Modeling and Interpreting Interactions

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Overview

• Interactions in Pol-Sci: ubiquitous, but should be more
  – In present context, concern re: coefficient heterogeneity is a call for
    interactions—i.e., models of conditional, i.e., context-variant, coefficients.

• From theory to empirical-model specification: Arguments
  that imply interactions (& some that don’t), & how to write.

• Interpretation:
  – Effects = derivatives & differences, not coefficients!
  – Std Errs (etc.): effects vary, so do std errs (etc.)!

• Presentation: Tables & Graphs, & Choosing between equivalent Specifications

• Use & abuse of some common-practice “rules”

• Extensions:
  – Split-sample v. dummy-interaction
  – Common 2nd-moment implications of interactions
  – Interactions with uncertainty = random coefficients = hierarchical...
Interactions in Pol-Sci Research

- Common. ‘96-‘01 *AJPS*, *APSR*, *JoP*:
  - 54% some stat meth (=s.e.’s), of which 24% = interax (so interax ≈ 12.5% or 1/8<sup>th</sup> total; more if exclude *CP*).
  - (N.b., most rest QualDep & frml thry, not counted, & “thry” in denom) so understate tech nature discipline)

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Interactions in Pol-Sci Theory

• Ubiquitous, but our Theories/Substance say should be even more; core classes of argument inherently interactive:
  
  – **INSTITUTIONAL**: institutions are inherently interactive variables:
    
    • Institutions funnel, moderate, shape, condition, constrain, refract, magnify, augment, dampen, mitigate political processes that…
      
      – …translate societal interest-structures into effective political pressures,
      – …&/or pressures into public-policy responses,
      – …&/or policies to outcomes.
    
    • I.e., they affect *(modify, condition, moderate…)* effects ≡ *interaction.*
Interactions in Pol-Sci Theory

• Views from across institutionalist perspectives:
  – Hall: “institutionalist model => policy more than sum countervailing pressure from soc grps; that press mediated by organizational dynamic.”
  – Ikenberry: “[Political struggles] mediated by inst’l setting where [occur]”
  – Steinmo & Thelen: “inst’s… constrain & refract politics… [effects of] macro-structures magnify or mitigated by intermediate-level inst’s… help us… explain the contingent nature of pol-econ development…”
  – Shepsle: “SIE clearly a move [to] incorporating inst’l features into R-C. Structure & procedure combine w/ preferences to produce outcomes.”
Interactions in Pol-Sci Theory

• Ubiquitous, but our Theories/Substance say should be even more; core classes of argument inherently interactive:
  – **INSTITUTIONAL**: …
  – **STRATEGIC**: actors’ choices (outcomes) conditional upon institutional/structural environ., opportunity set, & (interdep.) other actors’ choices.
  – **CONTEXTUAL**: actors’ choices (outcomes) conditional upon environment, opportunity set, & (interdep.) aggregates of other actors’ choices.
Interactions in Pol-Sci Theory\textsuperscript{4}

• Across subfields:
  – Comparative Politics \textit{examples}:
    • Electoral system & societal structure interact to produce party system.
    • Divided government & polarization retards legislative productivity, $\Rightarrow$ conditional dynamics.
    • Corruption interactive product of institutional & societal structures.
  – International Relations \textit{examples}:
    • System polarity & offense-defense balance $\Rightarrow$ war propensity.
    • Terrorist targeting & counterterrorism responses depend “grievance” & resources
  – American Politics …
Interactions in Pol-Sci Theory

- Political Economy:
  - Electoral & partisan cycles (i.e., effects) depend on political & economic institutional, structural, contextual conditions.

- Political Behavior:
  - Government's institutions shape voter behavior: balancing (Kedar, Alesina); economic voting (Powell & Whitten); etc.

- Legislative Studies:
  - Effects of divided government different in presidential vs. parliamentary systems.

- Political Development:
  - Effect of inequality on democratization depends on cleavage structure.
Theory & Substance: Everyone’s Favorite “Model”

Economics Affects Politics and Society
Politics Affects Economics and Society
Society Affects Politics and Economics

Picture & text seem relate more directly to the “ubiquitous endogeneity” of Society, Economy, & Polity. For “ubiquitous context-conditionality”, think “affects the effects of {Politics, Economics, Society} on {Society, Polity, Economy}.”
Theory & Substance: An Old (& still) Favorite “Model” of Mine

The Cycle ofPolitical Economy

Examples of the Elements at Each Stage:
(A) Interests:
- Sectoral Structure of Economy
- Income Distribution
- Age Distribution
- Trade Openness

Elections:
- Electoral Law
- Voter Participation

Government Formation:
- Fractionalization
- Polarization

(B) Representation:
- Partisanship

Policy:
- Fiscal Policy
- Monetary Policy
- Institutional Adjustment

Government Termination:
- Replacement Risk

(C) Outcomes:
- Unemployment
- Inflation
- Growth
- Sectoral Shift
- Debt
- Institutional Change

Again, the ubiquitous context-conditionality perhaps not fully overtly shown, but implication here that effect factors at each stage tend depend on others at that and other stages.
Theory & Substance:
An Newer Favorite “Model” of Mine

• **Complex Context-Conditionality:**
  – Effect of (almost) everything depends on (almost) everything else.
  – E.g., Principal-Agent Situations
    • If fully principal, \( y_1 = f(X) \); if fully agent, \( y_2 = g(Z) \); institutions: \( 0 \leq h(I) \leq 1 \).

\[
y = h(I) f(X) + \{1 - h(I)\} g(Z)
\]

\[
\Rightarrow \frac{\partial y}{\partial x} = h(I) \frac{\partial f(X)}{\partial x}; \quad \frac{\partial y}{\partial z} = -h(I) \frac{\partial g(Z)}{\partial z};
\]

\[
\frac{\partial y}{\partial i} = \frac{\partial h(I)}{\partial i} \left[ f(X) - g(Z) \right]
\]
(Complex) Context-Conditionality: (Hallmark of Modern Pol-Sci Theory?)

- Principal-Agent (Shared Control) Situations, for example:
  - If fully principal: \( y_1 = f(X) \);
  - If fully agent: \( y_2 = g(Z) \);
  - Institutions \(
    \Rightarrow \) Monitoring & Enforcement costs principal must pay to induce agent behave as principal would: \( 0 \leq h(I) \leq 1 \).
- RESULT:

\[
y = h(I) f(X) + \{1 - h(I)\} g(Z)
\]

\[
\Rightarrow \frac{\partial y}{\partial x} = h(I) \frac{\partial f(X)}{\partial x} ;
\]

\[
\frac{\partial y}{\partial z} = -h(I) \frac{\partial g(Z)}{\partial z} ;
\]

\[
\frac{\partial y}{\partial i} = \frac{\partial h(I)}{\partial i} [f(X) - g(Z)]
\]

...i.e., effect of anything depends on everything else!
Not Every Argument Is an Interactive Argument

• Not Interactive:
  - \( X \) affects \( Y \) through its effect on \( Z \): \( X \Rightarrow Z \Rightarrow Y \)
    • In (political) psychology / behavior, this called mediation. Interaction is called moderation in this literature.
  - \( X \) and \( Z \) affect each other: \( X \Leftrightarrow Z \).
    • I.e., \( X \) and \( Z \) endogenous to each other. Note: irrelevant to Gauss-Markov (OLS is BLUE); merely implies care to what partials (coefficients) mean.
  - \( Y \) depends on \( X \) controlling for \( Z \), or \( Y \) depends on \( X \) & \( Z \):
    \[
    E(Y|X,Z) = f(Z), \quad E(Y|X) = f(Z), \quad Y = f(X,Z)
    \]
    • I.e., e.g., showing outcomes differ across 2×2 of \( X \) & \( Z \) insufficient; issues is difference of differences across rows or down columns.

• Interactive: \textit{Effect of } \( X \) \textit{on } \( Y \) \textit{depends on } \( Z \) (\( \Rightarrow \) converse: Effect of \( Z \) on \( Y \) depends on \( X \)):

\[
\frac{\partial Y}{\partial X} = f(Z) \Leftrightarrow \frac{\partial Y}{\partial Z} = f(X)
\]
From Theory/Substance to Empirical-Model Specification

- Classic Comparative-Politics Example:
  - Societal Fragmentation, $S_{Frag}$, &
  - Electoral-System Proportionality, $D_{Mag}$,
  - $\Rightarrow$ Effective # Parliamentary Parties: $ENPP$

- “Theory”: $ENPP = f(S_{Frag}, D_{Mag}, \cdot, \varepsilon)$

- Hypotheses:
  \[
  \frac{\partial ENPP}{\partial S_{Frag}} \geq 0, \quad \frac{\partial ENPP}{\partial D_{Mag}} \geq 0
  \]

- Empirical Specification: Lots ways get there...
A Typical Linear-Interactive Specification

- Want linear $f(\cdot)$ w/ these properties; many ways to get there:

$$\begin{align*}
ENPP &= \beta_0 + \beta_1 S\text{Frag} + \beta_2 D\text{Mag} + \varepsilon \\
\frac{\partial ENPP}{\partial S\text{Frag}} &= \beta_1 \rightarrow f(D\text{Mag}) = \alpha_0 + \alpha_1 D\text{Mag} \\
\frac{\partial ENPP}{\partial D\text{Mag}} &= \beta_2 \rightarrow f(S\text{Frag}) = \gamma_0 + \gamma_1 S\text{Frag} \\
\Rightarrow ENPP &= \beta_0 + (\alpha_0 + \alpha_1 D\text{Mag}) S\text{Frag} + (\gamma_0 + \gamma_1 S\text{Frag}) D\text{Mag} + \varepsilon \\
&= \beta_0 + \alpha_0 S\text{Frag} + \alpha_1 D\text{Mag} S\text{Frag} + \gamma_0 D\text{Mag} + \gamma_1 S\text{Frag} D\text{Mag} + \varepsilon \\
&= \beta_0 + \alpha_0 S\text{Frag} + \gamma_0 D\text{Mag} + (\alpha_1 + \gamma_1) S\text{Frag} D\text{Mag} + \varepsilon \\
&= \beta_0 + \beta_{SF} S\text{Frag} + \beta_{DM} D\text{Mag} + \beta_{SFDM} S\text{Frag} D\text{Mag} + \varepsilon \\
\Rightarrow \frac{\partial ENPP}{\partial S\text{Frag}} &= \beta_{SF} + \beta_{SFDM} D\text{Mag} \\
\frac{\partial ENPP}{\partial D\text{Mag}} &= \beta_{DM} + \beta_{SFDM} S\text{Frag}
\end{align*}$$
Interpretation of *Effects*:
Derivatives & Differences, *Not* Coefficients

- Standard Linear Interactive Model:
  \[ EN = \beta_0 + \beta_{SF}SF + \beta_{DM}DM + \beta_{SFDM}SF \times DM + \ldots + \varepsilon \]

- Effect of *SFrag* on *ENPP* (is a function of *DMag*):
  \[
  \text{Effect}(SF) \equiv \frac{\partial EN}{\partial SF} = \beta_{SF} + \beta_{SFDM}DM
  \]
  \[ \Delta EN = \beta_{SF}\Delta SF + \beta_{SFDM}DM \cdot \Delta SF \]
  \[ \equiv \frac{\Delta EN}{\Delta SF} = \beta_{SF} + \beta_{SFDM}DM \]

- Effect of *DMag* on *ENPP* (is *f* of *SFrag*):
  \[
  \text{Effect}(DMag) \equiv \frac{\partial ENPP}{\partial DMag} = \beta_{DM} + \beta_{SFDM}SFrag
  \]
  \[ \equiv \Delta ENPP = \beta_{DM}\Delta DM + \beta_{SFDM}SFrag \cdot \Delta DM \]
  \[ \equiv \frac{\Delta ENPP}{\Delta DM} = \beta_{DM} + \beta_{SFDM}SFrag \]
Interpretation of *Effects*: NOTES

- “Main Effect” & “Interactive Effect”:
  - For example, $\beta_{SF} = \text{“main effect of SFrag”}$
  - ....*but* $\beta_{SF}$ is merely the effect of SFrag at other variable(s) involved in interaction with it=0, so:
    - Other-var(s)=0 may be extreme in the sample, or beyond sample range, or even logically impossible.
    - Other-var(s)=0 substantive meaning of 0 altered by rescaling
      - E.g., by “centering” (centering changes nothing, btw…)
    - Other-var(s)=0 may not have anything substantively main about it
  - Is no Main Effect or separately & Interactive Effect; is just the effect, which conditional, varies:

\[
\text{Effect}(SF) \equiv \frac{\partial EN}{\partial SF} = \beta_{SF} + \beta_{SFDM} DM \quad ; \quad \text{Effect}(DM) \equiv \frac{\partial EN}{\partial DM} = \beta_{DM} + \beta_{SFDM} SF
\]
Interpretation of Effects: NOTES²

\[ EN = \beta_0 + \beta_{SF} SF + \beta_{DM} DM + \beta_{SFDM} SF \times DM + \ldots + \varepsilon \]

- COEFFICIENTS ARE NOT EFFECTS. EFFECTS ARE DERIVATIVES &/OR DIFFERENCES.
  - Only in \textit{purely} linear-additive-separable model are they equal because only there do derivatives simply = coefficients.
  - \( \beta_{SF} \) is \textit{not} “effect of SFrag ‘independent of’…” & definitely not its “effect ‘controlling for’…other variable(s) in the interaction”

- Cannot substitute linguistic invention for understanding model’s logic (its simple math)
Interpretation of *Effects*: NOTES\textsuperscript{3}

- Interactions are logically symmetric:
  - For any function, not just lin-add.
  - If argue effect $x$ depends $z$, must also believe effect $z$ depends $x$.

- Interactions often have 2\textsuperscript{nd}-moment (variance, i.e., heteroskedasticity) implications too:
  - Larger district magnitudes, $DMag$, are “permissive” elect sys: allow more parties...
  - Fewer *Veto Actors* allow greater policy-change... (both need additional assumpts)

- All of this holds for any type of variable:
  - Measurement: binary, continuous...
  - *Level*: micro or macro; $i$, $j$, $k$, …
Frequent 2\textsuperscript{nd}-Moment Implications Interactions

- **DMag** permissive ele sys: \textit{allows} more parties...

  \[ NP = \beta_0 + \beta_1 DM + \varepsilon \; ; \; V(\varepsilon) = f(DM) \; , \; \text{e.g.,} \; \sigma_0 + \sigma_1 DM \]

  - Note: unmodeled interactions look like heteroskedasticity; that’s general, actually. Anything unmodeled gets into \( e^2 \)...

- Few **Veto Actors** \textit{allows} greater policy-change...

\[ y = \beta_0 + \beta_1 VP + \varepsilon \; ; \; V(\varepsilon) = f(VP) \; , \; \text{e.g.,} \; \sigma_0 + \sigma_1 VP \]

- I.e., these are Rndm-Coeff \&/or Het-sked Props...
Interpretation of Effects: Standard Errors for Effects

\[ ENPP = \beta_0 + \beta_{SF} S\text{Frag} + \beta_{DM} D\text{Mag} + \beta_{SFD} S\text{Frag} D\text{Mag} + \ldots + \varepsilon \]

- Std Errs reported with regression output are for coefficients, not for effects.
  - The s.e. (t-stat, p-level) for \( \hat{\beta}_{SF} \) regards the estimated effect of \( S\text{Frag} \) on \( ENPP \) at \( D\text{Mag}=0 \) (…which is logically impossible).
- Effect of \( x \) depends on \( z \) & v.v. (i.e., which was the point, remember?), so does the s.e.:

\[
\text{Effect}(x) \equiv \frac{\partial y}{\partial x} = \beta_x + \beta_{xz} z \quad \Rightarrow \quad \text{Est.Eff.}(x) \equiv E\left( \frac{\partial y}{\partial x} \right) = \hat{\beta}_x + \hat{\beta}_{xz} z
\]

\[
\text{Est.Var.}\{\text{Est.Eff.}(x)\} \equiv E\left[ \text{Var}\left\{ E\left( \frac{\partial y}{\partial x} \right) \right\} \right] = E\left[ \text{Var}\left\{ \hat{\beta}_x + \hat{\beta}_{xz} z \right\} \right] = V\left\{ \hat{\beta}_x + \hat{\beta}_{xz} z \right\} = V\left\{ \hat{\beta}_x \right\} + V\left\{ \hat{\beta}_{xz} \right\} \cdot z^2 + 2 \cdot C\left( \hat{\beta}_x, \hat{\beta}_{xz} \right) z
\]

- In words… More Generally:
  \[ \text{Var}(x'\hat{\beta}) = x' \left[ V(\hat{\beta}) \right] x \]
From Hypotheses to Hypotheses Tests:
Does $Y$ Depend on $X$ or $Z$?

$$ENPP = \beta_0 + \beta_{SF}S\text{Frag} + \beta_{DM}D\text{Mag} + \beta_{SFDM}S\text{FragDMag} + \ldots + \varepsilon$$

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<thead>
<tr>
<th>Hypothesis</th>
<th>Mathematical Expression</th>
<th>Statistical test</th>
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<tbody>
<tr>
<td>$x$ affects $y$, or $y$ is a function of (depends on) $x$</td>
<td>$y=f(x)$ $\frac{\partial y}{\partial x} = \beta_x + \beta_{xz}z \neq 0$</td>
<td>$F$-test: $H_0: \beta_x = \beta_{xz} = 0$</td>
</tr>
<tr>
<td>$x$ increases $y$</td>
<td>$\frac{\partial y}{\partial x} = \beta_x + \beta_{xz}z &gt; 0$</td>
<td>Multiple $t$-tests: $H_0: \beta_x + \beta_{xz}z \leq 0$</td>
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<td>$x$ decreases $y$</td>
<td>$\frac{\partial y}{\partial x} = \beta_x + \beta_{xz}z &lt; 0$</td>
<td>Multiple $t$-tests: $\beta_x + \beta_{xz}z \geq 0$</td>
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<tr>
<td>$z$ affects $y$, or $y$ is a function of (depends on) $z$</td>
<td>$y=g(z)$ $\frac{\partial y}{\partial z} = \beta_z + \beta_{zx}x \neq 0$</td>
<td>$F$-test: $H_0: \beta_z = \beta_{zx} = 0$</td>
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From Hypotheses to Hypotheses Tests:
Is Y’s Dependence on X Conditional on Z & v.v.? How?

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<td>The effect of x on y depends on z</td>
<td>( y = f(xz, \cdot) ) ( \frac{\partial y}{\partial x} = \beta_x + \beta_{xz} z = g(z) ) ( \frac{\partial (\partial y/\partial x)}{\partial z} = \frac{\partial^2 y}{\partial x \partial z} = \beta_{xz} = 0 )</td>
<td>t-test: ( H_0: \beta_{xz} = 0 )</td>
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<td>The effect of x on y increases in z</td>
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<td>seasonality</td>
<td>( y = f(xz, \cdot) ) ( \frac{\partial y}{\partial z} = \beta_z + \beta_{xz} x = h(x) ) ( \frac{\partial (\partial y/\partial z)}{\partial x} = \frac{\partial^2 y}{\partial z \partial x} = \beta_{xz} = 0 )</td>
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**Discussion:** Alternative views on how to explore this...

Does Y Depend on X, Z, or XZ?

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<td>y is a function of (depends on) z, z, and/or their interaction</td>
<td>( y = f(x, z, xz) )</td>
<td>F-test: ( H_0: \beta_x = \beta_z = \beta_{xz} = 0 )</td>
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</table>
Discussion re: determining if Z conditions effect of X, etc.

- Some suggest need find some Z0 & Z1 (in sample?) for which dY/dX differ significantly:
  - \( dY/dX|_{Z_0} = b_x + b_{xz}Z_0 \), \( dY/dX|_{Z_1} = b_x + b_{xz}Z_1 \)
  - So: \( dY/dX|_{Z_0} - dY/dX|_{Z_1} \)
    - \( = b_x + b_{xz}Z_0 - (b_x + b_{xz}Z_1) = b_{xz}(Z_0 - Z_1) \)
  - Wald test this signif diff zero =>
    - \( = b_{xz}(Z_0 - Z_1)/[\text{var}\{b_{xz}(Z_0 - Z_1)\}]^{.5} \)
    - \( = b_{xz}(Z_0 - Z_1)/[\text{var}\{b_{xz}\}(Z_0 - Z_1)^2]^{.5} \)
    - \( = b_{xz}(Z_0 - Z_1)/[\text{s.e.}\{b_{xz}\}(Z_0 - Z_1)] = b_{xz}/\text{s.e.}\{b_{xz}\} \)
  - I.e., just the standard (Wald) t-test on interactive coeff, and that’s because that coefficient is cross-derivative about which the hypothesis asks!
Use & Abuse of Some Common ‘Rules’

- **Centering to Redress Colinearity Concerns:**
  - Adds no info, so changes *nothing*; no help with colinearity or anything else; only moves substantive content of $x=0, z=0$.
  - Specifically, makes coeff. on $x$ ($z$), effect when $z$ ($x$) at sample-mean, the new 0. Do only if aids presentation.

- **Must Include All Components (if $x \cdot z$, then $x \& z$):**
  - Application of Occam’s Razor &/or scientific caution (e.g., greater flexibility to allow linear w/in lin-interax model), but
  - *Not* a logical or statistical requirement.
  - Safer rule than opposite & to check almost always, but
  - *Not* override theory & evidence, esp. if (insofar as strongly) agree to exclude...

- **Pet-Peeve: Linguistic Gymnastics to Dodge the Math**
  - “Main effect, Interactive effect”: *the* effect in model is $dy/dx$.
  - Discussion of [coefficients & s.e.’s] as if [effects & s.e.’s].
Presentation: Marginal-Effects / Differences Tables & Graphs

- Plot/Tabulate **Effects**, \(dy/dx\), over Meaningful &/or Illuminating Ranges of \(z\), with Conf. Int.’s

\[
d\hat{y} / dx \pm t_{df,p} \sqrt{Var(d\hat{y} / dx)} = \hat{\beta}_x + \hat{\beta}_{xz} z \pm t_{df,p} \sqrt{V(\hat{\beta}_x) + V(\hat{\beta}_{xz})z^2 + 2C(\hat{\beta}_x, \hat{\beta}_{xz})z}
\]

- Explain axes
- Explain shape
- Linear-interax:
  - Will cross 0 & be insig @ 0.
- Rescaling &
  - “main effect”
  - “centering”
  - Max(Asterisks)
**Presentation: Expected-Value/Predictions Tables & Graphs**

- **Predictions,** $E(y|x,z)$: notice how helpful in matrix form:

  \[
  \hat{y} \pm t_{df,p} \sqrt{Var(\hat{y})} = \\
  \hat{\beta}_0 + \hat{\beta}_x x + \hat{\beta}_z z + \hat{\beta}_{xz} xz \pm t_{df,p} \sqrt{\frac{V(\hat{\beta}_0) + V(\hat{\beta}_x)x^2 + V(\hat{\beta}_z)z^2 + V(\hat{\beta}_{xz})(xz)^2}{1}}
  \]

  \[
  + 2C(\hat{\beta}_0, \hat{\beta}_x)x + 2C(\hat{\beta}_0, \hat{\beta}_z)z + 2C(\hat{\beta}_0, \hat{\beta}_{xz})xz
  \]

  \[
  + 2C(\hat{\beta}_x, \hat{\beta}_z)xz + 2C(\hat{\beta}_x, \hat{\beta}_{xz})x^2z + 2C(\hat{\beta}_z, \hat{\beta}_{xz})xz^2
  \]

- Here’s one place a little matrix algebra would help:

  \[
  \hat{y} \pm t_{df,p} \sqrt{Var(\hat{y})} = \hat{\beta}_0 + \hat{\beta}_x x + \hat{\beta}_z z + \hat{\beta}_{xz} xz \pm t_{df,p} \sqrt{x'\hat{V}(\hat{\beta})x}
  \]

  \[
  = \hat{\beta}_0 + \hat{\beta}_x x + \hat{\beta}_z z + \hat{\beta}_{xz} xz \pm t_{df,p} \sqrt{[1 \ x \ z \ xz][\begin{array}{c}
  \hat{V}(\hat{\beta}_0) \\
  \hat{C}(\hat{\beta}_0, \hat{\beta}_x) \\
  \hat{C}(\hat{\beta}_0, \hat{\beta}_z) \\
  \hat{C}(\hat{\beta}_0, \hat{\beta}_{xz})
  \end{array}]}
  \]

  \[
  \begin{array}{c}
  \begin{array}{c}
  \hat{V}(\hat{\beta}_x) \\
  \hat{C}(\hat{\beta}_x, \hat{\beta}_z) \\
  \hat{C}(\hat{\beta}_x, \hat{\beta}_{xz})
  \end{array} \\
  \begin{array}{c}
  \hat{V}(\hat{\beta}_z) \\
  \hat{C}(\hat{\beta}_z, \hat{\beta}_{xz})
  \end{array} \\
  \hat{C}(\hat{\beta}_z, \hat{\beta}_{xz})
  \end{array}
  \begin{array}{c}
  \begin{array}{c}
  1 \\
  x \\
  z \\
  xz
  \end{array}
  \end{array}
  \]

- Use spreadsheet or stat-graph software (…list coming…)

Slide 27
Presentation:
Choose Illuminating Graphics & Base Cases

• Interpretation same regardless of “type” of interax: *effect* always \(\equiv \frac{dy}{dx}\), but present appropriately:
  - All combos Dummy, Discrete, or Continuous:
    • Dummy-Dummy\(\Rightarrow >4\) (or \(2\#\text{interacting variable}\)) points estimated, so box & whisker or histograms effective
    • Dummy-Continuous or Discrete(*few*)-Continuous\(\Rightarrow >2\) (or \# categories) slopes, so \(E(y|x,z)\) as line or \(dy/dx\) as box & whisker or histograms effective
    • Continuous – Continuous (or DiscMany)\(\Rightarrow \)Effect-lines best or (slices from) contour plot (i.e., slices from 3D)
  - Powers (e.g., \(X & X^2\Rightarrow \text{parabola}\)) viewable as interax w/ self; certain slope shifts too (e.g., \(dy/dx=a\) for \(x<x^0\) & \(b\) for \(x>x^0\) is \(x\) interact w/ dummy for condition)
Presentation²:
Choose Illuminating Graphics & Base Cases

• Interpretation same regardless of “type” of interax: effect always \( \equiv \frac{dy}{dx} \), but present appropriately…
  
  – Always plot over substantively revealing ranges.
  
  – Especially with sets of dummies, have several (identical) specification options:
    
    • (full-set or set-less-1): choose which (\& what base if use set-less-1) to abet presentation \& discussion
    
    • (overlapping or disjoint): choose to facilitate presentation \& discussion.

  – Scale Effectively: e.g., center only if \& to extent that aids presentation \& discussion (b/c centering does nothing else)
Presentation³: Choose Illuminating Graphics & Base Cases. Examples.

Dummy-Continuous Interaction: could also plot two $\text{E(}\text{Cands}\mid\text{Groups})$ lines, with c.i.’s, effectively.
Presentation: Choose Illuminating Graphics & Base Cases. Examples.

Dummy-Dummy Interaction: could also plot four $E(Supp|\text{gender,party})$ box-whiskers effectively.
Presentation: Choose Illuminating Graphics & Base Cases. Examples.

Presentation: Choose Illuminating Graphics & Base Cases. Examples.

Figure 14. Marginal Effect of Parliamentary Support for Government, Pairwise-Interaction Model, with 90% Confidence Intervals

\[ \text{GovDur} = \beta_0 + \beta_{np}NP + \beta_{ps}PS + \beta_{pd}PD + \beta_{nppe}NP \times PS + \beta_{npdp}NP \times PD + \beta_{psdp}PD \times PS + \varepsilon \]  [25]
Elaborations, Complications, & Extensions: Sample-Splitting v. (Dummy-)Interacting\(^1\)

- Split-sample (e.g., **unit-by-unit**) ≈ Full-Dum Interax:
  - Subsample by binary (or multinomial, e.g., by-unit in TSCS) category to estimate separately ≈ Include dummy for each category (or set-less-1) & interact each dummy with each \(x\) (and include \(x\) by itself also if set-less-1)
    - Coeff’s same (or equal substantive content if using set-1 dummies).
    - S.E.’s same except \(s^2\) part of OLS’s \(s^2(X'X)^{-1}\) versus \(s_i^2\) for splitting
    - Can make essentially exact by allow \(s_i^2\) (FWLS)
  - Subsample by hi/lo values of some non-nominal regressor is equiv to *nominalizing* the info in that var & dummy-interact
    - I.e., wasting information, when usually have too little (non-parametric or extreme-measurement-error arguments might justify)
    - So usually a bad idea… (*could discuss arg’s for it, under rare circ.*)
Elaborations, Complications, & Extensions: Sample-Splitting v. (Dummy-)Interacting

- Split-sample abets eyeballing, obfuscates statistical analysis, of the main point: the different effects by category.
  - What’s s.e/signif. of $b_{1i}-b_{1j}$? Need:
    \[
    s.e.(b_{1i} - b_{1j}) = \sqrt{V(b_{1i}) + V(b_{1j}) - 2C(b_{1i}, b_{1j})} = \sqrt{V(b_{1i}) + V(b_{1j})}
    \]
  - Luckily, cov=0, but, still, squaring 2 terms, sum, & root in head?
- Can choose full dummy set to mirror the split-sample estimates directly (& report that way, if wish) or the set-less-one to get significance of differences b/w samples directly (in the standard reported t-test)
  - Same thing, so choose form to optimize presentational efficacy.
- One advantage of hierarchical models (random coeff.) is how it affords, naturally, various positions b/w these extremes.
  - E.g., can “borrow strength” across units.
Typical 2\textsuperscript{nd}-Moment Implications of Interactions

- \textit{DMag} permissive ele sys: \textbf{allow} more parties…
  
  \[ NP = \beta_0 + \beta_1 DM + \varepsilon \ ; \ V(\varepsilon) = f(DM) \ , \text{e.g.,} \sigma_0 + \sigma_1 DM \]

- Few \textit{Veto Actors} \textbf{allow} greater policy-change…

\[ y = \beta_0 + \beta_1 VP + \varepsilon \ ; \ V(\varepsilon) = f(VP) \ , \text{e.g.,} \sigma_0 + \sigma_1 VP \]

- I.e., these are Rndm-Coeff &/or Het-sked Props…
  
  \[ NP = \beta_0 + \beta_1 DM + \beta_2 SF + \beta_3 DM \times SF + \varepsilon \ ; \ V(\varepsilon) = f(DM) \ , \text{e.g.,} \sigma_0 + \sigma_1 DM \]
Sandwich Estimators

\[ EN = \beta_0 + \beta_1 SF + \beta_2 DM + \varepsilon \]

\[ \frac{\partial EN}{\partial SF} = \beta_1 = \alpha_0 + \alpha_1 DM + \omega_1 \]

\[ \frac{\partial EN}{\partial DM} = \beta_2 = \gamma_0 + \gamma_1 SF + \omega_2 \]

\[ \Rightarrow EN = \beta_0 + (\alpha_0 + \alpha_1 DM + \omega_1) SF + (\gamma_0 + \gamma_1 SF + \omega_2) DM + \varepsilon \]

\[ = \beta_0 + \alpha_0 SF + (\alpha_1 + \gamma_1) DM \times SF + \gamma_0 DM + \{ \varepsilon + \omega_1 SF + \omega_2 DM \} \]

\[ = b_0 + b_1 SF + b_2 DM \times SF + b_3 DM + \varepsilon^* \]

- Notice the compound error term:
  - \( V(\varepsilon^*) \) will not be \( \sigma^2 I \) even if \( \varepsilon \) is, so \( V(b) \) doesn’t reduce to \( \sigma^2(X'X)^{-1} \), so OLS s.e.’s wrong.
  - Be OK on average (unbiased) & in limit (consistent) if element of \( V(\varepsilon^*) \) “orthogonal to \( xx' \)”
  - But def’ly not because \( \varepsilon^* \) includes \( x \) & \( z \), which part of \( X \)!
Sandwich Estimators

\[ V(b_{LS}) = (X'X)^{-1}X'[V(\varepsilon + \omega_1 SF + \omega_2 DM)]X(X'X)^{-1} \]

• Brilliant insight of ‘robust’ (i.e., consistent) “sandwich” estimators:
  - Only need formula that accounts relation \( V(\varepsilon^*) \) to “\( X'X \)”, i.e., regressors, squares, & cross-prod’s involved in \( X'[\cdot]X' \)

• ⇒ “, robust” (or, in HM: “, cluster”) can work:

\[ V(\varepsilon_i^*)_{RE} = \sigma^2 + \sigma_{\omega_1}^2 x_i^2 + \sigma_{\omega_2}^2 z_i^2 \]

so track \( e^2 \) rel \( xx' \) & \( zz' \) ⇒

White’s heteroskedasticity-consistent s.e.’s:

\[ [\cdot] = \frac{1}{n} \sum_{i=1}^{n} e_i^2 x_i x_i' \]
Cross-Level Interactions

• Nothing much different if interactions between variables that vary at different levels (note, e.g., not many subscripts used above):
  – If CLRM assumptions apply, then unbiased, consistent, and efficient.

• Two main issues of concern, though:
  – *Un- or insufficiently modeled* parameter heterogeneity (incl. intercept): can cause bias, if pattern un/insuff. het. relates to \( \mathbf{X} \),
  – Non-spherical error-covariance matrix:
    » An efficiency & proper s.e.’s issue, not bias/consistency.
    » As just seen, surely will arise, & likely in different specific forms depending assumed error-components structure.
  – As before: Effects, their variances, symmetry of interactive propositions, that neither micro- nor macro-level coefficients=effects...all that applies...
Cross-Level Interactions:

\[ \text{reg spend L.spend unem left growthpc depratio cdem trade lowwage fdi skand skand_unem} \]

\[
\begin{align*}
\text{spend}_{it} &= \beta_i^0 + \beta_i^l \text{left}_{it} + \ldots + \varepsilon_{it} \\
\beta_i^0 &= \alpha_0 + \alpha_1 \text{skand}_i + u_i^0 \\
\beta_i^l &= \gamma_0 + \gamma_1 \text{skand}_i + u_i^1 \\
\Rightarrow \text{spend}_{it} &= \alpha_0 + \alpha_1 \text{skand}_i + u_i^0 + \gamma_0 \text{left}_{it} \\
&\quad + \gamma_1 \text{left}_{it} \times \text{skand}_i + \text{left}_{it} u_i^1 + \ldots + \varepsilon_{it}
\end{align*}
\]

gathering terms:

\[
\begin{align*}
\text{spend}_{it} &= \alpha_0 + \ldots + \alpha_1 \text{skand}_i + \gamma_0 \text{left}_{it} \\
&\quad + \gamma_1 \text{left}_{it} \times \text{skand}_i + \left( u_i^0 + \text{left}_{it} u_i^1 + \varepsilon_{it} \right)
\end{align*}
\]

\[
\frac{\partial \text{spend}}{\partial \text{left}} = b_{\text{left}} + b_{\text{leftsk}} \text{skand} + u_i^1 \\
\frac{\partial \text{spend}}{\partial \text{skand}} = b_{\text{skand}} + b_{\text{leftsk}} \text{left}
\]

\[
V(\varepsilon_i^*)_{HM} = \sigma_0^2 + \sigma_1^2 x_i x_i' + \sigma_2^2 z_i z_i' \Rightarrow \text{cluster: } [\cdot] = \sum_{j=1}^{n} \left\{ (\sum_{i=1}^{n} e_{ij} x_{ij})' (\sum_{i=1}^{n} e_{ij} x_{ij}) \right\}
\]
From CLRM to Multilevel Model

\[ \text{spend}_{it} = \beta_i^0 + \beta_i^l \text{left}_{it} + ... + \varepsilon_{it} \]

\[ \beta_i^0 = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0 \]

\[ \beta_i^l = \gamma_0 + \gamma_1 \text{skand}_i + u_i^1 \]

- If CLRM assumptions apply, then OLS unbiased, consistent, and efficient.
  
  - Two main issues of concern:
    
    - Parameter heterogeneity: (see pictures on next slide)
      
      - systematic &/or stochastic (fixed v. rndm intercpt/coeff)
      
      - can cause bias if pattern unmodeled hetero relates to \( X \),
    
    - Non-spherical error cov-mat: an efficiency & proper s.e.’s issue, not a bias/consistency one
      
      - But “mere inefficiency” can be serious.
      
      - And accurate std err’s very important.
From the CLRM to HLM

- Examples of parameter heterogeneity that covaries w/ x values, so bias:

  - Note: FE v. RE both theoretically could cause bias if cov w/ x, but latter identified by orthogonality assumption, as we’ll see.
From the CLRM to RE Model

• Arbitrary R.E. Model: Odd that std. lin-interact model:
  - Assumes know \( y = f(X) + \text{error} \): \( y_i = \beta_0 + \beta_x x_i + \beta_z z_i + \beta_{xz} x_i z_i + \varepsilon_i \)
  - But \( \frac{dy}{dx} = f(z) \) w/o error!: \( \frac{dy}{dx} = \beta_x + \beta_{xz} xz \)
  - So, try:

\[
\begin{align*}
y & = \beta_0 + \beta_1 x + \beta_2 z + \varepsilon^0 \\
\frac{dy}{dx} & \equiv \beta_1 = \alpha_0 + \alpha_1 z + \varepsilon^1 \\
\frac{dy}{dz} & \equiv \beta_2 = \gamma_0 + \gamma_1 x + \varepsilon^2 \\
\Rightarrow y & = \beta_0 + \left( \alpha_0 + \alpha_1 z + \varepsilon^1 \right) x + \left( \gamma_0 + \gamma_1 x + \varepsilon^2 \right) z + \varepsilon^0 \\
& = \beta_0 + \alpha_0 x + \gamma_0 z + (\alpha_1 + \gamma_1) xz + (\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z)
\end{align*}
\]

  - \( \Rightarrow \) std. lin-interact...except compound error-term...

• Hierarchical/Multilevel/Mixed-Effect Model: Same model, except \( x_{ij} \) & \( z_j \), & specifically: \( \varepsilon^* = \varepsilon_{ij}^0 + \varepsilon_{ij}^1 x_{ij} + \varepsilon_{ij}^2 z_j \)
  - So also std lin-interact, but w/ diff compound-error structure

• These also called “error-component” models
From CLRM to Hierarchical Model

- Std. HLM: Same model, except \( x_{ij} \), \( z_j \), \( \mathcal{E} \)
  - So a std. lin-interact too, but with different compound-error stochastic properties.

\[
\text{spend}_{it} = \beta_i^0 + \beta_i^l \text{left}_{it} + \ldots + \varepsilon_{it}
\]

\[
\beta_i^0 = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0
\]

\[
\beta_i^l = \gamma_0 + \gamma_1 \text{skand}_i + u_i^1
\]

\[
\Rightarrow \text{spend}_{it} = \alpha_0 + \alpha_1 \text{skand}_i + u_i^0 + \gamma_0 \text{left}_{it} + \gamma_1 \text{left}_{it} \times \text{skand}_i + \text{left}_{it} u_i^1 + \ldots + \varepsilon_{it}
\]

Gathering terms:

\[
\text{spend}_{it} = \alpha_0 + \ldots + \alpha_1 \text{skand}_i + \gamma_0 \text{left}_{it} + \gamma_1 \text{left}_{it} \times \text{skand}_i + \left( u_i^0 + \text{left}_{it} u_i^1 + \varepsilon_{it} \right)
\]

\[
\Rightarrow \frac{\partial \text{spend}}{\partial \text{left}} = b_{left} + b_{lfisk} \text{skand} + u_i^1 \quad \& \quad \frac{\partial \text{spend}}{\partial \text{skand}} = b_{skand} + b_{lfisk} \text{left}
\]
Properties of OLS under HLM Conditions

• Properties of OLS Estimates of Linear-Interaction Model if truly RE/HLM:

\[ y = \beta_0 + \beta_x x + \beta_z z + \beta_{xz} xz + (\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z) = X\beta + (\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z) \]

• So, OLS coeff. est.'s still differ from truth by \( A\varepsilon^* \):

\[ \hat{\beta}_{LS} = (X'X)^{-1} X'y = (X'X)^{-1} X'[X\beta + (\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z)] \]
\[ = (X'X)^{-1} X'X\beta + (X'X)^{-1} X'(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z) = \beta + (X'X)^{-1} X'\varepsilon^* \]

• So, OLS coeff. est.'s unbiased & consistent:

\[ E(\hat{\beta}_{LS}) = E[\beta + (X'X)^{-1} X'\varepsilon^*] = E[\beta + (X'X)^{-1} X'(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z)] \]
\[ = \beta + (X'X)^{-1} X'E(\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z) = \beta + (X'X)^{-1} X'[E(\varepsilon^0) + E(\varepsilon^1)x + E(\varepsilon^2)z] \]
\[ = \beta + (X'X)^{-1} X'[0 + E(\varepsilon^1)x + E(\varepsilon^2)z] = \beta + (X'X)^{-1} X'[0 + 0 + 0] = \beta. \quad Q.E.D. \]

- Note: only works for models w/ additively separable stochastic component; not necessarily for others (e.g., logit/probit)
Properties of OLS under HLM Conditions

• But, OLS s.e.’s will be wrong; not \( s^2(X'X)^{-1} \), but:

\[
V \left( \hat{\beta}_{LS} \right) = V \left[ \beta + (X'X)^{-1} X' \varepsilon^* \right] \\
= V[\beta] + V \left[ (X'X)^{-1} X' \varepsilon^* \right] + 2C \left[ \beta, (X'X)^{-1} X' \varepsilon^* \right] \\
= 0 + V \left[ (X'X)^{-1} X' \varepsilon^* \right] + 0 \\
= (X'X)^{-1} X' V \left( \varepsilon^* \right) X (X'X)^{-1} \\
= (X'X)^{-1} X' \left[ V (\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z) \right] X (X'X)^{-1} \\
= (X'X)^{-1} X' \left[ V (\varepsilon^0) + V (\varepsilon^1 x) + V (\varepsilon^2 z) \right] X (X'X)^{-1} \\
(\text{the covariance terms are assumed zero})
Sandwich Estimators

\[ V(\hat{\beta}_{LS}) = (X'X)^{-1}X'[V(\varepsilon^0) + V(\varepsilon'x) + V(\varepsilon^2z)]X(X'X)^{-1} \]

- Not \( \sigma^2 I \) (even if each \( \varepsilon^* \) component is), so whole thing doesn’t reduce to \( \sigma^2 (X'X)^{-1} \), so OLS s.e.’s wrong.
- Be OK on avg (unbiased) & in limit (consistent) if that term varied in way “orthogonal to xx’”
  - But, as b4, def’ly not b/c \([\cdot]\) includes \( x \) & \( z \), which part of \( X \).
  - =brilliant insight of ‘robust’ (i.e., consistent) s.e. est’s:
    - Only need s.e. formula that accounts relation \( V(\varepsilon^*) \) to “xx’”, i.e., to the regressors, their squares, & cross-prod’s involved in \( X'[\cdot]X \)
- \( \Rightarrow \), robust” & “, cluster” can work (for RE & HLM, resp’ly)
  - \( \hat{V}(\hat{\beta})_{RE} = \sigma^2(I + xx' + zz') \) so track e^2 rel \( xx' \) & \( zz' \) \( \Rightarrow \)
    - i.e., White’s het-consistent s.e.’s
  - \( \hat{V}(\hat{\beta})_{HM} = \sigma_0^2 I + \sigma_1^2 xx' + \sigma_2^2 zz' \) sim but grpng \( \Rightarrow \)
    - i.e., het-cluster consistent s.e.’s
**From the CLRM to HLM**

\[
y = \beta_0 + \alpha_0 x + \gamma_0 z + (\alpha_1 + \gamma_1) xz + (\varepsilon^0 + \varepsilon^1 x + \varepsilon^2 z)
\]

\[
V(\hat{\beta}_{LS}) = (X'X)^{-1}X'[V(\varepsilon^0) + V(\varepsilon^1 x) + V(\varepsilon^2 z)]X(X'X)^{-1}
\]

• ⇒appropriate “, robust” & “, cluster” can work
  – I.e., *asymptotically* std errs right...
    - Note: generally need large \(n_j\), more than just large \(n\), for cluster.
    - I.e., *coefficients still inefficient*.
      - Want/need efficiency, or \(n_j\) low? HLM/RE or FGLS/FWLS.
      - Note: similarity RE and HLM, RE & FWLS. As suggests, RE only helps efficiency and only rightly does so if that’s all it does. (I.e., if the RE’s orthogonal to \(X\).)
  – I.e., “work” thusly for models with additively-separable stochastic components
    - As w/ all such “sandwich” estimators, logical disconnect in applying them to models w/o such separability.
Additional Materials

• Unlikely to cover
• Contents:
  – Interactions in nonlinear/QualDep models
  – Nonlinear Least Squares and complex context-conditionality. Applications:
    • “Multiple Hands on the Wheel”
    • “Veto Actors Bargaining in Common Pools” (Multiple Effects of Multiple Policymakers)
Elaborations, Complications, & Extensions:
Interax in QualDep (Inherently Interactive) Models

- **Probit/Logit Models w/ Interactions**
  - **Probit:** \[ p(y = 1) = \Phi(x'\beta) \]
  - **Logit:** \[ p(y = 1) = \frac{\exp(x'\beta)}{1 + \exp(x'\beta)} = [1 + \exp(-x'\beta)]^{-1} \]

- **Marginal Effects:** (nonlinear, so must specify @ what \( x \))
  - Start w/ \( x'\beta \) pure lin-add, model inherently inter. b/c S-shaped:
    - **Probit:**
      \[
      \frac{\partial p}{\partial x_k} = \frac{\partial \Phi(x'\beta)}{\partial x_k} = \phi(x'\beta) \cdot \frac{\partial x'\beta}{\partial x_k} = \phi(x'\beta) \cdot \beta_k
      \]
    - **Logit:**
      \[
      \frac{\partial p}{\partial x_k} = \frac{\partial \{e^{x'\beta}[1+e^{x'\beta}]^{-1}\}}{\partial x_k} = \frac{e^{x'\beta}}{1+e^{x'\beta}} \cdot \beta_k - \frac{e^{x'\beta}}{(1+e^{x'\beta})^2} \cdot e^{x'\beta} \cdot \beta_x
      \[
      = \left[ \frac{e^{x'\beta}(1+e^{x'\beta})}{(1+e^{x'\beta})^2} - \frac{(e^{x'\beta})^2}{(1+e^{x'\beta})^2} \right] \cdot \beta_k = \frac{e^{x'\beta}}{(1+e^{x'\beta})^2} \cdot \beta_k
      \[
      = \frac{e^{x'\beta}}{1+e^{x'\beta}} \cdot \frac{1}{1+e^{x'\beta}} \cdot \beta_k = p \cdot (1 - p) \cdot \beta_k
      \]

- If \( x'\beta = \ldots + \beta_x x + \beta_z z + \beta_{xz} x z \ldots \Rightarrow \) same except \( dx'\beta / dx = \beta_x x + \beta_{xz} x z \); underlying propensity, i.e., movement along S-shape also interact explicitly \( x \) & \( z \). [Discuss meaning inherent v. explicit interax...]
  - **Probit:**
    \[ \frac{\partial p}{\partial x} = \phi(x'\beta) \cdot (\beta_x + \beta_{xz} x z) \]
  - **Logit:**
    \[ \frac{\partial p}{\partial x} = p \cdot (1 - p) \cdot (\beta_x + \beta_{xz} x z) \]
Elaborations, Complications, & Extensions: Interax in Nonlin/Qual (Inherently Interax) Models

- **Standard Errors?**

  - **Delta Method:**

    \[
    \text{Asym.Var.}(f(\hat{\beta})) \\
    \approx \left[ \nabla_{\hat{\beta}} f(\hat{\beta}) \right]' V(\hat{\beta}) \left[ \nabla_{\hat{\beta}} f(\hat{\beta}) \right]
    \]

- **Probit Marginal-Effect s.e.:**

  \[
  \partial \left\{ \phi(x'\hat{\beta}) \frac{\partial x'\hat{\beta}}{\partial x_1} \right\} \hat{V}(\hat{\beta}_1) \ldots \hat{C}(\hat{\beta}_1, \hat{\beta}_k) \\
  \quad \partial \left\{ \phi(x'\hat{\beta}) \frac{\partial x'\hat{\beta}}{\partial x_2} \right\} \hat{V}(\hat{\beta}_2) \ldots \hat{C}(\hat{\beta}_2, \hat{\beta}_k) \\
  \quad \ldots \\
  \quad \partial \left\{ \phi(x'\hat{\beta}) \frac{\partial x'\hat{\beta}}{\partial x_k} \right\} \hat{V}(\hat{\beta}_k) \quad \hat{C}(\hat{\beta}_1, \hat{\beta}_k) \ldots \hat{C}(\hat{\beta}_k, \hat{\beta}_k)
  \]

- **Logit:** same, except \( \hat{p}(1 - \hat{p}) \frac{\partial x'\hat{\beta}}{\partial x} \) replaces \( \phi(x'\hat{\beta}) \frac{\partial x'\hat{\beta}}{\partial x} \)

- For first-difference effects, similar, but need specify from what \( x \) to what \( x \), and not just at what \( x \).

- Or you could **CLARIFY**… or **mfx**…
Complex Context-Conditionality and Nonlinear Least-Squares

- **Complex Context Conditionality**: The effect of anything depends on most everything else. E.g.:
  - **Policymaking**:
    - Socioeconomic-structure of interests
    - Party-system and internal party-structures
    - Electoral system & Governmental system
    - Socio-economic realities linking policies to outcomes
  - **Voting**:
    - Voter preferences & informational environment
    - Party/candidate locations & informational environment
    - Electoral & governmental system
  - **Institutions**: Sets of institutions; effect each depends configuration others present (e.g., that core of VoC claim).
  - **Strategic Interdependence**: each actors’ action depends on everyone else’s; complex feedback (see Franzese & Hays....)
Empirically \(\Rightarrow\) Multicollinear Nightmare: Options?

- Ignore context conditionality (stay linear-additive):
  - Inefficient at best, biased more usually, and, anyway, context-conditionality is our interest!
- Isolate one or some very few interactions for close study; ignore rest (stay linear-interactive):
  - Same, to degree lessened by amount interax allow, but demands on data rise rapidly w/ that amount.
- “Structured Case Analysis”:
- **EMTI\textsuperscript{TM}**: Lean harder on thry/subst to specify more precisely the nature interax: functional form, precise measures, etc.
  - Refines question put to the data (changes default tests also).
  - *GIVEN* thry/subst. specification into empirical model, can estimate complex interactivity. Side benefits. But must *give*. 
Nonlinear Least-Squares

Estimate NLS:
\[ y = f(X, \beta) + \epsilon \text{ with } \epsilon \sim g(\epsilon) \]
\[ \Rightarrow E(y) = f(X, \beta), \text{ so } y = f(X, \hat{\beta}) + \hat{\epsilon} \]
\[ \Rightarrow \text{Min } _{\beta} \hat{\epsilon}'\epsilon \Rightarrow \text{Min } _{\beta} [y - f(X, \hat{\beta})]'[y - f(X, \hat{\beta})] \]
\[ \Rightarrow \text{Min } _{\beta} SSE = y'y - y'f(X, \hat{\beta}) - f(X, \hat{\beta})'y + f(X, \hat{\beta})'f(X, \hat{\beta}) \]
\[ \Rightarrow \text{FOC: } \nabla _{\beta} SSE = 0 \Rightarrow -2\nabla _{\beta} f(X, \hat{\beta})'y + 2\nabla _{\beta} f(X, \hat{\beta})'f(X, \hat{\beta}) = 0 \]

So, if, e.g., \( f(X, \beta) = X\beta \), then: \( X'y = X'X\hat{\beta} \Rightarrow \hat{\beta} _{LS} = (X'X)^{-1}X'y \), and if
\[ V(\epsilon) \equiv \Omega = \sigma^2I, \text{ then } \hat{V}(\hat{\epsilon}) _{LS} = \frac{1}{n-k}[y - f(X, \hat{\beta} _{LS})]'[y - f(X, \hat{\beta} _{LS})] \] (also, as always).
That is, intuitively, writing \( \nabla _{\beta} f(X, \hat{\beta} _{LS}) \) as simply \( \nabla \), we have:
\[ \hat{\beta} _{LS} = (\nabla \nabla)^{-1}\nabla y \]
\[ \nabla (\hat{\beta} _{LS}) _{LS} = \nabla [(\nabla \nabla)^{-1}\nabla y] = (\nabla \nabla)^{-1}\nabla \nabla (y)(\nabla \nabla)^{-1}, \]
which if \( f(X, \hat{\beta} _{LS}) = X\hat{\beta} _{LS} \) meaning \( V = X \), & if \( \Omega = \sigma^2I \), gives the familiar
\[ \hat{\beta} _{LS} = (X'X)^{-1}X'y \& \nabla (\hat{\beta} _{LS}) _{LS} = \sigma^2(X'X)^{-1}, \text{ as always.} \]

- NLS is BLUE under same conditions OLS, w/ \( \nabla \) for \( X \).
- Interpreting NLS (already know how): Effects = deriv’s & 1st-diff’s; s.e.’s by Delta Method or simulation…
Generalized Nonlinear Least-Squares

- **GNLS:**
  \[ y = f(X, \beta) + \varepsilon \quad \text{with} \quad V(\varepsilon) = \sigma^2 \Omega \neq \sigma^2 I \]
  \[ \Rightarrow \hat{\beta}_{GNLS} = (\nabla' \Omega^{-1} \nabla)^{-1} \nabla' \Omega^{-1} y \]
  \[ \Rightarrow V(\hat{\beta}_{GNLS}) = (\nabla' \Omega^{-1} \nabla)^{-1} \nabla' \Omega^{-1} V(y) \Omega^{-1} \nabla (\nabla' \Omega^{-1} \nabla)^{-1} \]
  \[ = (\nabla' \Omega^{-1} \nabla)^{-1} \nabla' \Omega^{-1} \Omega \Omega^{-1} \nabla (\nabla' \Omega^{-1} \nabla)^{-1} \]
  \[ = (\nabla' \Omega^{-1} \nabla)^{-1} \nabla' \Omega^{-1} \nabla (\nabla' \Omega^{-1} \nabla)^{-1} = (\nabla' \Omega^{-1} \nabla)^{-1} \]

- **GNLS is BLUE in same cond’s NLS, but Ω for I.**
- **…don’t know Ω, so need consistent 1st stage (e.g., NLS)**
- **FGNLS is asymptotically BLUE:**
  \[ y = f(X, \beta) + \varepsilon \quad \text{with} \quad V(\varepsilon) = \sigma^2 \Omega \neq \sigma^2 I \]
  \[ \Rightarrow \hat{\beta}_{FGNLS} = (\nabla' \hat{\Omega}^{-1} \nabla)^{-1} \nabla' \hat{\Omega}^{-1} y \]
  \[ \Rightarrow V(\hat{\beta}_{FGNLS}) = (\nabla' \hat{\Omega}^{-1} \nabla)^{-1} \nabla' \hat{\Omega}^{-1} V(y) \hat{\Omega}^{-1} \nabla (\nabla' \hat{\Omega}^{-1} \nabla)^{-1} \]
  \[ = (\nabla' \hat{\Omega}^{-1} \nabla)^{-1} \]
Nonlinear Least-Squares & EMTI

- **EITM**: Empirical Implications of Theoretical Models
  - **Vision**: Theory ⇒ more, sharper predictions ⇒ better tests, which therefore inform theory more, which...

- **TMEI**: Theory-specified Models for Empirical Inference
  - **Vision**: Theory structures empirical models & relations b/w obs ⇒ specification & (causal) i.d. of empirical models

- **TIEM**: Theoretical Implications of Empirical Measures
  - **Vision**: Emp. regularities, findings, measures inform theory dev’p.

- **EMTI**: Empirical Models of Theoretical Intuitions
  - **Vision**: Intuitions derived from theoretical models specify empirical models. I.e., empirical specification to match intuitions, not model.

- **Note**: Strongly counter some alternative moves stats & econometrics, & related; there toward non-parametric, matching, & experimentation—there, “model-dependence” a 4-letter word. Alternative audiences & rhetorical purposes?
  - Convince skeptic some causal effect exists, vs.
  - For the convinced, give richer, portable model of how world works.
Nonlinear Least-Squares:
"Multiple Hands on the Wheel" Model (Franzese, PA ‘03)

- Monetary Policy in Open & Institutionalized Econ
  - Key C&IPE Insts/Struct: CBI, ER-Regime, Mon. Open
    - CBI ≡ Govt Delegated Mon Pol to CB
    - Peg ≡ Domestic (CB&Gov) Delegate to Peg-Curr (CB&Gov)
    - FinOp ≡ Dom cannot delegate b/c effectively del’d to globe
  - Effect of ev’thing to which for. & dom. mon. pol-mkrs would respond diff’ly depends on combo insts-structs & v.v., & through intl inst-structs, for. on dom. & v.v.

\[
\pi = \begin{cases} 
P \cdot E \cdot C \cdot \pi_1(X_1) + P \cdot E \cdot (1-C) \cdot \pi_2(X_2) \\
+P \cdot (1-E) \cdot C \cdot \pi_3(X_3) + P \cdot (1-E) \cdot (1-C) \cdot \pi_4(X_4) \\
(1-P) \cdot E \cdot C \cdot \pi_5(X_5) + (1-P) \cdot E \cdot (1-C) \cdot \pi_6(X_6) \\
+(1-P) \cdot (1-E) \cdot C \cdot \pi_7(X_7) + (1-P) \cdot (1-E) \cdot (1-C) \cdot \pi_8(X_8)
\end{cases}
\]

- Multicolinear Nightmare:
  - 2^3 = 8 inst-struct conds, \(i\) times \(k\) factors per \(\pi_1(X_i)\) if lin-interact
  - Exponentially more if all polynomials; \(k!/(k-2)!\) if all pairs.
  - Good thing can lean on some thry to specify more precisely!
Nonlinear Least-Squares: “Multiple Hands on the Wheel” Model

- CB & Govt Interaction (Franzese, AJPS ‘99):

\[
E(\pi) = c \cdot \pi_c(x_c) + (1 - c) \cdot \pi_g(x_g)
\]

\[
\pi_c = \pi \quad \pi_g(x_g) = \pi_g(GP, UD, BC, TE, EY, FS, AW, \pi_a)
\]

- Full Monetary Exposure & Atomistic \implies zero domestic autonomy \implies

\[
\begin{align*}
\pi_1(x_1) &= \pi_2(x_2) = \pi_5(x_5) = \pi_6(x_6) = \pi_a \\
\Rightarrow & \quad \begin{cases} 
E \cdot \pi_a + P \cdot (1 - E) \cdot C \cdot \pi_3(x_3) + P \cdot (1 - E) \cdot (1 - C) \cdot \pi_4(x_4) \\
+ (1 - P) \cdot (1 - E) \cdot C \cdot \pi_c + (1 - P) \cdot (1 - E) \cdot (1 - C) \cdot \pi_g(x_8)
\end{cases}
\end{align*}
\]

- s.t. that, full e.r.fix \implies CB&Govt match peg \implies

\[
\begin{align*}
\pi_3(x_3) &= \pi_4(x_4) = \pi_p \\
\Rightarrow & \quad \begin{cases} 
E \cdot \pi_a + P \cdot (1 - E) \cdot \pi_p \\
+ (1 - P) \cdot (1 - E) \cdot \left[ C \cdot \pi_c + (1 - C) \cdot \pi_g(x_8) \right]
\end{cases}
\end{align*}
\]
Nonlinear Least-Squares:
“Multiple Hands on the Wheel” Model

- Compact & intuitive, yet gives all theoretically expected interactions, in the form expected

\[
\pi = E \cdot \pi_a + (1 - E) \cdot \left\{ P \cdot \pi_p + (1 - P) \cdot \left[ C \cdot \pi_c + (1 - C) \cdot \pi_g (X_g) \right] \right\}
\]

\[
\frac{\partial \pi}{\partial E} = \pi_a \left( P^*, E^*, C^*, X^*, \pi_a^* \right) - \left\{ P \cdot \pi_p \left( P^*, E^*, C^*, X^*, \pi_p^* \right) + (1 - P) \cdot \left[ C \cdot \pi_c + (1 - C) \cdot \pi_g (X_g) \right] \right\}
\]

\[
\frac{\partial \pi}{\partial P} = (1 - E) \cdot \left\{ \pi_p \left( P^*, E^*, C^*, X^*, \pi_p^* \right) - \left[ C \cdot \pi_c + (1 - C) \cdot \pi_g (X_g) \right] \right\}
\]

\[
\frac{\partial \pi}{\partial C} = (1 - E) \cdot \left\{ (1 - P) \cdot \left[ \pi_c - \pi_g (X_g) \right] \right\}
\]

\[
\frac{\partial \pi}{\partial X} = (1 - E) \cdot \left\{ (1 - P) \cdot \left[ (1 - C) \cdot \frac{\partial \pi_g}{\partial X} \right] \right\}
\]

\[
\frac{\partial \pi}{\partial X^*} = E \cdot \frac{\partial \pi_a}{\partial X^*} + (1 - E) \cdot \left\{ P \cdot \frac{\partial \pi_p}{\partial X^*} + (1 - P) \cdot \left[ (1 - C) \cdot \frac{\partial \pi_g}{\partial \pi_a} \cdot \frac{\partial \pi_a}{\partial X^*} \right] \right\}
\]
Nonlinear Least-Squares:

“Multiple Hands on the Wheel” Model

- Effectively Estimable, yet gives all theoretically expected interactions, in the form expected

\[ E(\pi) = B_0 + \beta_e E \cdot \beta_{e},\pi_a + (1 - \beta_e E) \left[ \left( \beta_{gp} GP + \beta_{ey} EY + \beta_{up} UP + \beta_{bc} BC + \beta_{aw} AW + \beta_{fs} FS + \beta_{te} TE + \beta_a \pi_a \right) \right] \]

- Just 14 parameters (plus intercepts & dynamics, assuming those constant), just 3 more than lin-add!

- Parameters substantive meaning, too:
  - Degree to which...constrains certain set of actors.
  - Yields est. of inflation-target hypothetical fully indep CB
    - \( \Rightarrow \) general strategy for estimating/measuring unobservables
      - If know role factor will play & explanators of factor well enough, can estimate unobservables conditional on both those theories, if both powerful enough & enough empirical variation.
Nonlinear Least-Squares:
“Multiple Hands on the Wheel” Model

- Neat, but does it work? (Try it! Data online; stata: help nl). Estimated Equation, w/ Std. Errs.:

\[ E(\pi) = \begin{pmatrix}
0.53^{.05} + 0.55^{.05} \pi_{t-1} - 0.12^{.04} \pi_{t-2} + 0.44^{.14} E \cdot \pi_a + \\
0.1^{.05} SP \cdot 0.59^{.07} \pi_{sp} + 0.22^{.12} MP \cdot 0.59^{.07} \pi_{mp} + \\
(1 - 0.44^{.14} E) \begin{pmatrix}
1.0^{.05} SP & 0.22^{.12} MP \\
1.0^{.05} SP & 0.22^{.12} MP \\
(1 - 0.44) & (1 - 0.44) \\
(1 - 0.44) & (1 - 0.44)
\end{pmatrix}
\] + 1.0^{.11} C(-0.59^{.12}) + \\
\begin{pmatrix}
-0.60^{.30} GP + 2.6^{.13} EY + 16^{.46} UP - 11^{.24} BC \\
-0.60^{.30} GP + 2.6^{.13} EY + 16^{.46} UP - 11^{.24} BC \\
+ 1.2^{.49} AW - 1.1^{.30} FS - 8.2^{.49} TE + 0.64^{.24} \pi_a
\end{pmatrix} \]

- Estimated Effects (highly context-conditional):

\[ E\left(\frac{d\pi}{dC}\right) = (1 - 0.44 \cdot E) \cdot \left\{(1 - b_p \cdot P) \cdot [(0.6GP - 2.6EY - 16UP + 11BC - 1.2AW + 11FS + 8.2TE - 0.64\pi_a) - 0.59]\right\}
\]

\[ E\left(\frac{d\pi}{dx}\right) - (1 - 0.44E) \cdot \left\{(1 - SP - 0.22MP) (1 - C) b_x \right\}\]

\[ E\left(\frac{d\pi}{dP}\right) = (1 - 0.44E) b_p \cdot \left\{59\pi_p - ((1 - C)(-0.6GP + 2.6EY + 16UP - 11BC + 1.2AW - 11FS - 8.2TE + 0.64\pi_a) - 0.59)\right\}
\]

\[ E\left(\frac{d\pi}{dE}\right) = 0.44 \cdot \{\pi_a - \left\{b_p \cdot P \cdot 59\pi_p + (1 - b_p \cdot P) \cdot [(1 - C)(-0.6GP + 2.6EY + 16UP - 11BC + 1.2AW - 11FS - 8.2TE + 0.64\pi_a) - 0.59]\right\}\}
\]
### Nonlinear Least-Squares: “Multiple Hands on the Wheel” Model

**Table 2:** Estimated Effects of Domestic Political-Economic Conditions, $d\pi/x$, as Function of Central Bank Autonomy, CBA, International Monetary Exposure, $E$, and Exchange-Rate Regime, $P$

<table>
<thead>
<tr>
<th></th>
<th>Little Exposed (E Float)</th>
<th>Moderately Exposed (E Float)</th>
<th>Highly Exposed (E Float)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Basket Peg</td>
<td>Simple Peg</td>
<td>Basket Peg</td>
</tr>
<tr>
<td>central 0.26</td>
<td>+1.563$^{79}$</td>
<td>+1.224$^{61}$</td>
<td>+1.352$^{69}$</td>
</tr>
<tr>
<td></td>
<td>+0.000$^{09}$</td>
<td>+1.059$^{53}$</td>
<td>+0.000$^{07}$</td>
</tr>
<tr>
<td></td>
<td>+0.678$^{37}$</td>
<td>+0.531$^{29}$</td>
<td>+0.587$^{32}$</td>
</tr>
<tr>
<td></td>
<td>+0.000$^{04}$</td>
<td>+0.459$^{25}$</td>
<td>+0.000$^{03}$</td>
</tr>
<tr>
<td>bank 0.46</td>
<td>+1.120$^{57}$</td>
<td>+0.877$^{44}$</td>
<td>+0.970$^{50}$</td>
</tr>
<tr>
<td></td>
<td>+0.000$^{06}$</td>
<td>+0.759$^{39}$</td>
<td>+0.000$^{05}$</td>
</tr>
<tr>
<td></td>
<td>+0.495$^{28}$</td>
<td>+0.388$^{22}$</td>
<td>+0.000$^{05}$</td>
</tr>
<tr>
<td>auton. 0.66</td>
<td>+0.678$^{37}$</td>
<td>+0.531$^{29}$</td>
<td>+0.587$^{32}$</td>
</tr>
<tr>
<td></td>
<td>+0.000$^{04}$</td>
<td>+0.459$^{25}$</td>
<td>+0.000$^{03}$</td>
</tr>
</tbody>
</table>

**Estimated Impact of a Post-Election Year ($d\pi/d\varepsilon_Y$)**

<table>
<thead>
<tr>
<th></th>
<th>Estimated Impact of 10% Increase in Union Density ($0.1d\pi/dUP$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>central 0.26</td>
</tr>
<tr>
<td></td>
<td>+0.98$^{55}$</td>
</tr>
<tr>
<td></td>
<td>+0.00$^{05}$</td>
</tr>
<tr>
<td></td>
<td>+0.37$^{11}$</td>
</tr>
</tbody>
</table>

**Estimated Impact of 1% Increase in Financial-Sector Employment-Share ($d\pi/dFS$)**

|                  | -0.66$^{18}$        | -0.52$^{12}$    | -0.00$^{03}$     |
|                  | -0.57$^{16}$        | -0.45$^{11}$    | -0.00$^{03}$     |
|                  | -0.25$^{09}$        | -0.19$^{06}$    | -0.00$^{01}$     |

**Estimated Impact of 1% Increase in Average Inflation Abroad ($d\pi/d\pi$)**

|                  | +0.49$^{14}$        | +0.41$^{13}$    | +0.11$^{05}$     |
|                  | +0.50$^{12}$        | +0.43$^{11}$    | +0.17$^{07}$     |
|                  | +0.32$^{06}$        | +0.28$^{06}$    | +0.17$^{06}$     |

**Notes:** These are first-year effects, meaning before the estimated dynamics unfold. Standard errors noted in superscripts.
Nonlinear Least-Squares: “Multiple Hands on the Wheel” Model

Figure 1. Estimated Partisan Cycles in the Linear & Theoretically Informed Models at High & Low CBA, E, & MP
Nonlinear Least-Squares: 
“Multiple Hands on the Wheel” Model

*Figure 2*: Estimated Domestic-Inflation Effect of Actual or Counter-Factual $\Delta P$ in 21 Countries, 1957-90. Estimates plotted for $\Delta \text{INF}/\Delta \text{SP}$ at the values of all other variables in the equation actually occurring in that country-year. For counter-factual pegs, peg country assumed to have OECD-average inflation that year. Shading separates countries and extends from 1955 to 1990 in each country, left to right.
Multiple Policymakers: Veto Actors Bargaining in Common Pools

- Multiple implications for policy outcomes dispersal of policymaking-authority across diverse actors:
  - Veto-Actor Theory (Tsebelis ‘02) emphasizes:
    - Privileges S.Q., & so retards policy adjustment, reduces change.
  - Collective-Action/Common-Pool Theories (WSJ ‘81):
    - Externalities & so overexploit/underinvest public goods.
  - Bargaining & Delegation Theories rather stress:
    - Bargaining Strengths/Positions, yielding Weighted Compromise.

- This project attempts a synthesis:
  - Disting. theoretically/conceptually many effects of # (fragment.) & diversity (polar., partisan) policymakers.
  - Empirical model of many effects distinctly & effectively.
  - Preliminary application to evolution fiscal policy (pub debt) in developed democracies, 1950s-90s.
**Veto Actors: Deadlock, Delayed Stabilization, & Policy-Adjustment Retardation**

- Tsebelis (‘95b, ‘99, ‘00, ‘02): Essential Argument:
  - ↑ # &/or ideological/interest polarization of pol-mkng actors whose approval required to ΔSQ, i.e., *veto actors*, ⇒, loosely, ↓ probability &/or magnitude policy Δ.
  - I.e., strictly, as size W(SQ) ↓, which generally does as # &/or polarization VA ↑, range *possible* policy Δ(SQ) ↓.
  - ⇒ following empirical prediction (Tsebelis 2002, Fig. 1.7):
    - Suggests both mean/expected policy-Δ & variance pol & pol-Δ ↑↓ as size of W(SQ) ↑↓ (aside: why only suggests)
    - No prediction of pol-level or of direction pol-Δ, only of E(|Δp|), V(Δp).
Veto-Actor Implications

- ↑ # (Frag) & Polar of VA Privileges SQ ⇒
  - Retards policy-adjustment rates/delays stabilization,
  - ↓ range of possible policy-Δ, & so, possibly,
  - ↓ magnitude/variance policy- Δ (1st- & 2nd-order E(Δ)).

- Results, e.g. in fiscal policy, deficits & debts; originally mixed, but tighter specify thry into empirical analysis:
  - (F '00, '02) **How model:** policy-adjustment-rate effect =
    conditional coefficient on LDV in dynamic model, not level.
  - (F '00, '02) **How measure:** frag & polar in VA theory =
    - raw #, not eff. # (size-wtd) VA;
    - max range pref’s, not V(pref’s) or sd(pref’s), (size-wtd)

- ⇒ Model: \( y_t = \ldots + \theta(\#VA, \text{Range}\{\text{pref}(VA)\}) \times y_{t-1} \ldots \)
  &/or \( V(y_t) = f(\#, \text{Range}) \) ⇒ empirical support.
Common-Pool Theory (1)

  - Benefits concentrate district $i$: $B_i = f(C)$, $f' > 0$ & $f'' < 0$
  - Costs disperse across $n$ districts: $C_i = C/n$
  - **optimal project-size from $i$’s view ↑ in # districts**: $f'(C^*) = 1/n$  
    (...log-linearly?)

- **Alternative Decision Rules/Processes [...] ⇒**
  - [...] *Law of 1/n* is general, & stronger as legislative behavior more Universalistic & less Minimal-Winning, which tendency ↑ as rational ignorance, winning-coalition uncertainty, or legislative-rule closure to amend or veto ↑.
  - E.g., PubRev = common pool for $n$ reps, overused to distribute bens; this CA prob worsens “proportionally” by *law 1/n*, i.e. at rate b/w those at which $(n+1)/2n$ (MWC) & $1/n$ (uni) ↓ in $n$

![Graph showing Minimum-Winning-Coalition Decision-Making and Universalistic Decision-Making](Slide 68)
Manifestations of Common Pools

- Velasco (‘98, ‘99, ‘00): inter-temporal totality pub rev is C-P to today’s policymakers ⇒ deficits & debts also law of 1/n
- Peterson & co’s, Treisman: federalism ⇒ multiple tax authorities ⇒ several common-pool problems:
  - Inter-jurisdiction competition (w/ high factor mobility) ⇒ C-P of investment resources ⇒ over-fishing: taxes too low.
  - National govt as lender last resort ⇒ subnational jurisdictions see fed guarantee & funds as common pool ⇒ excessive borrowing by subnat’l units. (e.g., EU, EMU & Euro ⇒ common pools…)
- Again, should be quite general:
  - Anything that gains set of pol-makers credit ⇒ underinvested as ↑n
  - Anything that gains set of pol-makers blame ⇒ overexploited as ↑n
- E.g., (thry 2nd-best), ELECTIONEERING:
  - Magnitude incentive electioneer fades w/ n (see, e.g., Goodhart)
  - Control over electioneering diminishes w/ n.
- Notice: CP not arise in Tsebelis’ VA Theory b/c # & pref’s of VA’s exog & predetermined, whereas in CP theory: prefs=f(#).
Modeling Common-Pool Effects

- CP Effects distinguishable from VA Effects:
  - C-P Effects on *levels*, not (as in VA) in dynamics.
  - Proportional to $1/n$ for *equal-sized* actors. Standard Olsonian encompassingness logic $\Rightarrow$ proper $n$ here *is* size-weighted (effective & s.d./var.)
  - Fractionalization (#) & esp. polarization (het.) relate to VA effects; CP, conversely, relate primarily to #, although het. can exacerbate some CA probs.

- Suggests Proper Model of Policy-Response to some public demand for, $x_1 \beta_1$, or against, $x_2 \beta_2$:
  - $\ldots + (x_1 \beta_1)(1-f(ln(Eff#)) + (x_2 \beta_2)(1+f(ln(Eff#)) + \ldots$
  - *Same* $f(ln(Eff#))$, b/c overexploit/underinvest same
Bargaining, Delegation, & Compromise

- **Explicit extensive-form delegation & bargaining games:** huge theoretical & empirical literature

- **F (‘99, ‘02, ‘03):** less context-specific empirical strategy...
  - Because broad comparativist seek thry that *travels*, not that requires different model each context.

- **Offering is roughly equivalent Nash Bargaining.**
  - Most ext forms ⇒ eqbm bounded by actors’ ideal pts:
    - Convex set/hull, upper-contour set (=core of coop. game thry),
    - So like Tsebelis, but further, though short of explicit ext-form
  - Policy outside that range possible,
    - e.g., if uncertainty resolved unfavorably,
    - but that ⇒ highly unlikely that E(pol) outside this range
  - Thus, E(pol)=some convex-combo (wtd-avg) pol-mkrs’ ideals ⇒ convex-combo emp. models ≈ compromise
    - If Nash Bargain, e.g., (n.b. NB=coop. game-thry but equiv. sev. reasonable ext-form non-coop barg. games: Rubinstein ‘82), ⇒ (geometric) *wtd-influence pol-mkng*; i.e., simple wtd-avg.
Empirical Manifestations & Model of Compromise Policymaking

- **Re: def’s & debt, Cusack (‘99, ‘01; cf., Clark ‘03)**
  - *Arg:* left more Keynes-active counter-cyc; right less, even pro-cyc
  - *Add Nash-Barg Model ⇒ wtd-avg pol-mkr partisanship conditions ° Keynesian cntr-cyc fisc-pol response to macroecon.*

- **Empirical Implementation:**
  - **Ideally:**
    - Describe barg power party $i$ as $f(\text{charact’s } i \& \text{ barg envir, } j, \Rightarrow f(v_{ij})$
    - Desc. pol response to conditions $x_k$ if $i$ sole pol-mkng control: $q_i(x_k)$
    - Then embed Nash-Barg sol’n, $\Sigma_i f(v_{ij})q_i(x_k)$, in emp. model to est.
  - **Currently:**
    - Assume wtd-avg compromise outcome pre-estimation.
    - I.e., simply assume by measure & specification that Policy responds to $WtdPartisanship\times CurrGovt \times MacroeconomicConditions$. 
Empirical Model of the Theoretical Synthesis (1)

- Different aspects of policy-maker fragmentation, polarization, & partisanship:
  - V-A Effects: raw # (frag) and ideological ranges (polar)
  - C-P Effects: eff # (frag) &; maybe, ideol. s.d./var (polar)
  - D-B Effects: power-wtd mean ideologies (partisanship)

- Different ways these distinct effects manifest in pol:
  - V-A (primarily) to slow pol-adjust (delay stabilization);
  - C-P induces over-draw from common resources (incl. from future as in debt); under-invest in common properties (less electioneering), log-proportionately
  - D-B induces convex-combinatorial (compromise) policies, incl. greater left-activist/right-conservative Keynesian-countercyclical/conservative pro-cyclical, in proportion to degree left/right controls policymaking
Empirical Model of the Theoretical Synthesis (2)

...implies specification where:

- Abs # VA & ideol range modify pol-adjust rates
- (log) Eff # pol-mkrs & s.d. ideol (wtd measures) gauge C-P prob in electioneering (+debt-lvl effect?)
- Some barg process among partisan pol-mkrs (e.g., Nash \( \Rightarrow \) wtd-influence) determines combo reflected in net policy responsiveness to macro (° K-activism)

\[
D_{it} = \alpha_i + \left(1 + \rho_n \text{NoP}_{it} + \rho_{ar} \text{ARwiG}_{it}\right) \times \left(\rho_1 D_{i,t-1} + \rho_2 D_{i,t-2} + \rho_3 D_{i,t-3}\right) \\
+ \left(\beta_{\Delta Y} \Delta Y_{i,t} + \beta_{\Delta U} \Delta U_{i,t} + \beta_{\Delta P} \Delta P_{i,t}\right) \times \left(1 + \beta_{cg} \text{CoG}_{it}\right) \\
+ \left(\gamma_{e1} \text{E}_{it} + \gamma_{e2} \text{E}_{i,t-1}\right) \times \left(1 + \gamma_{en} \text{ENoP}_{it} + \gamma_{sd} \text{SDwiG}_{it}\right) + x'_i \eta + z'_i \omega + \epsilon_{it}
\]
Empirical Model Specification & Data

\[ D_{it} = \alpha_i + (1 + \rho_n \text{NoP}_{it} + \rho_{ar} \text{ARwiG}_{it}) \times (\rho_1 D_{i,t-1} + \rho_2 D_{i,t-2} + \rho_3 D_{i,t-3}) + x' \eta + z' \omega + \varepsilon_{it} \]

- \( D_{it} \) = Debt (%GDP);
- \( \text{NoP} & \text{ARwiG} = \) raw Num of Prtys in Govt & Abs Range w/i Govt:
  - VA conception, so modify dynamics. Expect \( \rho_n \) & \( \rho_{ar} > 0 \).
  - By thry & for efficiency: modify all lag dynamics same.
- CoG (govt center, left to right, 0-10):
  - Modifies response to macroecon (equally, by thry & for eff’cy) : \( \beta_{cg} < 0 \).
  - Macroec: \( \Delta Y = \) real GDP growth; \( \Delta U = \) \( \Delta \) unemp rate; \( \Delta P = \) infl rate.
- \( x' \eta \) = controls: set pol-econ cond’s response to which not partisan-differentiated or gov comm-pool: (e.g., E(real-int)-E(real-grow), \( \text{T}o\text{T} \))
- ENoP & SDwiG = Effective Num of Prtys in govt & Std Dev w/i Govt:
  - Frag & Polar by \( \text{wtd-influence} \) concept. CP lvl-effects modify (at same rate) electioneering, \( E_t \), pre-elect-year, & \( E_{t-1} \), post-elect-yr.: \( \gamma_{en} \) & \( \gamma_{sd} < 0 \).
- \( z' \omega \) = set of constituent terms in the interactions:
  - ENoP, SDwiG \( \text{may} \) have positive coeff’s by CP effect lvl debt, but issue is \( \text{temporal fract} \) more than curr. govt fract. Thry o/w says omit.
|                          | Coeff. | Std. Err. | t-Stat. | Pr($T>|t|$) |
|--------------------------|--------|-----------|---------|-------------|
| **Lagged**               |        |           |         |             |
| $D_{t-1}$                | 1.212  | 0.060     | 20.112  | 0.000       |
| $D_{t-2}$                | -0.153 | 0.085     | -1.792  | 0.074       |
| $D_{t-3}$                | -0.121 | 0.045     | -2.677  | 0.008       |
| **Dependent Variables**  |        |           |         |             |
| $\rho_n$ (veto-actor effect: fractionalization) | **0.007** | **0.006** | **1.089** | **0.277** |
| $\rho_{ar}$ (veto-actor effect: polarization)   | **-0.000** | **0.006** | **-0.013** | **0.990** |
| **Macroeconomic Conditions** |        |           |         |             |
| $\Delta Y$              | -0.336 | 0.111     | -3.033  | 0.003       |
| $\Delta U$              | 0.992  | 0.308     | 3.219   | 0.001       |
| $\Delta P$              | -0.188 | 0.063     | -2.965  | 0.003       |
| $\beta_{cg}$ (partisan-compromise bargaining) | **-0.037** | **0.037** | **-0.988** | **0.323** |
| $x_1$ (open)            | 15.891 | 5.279     | 3.010   | 0.003       |
| $x_2$ (ToT)             | 0.388  | 1.744     | 0.222   | 0.824       |
| $x_3$ (open · ToT)      | -10.681| 5.156     | -2.072  | 0.039       |
| $x_4$ (dxrig)           | -0.036 | 0.066     | -0.544  | 0.587       |
| $x_5$ (gy)              | 2.064  | 1.094     | 1.886   | 0.060       |
| **Pre- and Post-Electoral Indicators** |        |           |         |             |
| $E_t$                   | 0.687  | 0.568     | 1.210   | 0.227       |
| $E_{t-1}$               | 1.490  | 0.645     | 2.310   | 0.021       |
| $\gamma_{en}$ (common-pool effect: fractionalization) | **-0.547** | **0.182** | **-3.001** | **0.003** |
| $\gamma_{sd}$ (common-pool effect: polarization) | **0.573** | **0.486** | **1.179** | **0.239** |
| $z_1$ (CoG)             | 0.051  | 0.131     | 0.390   | 0.697       |
| $z_2$ (ENoP)            | 0.281  | 0.446     | 0.629   | 0.530       |
| $z_3$ (SDwG)            | 0.542  | 0.437     | 1.242   | 0.215       |
| $z_4$ (NoP)             | 0.181  | 0.277     | 0.654   | 0.514       |
| $z_5$ (ARwG)            | -0.312 | 0.259     | -1.205  | 0.228       |

### Summary Statistics

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N (Deg. Free)</strong></td>
<td>735</td>
<td>(691)</td>
<td>s_e^2</td>
<td>2.525</td>
</tr>
<tr>
<td><strong>R^2 (\bar{R}^2)</strong></td>
<td>0.991</td>
<td>(0.990)</td>
<td>DW-Stat.</td>
<td>2.101</td>
</tr>
</tbody>
</table>
Pace Brambor et al. (‘06), but joint-significance of multiple-policymaker conditioning effects \((\gamma_{en}, \gamma_{sd}, \rho_{n}, \rho_{ar}, \beta_{cg})\) overwhelmingly rejects excluding \((p \approx .001)\), whereas joint-sig coeff’s on constit. terms, \(z\), clearly fails reject \((p \approx .602)\) exclusion. (Almost) All theory says should be zero, so...

| Lagged Variables | Coeff. | Std. Err. | \(t\)-Stat. | \(Pr(T > |t|)\) |
|------------------|--------|-----------|-------------|----------------|
| \(D_{t-1}\)     | 1.207  | 0.060     | 20.290      | 0.000          |
| \(D_{t-2}\)     | -0.158 | 0.085     | -1.851      | 0.065          |
| \(D_{t-3}\)     | -0.117 | 0.045     | -2.577      | 0.010          |

| Dependent Variables | Coeff. | Std. Err. | \(t\)-Stat. | \(Pr(T > |t|)\) |
|---------------------|--------|-----------|-------------|----------------|
| \(\rho_{n}\) (veto-actor effect: fractionalization) | 0.011  | 0.005     | 2.369       | 0.018          |
| \(\rho_{ar}\) (veto-actor effect: polarization)  | -0.002 | 0.004     | -0.437      | 0.662          |

| Macroeconomic Conditions | Coeff. | Std. Err. | \(t\)-Stat. | \(Pr(T > |t|)\) |
|--------------------------|--------|-----------|-------------|----------------|
| \(\Delta Y\)            | -0.375 | 0.087     | -4.332      | 0.000          |
| \(\Delta U\)            | 1.095  | 0.286     | 3.829       | 0.000          |
| \(\Delta P\)            | -0.207 | 0.053     | -3.889      | 0.000          |

| \(\beta_{cg}\) (partisan-compromise bargaining) | Coeff. | Std. Err. | \(t\)-Stat. | \(Pr(T > |t|)\) |
|--------------------------------------------------|--------|-----------|-------------|----------------|
| \(x_{1}\) (open)                                | 16.128 | 5.314     | 3.035       | 0.002          |
| \(x_{2}\) (ToT)                                 | 0.414  | 1.728     | 0.239       | 0.811          |
| \(x_{3}\) (open \cdot ToT)                      | -10.780| 5.194     | -2.076      | 0.038          |
| \(x_{4}\) (dxrig)                               | -0.038 | 0.066     | -0.578      | 0.563          |
| \(x_{5}\) (ov)                                  | 1.898  | 1.100     | 1.724       | 0.085          |

| Pre- and Post-Electoral Indicators | Coeff. | Std. Err. | \(t\)-Stat. | \(Pr(T > |t|)\) |
|-----------------------------------|--------|-----------|-------------|----------------|
| \(E_{t}\)                         | 0.475  | 0.420     | 1.133       | 0.258          |
| \(E_{t-1}\)                       | 1.146  | 0.562     | 2.040       | 0.042          |

| Summary Statistics | Coeff. | Std. Err. | \(t\)-Stat. | \(Pr(T > |t|)\) |
|--------------------|--------|-----------|-------------|----------------|
| \(N\) (Deg. Free)  | 735 (696) |          |             |                |
| \(R^{2}\) (\(R^{2}\)) | 0.991 (0.990) |       |             |                |

\(\gamma_{en}\) (common-pool effect: fractionalization)  
\(\gamma_{sd}\) (common-pool effect: polarization)
### Veto-Actor Effects: Estimates of Policy-Adjustment Rate

<table>
<thead>
<tr>
<th>Adjustment Rates</th>
<th>NoP=1</th>
<th>NoP=2</th>
<th>NoP=3</th>
<th>NoP=4</th>
<th>NoP=5</th>
<th>NoP=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag Coefficient</td>
<td>0.943</td>
<td>0.952</td>
<td>0.960</td>
<td>0.969</td>
<td>0.978</td>
<td>0.986</td>
</tr>
<tr>
<td>Policy-Adjust/Yr</td>
<td>0.057</td>
<td>0.048</td>
<td>0.040</td>
<td>0.031</td>
<td>0.022</td>
<td>0.014</td>
</tr>
<tr>
<td>Long-Run Mult.</td>
<td>17.498</td>
<td>20.639</td>
<td>25.154</td>
<td>32.200</td>
<td>44.727</td>
<td>73.208</td>
</tr>
<tr>
<td>90%-Life</td>
<td>39.127</td>
<td>46.362</td>
<td>56.761</td>
<td>72.985</td>
<td>101.832</td>
<td>167.415</td>
</tr>
</tbody>
</table>

### Bargaining Effects: Estimates of Keynesian Fiscal Responsiveness

<table>
<thead>
<tr>
<th>Mean Econ. Performance</th>
<th>Mean Econ. Performance</th>
<th>Mean Econ. Performance</th>
<th>Mean Econ. Performance</th>
<th>Mean Econ. Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2 std. dev.</td>
<td>-1 std. dev.</td>
<td>+1 std. dev.</td>
<td>+2 std. dev.</td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>-2.354</td>
<td>0.454</td>
<td>3.261</td>
<td>6.069</td>
</tr>
<tr>
<td>d(UE)</td>
<td>1.915</td>
<td>1.034</td>
<td>0.153</td>
<td>-0.728</td>
</tr>
<tr>
<td>Infl</td>
<td>-3.593</td>
<td>1.230</td>
<td>6.054</td>
<td>10.877</td>
</tr>
</tbody>
</table>

### Collective-Action/Common-Pool Effects: Estimates of Electoral Debt-Cycle Magnitude

<table>
<thead>
<tr>
<th>Electoral-Cycle Magnitude</th>
<th>ENoP=1</th>
<th>ENoP=2</th>
<th>ENoP=3</th>
<th>ENoP=4</th>
<th>ENoP=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electoral-Cycle Magnitude</td>
<td>1.07410</td>
<td>0.86454</td>
<td>0.65497</td>
<td>0.44541</td>
<td>0.23585</td>
</tr>
</tbody>
</table>
Extension & Refinement

\[ E(y_t) = \delta^0 + x_t^0 b^0 + \left( \rho_0 + \rho_1 \ln(NoP_t) + \rho_2 \ln(1 + ARwiG_t) \right) y_{t-1} \]

\[ + \left[ x_t^1 b^1 + \sum_{i=1}^{l} p(c_{it}) \times q_i(x_t^2) \right] \times [1 + \alpha_1 \ln(NoP_t) + \alpha_2 \ln(1 + ARwiG_t)] \]

\[ \times [1 + \gamma_1 \ln(ENoP_t) + \gamma_2 \ln(1 + SDwiG_t)] \]

- \( x^0 \) = factors that affect policy-outcomes unless pol-mkrs act to change status quo, i.e., that have effect on pol-out directly.
- \( x^1 \) = factors affecting policy-outcomes if policymakers act to change status quo, without partisan-differentiated response
- \( x^2 \) = factors affecting policy-outcomes if policymakers act to change status quo, with partisan-differentiated response
- \( \{NoP,ARwiG\} \) = sources of veto-actor effects; as before
- \( \{ENoP,SDwiG\} \) = sources of common-pool effects; as before
- \( \{p(c_{it}),q_j(x_t)\} \) = sources of bargaining & delegation effects:
  - \( p(c_{it}) \): Effective policy-influence of party \( i \) in context \( t \). (E.g., as now: cabinet seat-shares, but could become richer model.)
  - \( q_j(x_t) \): Model of response of party \( i \) to pol-econ conditions \( x_t \). (E.g., as now: \( CoG_i \times Macroecon_t \), but could become richer model.)
## Results of Fuller Model

| Temporal Dynamics | Coeff. | Std. Err. | t-Stat. | Pr(>|t|) |
|------------------|--------|-----------|---------|----------|
| D(t-1)           | 1.197  | 0.059     | 20.144  | 0.000    |
| D(t-2)           | -0.139 | 0.085     | -1.629  | 0.104    |
| D(t-3)           | -0.121 | 0.045     | -2.698  | 0.007    |

| Veto-Actor Effect on Outcome-Adjustment Rate | Coeff. | Std. Err. | t-Stat. | Pr(>|t|) |
|---------------------------------------------|--------|-----------|---------|----------|
| NoP                                         | 0.008  | 0.004     | 1.883   | 0.060    |

| Variable | Coeff. | Std. Err. | t-Stat. | Pr(>|t|) |
|----------|--------|-----------|---------|----------|
| Open     | 16.624 | 3.758     | 4.423   | 0.000    |
| Open*ToT | -11.190| 3.135     | -3.569  | 0.000    |

| Variable | Coeff. | Std. Err. | t-Stat. | Pr(>|t|) |
|----------|--------|-----------|---------|----------|
| Ele(t)   | 0.315  | 0.363     | 0.867   | 0.386    |
| Ele(t-1) | 0.873  | 0.399     | 2.186   | 0.029    |
| OY       | 2.833  | 1.295     | 2.187   | 0.029    |
| DXRIG3   | -0.073 | 0.072     | -1.009  | 0.314    |

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>ln(ENoP)</td>
<td>-0.277</td>
<td>0.071</td>
<td>-3.903</td>
<td>0.000</td>
</tr>
</tbody>
</table>

| Variable | Coeff. | Std. Err. | t-Stat. | Pr(>|t|) |
|----------|--------|-----------|---------|----------|
| Growth   | -0.238 | 0.084     | -2.815  | 0.005    |
| d(UE)    | 0.749  | 0.228     | 3.289   | 0.001    |
| Inflation| -0.137 | 0.047     | -2.947  | 0.003    |

| Bargaining-Compromise Effects on Partisan Policy-Responses | Coeff. | Std. Err. | t-Stat. | Pr(>|t|) |
|----------------------------------------------------------|--------|-----------|---------|----------|
| CoG                                                      | -0.049 | 0.026     | -1.893  | 0.059    |

| Veto-Actor Effect on Policy-Adjustment Rate | Coeff. | Std. Err. | t-Stat. | Pr(>|t|) |
|--------------------------------------------|--------|-----------|---------|----------|
| NoP                                        | 0.215  | 0.121     | 1.773   | 0.077    |

| Common-Pool Effect on Debt Level | Coeff. | Std. Err. | t-Stat. | Pr(>|t|) |
|---------------------------------|--------|-----------|---------|----------|
| ln(ENoP)                        | 1.123  | 0.486     | 2.320   | 0.021    |

| Summary Statistics | Coeff. | Std. Err. | t-Stat. | Pr(>|t|) |
|--------------------|--------|-----------|---------|----------|
| N (Deg. Free)      | 735(697)| 4.86      | 2.320   | 0.021    |
| R² (R²)            | 0.991(0.990)| 2.510     | 2.090   |          |