhelmingly rejects excluding (p≈.001), whereas joint-sig coeff's on constit. terms, **z**, clearly fails reject (p≈.602) exclusion. (Almost) All theory says should be zero, so…

		Coeff.	**Std. Err.**	***t*-Stat.**	**Pr(*T*>|*t*|)**
Lagged Dependent Variables	D_{t-1}	1.207	0.060	20.290	0.000
	D_{t-2}	-0.158	0.085	-1.851	0.065
	D_{t-3}	-0.117	0.045	-2.577	0.010
$\boldsymbol{\rho_n}$ (*veto-actor effect: fractionalization*)		**0.011**	**0.005**	**2.369**	**0.018**
$\boldsymbol{\rho_{ar}}$ (*veto-actor effect: polarization*)		**-0.002**	**0.004**	**-0.437**	**0.662**
Macroeconomic Conditions	ΔY	-0.375	0.087	-4.332	0.000
	ΔU	1.095	0.286	3.829	0.000
	ΔP	-0.207	0.053	-3.889	0.000
$\boldsymbol{\beta_{cg}}$ (*partisan-compromise bargaining*)		**-0.051**	**0.020**	**-2.484**	**0.013**
Controls	x_1 (*open*)	16.128	5.314	3.035	0.002
	x_2 (*ToT*)	0.414	1.728	0.239	0.811
	x_3 (*open·ToT*)	-10.780	5.194	-2.076	0.038
	x_4 (*dxrig*)	-0.038	0.066	-0.578	0.563
	x_5 (*oy*)	1.898	1.100	1.724	0.085
Pre- and Post-Electoral Indicators	E_t	0.475	0.420	1.133	0.258
	E_{t-1}	1.146	0.562	2.040	0.042
$\boldsymbol{\gamma_{en}}$ (*common-pool effect: fractionalization*)		**-0.570**	**0.209**	**-2.727**	**0.007**
$\boldsymbol{\gamma_{sd}}$ (*common-pool effect: polarization*)		**0.881**	**0.586**	**1.503**	**0.133**
Summary Statistics					
N (Deg. Free)		735 (696)		s_e^2	2.522
R^2 ($\bar{R}^2$)		0.991 (0.990)		**DW-Stat.**	2.099

Veto-Actor Effects: Estimates of Policy-Adjustment Rate						
Adjustment Rates	***NoP=1***	***NoP=2***	***NoP=3***	***NoP=4***	***NoP=5***	***NoP=6***
Lag Coefficient[a]	0.943	0.952	0.960	0.969	0.978	0.986
Policy-Adjust/Yr[b]	0.057	0.048	0.040	0.031	0.022	0.014
Long-Run Mult.[c]	17.498	20.639	25.154	32.200	44.727	73.208
½-Life[d]	11.778	13.956	17.087	21.971	30.654	50.397
90%-Life[e]	39.127	46.362	56.761	72.985	101.832	167.415

Bargaining Effects: Estimates of Keynesian Fiscal Responsiveness						
	Mean Econ. Performance -2 std. dev.	***Mean Econ. Performance -1 std. dev.***	***Mean Economic Performance***	***Mean Econ. Performance +1 std. dev.***	***Mean Econ. Performance +2 std. dev.***	
Growth	-2.354	0.454	3.261	6.069	8.877	
d(UE)	1.915	1.034	0.153	-0.728	-1.608	
Infl	-3.593	1.230	6.054	10.877	15.701	
CoG	***E(D|Econ)***[f]	***E(D|Econ)***	***E(D|Econ)***	***E(D|Econ)***	***E(D|Econ)***	***Fiscal-Cycle Magnitude***[g]
3.0	3.157	0.599	-1.959	-4.516	-7.074	10.231
4.2	2.930	0.556	-1.818	-4.192	-6.566	9.496
5.4	2.703	0.513	-1.677	-3.867	-6.058	8.761
6.6	2.476	0.470	-1.536	-3.543	-5.549	8.026
7.8	2.250	0.427	-1.396	-3.218	-5.041	7.291
9.0	2.023	0.384	-1.255	-2.894	-4.533	6.555

Collective-Action/Common-Pool Effects: Estimates of Electoral Debt-Cycle Magnitude					
	ENoP=1	***ENoP=2***	***ENoP=3***	***ENoP=4***	***ENoP=5***
Electoral-Cycle Magnitude[h]	1.07410	0.86454	0.65497	0.44541	0.23585

Extension & Refinement

$$E(y_t) = \delta^0 + \mathbf{x}_t^0\mathbf{b}^0 + \left(\rho_0 + \rho_1 \ln(NoP_t) + \rho_2 \ln(1 + ARwiG_t)\right) y_{t-1}$$
$$+ \left\{ \begin{array}{l} \left[\mathbf{x}_t^1\mathbf{b}^1 + \sum_{i=1}^{I} p(\mathbf{c}_{it}) \times q_i(\mathbf{x}_t^2) \right] \\ \times \left[1 + \gamma_1 \ln(ENoP_t) + \gamma_2 \ln(1 + SDwiG_t)\right] \end{array} \right\} \times \left[1 + \alpha_1 \ln(NoP_t) + \alpha_2 \ln(1 + ARwiG_t)\right]$$

- $\mathbf{x}^0$ = factors that affect policy-outcomes *unless* pol-mkrs act to change *status quo*, i.e., that have effect on pol-out directly.
- $\mathbf{x}^1$ = factors affecting policy-outcomes *if* policymakers act to change status quo, *without* partisan-differentiated response
- $\mathbf{x}^2$ = factors affecting policy-outcomes *if* policymakers act to change status quo, *with* partisan-differentiated response
- $\{NoP, ARwiG\}$ = sources of **veto-actor effects**; as before
- $\{ENoP, SDwiG\}$ = sources of **common-pool effects**; as before
- $\{p(\mathbf{c}_{it}), q_j(\mathbf{x}_t)\}$ = sources of **bargaining & delegation effects**:
  - $p(\mathbf{c}_{it})$: Effective policy-influence of party i in context t. (E.g., as now: cabinet seat-shares, but could become richer model.)
  - $q_j(\mathbf{x}_t)$: Model of response of party i to pol-econ conditions $\mathbf{x}_t$. (E.g., as now: $CoG_i \times Macroecon_t$, but could become richer model.)

Results of Fuller Model

		Coeff.	Std. Err.	*t*-Stat.	Pr(*T*>|*t*|)
Temporal Dynamics	D(t-1)	1.197	0.059	20.144	0.000
	D(t-2)	-0.139	0.085	-1.629	0.104
	D(t-3)	-0.121	0.045	-2.698	0.007
Veto-Actor Effect On Outcome-Adjustment Rate	**NoP**	**0.008**	**0.004**	**1.883**	**0.060**
$\mathbf{x}_0$: Variables with "Direct" Effect on Outcome	Open	16.624	3.758	4.423	0.000
	Open*ToT	-11.190	3.135	-3.569	0.000
$\mathbf{x}_1$: Variables with Effects via Non-Partisan-Differentiated Policy Response	Ele(t)	0.315	0.363	0.867	0.386
	Ele(t-1)	0.873	0.399	2.186	0.029
	OY	2.833	1.295	2.187	0.029
	DXRIG3	-0.073	0.072	-1.009	0.314
Common-Pool Effect on Policy Response	**ln(ENoP)**	**-0.277**	**0.071**	**-3.903**	**0.000**
$\mathbf{x}_2$: Variables with Effects via Partisan-Differentiated Policy Response	Growth	-0.238	0.084	-2.815	0.005
	d(UE)	0.749	0.228	3.289	0.001
	Inflation	-0.137	0.047	-2.947	0.003
Bargaining-Compromise Effects on Partisan Policy-Responses	**CoG**	**-0.049**	**0.026**	**-1.893**	**0.059**
Veto-Actor Effect On Policy-Adjustment Rate	**NoP**	**0.215**	**0.121**	**1.773**	**0.077**
Common-Pool Effect on Debt Level	*ln(ENoP)*	*1.128*	*0.486*	*2.320*	*0.021*
Summary Statistics					
N (Deg. Free)		735 (697)		s_e^2	2.510
R^2 ($\overline{R}^2$)		0.991 (0.990)		**DW-Stat.**	2.090