PROBLEM 8.1 P&T

- Three types of voters, $J \in \{1,2,3\}$, each of size 1
- VOTER PREFS:

$$W^J = \underset{\text{private cons.}}{u(c^J)} + \underset{\text{pub. good}}{H(g)} = u(\underset{\text{income: exog. \& equal}}{1} - \underset{\text{lump-sum tax} \Rightarrow \text{no distortion}}{\tau} + \underset{\text{Group-specific Transfer}}{f^J}) + H(g)$$

$u(\cdot)$ & $H(\cdot)$
both increasing & concave
$\Rightarrow u', H' > 0 \; ; \; u'', H'' < 0$

- POLICY: $q = [\underset{\text{tax}}{\tau}, \underset{\text{pub-good}}{g}, \underset{\text{rents}}{r}, \underset{\text{transfers}}{\{f^J\}}] \geq 0$ all weakly positive

$\Rightarrow$ BUDGET CONSTRAINT: $\underset{\text{revenue}}{3\tau} = \sum_J f^J + g + r$

- OFFICE-SEEKING, RENT-SEEKING PARTIES:

$$E(v^P) = \underset{\text{prob. p wins}}{p_P} \cdot (\underset{\text{exog. "ego" rents}}{R} + \gamma \underset{\text{endog. "corruption" rents, inefficiently gleaned from budget } (\gamma)}{r})$$

- CREDIBLE COMMITMENT TO PLATFORMS IN "PR" = Single District

$$\Rightarrow p_P = Prob\left[\frac{1}{3} \sum_J \pi_{PJ} \geq \frac{1}{2}\right]$$

vote-share of party P in group J

- P&T-Style PROBABILISTIC VOTING

$\Rightarrow i \in J$ votes P iff $W^J(q_P) > W^J(q_{P'}) + (\delta + \sigma^{iJ})$

- where δ common "ideology" shock, $\delta \sim U[-\frac{1}{2\psi}, \frac{1}{2\psi}]$
- σ^{iJ} = individual, idiosyncratic shock,

$$\sigma^{iJ} \sim U\left[-\frac{1}{2\phi^J} + \bar{\sigma}^J, \frac{1}{2\phi^J} + \bar{\sigma}^J\right]$$

Group J's Bias

(J's density)

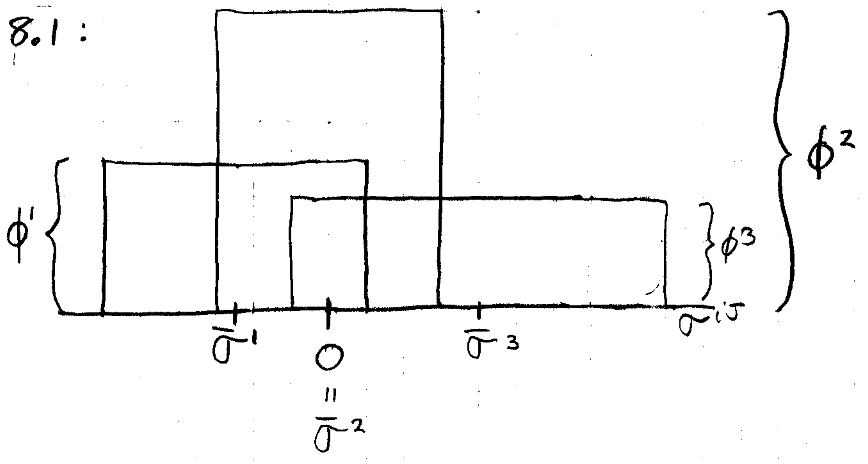
zero-mean

pro-P' shock

Global Uncertainty

- ASSUMPTIONS REGARDING GROUP CHARACTERISTICS
  - $\bar{\sigma}^1 < \bar{\sigma}^2 = 0 < \bar{\sigma}^3 \Rightarrow$ (J=2) on average neutral; (J=1) pro-P ; (J=3) pro-P'
  - $\phi^2 > \phi^1, \phi^3 \Rightarrow$ Middle grp higher density ($\approx$ "clout" in DL)
  - $\bar{\sigma}^1\phi^1 + \bar{\sigma}^3\phi^3 = 0 \Rightarrow \sum_J \bar{\sigma}^J\phi^J = 0$ } An sneakily useful assumption whose meaning is that individualistic shocks cancel out!

$\Rightarrow$ FIG 8.1:



- consider a deviation by A from $q_A = q_B$
  - $\downarrow\tau$ or $\uparrow g$ each would $\Rightarrow$ rightward shift of each group's cutpoint by same amount, say σ', $\Rightarrow$ gain $\phi^J\sigma'$ voters in each
  - $\uparrow r \Rightarrow$ lose votes in all groups by opposite of above.
  - $\uparrow f^J \downarrow f^{J'} \Rightarrow$ shift J cutpoint right, gaining $\phi^J\sigma''$, and J' left, losing $\phi^{J'}\sigma''$

$\Rightarrow$ VOTE-SHARE PARTY A IN GROUP J : $\Pi_{A,J}$

$$\Pi_{A,J} = \phi^J\left[W^J(q_A) - W^J(q_B) - \delta - \bar{\sigma}^J\right] + \frac{1}{2}$$

- Density $\cdot$ [Net Advantage] + starting vote

- Net Advantage: ① Policy Advantage: $\{W^J(q_A) - W^J(q_B)\}$
  ② (Common) Ideology Shock Advantage
  ③ Mean Group-Advantage

uncertainty part

• What's P_A?

$$\pi_{A,J} = \phi^J\left[W_A^J - W_B^J - \delta - \bar{\sigma}^J\right] + \tfrac{1}{2}$$

$$P_A = Prob\left[\tfrac{1}{3}\sum_J \pi_{A,J} \geq \tfrac{1}{2}\right]$$

$$\Rightarrow Prob\left[\tfrac{1}{3}\sum_J \phi^J\left[W_A^J - W_B^J - \delta - \bar{\sigma}^J\right] > 0\right]$$ the ½'s cancel

$$\Rightarrow Prob\left[\tfrac{1}{3}\sum_J \phi^J\left[W_A^J - W_B^J - \bar{\sigma}^J\right]\right] > \tfrac{1}{3}\sum_J \phi^J \delta$$ bringing δ out

$$\Rightarrow Prob\left[\frac{\sum_J \phi^J\left[W_A^J - W_B^J - \bar{\sigma}^J\right]}{\sum_J \phi^J}\right] > \delta$$

$\Rightarrow$ (sneaky Assumption):

$$P_A = Prob\left[\frac{1}{3\phi}\sum_J \phi^J\{W_A^J - W_B^J\}\right] > \delta$$

• For any $X \sim U[a,b]$, Prob $X < C = \frac{C-a}{b-a}$

$$\Rightarrow Prob(\delta < C) = \frac{C - (-\frac{1}{2\psi})}{\frac{1}{2\psi} - (-\frac{1}{2\psi})} = \frac{C + \frac{1}{2\psi}}{\frac{1}{2\psi} + \frac{1}{2\psi}}$$

$$= \frac{C + \frac{1}{2\psi}}{\frac{1}{\psi}} = \psi C + \tfrac{1}{2}$$

$$\Rightarrow P_A = \frac{\psi}{3\phi}\sum_J \phi^J\{W_A^J - W_B^J\} + \tfrac{1}{2}$$

So, question ⓐ: $\underset{q = \{\tau, \{f^J\}, g, r\}}{Max} \sum_J W^J(q)$ s.t. $\underbrace{q \geq 0 \;\&\; 3\tau = \sum_J f^J + g + r}$

$\Downarrow$

• n.b., income equal $\Rightarrow$ no redist.

• n.b., r = pure loss $\Rightarrow$ no rents

$\Rightarrow$ find optimal $\{\tau, g\}$

$$\Rightarrow \underset{\{\tau, g\}}{Max} \sum W^J(q) \text{ s.t. } 3\tau = g \text{ or } \tau = \tfrac{1}{3}g$$

$$\Rightarrow \underset{\{\tau, g\}}{Max} \sum\left[U(c^J) + H(g)\right] \text{ s.t. } \tau = \tfrac{1}{3}g$$

$$\Rightarrow \underset{g}{Max} \sum_J U(1 - \tfrac{1}{3}g) + H(g)\}$$

$$\Rightarrow \tfrac{1}{3}U'(1 - \tfrac{1}{3}g) = H_g(g)$$

$\Rightarrow$ not very interesting; size of gov't $\Rightarrow f$(rel. marg. util. priv/pub.)

(b) Now Parties Maximizing $E(V^P) = p_P \cdot (R + \gamma r)$

$$\Rightarrow \underset{q_A = \{\tau_A, \{f_A^J\}, g_A, r_A\}}{Max} \; E(V^P) \text{ s.t. } 3\tau_A = \sum_J f_A^J + g_A + r_A$$

$$= \underset{q_A}{Max} \; p_A \cdot (R+\gamma r) = \left[\tfrac{1}{2} + \tfrac{\psi}{3\phi} \sum_J \phi^J \{W_A^J - W_B^J\}\right](R+\gamma r) \text{ s.t. } \ldots$$

$$= \underset{q_A}{Max} \left[\tfrac{1}{2} + \tfrac{\psi}{3\phi} \sum_J \phi^J \{U(1-\tau_A + f_A^J) + H(g_A) - W_B^J\}\right](R+\gamma r)$$
$$\text{s.t. } 3\tau_A = \sum_J f_A^J + g_A + r_A$$

So, $\mathcal{L}$

$$\underset{q}{Max} \left[\tfrac{1}{2} + \tfrac{\psi}{3\phi} \sum_J \phi^J \{U(1-\tau_A + f_A^J) + H(g_A) - W_B^J\}\right](R+\gamma r) - \lambda\left(\sum_J f_A^J + g_A + r - 3\tau_A\right)$$

$$\frac{\partial \mathcal{L}}{\partial f^J} = (R+\gamma r) \cdot \frac{\psi}{3\phi} \cdot \phi^J \cdot U'(C^J) = \lambda \quad \forall J$$
$$\Rightarrow \phi^J U'(C^J) = \lambda \cdot \frac{3\phi}{\psi} \cdot (R+\gamma r)^{-1} \quad \forall J$$

① $\phi^1 U'(C^1) = \phi^2 U'(C^2) = \phi^3 U'(C^3) \Rightarrow f_A^J$ increasing in DL cbut

② All $f_A^J \uparrow$ proportionately in $f(\psi, R, \gamma, r^*)$ but $r^* \ldots$

$$\frac{\partial \mathcal{L}}{\partial r} = \gamma p_A - \lambda = 0 \Rightarrow \gamma p_A = \phi^J U'(C^J) \frac{\psi}{3\phi} \cdot (R+\gamma r)$$
$$\Rightarrow R + \gamma r = 3\phi \gamma p_A / \phi^J U'(C^J)\psi$$
$$\Rightarrow r^* = \frac{3\phi \, p_A}{\phi^J U'(C^J)\psi} - R/\gamma$$

(n.b., in eqbm $p_A = \frac{1}{2}$)

① $r^* \downarrow$ in ψ, R, γ

$$\frac{\partial \mathcal{L}}{\partial g} = \cancel{(R+\gamma r)} \cdot \cancel{\frac{\psi}{3\phi}} \cdot \sum_J \phi^J \cdot H_g(g) = \lambda = \cancel{(R+\gamma r)} \cdot \cancel{\frac{\psi}{3\phi}} \cdot \phi^J \cdot U'(C^J)$$

① $H_g(g) = \dfrac{\phi^J \cdot U'(C^J)}{3\phi}$

② $\sum_J \phi^J H_g(g) = \lambda \cdot 3\phi \cdot \psi^{-1} \cdot (R+\gamma r)^{-1}$

$$\Rightarrow H_g(g) = \lambda \cdot \psi^{-1} \cdot (R+\gamma r)^{-1}$$
$$\Rightarrow g \uparrow \text{ proportionately in } f(\psi, R, \gamma, r^*)$$

but r^*

F.O.C.'s

$$0 = \frac{\partial \mathcal{L}}{\partial f^J} \Rightarrow (R + \gamma r_A)\frac{\psi}{3\phi}\,\theta^J u'(C_A^J) = \lambda$$

$$0 = \frac{\partial \mathcal{L}}{\partial r} \Rightarrow \gamma p_A = \lambda$$

$$0 = \frac{\partial \mathcal{L}}{\partial g} \Rightarrow (R + \gamma r_A)\,\psi\, H_g(g_A) = \lambda$$

$$0 = \frac{\partial \mathcal{L}}{\partial J} \Rightarrow \frac{1}{3}(R + \gamma r_A)\cdot\frac{\psi}{3\phi}\cdot\sum_J \phi^J u'(C_A^J) = \lambda$$

$$0 = \frac{\partial \mathcal{L}}{\partial J} \Rightarrow 3\tau_A = \sum_J f_A^J + g_A + r_A$$

$\Rightarrow$ Need More Structure on $U(\cdot)$ & $H(\cdot)$ to get interesting...

NLS: $$y_i = h(X_i, \beta) + \varepsilon_i$$

$$\Rightarrow \frac{1}{2}\sum \varepsilon_i^2 = \frac{1}{2}\sum [y_i - h(X_i, \beta)]^2 \equiv SSE(\beta)$$

$\underset{\beta}{\text{Min}}\, SSE \Rightarrow$ $$\frac{\partial SSE(\beta)}{\partial \beta} = \sum_{i=1}^{n} [y_i - h(X_i, \beta)]\,\frac{\partial h(X_i, \beta)}{\partial \beta} = 0$$

• The Normal Equation from LS; e.g., if $h(\cdot)$ linear

$$\Rightarrow \sum_{i=1}^{n} (y_i - X_i\beta)\cdot X_i = \underset{\sim}{0} \quad \text{i.e., } \sum_{i=1}^{n} e_i X_i = \underset{\sim}{0}$$

NLS: Use Taylor Series Approx. to $h(X, \beta)$

$$\Rightarrow h(X, \beta) \approx h(X, \beta^0) + \sum_{k=1}^{K} \frac{\partial h(X, \beta)}{\partial \beta_k^0}\cdot(\beta_k - \beta_k^0)$$

$$\Rightarrow h(X, \beta) \approx \left[h(X, \beta^0) - \sum_{k=1}^{K} \frac{\partial h(X, \beta^0)}{\partial \beta_k^0}\cdot \beta_k^0\right] + \sum_{k=1}^{K} \beta_k \cdot \frac{\partial h(X, \beta^0)}{\partial \beta^0}$$

letting $X_k^0 = \dfrac{\partial h(X, \beta^0)}{\partial \beta_k^0}$ $\Rightarrow h(X, \beta) \approx [h^0 - \sum X_k^0 \beta_k^0] + \sum X_k^0 \beta_k$

$$\underset{\sim}{y} = h(\underset{\sim}{X}, \underset{\sim}{\beta}) + \underset{\sim}{\varepsilon} \approx h^0 - \underset{\sim}{X}^{0\prime}\underset{\sim}{\beta}^0 + \underset{\sim}{X}^{0\prime}\underset{\sim}{\beta} + \varepsilon$$

$\Rightarrow$ Replace usual regressors, X, w/ pseudoregressors X^0 & β^0 & iterate until β^0 not updated noticeably

$\Rightarrow$ If pseudoregressors well-behaved, $\hat{\beta}$ all usual properties.