Marginal Effect of Shocks and Shifts in Dynamic Models

William D. MacMillan

5/21/09

Suppose we have a model $Y_t = \lambda Y_{t-1} + \beta X_t + \epsilon_t$. A shock is $X_1 = 1$ for the first period, with $X_{t\neq 1} = 0$. For this case, in expectation,

$$Y_1 = \beta X_1 \tag{1}$$
$$Y_2 = \lambda(\beta X_1) \tag{2}$$
$$Y_3 = \lambda\lambda(\beta X_1) \tag{3}$$
$$= \lambda^{t-1}\beta X_1 \Rightarrow \frac{\partial Y}{\partial X} = \lambda^{t-1}\beta \tag{4}$$

For a shift, where $X_t = 1$

$$Y_1 = \beta X_1 \Rightarrow \frac{\partial Y}{\partial X} = \beta \tag{5}$$
$$Y_2 = \lambda(\beta X) + \beta X \Rightarrow \frac{\partial Y}{\partial X} = \lambda\beta + \beta \tag{6}$$
$$Y_3 = \lambda(\lambda(\beta X) + \beta X) + \beta X \Rightarrow \frac{\partial Y}{\partial X} = \lambda^2\beta + \lambda\beta + \beta \tag{7}$$
$$\frac{\partial Y}{\partial X} = \sum_{t=1}^{T} \beta\lambda^{t-1} \tag{8}$$

The marginal effect for either a shock or a shift is equations 4 and 8 evaluated at t, respectively. The associated .do file creates visualizations of the math presented here for a dynamic model.[1]

[1] available at `http://www-personal.umich.edu/~franzese/ICPSR/icpsr_dynamic_counterfactual.do`