LAB2 - Heterogeneity: FE, RE, Interaction, & RC/HLM/Multilevel/Mixed Models and Estimators

—Start a log file:

```
. log using "H:\HeterogeneityLab.log", append
```

—Point Stata to the directory where we've put some useful add-on (ADO) command files:

```
. adopath + "T:\CCB Released Files\Statistics Workshop Sep 2009\ado\plus/"
```

—We'll need to expand the base memory for Stata to use the following (large) dataset:

```
set memory 32m
```

—We'll need to expand the base memory for Stata to use the following (large) dataset:

```
use "H:\WS_Data_level1_merge_level2.dta", clear
```

—This multilevel dataset has already been **xtset** (although technically, it is *ij*, not *it*, data):

```
. xtset
       panel variable:  group (unbalanced)
```

—These are survey data on, *inter alia*, EU support in differently size samples from 17 European countries.

—Another way to see which variable is the panel identifies is:

```
. iis
i() is group
```

COMPARING THE *UNIT-BY-UNIT*, *WITHIN*, *POOLED*, *BETWEEN, RANDOM-EFFECT* MODEL ESTIMATES:

Unit-by-unit:

(by menus: Linear models and related—Linear regession—[then enter DepVar IndVar1 IndVar2 IndVar3]—[then by/if/in tab: check repeat command by groups, then enter `group` *in the space]*

`. by group, sort : regress eu_support education leftright edlr`

(Comment: just notice how much or little heterogeneity you see across units in coefficient estimates, R^2's, ...)

(This gets pretty noisy as the number of groups grows, as you can see ...)

```
-> group = 1

      Source |       SS       df       MS              Number of obs =     489
-------------+------------------------------           F(  3,   485) =   17.13
       Model |  465.652747     3  155.217582           Prob > F      =  0.0000
    Residual |  4394.23069   485  9.06026946           R-squared     =  0.0958
-------------+------------------------------           Adj R-squared =  0.0902
       Total |  4859.88344   488  9.95877753           Root MSE      =    3.01

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   education |   1.987964   .5057722     3.93   0.000     .9941884    2.981739
   leftright |   .1780868   .2423311     0.73   0.463    -.2980617    .6542352
        edlr |  -.1818668   .0976701    -1.86   0.063    -.3737757     .010042
       _cons |   1.619286   1.266475     1.28   0.202    -.8691691     4.10774
------------------------------------------------------------------------------
 [...]
-> group = 17

      Source |       SS       df       MS              Number of obs =     476
-------------+------------------------------           F(  3,   472) =   16.99
       Model |  355.764476     3  118.588159           Prob > F      =  0.0000
    Residual |  3295.33374   472  6.98163928           R-squared     =  0.0974
```

```
-------------+------------------------------           Adj R-squared =  0.0917
       Total |  3651.09821   475  7.68652256           Root MSE      =  2.6423

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   education |   1.129315    .234168     4.82   0.000     .6691739    1.589455
   leftright |   .2023659   .1185598     1.71   0.089    -.0306045    .4353363
        edlr |  -.0795068    .046379    -1.71   0.087    -.1706416     .011628
       _cons |   3.508603    .601203     5.84   0.000     2.327238    4.689969
------------------------------------------------------------------------------
```

Within (also known as: Fixed-Effect or Least-Squares-Dummy-Variable (LSDV) Estimator):

. xtreg eu_support education leftright edlr , fe

(Comments:

—note the shares of within, between, & total variation the model explains*: perhaps substantively interesting.*

—note the corr(u_i,Xb): this is the correlation of the estimated fixed effects and the Xb's; its magnitude gives some sense of the potential for omitted-variable bias were one to omit the fixed (or random) macro-level factor.

—note the F-test that all u_i=0; this is the test of whether you the units' intercepts are equal.)

```
Fixed-effects (within) regression               Number of obs      =      8536
Group variable (i): group                       Number of groups   =        17

R-sq:  within  = 0.0307                         Obs per group: min =       476
       between = 0.0070                                        avg =     502.1
       overall = 0.0257                                        max =       561

                                                F(3,8516)          =     89.76
corr(u_i, Xb)  = -0.0375                        Prob > F           =    0.0000

------------------------------------------------------------------------------
```

```
 eu_support |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   education |   .5978593   .0654331     9.14   0.000     .4695945    .7261242
   leftright |   .0607306   .0318553     1.91   0.057    -.0017135    .1231747
        edlr |  -.0263868   .0117219    -2.25   0.024    -.0493646   -.003409
       _cons |   5.091498   .1802538    28.25   0.000     4.738157     5.44484
-------------+----------------------------------------------------------------
     sigma_u |  .80303848
     sigma_e |  2.5503461
         rho |  .09020262   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(16, 8516) =    49.04            Prob > F = 0.0000
```

—Lets save these estimates for use later…

```
. estimates store FEmodel
```

—`xtreg` *estimates the fixed-effects model by regressing unit-mean-differenced DepVar on IndVars; the same model can be estimated by pooled regression with dummy-variable indicators for each unit (or N-1 units & the constant):*

```
. reg eu_support ctry1 ctry2 ctry3 ctry4 ctry5 ctry6 ctry7 ctry8 ctry9 ctry10
ctry11 ctry12 ctry13 ctry14 ctry15 ctry16 ctry17 education leftright edlr ,
nocons
```

—Note: unfortunately, I named some other variables starting with ctry, so I couldn't just use **ctry***

```
      Source |       SS       df       MS              Number of obs =    8536
-------------+------------------------------           F( 20,  8516) = 2916.51
       Model |  379395.179    20  18969.759            Prob > F      =  0.0000
    Residual |  55390.3207  8516   6.504265            R-squared     =  0.8726
-------------+------------------------------           Adj R-squared =  0.8723
       Total |    434785.5  8536  50.9355084           Root MSE      =  2.5503

------------------------------------------------------------------------------
```

eu_support	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
ctry1	3.720617	.2087149	17.83	0.000	3.311485	4.129749
ctry2	5.106873	.2129541	23.98	0.000	4.689432	5.524315
ctry3	5.444258	.2122778	25.65	0.000	5.028142	5.860374
ctry4	4.385817	.2134096	20.55	0.000	3.967482	4.804152
ctry5	5.408641	.2131427	25.38	0.000	4.99083	5.826453
ctry6	3.447474	.2168233	15.90	0.000	3.022448	3.8725
ctry7	5.830927	.2176972	26.78	0.000	5.404187	6.257666
ctry8	6.033122	.2127423	28.36	0.000	5.616096	6.450149
ctry9	5.895191	.2066391	28.53	0.000	5.490128	6.300253
ctry10	6.245298	.2079601	30.03	0.000	5.837645	6.65295
ctry11	4.96982	.2051506	24.23	0.000	4.567675	5.371965
ctry12	4.871941	.2129164	22.88	0.000	4.454573	5.289309
ctry13	4.190387	.2095116	20.00	0.000	3.779693	4.60108
ctry14	5.88373	.2165555	27.17	0.000	5.459229	6.308232
ctry15	5.024483	.2048883	24.52	0.000	4.622852	5.426114
ctry16	5.260357	.2174597	24.19	0.000	4.834083	5.686631
ctry17	4.833958	.2079648	23.24	0.000	4.426296	5.241619
education	.5978593	.0654331	9.14	0.000	.4695945	.7261242
leftright	.0607306	.0318553	1.91	0.057	-.0017135	.1231747
edlr	-.0263868	.0117219	-2.25	0.024	-.0493646	-.003409

—The same F-test that all u_i=0 as before is now the test that the units' intercepts are equal. Notice it's identical.

```
. testparm ctry* , equal

 ( 1) - ctry1 + ctry2 = 0
 ( 2) - ctry1 + ctry3 = 0
 ( 3) - ctry1 + ctry4 = 0
 ( 4) - ctry1 + ctry5 = 0
```

```
 ( 5) - ctry1 + ctry6 = 0
 ( 6) - ctry1 + ctry7 = 0
 ( 7) - ctry1 + ctry8 = 0
 ( 8) - ctry1 + ctry9 = 0
 ( 9) - ctry1 + ctry10 = 0
 (10) - ctry1 + ctry11 = 0
 (11) - ctry1 + ctry12 = 0
 (12) - ctry1 + ctry13 = 0
 (13) - ctry1 + ctry14 = 0
 (14) - ctry1 + ctry15 = 0
 (15) - ctry1 + ctry16 = 0
 (16) - ctry1 + ctry17 = 0

       F( 16,  8516) =   49.04
            Prob > F =    0.0000
```

—Mathematically the same model using N-1 unit-dummies & the constant:

```
. reg eu_support ctry1 ctry2 ctry3 ctry4 ctry5 ctry6 ctry7 ctry8 ctry9 ctry10
ctry11 ctry12 ctry13 ctry14 ctry15 ctry16 education leftright edlr
      Source |       SS       df       MS              Number of obs =    8536
-------------+------------------------------           F( 19,  8516) =   54.24
       Model |  6703.23819    19  352.80201            Prob > F      =  0.0000
    Residual |  55390.3207  8516   6.504265            R-squared     =  0.1080
-------------+------------------------------           Adj R-squared =  0.1060
       Total |  62093.5589  8535  7.27516801           Root MSE      =  2.5503

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ctry1 |  -1.113341    .164225    -6.78   0.000    -1.435262   -.7914197
       ctry2 |   .2729158   .1635719     1.67   0.095    -.0477247    .5935563
```

```
       ctry3 |   .6103001   .1628892     3.75   0.000     .2909977    .9296025
       ctry4 |  -.4481405   .1639987    -2.73   0.006    -.7696178   -.1266632
       ctry5 |   .5746836   .1646154     3.49   0.000     .2519974    .8973698
       ctry6 |  -1.386483   .1654062    -8.38   0.000     -1.71072   -1.062247
       ctry7 |    .996969   .1661055     6.00   0.000      .671362    1.322576
       ctry8 |   1.199165   .1645178     7.29   0.000     .8766701     1.52166
       ctry9 |   1.061233   .1619568     6.55   0.000     .7437585    1.378708
      ctry10 |    1.41134   .1624413     8.69   0.000     1.092916    1.729765
      ctry11 |   .1358625   .1618632     0.84   0.401    -.1814286    .4531535
      ctry12 |   .0379833   .1639436     0.23   0.817    -.2833859    .3593525
      ctry13 |   -.643571   .1617221    -3.98   0.000    -.9605855   -.3265565
      ctry14 |   1.049773   .1652395     6.35   0.000     .7258634    1.373682
      ctry15 |   .1905258   .1590045     1.20   0.231    -.1211617    .5022132
      ctry16 |   .4263994   .1641516     2.60   0.009     .1046224    .7481764
   education |   .5978593   .0654331     9.14   0.000     .4695945    .7261242
   leftright |   .0607306   .0318553     1.91   0.057    -.0017135    .1231747
        edlr |  -.0263868   .0117219    -2.25   0.024    -.0493646    -.003409
       _cons |   4.833958   .2079648    23.24   0.000     4.426296    5.241619
------------------------------------------------------------------------------
```

—Notice the constant is now the intercept for group 17, the other unit-dummies are deviations from that intercept.
—In this last formulation, the test that the 16 included country-indicators jointly have coefficients (intercept-shifts) of zero is identical *to the test above that the u_i from the unit-mean-differenced formulation all equal zero:*

```
. testparm ctry*
 ( 1)  ctry1 = 0
 ( 2)  ctry2 = 0
 ( 3)  ctry3 = 0
 ( 4)  ctry4 = 0
 ( 5)  ctry5 = 0
 ( 6)  ctry6 = 0
 ( 7)  ctry7 = 0
 ( 8)  ctry8 = 0
```

```
( 9)  ctry9 = 0
(10)  ctry10 = 0
(11)  ctry11 = 0
(12)  ctry12 = 0
(13)  ctry13 = 0
(14)  ctry14 = 0
(15)  ctry15 = 0
(16)  ctry16 = 0

      F( 16,  8516) =   49.04
           Prob > F =    0.0000
```

Fully pooled:

—Note: the pooled estimator can be shown to be a weighted average of the within *and the (below)* between *estimators. The weights relate directly to the shares of total variation, V, at these levels* V_{within} & $V_{between}$ *(n.b.,* $V=V_{within}+V_{between}$*).*

. reg eu_support education leftright edlr

```
      Source |       SS       df       MS              Number of obs =    8536
-------------+------------------------------           F(  3,  8532) =   75.19
       Model |  1599.42779     3  533.142597           Prob > F      =  0.0000
    Residual |  60494.1311  8532  7.09026385           R-squared     =  0.0258
-------------+------------------------------           Adj R-squared =  0.0254
       Total |  62093.5589  8535  7.27516801           Root MSE      =  2.6628

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   education |   .6038181   .0677221     8.92   0.000     .4710664    .7365697
   leftright |   .0808447    .033066     2.44   0.015     .0160274     .145662
        edlr |  -.0340327   .0121919    -2.79   0.005    -.0579317   -.0101336
       _cons |   5.074065   .1862762    27.24   0.000     4.708919    5.439212
```

```
------------------------------------------------------------------------------
```

Between:

—This model regresses the unit-means of the DepVar on the unit-means of the IndVars. Note: if we actually wanted to estimate this model optimally, we could take advantage of the fact that unit means are based on different numbers of observations. Knowing that the variance of an average is proportional to 1/n_j, we want to weight (FWLS) by $1/\sqrt{n_j}$.

```
. xtreg eu_support education leftright edlr , be

Between regression (regression on group means)  Number of obs      =      8536
Group variable (i): group                       Number of groups   =        17

R-sq:  within  = 0.0005                         Obs per group: min =       476
       between = 0.1517                                        avg =     502.1
       overall = 0.0002                                        max =       561

                                                F(3,13)            =      0.78
sd(u_i + avg(e_i.))=  .8075944                  Prob > F           =    0.5285

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   education |   6.405605   4.847655     1.32   0.209    -4.067118    16.87833
   leftright |   3.735945   2.547357     1.47   0.166    -1.767286    9.239175
        edlr |   -1.47684   1.017944    -1.45   0.171    -3.675974    .7222941
       _cons |   -9.66531    12.1824    -0.79   0.442    -35.98378    16.65316
------------------------------------------------------------------------------
```

Random-Effect:

—Recall that RE a weighting fixed-effect and pooled estimates. Notice how similar these estimates are to the FE ests…

```
. xtreg eu_support education leftright edlr , re
Random-effects GLS regression                   Number of obs      =      8536
```

```
Group variable: group                           Number of groups   =        17

R-sq:  within  = 0.0307                         Obs per group: min =       476
       between = 0.0070                                        avg =     502.1
       overall = 0.0257                                        max =       561

Random effects u_i ~ Gaussian                   Wald chi2(3)       =    268.80
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   education |   .5979731   .0654208     9.14   0.000     .4697508    .7261955
   leftright |   .0611512   .0318512     1.92   0.055    -.0012759    .1235784
        edlr |  -.0265411   .0117209    -2.26   0.024    -.0495137   -.0035686
       _cons |   5.090943   .2647705    19.23   0.000     4.572002    5.609883
-------------+----------------------------------------------------------------
     sigma_u |  .79952133
     sigma_e |  2.5503461
         rho |  .08948476   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

—Lets save these estimates also…

. estimates store REmodel

Hausman Test: $Haus = \left(\hat{\boldsymbol{\theta}}_c - \hat{\boldsymbol{\theta}}_e\right)' \left[{}^{A}\widehat{\text{var}}\left(\hat{\boldsymbol{\theta}}_c\right) - {}^{A}\widehat{\text{var}}\left(\hat{\boldsymbol{\theta}}_e\right)\right]^{-1} \left(\hat{\boldsymbol{\theta}}_c - \hat{\boldsymbol{\theta}}_e\right)' \sim^{A} \chi^2_k$

. hausman FEmodel REmodel

```
                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |    FEmodel      REmodel       Difference          S.E.
```

```
------------+----------------------------------------------------------------
   education |    .5978593     .5979731       -.0001138        .0012723
   leftright |    .0607306     .0611512       -.0004207        .0005114
        edlr |   -.0263868    -.0265411        .0001543        .0001549
-----------------------------------------------------------------------------
                              b = consistent under Ho and Ha; obtained from xtreg
               B = inconsistent under Ha, efficient under Ho; obtained from xtreg

    Test:  Ho:  difference in coefficients not systematic

                  chi2(3) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =        2.48
                Prob>chi2 =      0.4782
```

—*Note: syntax for* `Hausman` *is* `Hausman` *[always-consistent] [null-efficient, alt-biased]*

—*Note: This time we fail to reject. Not surprising since estimates so similar.*

SEEING UNIT-BY-UNIT & FULL-DUMMY-INTERACTION ARE (NEARLY) IDENTICAL:

—*To see this, it suffices to consider just two groups. It generalizes to n-groups, but would be (even more) tedious to do.*

INTERACTIONS OF INDICATOR-INDVAR ALREADY GENERATED:

```
. gen ctry1_edu=education*ctry1
. gen ctry2_edu=education*ctry2
. gen ctry1_lr=leftright*ctry1
. gen ctry2_lr=leftright*ctry2
. gen ctry1_edlr=edlr*ctry1
. gen ctry2_edlr=edlr*ctry2
```

UNIT-BY-UNIT:

```
. reg eu_support education leftright edlr if ctry1==1
      Source |       SS       df       MS              Number of obs =     489
-------------+------------------------------           F(  3,   485) =   17.13
       Model |  465.652747     3  155.217582           Prob > F      =  0.0000
    Residual |  4394.23069   485  9.06026946           R-squared     =  0.0958
-------------+------------------------------           Adj R-squared =  0.0902
       Total |  4859.88344   488  9.95877753           Root MSE      =    3.01

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   education |   1.987964   .5057722     3.93   0.000     .9941884    2.981739
   leftright |   .1780868   .2423311     0.73   0.463    -.2980617    .6542352
        edlr |  -.1818668   .0976701    -1.86   0.063    -.3737757     .010042
       _cons |   1.619286   1.266475     1.28   0.202    -.8691691     4.10774
------------------------------------------------------------------------------
. reg eu_support education leftright edlr if ctry2==1
      Source |       SS       df       MS              Number of obs =     504
-------------+------------------------------           F(  3,   500) =   10.99
       Model |  210.759687     3   70.253229           Prob > F      =  0.0000
    Residual |  3196.06125   500  6.39212249           R-squared     =  0.0619
-------------+------------------------------           Adj R-squared =  0.0562
```

```
       Total |  3406.82093   503  6.77300384           Root MSE      =  2.5283

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   education |   .6032422   .2851468     2.12   0.035     .0430086    1.163476
   leftright |  -.0744197   .1508333    -0.49   0.622    -.3707649    .2219256
        edlr |  -.0006216   .0510908    -0.01   0.990    -.1010008    .0997576
       _cons |   5.409136   .8423847     6.42   0.000     3.754086    7.064186
------------------------------------------------------------------------------
```

FULL-DUMMY-INTERACTION:

. reg eu_support ctry1 ctry1_edu ctry1_lr ctry1_edlr ctry2 ctry2_edu ctry2_lr ctry2_edlr if group<2.5 , nocons

```
      Source |       SS       df       MS              Number of obs =     993
-------------+------------------------------           F(  8,   985) =  588.09
       Model |  36253.9581     8  4531.74476           Prob > F      =  0.0000
    Residual |  7590.29193   985  7.70588014           R-squared     =  0.8269
-------------+------------------------------           Adj R-squared =  0.8255
       Total |    43844.25   993  44.1533233           Root MSE      =  2.7759

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ctry1 |   1.619286   1.167985     1.39   0.166    -.6727383     3.91131
   ctry1_edu |   1.987964   .4664397     4.26   0.000     1.072634    2.903293
    ctry1_lr |   .1780868   .2234857     0.80   0.426    -.2604761    .6166496
  ctry1_edlr |  -.1818668   .0900746    -2.02   0.044     -.358627   -.0051067
       ctry2 |   5.409136   .9249091     5.85   0.000     3.594118    7.224155
   ctry2_edu |   .6032422   .3130812     1.93   0.054    -.0111407    1.217625
    ctry2_lr |  -.0744197   .1656098    -0.45   0.653    -.3994082    .2505688
  ctry2_edlr |  -.0006216    .056096    -0.01   0.991     -.110703    .1094597
```

```
-------------------------------------------------------------------------------
```

—Notice that the coefficients are the same, but the standard errors are a little different. This is because the pooled full-dummy-interaction case assumes one, common σ_ε^2 for both sub-samples. Separate estimation allows them to differ. We can allow σ_ε^2 to vary by unit in the pooled full-dummy-interaction case by applying panel-weighted least-squares*, which is just FWLS with the weights given by the (inverse of the square root of the) unit-by-unit estimated variance, yielding more-nearly identical results (w/i rounding error appearing at 3^{rd} digit):*

. xtgls eu_support ctry1 ctry1_edu ctry1_lr ctry1_edlr ctry2 ctry2_edu ctry2_lr ctry2_edlr if group<2.5 , nocons panels(hetero)

```
Cross-sectional time-series FGLS regression

Coefficients:  generalized least squares
Panels:        heteroskedastic
Correlation:   no autocorrelation

Estimated covariances      =         2          Number of obs      =       993
Estimated autocorrelations =         0          Number of groups   =         2
Estimated coefficients     =         8          Obs per group: min =       489
                                                               avg =     496.5
                                                               max =       504
                                                Wald chi2(8)       =   5098.42
Log likelihood             =  -2411.32          Prob > chi2        =    0.0000

-------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+-----------------------------------------------------------------
       ctry1 |   1.619286   1.261284     1.28   0.199     -.852786    4.091357
   ctry1_edu |   1.987964   .5036993     3.95   0.000     1.000731    2.975196
    ctry1_lr |   .1780868   .2413379     0.74   0.461    -.2949269    .6511004
```

ctry1_edlr	-.1818668	.0972698	-1.87	0.062	-.3725122	.0087785
ctry2	5.409136	.8390353	6.45	0.000	3.764657	7.053615
ctry2_edu	.6032422	.284013	2.12	0.034	.0465869	1.159897
ctry2_lr	-.0744197	.1502336	-0.50	0.620	-.3688721	.2200328
ctry2_edlr	-.0006216	.0508877	-0.01	0.990	-.1003597	.0991164

Interpreting Interaction Effects

- Basic Linear-Interactive Model:

$$\text{eusup} = b_0 + b_{edu}\text{edu} + b_{lftrt}\text{lftrt} + b_{edlr}\text{edu}\times\text{lftrt} + \ldots + \varepsilon$$

- Effect of *edu*? $\frac{\partial \text{eusup}}{\partial \text{edu}} = b_{edu} + b_{edlr}\text{lftrt}$

  » For the record, the effect of *lftrt*: $\frac{\partial \text{eusup}}{\partial \text{lftrt}} = b_{lftrt} + b_{edlr}\text{edu}$

- Std Error of that Effect (of *edu* on *eusupp*)?

$$\hat{V}\left(\hat{b}_{edu} + \hat{b}_{edlr}\text{lftrt}\right) = \hat{V}\left(\hat{b}_{edu}\right) + \hat{V}\left(\hat{b}_{edlr}\right)\text{lftrt}^2 + 2\cdot\hat{C}\left(\hat{b}_{edu}, \hat{b}_{edlr}\right)\text{lftrt}$$

  – Std. Err. effect of *edu*: $\hat{V}\left(\hat{b}_{lftrt} + \hat{b}_{edlr}\text{edu}\right) = \hat{V}\left(\hat{b}_{lftrt}\right) + \hat{V}\left(\hat{b}_{edlr}\right)\text{edu}^2 + 2\cdot\hat{C}\left(\hat{b}_{edu}, \hat{b}_{edlr}\right)\text{edu}$

```
. reg eu_support education leftright edlr
      Source |       SS       df       MS              Number of obs =   42680
-------------+------------------------------           F(  3, 42676) =  371.33
       Model |  7869.51854     3  2623.17285           Prob > F      =  0.0000
    Residual |  301474.935 42676  7.06427347           R-squared     =  0.0254
-------------+------------------------------           Adj R-squared =  0.0254
       Total |  309344.453 42679  7.24816545           Root MSE      =  2.6579

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   education |   .5918654   .0302344    19.58   0.000     .5326053    .6511255
   leftright |    .080127   .0147551     5.43   0.000     .0512068    .1090472
        edlr |  -.0320928   .0054447    -5.89   0.000    -.0427646    -.021421
       _cons |   5.093097   .0831665    61.24   0.000     4.930089    5.256105
------------------------------------------------------------------------------
```

- What's effect of *edu*? Of *lftrt*? How moderate each other?

$$\frac{\partial \text{eusup}}{\partial \text{edu}} = .5919 - .0321(\text{lftrt}) \qquad \frac{\partial \text{eusup}}{\partial \text{lftrt}} = .0801 - .0321(\text{edu}) \qquad \frac{\partial^2 \text{eusup}}{\partial \text{edu}\,\partial \text{lftrt}} = b_{edlr} = -.0321$$

  - Only for modifying effect does standard regression output tell us directly.
- What are the standard errors of these effects?
  - Only for modifying effect does standard regression output tell us directly.
- Recall: *Main Effects* refer to beyond-range values; they not direct evidence on whether effect (generally) positive

- What are the standard errors of these effects? Need this:

```
•   .vce
•   Covariance matrix of coefficients of regress model
•           e(V) |  education   leftright        edlr       _cons
•   -------------+------------------------------------------------
•      education |   .00091412
•      leftright |   .00037584   .00021771
•           edlr |  -.00014825  -.00007456   .00002965
•          _cons |  -.00234221  -.00110293   .00037572   .00691666
```

- I prefer spreadsheet at this point:
  - Copy regression-estimation results; Paste into spreadsheet.
  - Ditto for estimated v-cov of estimated coefficients (as text or table)
  - Finally, you'll want summary stats for vars in your model (as text best):

```
-   . tabstat dgovpw psupgpw npgovpw PD NPPD NPGS NPPDGS, statistics( mean max min
    median iqr skewness sd ) columns(variables)
-      stats |    dgovpw   psupgpw   npgovpw        PD      NPPD      NPGS    NPPDGS
-   ---------+----------------------------------------------------------------------
-       mean |  25.24783  57.38261  1.965217  .6521739  1.369565  116.3004  765.9826
-        max |      45.1      80.4       4.3         1       3.8    305.52    2181.2
-        min |        11      41.1         1         0         0        49         0
-        p50 |      26.6      57.2       1.8         1       1.6        96     916.2
-        iqr |      13.7      10.8       1.7         1       2.2     83.47      1108
-   skewness |  .1587892   .821165   .907747 -.6390097  .2729543  1.218438  .1633625
-         sd |  9.655468  9.103879  .9599284  .4869848  1.213723  69.04308  644.2565
-   --------------------------------------------------------------------------------
```

- Spreadsht Formula to Plot Effect Lines w/ C.I.
  - ***Col 1:*** Conditioning Var (lftrt in *d*eusup/*d*edu)
    - *1st Cell (A29):* Enter min of range to consider (smpl min)
    - *2nd Cell:* =A29+1
      - Or sub "+(max-min)/(#steps)" for +1 (choose big# to smooth)
    - Copy down until reach max value you want plot to cover.
  - ***Column Two:*** Effect (*dGovDur/dNP* here)
    - *1st Cell (B29):* Enter =$B$3+$B$5*$A29, where:
      - $B$3 is absolute reference to cell containing coefficient on *edu*
      - $B$5 is absolute reference to cell containing coefficient on *edlr*
      - $A29 is reference to cell containing value of *lftrt* for that row
        - » $ optional, but helps if want copy whole block later for other effects
    - Copy Down

- Spreadsheet Formula to Plot Effect Lines w/ C.I.
  - ***Column Three:*** standard error (or can skip to bounds)
    - *1st Cell (A29):* =($B$11+$D$13*$A29^2+2*$B$13*$A29)^0.5
      - $B$11 is absolute reference to cell containing variance of *edu* coefficient
      - $D$13*$A29^2 is absolute reference to cell containing variance of *edlr* coefficient, times the square of the value of *lftrt* for that row.
      - 2*$B$13*$A29 is absolute reference to cell containing 2 times the covariance of *edu* and *edlr* coefficients times value of *lftrt* for that row.
      - ^0.5 turns that estimated variance into a standard error.
    - Copy down.
  - ***Column Four:*** Lower bound of 95% C.I.
    - *1st Cell (B29):* Enter =+$B29-1.96*$C29, where:
      - $B29 is (column-absolute opt.) reference to cell containing effect est
      - $C29 is (column-absolute opt.) reference to cell containing std err est
      - 1.96 is critical value for 95% C.I. on large-N t-dist or std-norm dist;
        - » can sub crit.val. for diff. C.I. % or smpl-size or use sprdsht formula
    - Copy Down
  - ***Column Five:*** Upper bound (analogous, but +1.96*$C29)

- In Stata, plot dY/dX w/ c.i. from smpl min-max:
  - `egen zmin = min(z)` ; `egen zmax = max(z)` finds those sample min & max for variable z. (z=leftright in our case, i.e., GS)
  - `gen z0 = (_n-1)/(v-1)*(zmax-zmin)+zmin in 1/v` creates var counting v equal-size steps from sample min to max.
  - `gen dyhatdx=_b[x]+_b[xz]*z0` creates var of dY/dX ests (x=education)
  - Stata code tedious to get to s.e.'s & c.i. plots (bit better in matrix form)
    - First have to work in matrices for bit, then back to vars:
      - `matrix V = get(VCE)` (makes matrix of v-cov mat)
      - `matrix C= V[3,1]` (grabs 3,1 element as covar)
      - `gen column1 = 1 in 1/v` (makes a variable equal to all ones)
      - `mkmat column1, matrix(col1)` (makes vector called col1 of that var)
      - `matrix cov_x_xz = C*col1` (makes a constant vector of covar)
      - `svmat cov_x_xz, name(cov_x_xz)` (makes that vector a variable)
    - Finally, you can generate variances & std errors, which you could tabulate:
      - `gen vardyhatdx=(_se[x])^2+(z0*z0)*(_se[xz]^2)+2*z0*cov_x_xz`
      - `gen sedyhatdx=sqrt(vardyhatdx)` (makes variable equal to s.e. of effect)
      - `tabdisp z0, cellvar(dyhatdx sedyhatdx)` (makes table effects & s.e.'s)
    - Or you can generate the confidence interval bounds & plot:
      - `gen LBdyhatdx=dyhatdx-invttail(e(df_r),.05)*sedyhatdx`
      - `gen UBdyhatdx=dyhatdx+invttail(e(df_r),.05)*sedyhatdx`
      - `twoway connected dyhatdx LBdyhatdx UBdyhatdx z0`

Calculating Predicted-Values & Standard Errors

Procedures	Command syntax

Create the variables which set the values of the variables *x*, *z*, and *xz* (& other variables, if any) for which $\hat{y}$ will be calculated.	`egen xmin = min(x)`
	`egen xmax = max(x)`
	`gen xh = ((_n-1)/(v-1))*(xmax-xmin) in 1/v`
	`egen zh=mean(z)`
	`gen xhzh=xh*zh`
	`gen col1h=1`

Assemble the variables into a matrix, **Mh**	`mkmat xh zh xhzh col1h in 1/v, matrix(Mh)`
Create **betas**, a column vector with *k x 1* dimensions.	`matrix betas=e(b)'`
Calculate the predicted values.	`matrix yhat=Mh*betas`
Convert the vector into a variable.	`svmat yhat, name(yhat)`
Create a matrix from the estimated covariance matrix of the coefficient estimates.	`matrix V = get(VCE)`
Calculate the variance of the predicted values.	`matrix VYH=Mh*V*Mh'`
Extract the diagonal elements into a row vector.	`matrix DVYH= vecdiag(VYH)`
Transpose elements into a column vector.	`matrix VARYHAT=DVYH'`
Convert the vector into a variable.	`svmat VARYHAT, name(varyhat)`
Calculate the estimated standard error of each predicted probability.	`gen seyhat1 = sqrt(varyhat1)`
Present a table of predicted values with corresponding standard errors of the predicted values.	`tabdisp xh, cellvar(yhat1 seyhat1)`

Generate lower and upper confidence interval bounds. Graph the predicted probabilities and the upper and lower confidence intervals.	`gen LByhat1=yhat1-invttail(e(df_m),.05)*seyhat1`
	`gen UByhat1=yhat1+invttail(e(df_m),.05)*seyhat1`
	
	`twoway connected yhat1 LByhat1 UByhat1 xh`

Google

'kam franzese michigan press'

for that data, and

stata & excel resources

or go directly to

http://www.press.umich.edu
/KamFranzese/Interactions.html

Google

'Matt Golder interaction'

or go directly to

http://homepages.nyu.edu
/~mrg217/interaction.html

Stata

help mfx

help predictnl

A MODEL WITH MACRO-LEVEL REGRESSORS &/OR CROSS-LEVEL INTERACTIONS:

```
. reg eu_support leftright GENGOV06 lrGS
      Source |       SS       df       MS              Number of obs =    8536
-------------+------------------------------           F(  3,  8532) =    8.42
       Model |  183.363072     3  61.1210239           Prob > F      =  0.0000
    Residual |  61910.1959  8532  7.25623486           R-squared     =  0.0030
-------------+------------------------------           Adj R-squared =  0.0026
       Total |  62093.5589  8535  7.27516801           Root MSE      =  2.6937

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   leftright |   .3102197   .0692995     4.48   0.000     .1743759    .4460636
    GENGOV06 |   .0310353   .0085924     3.61   0.000     .0141921    .0478785
        lrGS |  -.0074389    .001577    -4.72   0.000    -.0105302   -.0043475
       _cons |   5.326216   .3764811    14.15   0.000     4.588222     6.06421
------------------------------------------------------------------------------
```

—*Recall: If this model adequately captures E(**y**) in its* **Xb**, *the OLS coefficients estimates are unbiased and consistent, although potentially inefficient. However, we are concerned, especially in the context of this kind of data (although these would always be among our concerns), about two things: possible parameter (intercept & coefficients) heterogeneity and possible non-sphericity in V(ε). If what our model fails to capture regarding the parameter variability, across contexts for example, is unrelated to what we have included in our model as regressors, OLS will still produce unbiased estimates of those coefficients that we have estimated. In this case, though, V(ε) will be non-spherical, so our coefficient estimates will be inefficient, and non-spherical in a way that relates to our **xx'** matrix of regressors regardless, so OLS standard errors will be* wrong *(biased, inconsistent, & inefficient) because they derive from the wrong formula.*[1] *If what our model fails to capture regarding the parameter variability (for example) across contexts is* related *to what we have included in our model as regressors, OLS will produce biased estimates of our coefficients as well as having these problems with standard errors. (Our first line of defense against parameter heterogeneity in our model of E(**y**), of course, is to improve the model!) With non-spherical V(ε), which, even if our model of E(**y**) were perfect, we suspect for two reasons in particular in this kind of data—panel heteroscedasticity and*

[1] OLS standard errors are inefficiently estimated in that case where errors are non-spherical but in a manner unrelated to ***xx'***, but not biased.

within-unit clustering—OLS coefficient estimates will be inefficient and standard errors will be at least inefficient and almost surely biased and inconsistent as well, likely badly in this context: again because they apply the wrong formula. Thus, having estimated a model like the one above, in multilevel data perhaps especially (but also in general), we will want to explore possibilities of remaining (un- or inadequately modeled) parameter het &/or error non-sphericity.

Model E(**y**) Sufficient	V(**e**) Spherical		OLS is BLUE
	V(**e**) Nonspherical	Nonsphericity Unrelated **xx'**	OLS **b** unbiased, inefficient; V(**b**) unbiased, inefficient
		Nonsphericity Related **xx'**	OLS **b** unbiased, inefficient; V(**b**) biased, inefficient
Model E(**y**) Insufficient	Unmodeled **b** het unrelated **X**	V(**e**) Will be Nonspherical & Related **xx'**	OLS **b** unbiased, inefficient; V(**b**) biased, inefficient
	Unmodeled **b** het related **X**	V(**e**) Will be Nonspherical & Related **xx'**	OLS **b** biased, inefficient; V(**b**) biased, inefficient

We could/should return here to hettests, parameter-homogeneity tests, and graphical and other methods to explore model specification as in Lab 1.

Recall also these series of estimation options for various sandwich &/or FGLS estuimators:

```
reg spend unem growthpc depratio left cdem trade lowwage fdi
reg spend unem growthpc depratio left cdem trade lowwage fdi , vce(robust)
reg spend unem growthpc depratio left cdem trade lowwage fdi , vce(cluster cc)
```

What are these? What happens to your estimates of **β**? What about of V(**b**)?

Try this series of estimates:

```
xtpcse spend unem growthpc depratio left cdem trade lowwage fdi , independent
xtpcse spend unem growthpc depratio left cdem trade lowwage fdi , hetonly
xtpcse spend unem growthpc depratio left cdem trade lowwage fdi
```

What are these? What happens to your estimates of **β**? What about of V(**b**)?

One last set of estimates:

```
xtgls spend unem growthpc depratio left cdem trade lowwage fdi, p([i,h,c]) c([i,a,p])
```

What are these? What happens to your estimates of **β**? What about of V(**b**)?

We might very well wish to combine various of these options in particular ways. For instance:

```
. xtgls eu_support leftright GENGOV06 lrGS , panels(hetero)
Cross-sectional time-series FGLS regression

Coefficients:  generalized least squares
Panels:        heteroskedastic
Correlation:   no autocorrelation

Estimated covariances      =        17          Number of obs      =      8536
Estimated autocorrelations =         0          Number of groups   =        17
Estimated coefficients     =         4          Obs per group: min =       476
                                                               avg =  502.1176
                                                               max =       561
                                                Wald chi2(3)       =     28.65
Log likelihood             = -20386.67          Prob > chi2        =    0.0000

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   leftright |   .3561629   .0679653     5.24   0.000     .2229533    .4893725
    GENGOV06 |    .043021   .0086107     5.00   0.000     .0261444    .0598976
        lrGS |  -.0083882   .0015725    -5.33   0.000    -.0114701   -.0053062
       _cons |   4.887677   .3710113    13.17   0.000     4.160508    5.614846
------------------------------------------------------------------------------
```

Yes, but these now do not account the heteroscedasticity related to the regressors (except the country indicators). We can combine FGLS and robust v-cov estimation:

```
. xtgls eu_support leftright GENGOV06 lrGS , panels(hetero) robust
option robust not allowed
```

Well, Stata's `xtgls` *may not allow it, but it is allowable, and a perfectly reasonable option. Need to do it manually.*

MULTILEVEL MODELS:

Random-Intercepts Model:

```
. xtreg eu_support leftright GENGOV06 lrGS , re
Random-effects GLS regression                   Number of obs      =      8536
Group variable (i): group                       Number of groups   =        17

R-sq:  within  = 0.0027                         Obs per group: min =       476
       between = 0.0049                                        avg =     502.1
       overall = 0.0029                                        max =       561

Random effects u_i ~ Gaussian                   Wald chi2(3)       =     23.32
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.       z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   leftright |   .2964085   .0672896     4.40   0.000     .1645233    .4282936
    GENGOV06 |   .0302652    .027957     1.08   0.279    -.0245294    .0850599
        lrGS |  -.0071676   .0015292    -4.69   0.000    -.0101647   -.0041705
       _cons |   5.370431   1.228962     4.37   0.000      2.96171    7.779151
-------------+----------------------------------------------------------------
     sigma_u |    .865315
     sigma_e |  2.5866757
         rho |  .10064572   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

Random-Intercepts Model by Stata's more-general `xtmixed` command:

```
. help xtmixed
. xtmixed eu_support leftright GENGOV06 lrGS || group:
Performing EM optimization:
Performing gradient-based optimization:
```

```
Iteration 0:   log restricted-likelihood = -20268.179
Iteration 1:   log restricted-likelihood = -20268.179
Computing standard errors:
Mixed-effects REML regression                   Number of obs      =      8536
Group variable: group                           Number of groups   =        17
                                                Obs per group: min =       476
                                                               avg =     502.1
                                                               max =       561
                                                Wald chi2(3)       =     23.32
Log restricted-likelihood = -20268.179          Prob > chi2        =    0.0000
------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   leftright |   .2964487   .0672952     4.41   0.000     .1645525     .428345
    GENGOV06 |   .0302674   .0262164     1.15   0.248    -.0211159    .0816507
        lrGS |  -.0071683   .0015293    -4.69   0.000    -.0101657   -.004171
       _cons |   5.370298    1.15242     4.66   0.000     3.111596       7.629
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
group: Identity              |
                   sd(_cons) |   .8059319    .150207      .5593119    1.161295
-----------------------------+------------------------------------------------
                sd(Residual) |   2.586676   .0198191      2.548122    2.625814
------------------------------------------------------------------------------
LR test vs. linear regression: chibar2(01) =   633.50 Prob >= chibar2 = 0.0000
```

The emboldened line is the test of the random-intercepts model against OLS, to which I referred earlier as our test of clustering.

Random-Intercepts-and-Slopes Model:

```
. xtmixed eu_support leftright GENGOV06 lrGS || group: leftright GENGOV06

Performing EM optimization:
Performing gradient-based optimization:
Iteration 0:   log restricted-likelihood = -20243.882
Iteration 1:   log restricted-likelihood = -20243.882
Iteration 2:   log restricted-likelihood = -20243.882
Computing standard errors:
Mixed-effects REML regression                   Number of obs      =      8536
Group variable: group                           Number of groups   =        17
                                                Obs per group: min =       476
                                                               avg =     502.1
                                                               max =       561
                                                Wald chi2(3)       =      6.28
Log restricted-likelihood = -20243.882          Prob > chi2        =    0.0985
------------------------------------------------------------------------------
  eu_support |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   leftright |   .3007135   .1660177     1.81   0.070    -.0246752    .6261022
    GENGOV06 |   .0350085   .0226109     1.55   0.122    -.0093081    .0793251
        lrGS |  -.0075158   .0037763    -1.99   0.047    -.0149171   -.0001145
       _cons |    5.20807   .9713808     5.36   0.000     3.304199    7.111941
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
group: Independent           |
                sd(leftri~t) |   .1117327   .0249354      .0721469    .1730386
                sd(GENGOV06) |   .0114748   .0226007      .0002417    .5448443
                   sd(_cons) |   .5572208     .89791      .0236807    13.11174
```

```
------------------------------+------------------------------------------------
                sd(Residual) |   2.575385   .0197518      2.536961     2.61439
-------------------------------------------------------------------------------
LR test vs. linear regression:       chi2(3) =   682.09   Prob > chi2 = 0.0000
Note: LR test is conservative and provided only for reference
```

$$\frac{d(\text{eu\_sup})}{d(\text{lftrt})} = b_{lr} + b_{lrGS}\text{GS} + \gamma_{lr} \quad ; \quad \frac{d(\text{eu\_sup})}{d(\text{GS})} = b_{GS} + b_{lrGS}(\text{lftrt}) + \gamma_{GS}$$

$$\frac{d(\text{eu\_sup})}{d(\text{lftrt})} = .301 - .0075(\text{GS}) + \gamma_{lr} \quad ; \quad \frac{d(\text{eu\_sup})}{d(\text{GS})} = .0350 - .0075(\text{lftrt}) + \gamma_{GS}$$

$$E\left(\frac{d(\text{eu\_sup})}{d(\text{lftrt})}\right) = .301 - .0075(\text{GS}) \quad ; \quad \text{E}\left(\frac{d(\text{eu\_sup})}{d(\text{GS})}\right) = .0350 - .0075(\text{lftrt})$$

$$V\left(\frac{d(\text{eu\_sup})}{d(\text{lftrt})}\right) = V\left(b_{lr}\right) + V\left(b_{lrGS}\right)\text{GS}^2 + 2\times\text{C}\left(b_{lr}, b_{lrGS}\right)\text{GS} + V\left(\gamma_{lr}\right)$$

$$V\left(\frac{d(\text{eu\_sup})}{d(\text{GS})}\right) = V\left(b_{GS}\right) + V\left(b_{lrGS}\right)\text{lftrt}^2 + 2\times\text{C}\left(b_{GS}, b_{lrGS}\right)\text{lftrt} + V\left(\gamma_{GS}\right)$$

Need the covariances:

```
. vce
Covariance matrix of coefficients of xtmixed model
             | eu_support
        e(V) |  leftright   GENGOV06        lrGS       _cons
-------------+------------------------------------------------
```

```
eu_support   |
   leftright |  .02756188
    GENGOV06 |  .00050257   .00051125
        lrGS | -.00061675  -.00001165   .00001426
       _cons | -.02245402  -.02152717     .000503   .94358064
```

$$V\left(\frac{d(\text{eu\_sup})}{d(\text{lftrt})}\right) = .02756188 + .00001426\left(\text{GS}^2\right) + 2\times\left(-.00061675\right)\text{GS} + .012484$$