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Complete Replication of Persson and Tabellini Capital-Tax Competition Model (Ch. 12)

1 The Model

1.1 Variables and Parameters

We have the following paramaters:

- $\tau_k \equiv$ domestic capital tax rate
- $\tau_k^* \equiv$ foreign capital tax rate
- $\tau_L \equiv$ domestic tax rate on labor
- $\tau_L^* \equiv$ foreign tax rate on labor
- $e^i \equiv$ labor-to-capital endowment for individual i
- $l^i \equiv$ labor of individual i
- $x^i \equiv$ leisure of individual i
- $s^i \equiv$ savings of individual i
- $k^i + f^i \equiv$ domestic and foreign investment of individual i
- $c_1 \equiv$ consumption in period 1
- $c_2 \equiv$ consumption in period 2

1.2 Constraints

We initially have the following constraints:

- The general "unitary" time constraint is $1 = l^i + x^i$, indicating that individuals can spend their time working or in leisure activities.

- The activities an individual can carry out (consumption and investment) are normalized so that $c_1^i + k^i + f^i = 1$.
- $c_2^i = (1-\tau_k)k^i + (1-\tau_k^*)f^i - M(f^i)_(1-\tau_k)l^i$ is the period 2 budget constraint (where the function M captures the cost of moving capital between countries). Since $s^i = f^i + k^i$, then we can rewrite c_2^i as

$$
\begin{aligned}
c_2^i &= (1-\tau_k) + k^i(1-\tau_k^*)f^i - M(f^i) + (1-\tau_L)l^i \\
&= (1-\tau_k)(s^i - f^i) + (1-\tau_k^*)f^i - M(f^i) + (1-\tau_L)l^i \\
&= (1-\tau_k)s^i - (1-\tau_k)f^i + (1-\tau_k^*)f^i + M(f^i)_(1-\tau_L)l^i \\
&= (1-\tau_k)s^i - f^i + \tau_k f^i + f^i - \tau_k^* f^i + M(f^i)_(1-\tau_L)l^i \\
&= (1-\tau_k)s^i + \tau_k f^i - \tau_k^* f^i - M(f^i) + (1-\tau_L)l^i \\
&= (1-\tau_k)s^i + (\tau_k - \tau_k^*)f^i - M(f^i) + (1-\tau_L)l^i
\end{aligned}
$$

where the final line is equation (12.17)

1.3 Objective Function

The basic objective function for individual i is the following quasi-linear utility function

$$\omega = U(c_1^i) + c_2^i + V(x^i)$$

Substituting our values for c_1^i and c_2^i gives us

$$\omega = U(1-s^i) + (1-\tau_k)s^i + (\tau_k - \tau_k^*)f^i - M(f^i) + (1-\tau_L)l^i + V(x^i)$$

Substituting in our value for x_i gives

$$\omega = U(1-s^i) + (1-\tau_k)s^i + (\tau_k - \tau_k^*)f^i - M(f^i) + (1-\tau_L)l^i + V(1-l^i) \tag{1}$$

2 Best Response Functions

The best response functions are:

$$
\begin{aligned}
\frac{\partial\omega}{\partial s^i} &= 0 \\
U_c(1-s^i)(-1) + (1-\tau_k) &= 0 \\
U_c(1-s^i) &= (1-\tau_k) \\
1-s^i &= U_c^{-1}(1-\tau_k) \\
1-s^i &= U_c^{-1}(1-\tau_k) \\
s^{i*} &= 1 - U_c^{-1}(1-\tau_k) \\
s^{i*} &= S(\tau_k)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \omega}{\partial f^i} &= 0 \\
(\tau_k - \tau_k^*) - M_f(f^i) &= 0 \\
(\tau_k - \tau_k^*) &= M_f(f^i) \\
f^{i*} &= M_f^{-1}(\tau_k - \tau_k^*) \\
f^{i*} &= F(\tau_k,\ \tau_k^*)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \omega}{\partial l^i} &= 0 \\
(1-\tau_k) + V_x(1-l^i)(-1) &= 0 \\
(1-\tau_k) &= V_x(1-l^i) \\
V_x^{-1}(1-\tau_k) &= 1-l^i \\
l^i &= 1 - V_x^{-1}(1-\tau_k) \\
l^i &= L(\tau_l)
\end{aligned}
$$

And since $s = k + f \Rightarrow\ k = s - f$, then

$$k^i \equiv K(\tau_k,\ tau_k^*) = S(\tau_k) - F(\tau_k,\ \tau_k^*)$$

Together, these are our supply functions.

3 Indirect Utility Function

We can now plug these best response functions into the objective function to create the indirect utility function (where the superscript "i" is dropped for ease of notation).[1]

$$W^i(\tau_k, \tau_k^*) \equiv U(1-s^*) + (1-\tau_k)s^* + (\tau_k - \tau_k^*)f^* - M(f^*) + (1-\tau_l)l^* + V(1-l^*) \qquad (2)$$

4 Allowing for Heterogenity Between Individuals

4.1 Representing Heterogenity

Next, if we allow for heterogenity between individuals. Specifically, this means we modify the constraints to allow for heterogenity of individual endowments. Specifically:

- $1 + e^i = l^i + x^i$
- $(f^i + k^i) + c_1^i = 1 - e^i\ \Rightarrow\ s^i + c_1^i = 1 - e^i$ is the period 1 budget constraint

[1] It is "indirect" because it will show the utility from taxation, which impacts consumption and leisure. Since it is consumption and leisure that gives individuals utility, then taxation only "indirectly" impacts utility.

Let's return for a moment to the original objective function, equation (1). Substituting our values for c_1^i and c_2^i gives us

$$\omega = U(1 - e^i - s^i) + (1 - \tau_k)s^i + (\tau_k - \tau_k^*)f^i - M(f^i)_(1 - \tau_L)l^i + V(x^i)$$

Substituting in our value for x_i gives

$$\omega = U(1 - e^i - s^i) + (1 - \tau_k)s^i + (\tau_k - \tau_k^*)f^i - M(f^i)_(1 - \tau_L)l^i + V(1 + e^i - l^i) \tag{3}$$

The best response functions are:

$$\begin{aligned}
\frac{\partial\omega}{\partial s^i} &= 0 \\
U_c(1 - e^i - s^i)(-1) + (1 - \tau_k) &= 0 \\
U_c(1 - e^i - s^i) &= (1 - \tau_k) \\
1 - e^i - s^i &= U_c^{-1}(1 - \tau_k) \\
1 - e^i - s^i &= U_c^{-1}(1 - \tau_k) \\
s^{i*} &= 1 - e^i - U_c^{-1}(1 - \tau_k) \\
s^{i*} &= S(\tau_k) - e^i
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\omega}{\partial f^i} &= 0 \\
(\tau_k - \tau_k^*) - M_f(f^i) &= 0 \\
(\tau_k - \tau_k^*) &= M_f(f^i) \\
f^{i*} &= M_f^{-1}(\tau_k - \tau_k^*) \\
f^{i*} &= F(\tau_k,\ \tau_k^*)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\omega}{\partial l^i} &= 0 \\
(1 - \tau_k) + V_x(1 + e^i - l^i)(-1) &= 0 \\
(1 - \tau_k) &= V_x(1 + e^i - l^i) \\
V_x^{-1}(1 - \tau_k) &= 1 + e^i - l^i \\
l^i &= 1 + e^i - V_x^{-1}(1 - \tau_k) \\
l^i &= L(\tau_l) + e^i
\end{aligned}$$

And since $s = k + f \Rightarrow\ k = s - f$, then

$$k^i \equiv= S(\tau_k) - e^i - F(\tau_k,\ \tau_k^*) = S(\tau_k) - F(\tau_k,\ \tau_k^*) - e^i \equiv K(\tau_k,\ tau_k^*) - e^i$$

Together, these are our supply functions.

4.2 Including these in the Utility Function

These are incorporated into the utlity function as follows

$$W^i(\tau_k, \tau_k^*) \equiv U(1 - s - e) + (1 - \tau_k)(s - e) + (\tau_k - \tau_k^*)f - M(f) + (1 - \tau_l)(l + e) + V(1 - l - e)$$

which simplifies to

$$\begin{aligned} W^i(\tau_k, \tau_k) &\equiv U(1-s-e) + (1-\tau_k)(s-e) + (\tau_k - \tau_k^*)f - M(f) + (1-\tau_l)(l+e) + V(1-l-e) \\ &\equiv U(1-s-e) + (1-\tau_k)s - e + \tau_k e + (\tau_k - \tau_k^*)f - M(f) + (1-\tau_L)l + e - \tau_L e + V(1-l-e) \\ &\equiv U(1-s-e) + (1-\tau_k)s + \tau_k e + (\tau_k - \tau_k^*)f - M(f) + (1-\tau_L)l - \tau_l e + V(1-l-e) \\ &\equiv U(1-s-e) + (1-\tau_k)s + (\tau_k - \tau_k^*)f - M(f) + (1-\tau_L)l + V(1-l-e) + \tau_k e - \tau_L e \end{aligned}$$

$$\equiv U(1-s-e) + (1-\tau_k)s + (\tau_k - \tau_k^*)f - M(f) + (1-\tau_L)l + V(1-l-e) + (\tau_k - \tau_L)e \quad (4)$$

which is given on page 319.[2]

4.3 Accounting for Voters and Elections

Persson and Tabellini use a Besley and Coate's (1997) model of representative democracy wherein running for office is costly and citizens choose whether to enter the race by an expected-utility calculation. Because W^i is liner in the parameter e^i, then there exists a well-defined Condorcet winner[3] Specifically, this winner will implement the tax policy of the median voter who has endowment, e^m.

5 Government Budget Constraint

We now introduce the government. The government only taxes and consumes in the second period of the model. Government second period consumption, G, is funded through labor and capital tax revenue (note that capital from domestic and foreign sources are taxed). Specifically,

$$G = \tau_L l^* + \tau_k(k^* + f^{F*})$$

which can be rewritten as

$$G = \tau_L l^* + \tau_k(s^* - f^* + f^{F*}) \quad (5)$$

It is important to realize that government consumption is entirely wasted in this model. In other words, it enters into no utility functions.

[2] It is worth noting that since the functions U and V are not impacted by tax rates, then we can ignore the e inside both when we optimize this function.

[3] i.e. a single winner of an election that, though not necessarily the first preference of any single voter, is the least disagreeable choice of all voters.

6 Conditions for Optimal Tax Policy

There are two countries in this model, each with an indirect utility function. The optimal tax policy in this model will maximize

$$W^H + W^F \tag{6}$$

subject to the government budget constraints of both the home, H, and foreign, F, countries.

Combining (4), (5), and (6) gives us the following Lagrangian

$$\begin{array}{l} U(1-s^H) + (1-\tau_k)s^H + (\tau_k - \tau_k^*)f^H - M(f^H) + (1-\tau_L)l^H + V(1-l^H) + (\tau_k - \tau_L)e^H \\ U(1-s^F) + (1-\tau_k)s^F + (\tau_k - \tau_k^*)f^F - M(f^F) + (1-\tau_L^*)l^F + V(1-l^F) + (\tau_k - \tau_L)e^F \\ \lambda[\tau_L l^H + \tau_k(s^H - f^H + f^F) + \tau_L^* l^F + \tau_k^*(s^F - f^F + f^H)] \end{array} \tag{7}$$

where each variable in the government objective function is treated as a function of τ_k. Why? Because in the government constraint (i.e., from the perspective of the government), savings and investment are exogenous actions. To make the differentation easy, let's rewrite (6) as

$$\begin{array}{l} U(1-s^H) + (1-\tau_k)s^H + (\tau_k - \tau_k^*)f^H - M(f^H) + (1-\tau_L)l^H + V(1-l^H) + (\tau_k - \tau_L)e^H \\ U(1-s^F) + (1-\tau_k^*)s^F + (\tau_k^* - \tau_k)f^F - M(f^F) + (1-\tau_L^*)l^F + V(1-l^F) + (\tau_k - \tau_L)e^F \\ \lambda[\tau_L l^H + \tau_k s^H - \tau_k f^H + \tau_k f^F + \tau_L^* l^F + \tau_k^* s^F - \tau_k^* f^F + \tau_k^* f^H] \end{array} \tag{8}$$

and note that $F_{\tau_k} = -F_{\tau_k^*}$.

7 First Order Conditions

$$\begin{aligned} \frac{\partial W}{\partial \tau_k} &: \\ -s^H + e^H + f^H - f^F + \lambda[s^H + \tau_k S_{\tau_k} - f^H + f^F - \tau_k f^H + \tau_k f^F &= 0 \\ -s^H + e^H + \lambda[s^H + \tau_k S_{\tau_k} - \tau_k f^H + \tau_k f^F] &= 0 \quad \text{by symmetry of countries} \\ \lambda[s^H + \tau_k S_{\tau_k} - \tau_k f^H + \tau_k f^F] &= s^H - e^H \\ \lambda &= \frac{s^H - e^H}{s^H + \tau_k S_{\tau_k} - \tau_k f_\tau^H + \tau_k f_\tau^F} \end{aligned}$$

$$\begin{aligned} \frac{\partial W}{\partial \tau_L} &: \\ -l^H - e^H + \lambda[l^H + \tau_L l_{\tau_L}^H] &= 0 \\ \lambda[l^H + \tau_L l_{\tau_L}^H] &= l^H + e^H \\ \lambda &= \frac{l^H + e^H}{l^H + \tau_L l_{\tau_L}^H} \end{aligned}$$

Next, set $\lambda = \lambda$

$$\begin{aligned}
\frac{l^H+e^H}{l^H+\tau_L l^H_{\tau_L}} &= \frac{s^H-e^H}{s^H+\tau_k S_{\tau_k}-\tau_k f^H_\tau+\tau_k f^F_\tau} \\
\frac{l^H+e^H}{l^H+\tau_L l^H_{\tau_L}} &= \frac{s^H-e^H}{s^H+\tau_k S_{\tau_k}+2\tau_k f^F_\tau} \\
\text{by } F^H_\tau = -F^F_\tau \text{ then} & \\
(l^H+e^H)(s^H+\tau_k S_{\tau_k}+2\tau_k f^F_\tau) &= (s^H-e^H)(l^H+\tau_L l^H_{\tau_L}) \\
\frac{s^H}{s^H}(l^H+e^H)(s^H+\tau_k S_{\tau_k}+2\tau_k f^F_\tau) &= \frac{l^H}{l^H}(s^H-e^H)(l^H+\tau_L l^H_{\tau_L}) \\
s^H(l^H+e^H)(\frac{s^H}{s^H}+\frac{\tau_k S_{\tau_k}+2\tau_k f^F_\tau}{s^H}) &= l^H(s^H-e^H)(\frac{l^H}{l^H}+\frac{\tau_L l^H_{\tau_L}}{l^H}) \\
s^H(l^H+e^H)(1+\epsilon^N_S) &= l^H(s^H-e^H)(1+\epsilon_L) \\
\frac{l^H+e^H}{l^H}(1+\epsilon^N_S) &= \frac{s^H-e^H}{s^H}(1+\epsilon_L)
\end{aligned}$$