EXPLICIT FORMAL-THEORETICAL MODELS of INTERDEPENDENCE in TAX COMPETITION and of INTERNATIONAL CONFLICT

Essex Summer School Social Science Data Collection and Analysis (Franzese, *Specification*, Lecture 7b)

drawn from the joint work of

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OVERVIEW

- Explicit models of interdependence:
  - *Tax Competition*
  - *International Conflict*
    - [*Learning & Diffusion* and *Micro-Preferences* also, but we'll not cover here.]
- Using the theoretical models…
  - …to demonstrate interdependence;
  - …to establish/illustrate its explanators, sources, and mechanisms;
  - …to specify empirical models: esp., *how* covariates enter—in **X**, in **W**, in ρ…
- Outline of this session:
  - Tax Competition:
    - Recall the generic spillover/resource-flow models (Brueckner '03).
    - Brueckner ('03) / Brueckner & Saavedra ('01) simple model.
    - Persson & Tabellini ('00: ch. 12.4) model.
    - P&T Extensions (problems 3 & 4)
  - International Conflict:
    - As illustration of how general strategic interdependence in S-Econometric frame.
    - Follow Signorino ('99) and Signorino & Tarar ('06).

Brueckner's Resource-Flow Theoretical Model[1]

- Interdependence: i's utility depends on p_i <u>&</u> $\mathbf{p}_j$:

$$U^i = U^i\left(p_i, H^i(p_i, \mathbf{p}_j; \mathbf{x}_i); \mathbf{x}_i\right)$$

- Accordingly, i's optimal p_i^* depends j's, $\mathbf{p}_j$:

$$\underset{p_i}{Max}\Big|_{\mathbf{p}_j} U^i\left(p_i, H^i(p_i, \mathbf{p}_j; \mathbf{x}_i); \mathbf{x}_i\right)$$

$$\Rightarrow \; U^i_{p_i}(\bullet) + U^i_H(\bullet) H^i_{p_i}(\bullet) = 0$$

$$\Rightarrow \; p_i^* = U^{i\;-1}_{p_i}\left(-U^i_H(\bullet) H^i_{p_i}(\bullet)\right) \equiv R\left(\mathbf{p}_j; \mathbf{x}_i\right)$$

- Slope of this reaction-function depends on how $\mathbf{p}_j$ affects i's marginal utility (2nd line of above):

$$\frac{\partial U^i}{\partial p_i} = U^i_{p_i}(\bullet) + U^i_H(\bullet) H^i_{p_i}(\bullet) \Rightarrow \frac{\partial \frac{\partial U^i}{p_i}}{\partial \mathbf{p}_j} = U^i_{p_i H} H^i_{\mathbf{p}_j} + U^i_{HH} H^i_{\mathbf{p}_j} H^i_{p_i} + U^i_H H^i_{p_i \mathbf{p}_j}$$

Brueckner's Resource-Flow Theoretical Model[2]

- A bit more specifically in tax-competition context:

$$U^i = U^i\left(\tau_i, k^i(\tau_i, \boldsymbol{\tau}_j; \mathbf{x}_i); \mathbf{x}_i\right)$$

- First-order Condition:

$$U^i_{\tau_i}(\bullet) + U^i_k(\bullet)k^i_{\tau_i}(\bullet) = 0 \;\;\Rightarrow\;\; \tau_i^* = {U^i_{\tau_i}}^{-1}\left(-U^i_k(\bullet)k^i_{\tau_i}(\bullet)\right) \equiv R\left(\boldsymbol{\tau}_j; \mathbf{x}_i\right)$$

- Slope reaction-function:

$$\frac{\partial U^i}{\partial \tau_i} = U^i_{\tau_i} + U^i_k k^i_{\tau_i} \Rightarrow \frac{\partial \frac{\partial U^i}{\tau_i}}{\partial \boldsymbol{\tau}_j} = U^i_{\tau_i \tau_j} + U^i_{k\boldsymbol{\tau}_j} k_{\tau_i} + U^i_k k^i_{\tau_i \boldsymbol{\tau}_j}$$

- Notes:
  - In reaction-function, $\mathbf{x}_i$ gen'ly enters intercept & slope. This should inform empirical-model specification.
  - $R(\cdot)$ depends $U(\cdot)$ & $k(\cdot)$, but intuitions re: pos/neg externalities hold if care to sign reversals via $k(\cdot)$
    - E.g., tax-competition: negative externalities, so strategic complements…

$$\downarrow \boldsymbol{\tau}_j \Rightarrow \uparrow \text{marg cost } \tau_i\text{: i.e., } \downarrow U^i_{\tau_i}\text{, i.e., } U^i_{\tau_i \boldsymbol{\tau}_j} > 0 \;\Rightarrow \downarrow \tau_i$$

Brueckner & Saavedra Model of Municipal Tax-Competition[1]

- Production:
  - $F(K_i,P_i)=f(k_i)$, K perfectly mobile, P perfectly immobile
- Perfect competition for fixed total $K \Rightarrow f'(k_i)-t_i=\rho$
  - $\Rightarrow k_1, k_2, \rho$ as functions t_1 & t_2, with $\partial k_i/\partial t_i$ & $\partial \rho/\partial t_i <0$
- Residential land, L_i, fixed: $q_i^*=L_i/P_i$. Mrkt-clear $\Rightarrow q_i=q_i^*$
- Private good, x_i, public, z_i, housing, q_i; perfect Tiebot sorting $\Rightarrow$ cities of homogenous citizens with $U(x_i, q_i, z_i)$.
- Income:
  - $w_i+\rho k^*+r_i q_i^*$, where $w_i=f(k_i)-k_i f'(k_i)$ & (assume even-distrib cap) $k^*=(K_1+K_2)/(P_1+P_2)$
- Tax: t_i levied on housing and capital; housing price: r_i+t_i

Brueckner & Saavedra Model of Municipal Tax-Competition[2]

- Budget Constraints:
  - Citizens' BC:
    - *Consumption=wage inc + net cap inc + net house inc:*
    - $x_i=f(k_i)-k_if'(k_i)+\rho k^*+r_iq_i^*-(r_i+t_i)q_i$, which, since $q_i=q_i^*$, is
    - $x_i=f(k_i)-k_if'(k_i)+\rho k^*-t_iq_i^*$
  - Governments' BC: $z_i=t_i(q_i^*+k_i)$
- Utilities: $U(x_i, q_i, z_i)=U[f(k_i)-k_if'(k_i)+\rho k^*-t_iq_i^* , q_i^* , z_i, t_i(q_i^*+k_i)]$
- Rep cit max's U[·] over t_i, taking t_j as fixed (Nash)

$$U_x\times\left[f'\frac{\partial k_i}{\partial t_i}-f'\frac{\partial k_i}{\partial t_i}-k_if''\frac{\partial k_i}{\partial t_i}+\frac{\partial\rho}{\partial t_i}k^*+\rho\frac{\partial k^*}{\partial t_i}-q^*\right]+U_z\times\left[\left(q^*+k_i\right)+t_i\frac{\partial k_i}{\partial t_i}\right]=0,$$

and recall & note: $k^*=\bar{K}\big/\left(P_1+P_2\right)$ is fixed, and $f'=\rho+t_i\Rightarrow f''=\dfrac{\partial\rho}{\partial k_i}+\dfrac{\partial t_i}{\partial k_i}$,

so reduces to: $U_x\times\left[-k_i\left(\dfrac{\partial\rho}{\partial t_i}+1\right)+\dfrac{\partial\rho}{\partial t_i}k^*-q^*\right]+U_z\times\left[\left(q^*+k_i\right)+t_i\dfrac{\partial k_i}{\partial t_i}\right]=0$

$$\frac{U_z}{U_x}=\frac{k_i+q^*+\left(k_i-k^*\right)\dfrac{\partial\rho}{\partial t_i}}{k_i+q^*+t_i\dfrac{\partial k_i}{\partial t_i}}.$$ Call this equilibrium: equation [6].

- In symmetric case, $t_1=t_2$, so $k_1=k_2=k^*$, implying that:

$$\frac{U_z}{U_x}=\frac{k^*+q^*}{k^*+q^*+t_i\dfrac{\partial k_i}{\partial t_i}}>1$$

- I.e., z is under-provided, & t too low. Classic result.

Brueckner & Saavedra Model of Municipal Tax-Competition[3]

- Consider also an asymmetric eqbm, with city 1 having greater relative preference public good, z.
  - City 1 then has incentive increase t_1 to raise z_1.
  - Capital flows 1 to 2, but, at margin, revenue & z_1 in city 1 rises.
  - (Nash) Equilibrium has $t_1>t_2$ and $k_1<k_2$.
- Perhaps more critically for our concerns:

$$\frac{U_z}{U_x} = \frac{k_i + q^* + \left(k_i - k^*\right)\frac{\partial \rho}{\partial t_i}}{k_i + q^* + t_i \frac{\partial k_i}{\partial t_i}} \text{ implicitly defines reaction functions, } t_i^R = R\left(\mathbf{t}_j\right)$$

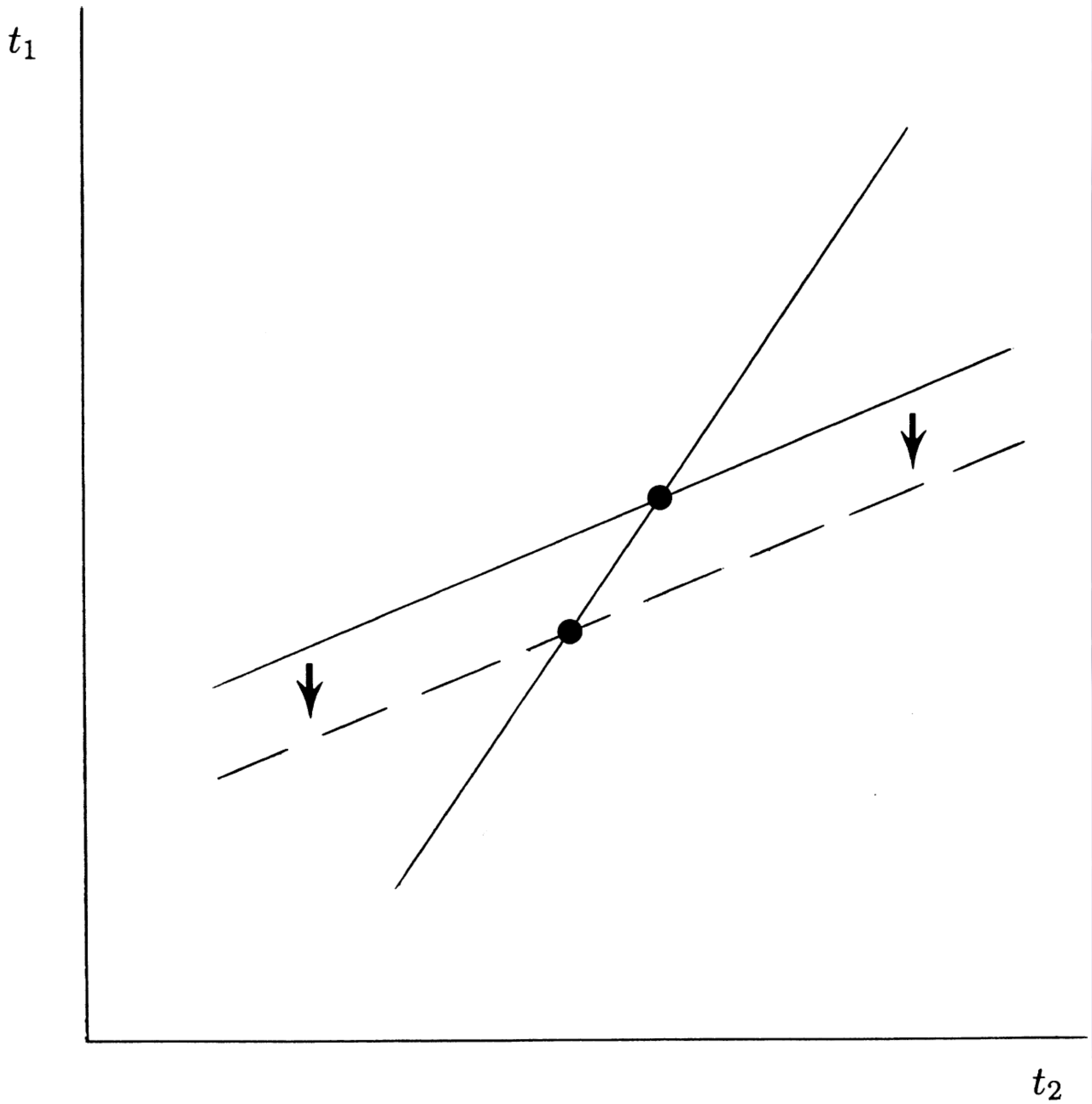
because $\frac{\partial \rho}{\partial t_i} = \frac{\partial\left(f'(k_i) - t_i\right)}{\partial t_i}$ and $\frac{\partial k_i}{\partial t_i}$ depend on $\mathbf{t}_j$ $\left[\text{...because } k_i = g\left(t_i, \mathbf{t}_j\right)\right]$.

- Substantively: the effects of tax competition derive from what competitors are doing (*duh!*). So…

Brueckner & Saavedra Model of Municipal Tax-Competition[4]

- The effects of tax competition derive from what competitors are doing (*duh!*). So…
  - Standard empirical strategies problematic:
    - Nonspatial LS or ML or Bayesian estimation inappropriate.
    - SUTVA violated, so ditto std 'causal inference' strategies.
  - Need to model the interdependence:
    - However, Galton's problem (+): easy to mistake common exposure (or selection) for contagion because look similar.
    - Specification (incl. measurement, etc.) is everything! EITM.
- The first-order conditions in [6] give reaction functions $t_i = R(\mathbf{t}_j)$ implicitly. (Nash) Equilibrium occurs at intersection of those $R(\cdot)$. (See next.)

Brueckner & Saavedra Model of Municipal Tax-Competition[5]



- $R(\cdot)$ may slope up or down.
  - Former is typical assumed 'race to [?]' case.
  - Latter: i prefers to use cap tax-base ↑ from ↑ $\mathbf{t}_j$ to ↓t_i yet maintain or even ↑ revenue.
- Can derive some EI of TM by comp statics on gen model.
  - Illustrated might be ↓ 1's pref for z, for instance.
  - Galton's Problem (+) will plague empirical eval though.
  - Need specify theoretical model adequately for *powerful* specify of empirical.
- Fuller aim is often to estimate these $R(\cdot)$. Call it EM of TM.
  - For that especially, & esp. given Galton's Problem (+), want/need specify theoretical model for powerful specification empirical…

Brueckner & Saavedra Model of Municipal Tax-Competition[6]

- B&S's Example:
  - Let *f* be quadratic…
    - $f(k_i)=\beta k_i+½(\gamma k_i^2)$, where $\beta,\gamma>0$
  - …and *U* linear
    - $U(x_i,q_i,z_i)=x_i+\lambda_i q_i+\eta_i z_i$, where $\lambda_i,\eta_i>0$
  - With these specifications:

$$f'(k_1)-t_1=\rho=f'(k_2)-t_2 \text{ becomes } \beta - \gamma k_1 - t_1=\rho=\beta - \gamma k_2 - t_2,$$

$$\text{which, with } k_1=k_2=k^*, \text{ becomes } k_1=k^*+(t_2-t_1)/2\gamma, \text{ so that } \frac{\partial k_1}{\partial t_1}=-\frac{1}{2}\gamma.$$

  - Substituting this into [6] produces…

Brueckner & Saavedra Model[7]

$$\frac{U_z}{U_x}=\frac{k_i+q^*+\left(k_i-k^*\right)\frac{\partial\rho}{\partial t_i}}{k_i+q^*+t_i\frac{\partial k_i}{\partial t_i}}\Rightarrow\frac{\eta_1}{1}=\frac{k^*+\frac{(t_2-t_1)}{2\gamma}+q^*+\left(k^*+\frac{(t_2-t_1)}{2\gamma}-k^*\right)\left(-\gamma(-\frac{1}{2\gamma})-1\right)}{k^*+\frac{(t_2-t_1)}{2\gamma}+q^*-t_1\frac{1}{2\gamma}}$$

$$\text{(substing above } k_i=k^*+\frac{(t_2-t_1)}{2\gamma},\ \frac{\partial k_i}{\partial t_i}=-\frac{1}{2\gamma},\ \frac{\partial\rho}{\partial t_i}=-\gamma(-\frac{1}{2\gamma})-1=-\frac{1}{2}\text{; simplify)}\Rightarrow$$

$$\eta_1=\frac{k^*+q^*+\frac{(t_2-t_1)}{2\gamma}-\frac{1}{2}\left(\frac{(t_2-t_1)}{2\gamma}\right)}{k^*+q^*+\frac{(t_2-t_1)}{2\gamma}-\frac{t_1}{2\gamma}}\text{, rearrange}\Rightarrow k^*+q^*+\frac{(t_2-t_1)}{2\gamma}-\frac{t_1}{2\gamma}=\frac{k^*+q^*+\frac{(t_2-t_1)}{2\gamma}-\frac{1}{2}\left(\frac{(t_2-t_1)}{2\gamma}\right)}{\eta_1}$$

$$\text{(multiply both sides by } 2\gamma)\Rightarrow 2\gamma\left(k^*+q^*\right)+t_2-2t_1=\frac{2\gamma}{\eta_1}\left(k^*+q^*\right)+\frac{1}{2\eta_1}(t_2-t_1)$$

$$\text{(isolate } t_1 \text{ and gather terms)}\Rightarrow -2t_1+\frac{1}{2\eta_1}t_1=\frac{2\gamma}{\eta_1}\left(k^*+q^*\right)-2\gamma\left(k^*+q^*\right)+\frac{1}{2\eta_1}t_2-t_2$$

$$\Rightarrow t_1\left(\frac{1-4\eta_1}{2\eta_1}\right)=2\gamma\left(k^*+q^*\right)\left(\frac{1-\eta_1}{\eta_1}\right)+t_2\left(\frac{1-2\eta_1}{2\eta_1}\right)\Rightarrow t_1=\left(\frac{2\eta_1}{1-4\eta_1}\right)\left(\frac{1-\eta_1}{\eta_1}\right)2\gamma\left(k^*+q^*\right)+\left(\frac{2\eta_1}{1-4\eta_1}\right)\left(\frac{1-2\eta_1}{2\eta_1}\right)t_2$$

$$\Rightarrow t_1=\left(\frac{4\gamma(1-\eta_1)}{1-4\eta_1}\right)\left(k^*+q^*\right)+\left(\frac{1-2\eta_1}{1-4\eta_1}\right)t_2$$

- For our purposes, especially key to note is how the sensitivity of t_1 depends on the marginal utility of public goods (relative to private). This is w_{12} in the theoretical model. It also enters the (non-spatial) intercept, where also go terms related to capital and housing endowments and to the diminishing returns in the production function.

Persson & Tabellini's (2000: 12.4) *International Tax-Competition Model*

- Two jurisdictions, domestic & foreign, levy capital taxes, τ_k & τ_k^*, to fund exogenously fixed (and wasted) public spend.
- Individuals:
  - have labor-capital endowment, e^i, and
  - choose labor, l, vs. leisure, x, and
  - make savings-investment decisions, $s=k+f$, with
    - foreign investment, f, paying mobility costs, $M(f)$,
  - to max $\omega=U(c^1)+c^2+V(x)$, over l, c^1, & c^2,
    - s.t. time-constraint: $1+e^i=l+x$, and
    - BC1 & BC2: $1-e^i = c^1+k+f \equiv c^1+s$ and $c^2=(1-\tau_k)k+(1-\tau_k^*)f-M(f)+(1-\tau_l)l$.
- $\Rightarrow$ equilibrium economic choices of citizens […], which $\Rightarrow$ indirect utility, W, defined over policy variables, τ_l, τ_k, & τ_k^*, […as indicated on next slide…]

Persson & Tabellini's (2000: 12.4) *International Tax-Competition Model*

- $\Rightarrow$ equilibrium economic choices of citizens:

$$s = S(\tau_k) = 1 - U_c^{-1}(1-\tau_k)$$

$$f = F(\tau_k, \tau_k^*) = M_f^{-1}(\tau_k - \tau_k^*)$$

$$k = K(\tau_k, \tau_k^*) = S(\tau_k) - F(\tau_k, \tau_k^*)$$

- $\Rightarrow$ indirect utility, *W*, over policy vars, τ_l, τ_k, & τ_k^*:
  - Utility of c^1…
  - Utility of c^2 via S & L…
  - Leisure value…
  - Convenient way write *f* costs…

$$\begin{aligned} W(\tau_l, \tau_k) = U\left(1 - S(\tau_k)\right) \\ + \left(1-\tau_k\right) S(\tau_k) + \left(1-\tau_l\right) L(\tau_l) \\ + V\left(1 - L(\tau_l)\right) \\ + \left(\tau_k - \tau_k^*\right) F(\tau_k, \tau_k^*)\, M\left(F(\tau_k, \tau_k^*)\right) \end{aligned}$$

- Besley-Coate (97) cit-cands face voters w/ these pref
  - Stages: 1) elects, 2) wins set taxes, 3) private actors go
  - Ebm winner e^P s.t. desires enact *Modified Ramsey Rule*…

Persson & Tabellini's (2000: 12.4) *International Tax-Competition Model*

- Besley-Coate ('97) cit-cands winner has endowment e^P s.t. desires implement this *Modified Ramsey Rule*:

$$\frac{S(\tau_k^p)-e^p}{S(\tau_k^p)}\left[1+\varepsilon_l(\tau_k^p)\right]=\frac{L(\tau_l^p)+e^p}{L(\tau_l^p)}\left[1+\frac{S_\tau(\tau_k^p)+2F_\tau^*(\tau_k^{p*},\tau_k^p)\tau_k}{S_\tau(\tau_k^p)}\right]$$

  - Ramsey Rule: choose τ_l & τ_k to equate elasticities lab & cap supply
    - Modification 1: Because noncoop b/w domestic & foreign, elasticity of savings on RHS inflated by the $2F$ term. Shift τ toward immobile: $\tau_l \uparrow$.
    - Modification 2: Because cit-cand has distributional pref's regarding τ_l vs. τ_k, she shifts τ toward τ_k by a Meltzer-Richard type consideration in e^P.
- $\Rightarrow$ **Best-Response Functions**: $\tau_k=T(e^P,\tau_k^*)$ & $\tau_k^*=T^*(e^{P*},\tau_k)$ for domestic & foreign policymakers.
  - $\partial T/\partial\tau_k^*$ & $\partial T^*/\partial\tau_k$, positive or negative as in B&S…
  - $F_\tau=M_f^{-1}(\tau_k-\tau_k^*)$, so $\downarrow$ mobility-costs likely $\downarrow \tau_k$ and $\uparrow |\partial T/\partial\tau_k^*|$.
  - Likewise, e^P seems to enter both as **x**, shift $R(\cdot)$, and as $\uparrow w_{ij}$.
    - Next illustrates reaction-functions & "leftward" shift domestic policymaker

Persson & Tabellini's (2000: 12.4) *International Tax-Competition Model*

- Globalization as ↓$M(f)$ example: Let $M(f)=\mu^B(f^A)^2$ express ↑/↓ mobility (from B) costs by ↑/↓ μ^B
  - Rest as before:

$$c_1^A = U_c^{-1}\left(1-\tau^A\right) \equiv \mathrm{C}^A\left(\tau^A\right) \quad ; \quad f^A = M_f^{-1}\left(\tau^A - \tau^B; \mu^B\right) \equiv F^A\left(\tau^A, \tau^B; \mu^B\right)$$

$$k = 1 + e^A - F^A\left(\tau^A, \tau^B; \mu^B\right) - U_c^{-1}\left(1-\tau^A\right) \equiv D^A\left(\tau^A, \tau^B; \mu^B\right) + e^A$$

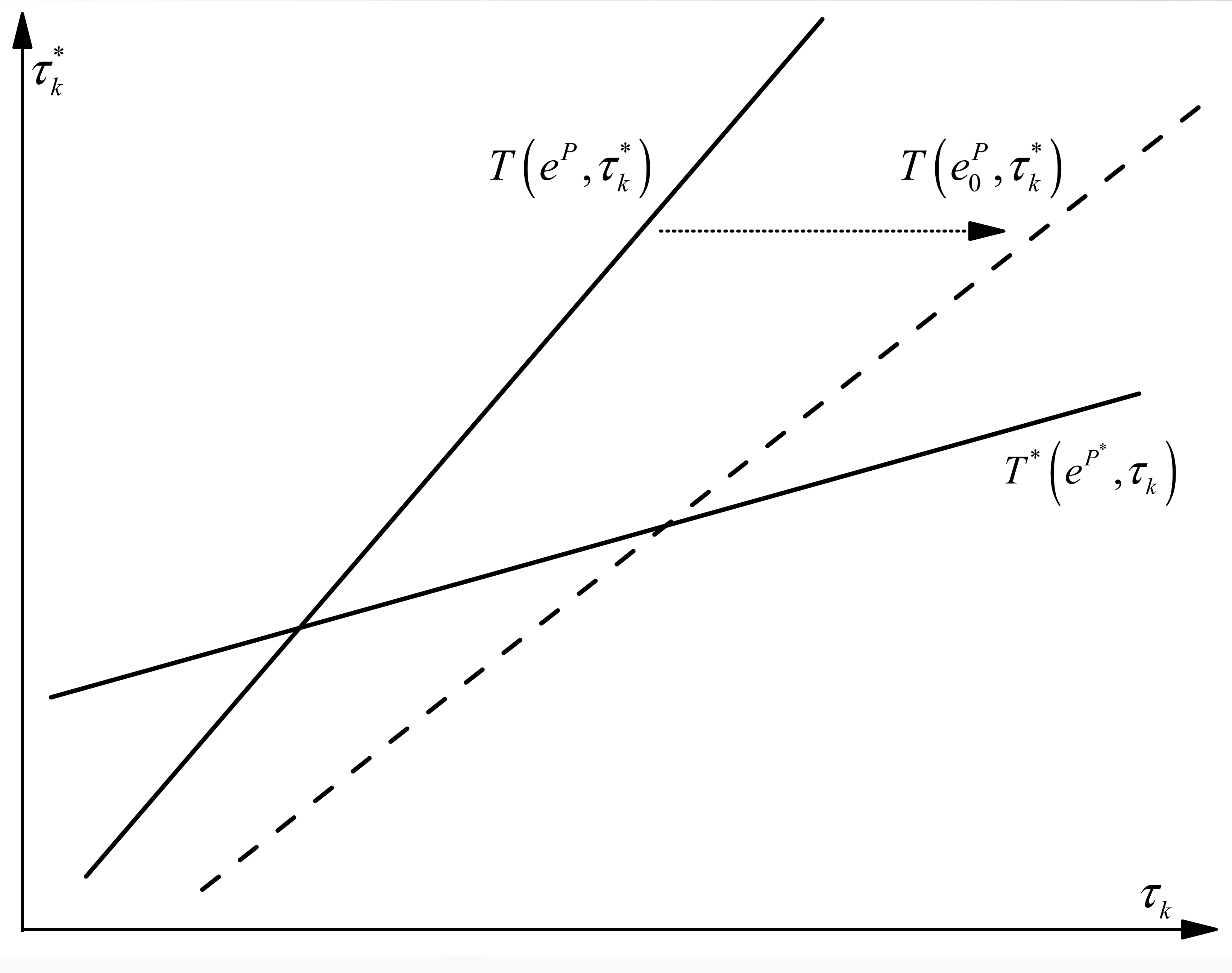
  - ⇒ Reaction functions like this:

$$\tau^A\left(C_{\tau^A}^A + F_{\tau^A}^A - F_{\tau^A}^B\right) = F^B - e^A$$

$$\tau^B\left(C_{\tau^B}^B + F_{\tau^B}^B - F_{\tau^B}^A\right) = F^A - e^B$$

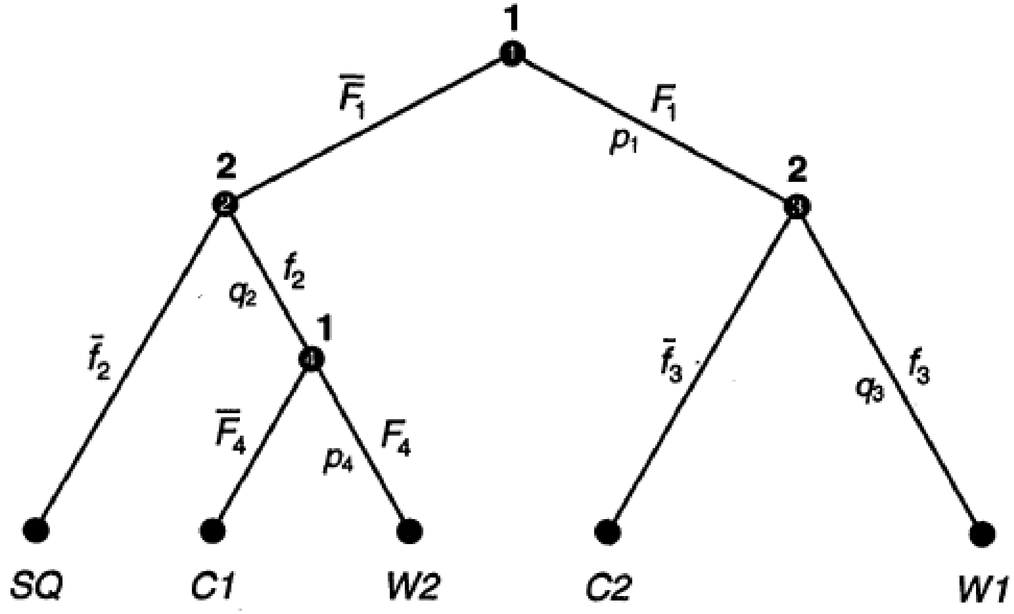
  - ⇒ μ^B & μ^A (and e^B & e^A) in two places:
    - Intercept shift (i.e., presence in **x**) essentially RHS of $R(\cdot)$
    - Responsiveness of A to B (i.e., in w_{ij}).

Persson & Tabellini's (2000: 12.4) *International Tax-Competition Model*



Signorino's (1999) Model of Strategic Interaction in International Conflict

FIGURE 2. A Typical Bilateral Crisis Game

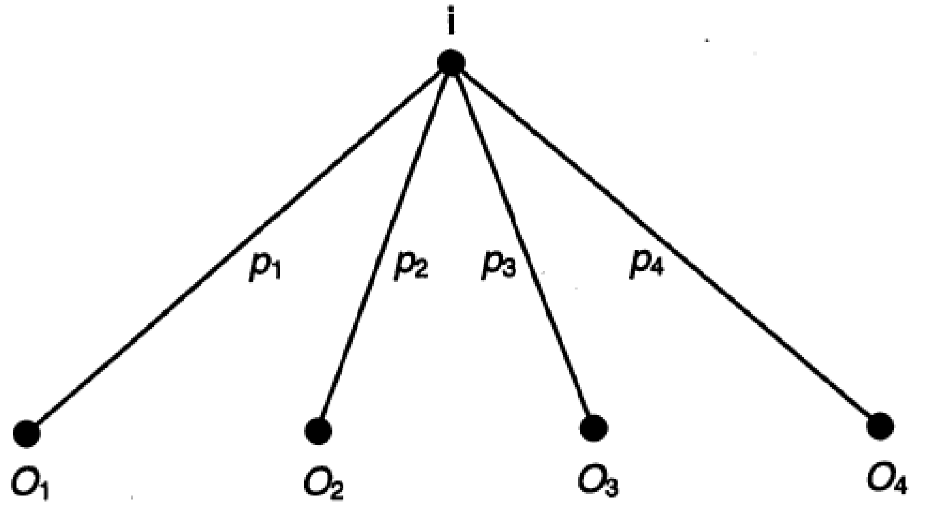


Note: States 1 and 2 alternate moves at decision nodes. Actions by state 1 are shown in uppercase, those by state 2 in lowercase. A bar over an action refers to the opposite of the action (e.g., not using force). The equilibrium choice probabilities used in the statistical model are denoted p_i for state 1 and q_j for state 2. Nonterminal nodes are numbered to simplify the expected utility notation and to index choice probabilities.

(Strategic Logit with Agent Errors)

The Underlying Behavioral Model: Random Utility via Multinomial Logit

FIGURE 1. Discrete Choice Model with Exogenously Determined Choice Probabilities



$$U^*(O_j) = U(O_j) + \varepsilon_j$$

Choose O_j such that $U^*(O_j) > U^*(O_k)$ for all $k \neq j$.

Assuming **ε** i.i.d. type-I extreme value gives

$$p_j = \frac{e^{U(O_j)}}{\sum_k e^{U(O_k)}},$$ where the $U(O_j)$ are "observed" utilities.

Quantal Response Equilibrium

Actors play best responses with mistakes from a known distribution of errors.

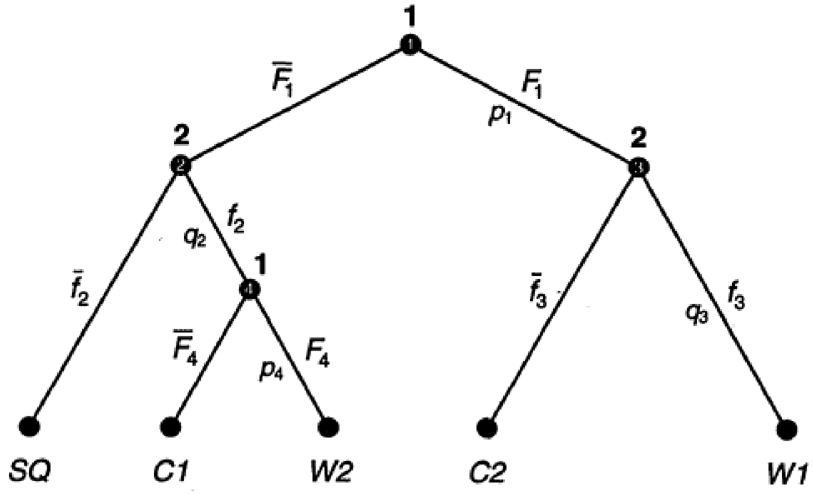
$$p_4 = \frac{e^{\lambda U_1(W2)}}{e^{\lambda U_1(W2)} + e^{\lambda U_1(C2)}}$$

$$q_3 = \frac{e^{\lambda U_2(W2)}}{e^{\lambda U_2(W2)} + e^{\lambda U_2(C2)}}$$

$$q_2 = \frac{e^{\lambda[p_4 U_2(W2)+(1-p_4)U_2(C1)]}}{e^{\lambda[p_4 U_2(W2)+(1-p_4)U_2(C1)]} + e^{\lambda U_2(SQ)}}$$

$$p_1 = \frac{e^{\lambda[q_3 U_1(W1)+(1-q_3)U_1(C2)]}}{e^{\lambda[q_3 U_1(W1)+(1-q_3)U_1(C2)]} + e^{\lambda\{q_2[p_4 U_1(W2)+(1-p_4)U_1(C1)]+(1-q_2)U_1(SQ)\}}}$$

FIGURE 2. A Typical Bilateral Crisis Game



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Best response functions are interdependent and probabilistic. The probability that State 1 chooses to fight depends on the probability that State 2 will fight.

We can make the observed utilities (and the best responses) a parameterized function of covariates.

$$U_i(SQ) = \beta_d D_{ij}$$

$$U_i(Ci) = -\beta_{ai} A_i$$

$$U_i(Cj) = \beta_{aj} A_j$$

$$U_i(W) = P_i \beta_{aj} A_j + (1 - P_i)(-\beta_{ai} A_i - \beta_{mi} M_i)$$

where $P_i = \dfrac{M_i}{M_i + M_j}$

Reduced-form probabilities get a bit complicated...

$$p_1 = \frac{e^{\left[\left\{\frac{e^{U_2(W2)}}{e^{U_2(W2)}+e^{U_2(C2)}}\right\}U_1(W1)+\left(1-\left\{\frac{e^{U_2(W2)}}{e^{U_2(W2)}+e^{U_2(C2)}}\right\}\right)U_1(C2)\right]}}{\text{EXP1}+e^{\text{EXP2}+\text{EXP3}}}, \text{ where:}$$

$$\text{EXP1} \equiv e^{\left[\left\{\frac{e^{U_2(W2)}}{e^{U_2(W2)}+e^{U_2(C2)}}\right\}U_1(W1)+\left(1-\left\{\frac{e^{U_2(W2)}}{e^{U_2(W2)}+e^{U_2(C2)}}\right\}\right)U_1(C2)\right]}$$

$$\text{EXP2} \equiv \left(\frac{e^{\left[\left\{\frac{e^{U_1(W2)}}{e^{U_1(W2)}+e^{U_1(C2)}}\right\}U_2(W2)+\left(1-\left\{\frac{e^{U_1(W2)}}{e^{U_1(W2)}+e^{U_1(C2)}}\right\}\right)U_2(C1)\right]}}{e^{\left[\left\{\frac{e^{U_1(W2)}}{e^{U_1(W2)}+e^{U_1(C2)}}\right\}U_2(W2)+\left(1-\left\{\frac{e^{U_1(W2)}}{e^{U_1(W2)}+e^{U_1(C2)}}\right\}\right)U_2(C1)\right]}+e^{U_2(SQ)}}\right)\left[\left\{\frac{e^{U_1(W2)}}{e^{U_1(W2)}+e^{U_1(C2)}}\right\}U_1(W2)+\left(1-\left\{\frac{e^{U_1(W2)}}{e^{U_1(W2)}+e^{U_1(C2)}}\right\}\right)U_1(C1)\right]$$

$$\text{EXP3} \equiv \left(1-\left(\frac{e^{\left[\left\{\frac{e^{U_1(W2)}}{e^{U_1(W2)}+e^{U_1(C2)}}\right\}U_2(W2)+\left(1-\left\{\frac{e^{U_1(W2)}}{e^{U_1(W2)}+e^{U_1(C2)}}\right\}\right)U_2(C1)\right]}}{e^{\left[\left\{\frac{e^{U_1(W2)}}{e^{U_1(W2)}+e^{U_1(C2)}}\right\}U_2(W2)+\left(1-\left\{\frac{e^{U_1(W2)}}{e^{U_1(W2)}+e^{U_1(C2)}}\right\}\right)U_2(C1)\right]}+e^{U_2(SQ)}}\right)\right)U_1(SQ)$$

$$q_3 = e^{\left[\left\{\frac{e^{U_2(W2)}}{e^{U_2(W2)}+e^{U_2(C2)}}\right\}U_1(W1)+\left(1-\left\{\frac{e^{U_2(W2)}}{e^{U_2(W2)}+e^{U_2(C2)}}\right\}\right)U_1(C2)\right]}$$

$$U_i(SQ) = \beta_d D_{ij} \quad ; \quad U_i(Ci) = -\beta_{ai} A_i \quad ; \quad U_i(Cj) = \beta_{aj} A_j$$

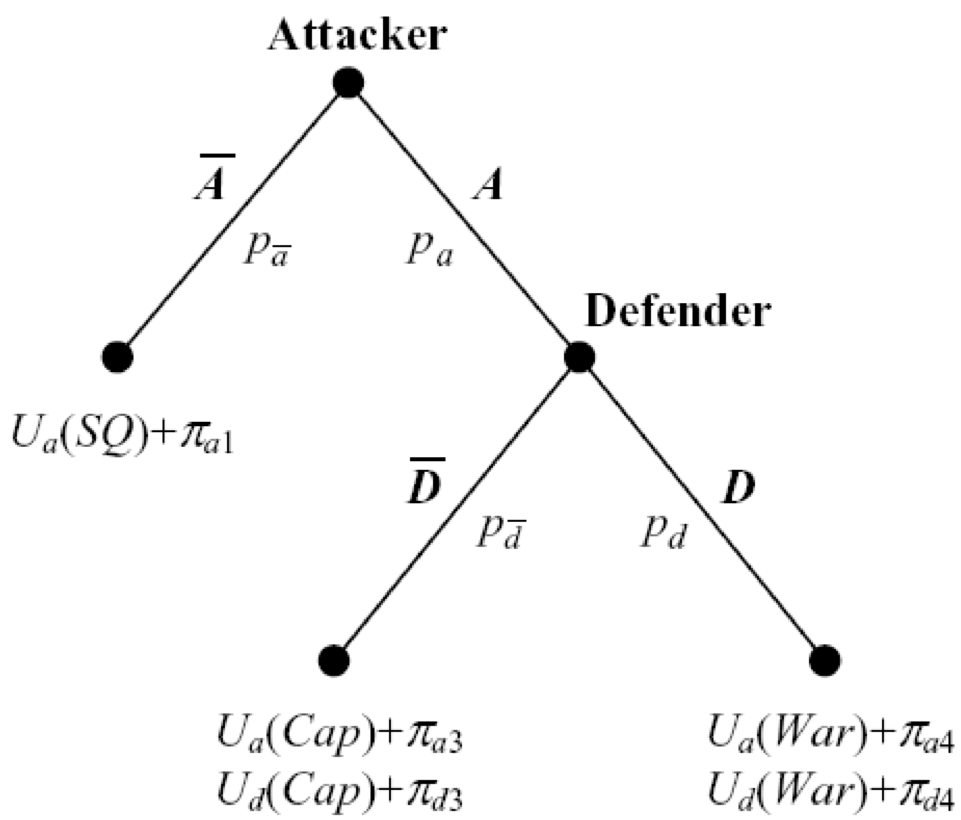
$$U_i(W) = P_i \beta_{aj} A_j + (1-P_i)(-\beta_{ai} A_i - \beta_{mi} M_i)$$

$$\Pr(W1) = p_1 \times q_3$$

$$\Pr(War) = \Pr(W1) + \Pr(W2)$$

Signorino and Tarar's (2006) Model of Extended Deterrence

FIGURE 1 The Deterrence Model with Uncertainty Concerning Utilities



(Strategic Probit with Private Information)

The Underlying Behavioral Model: Random Utility via Multinomial Probit

The defender chooses D iff $U_d^*(War) > U_d^*(Cap)$.

The attacker chooses A iff $p_d \times U_a^*(War) + (1 - p_d) \times U_a^*(Cap) > U_a^*(SQ)$.

Assuming $\pi_{ij} \sim$ i.i.d. $N(0, \sigma^2)$ gives us

$$p_d = \Phi\left[\frac{U_d(War) - U_d(Cap)}{\sqrt{2\sigma^2}}\right], \text{ and}$$

Only probabilistic from the analyst's and attacker's perspectives.

Two sources of uncertainty.

$$p_a = \Phi\left[\frac{p_d U_a(War) + (1 - p_d) U_a(Cap) - U_a(Cap)}{\sqrt{\sigma^2\left(p_d^2 + (1 - p_d)^2 + 1\right)}}\right]$$

Three sources of uncertainty here.

Deriving the Probabilities

$$\begin{aligned} p_d &= \Pr\left[U_d^*(War) > U_d^*(Cap)\right] \\ &= \Pr\left[U_d(War) + \pi_{d4} > U_d(Cap) + \pi_{d3}\right] \\ &= \Pr\left[U_d(War) - U_d(Cap) > \pi_{d3} - \pi_{d4}\right] \end{aligned}$$

$$\begin{aligned} p_d &= \Phi\left[\frac{U_d(War) - U_d(Cap)}{\sqrt{\mathrm{var}(\pi_{d3}) + \mathrm{var}(\pi_{d4})}}\right] \\ &= \Phi\left[\frac{U_d(War) - U_d(Cap)}{\sqrt{2\sigma^2}}\right] \end{aligned}$$

Deriving the Probabilities

$$p_a = \Pr\left[p_d U_a^*(War) + (1 - p_d) U_a^*(Cap) > U_a^*(SQ) \right]$$
$$= \Pr\left[p_d \left[U_a(War) + \pi_{a4} \right] + (1 - p_d) \left[U_a(Cap) + \pi_{a3} \right] > U_a(SQ) + \pi_{a1} \right]$$
$$= \Pr\left[p_d U_a(War) + (1 - p_d) U_a(Cap) - U_a(SQ) > \pi_{a1} - p_d \pi_{a4} - (1 - p_d) \pi_{a3} \right]$$

$$p_d = \Phi\left[\frac{U_d(War) - U_d(Cap)}{\sqrt{\mathrm{var}(\pi_{a1}) + p_d^2 \,\mathrm{var}(\pi_{a4}) + (1 - p_d)^2 \,\mathrm{var}(\pi_{a3})}} \right]$$
$$= \Phi\left[\frac{U_d(War) - U_d(Cap)}{\sqrt{\sigma^2 \left(p_d^2 + (1 - p_d)^2 + 1 \right)}} \right]$$

Note that the source of uncertainty matters…

If the uncertainty were due to agent error,

the denominators in p_d and p_a would be the same.

(Attackers and defenders make mistakes at same rate.)

$$p_d = \Phi\left[\frac{U_d(War) - U_d(Cap)}{\sqrt{2\sigma^2}}\right], \text{ and}$$

Uncertainty due to defender's mistakes.

$$p_a = \Phi\left[\frac{p_d U_a(War) + (1 - p_d) U_a(Cap) - U_a(Cap)}{\sqrt{2\sigma^2}}\right]$$

Uncertainty due to attacker's mistakes.

Well…Signorino's games don't exactly lead to spatial lag models. Do they? Could they? What about multilateral crises and war-joining behavior?

Some interdependent discrete choice models do lead directly to this kind of empirical representation.

E.g.,…

Beron et al.'s (2003) Model of International Environmental Cooperation

The Behavioral Model under ***Independence***

$$NB_i = \mathbf{x}_i \boldsymbol{\beta} + \varepsilon_i$$

Choose to cooperate if $NB_i > 0$ and not cooperate if $NB_i \leq 0$.

$$\Pr[NB_i > 0] = \Pr\left[\boldsymbol{\varepsilon}_i \; < \; \mathbf{x}_i \boldsymbol{\beta}\right]$$

$$= \Phi\left(\mathbf{x}_i \boldsymbol{\beta}\right)$$

Beron et al.'s (2003) Model of International Cooperation

Model under ***Interdependence***: The Role of Power

$Power = \mathbf{P} \times \mathbf{NB}$, where the p_{ij} are measures of export dependence.

$$\mathbf{NB} = \alpha\mathbf{P} \times \mathbf{NB} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \;\;\Rightarrow\;\; \mathbf{NB} = (\mathbf{I} - \alpha\mathbf{P})^{-1}\mathbf{X}\boldsymbol{\beta} + (\mathbf{I} - \alpha\mathbf{P})^{-1}\boldsymbol{\varepsilon}$$

$$\Pr[NB_i > 0] = \Pr\left[\left[(\mathbf{I} - \alpha\mathbf{P})^{-1}\boldsymbol{\varepsilon}\right]_i < \left[(\mathbf{I} - \alpha\mathbf{P})^{-1}\mathbf{X}\boldsymbol{\beta}\right]_i\right]$$

$$= \Pr\left[\boldsymbol{\varepsilon}_i < \frac{\left[(\mathbf{I} - \alpha\mathbf{P})^{-1}\mathbf{X}\boldsymbol{\beta}\right]_i}{\sigma_i}\right]$$

$$= \Phi\left[\frac{\left[(\mathbf{I} - \alpha\mathbf{P})^{-1}\mathbf{X}\boldsymbol{\beta}\right]_i}{\sigma_i}\right]$$

← C.d.f. for the marginal distribution after integrating out the other n-1 dimensions.

The Estimation Problem

$$\boldsymbol{\varepsilon}^* = (\mathbf{I} - \alpha\mathbf{P})^{-1}\boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon}^* \sim N(\mathbf{0}, \boldsymbol{\Sigma}), \text{ with } \boldsymbol{\Sigma} = \left[(\mathbf{I} - \alpha\mathbf{P})'(\mathbf{I} - \alpha\mathbf{P})\right]^{-1}$$

$$\boldsymbol{\varepsilon}^{**} = \boldsymbol{\Theta}\boldsymbol{\varepsilon}^*, \text{ with elements } \theta_{ij} = 1 - 2y_i \text{ for } i = j \text{ and } \theta_{ij} = 0 \text{ for } i \neq j$$

$$\boldsymbol{\varepsilon}^{**} \sim N(\mathbf{0}, \boldsymbol{\Theta\Sigma\Theta})$$

$$\mathbf{z} = \boldsymbol{\Theta}(\mathbf{I} - \alpha\mathbf{P})^{-1}\mathbf{X}\boldsymbol{\beta}$$

$$\Pr[\mathbf{y}] = \Pr\left[\boldsymbol{\varepsilon}^{**} < \mathbf{z}\right]$$

$$= \int_{-\infty}^{\mathbf{z}} \phi_N\left(\boldsymbol{\varepsilon}^{**}; \mathbf{0}, \boldsymbol{\Theta\Sigma\Theta}\right) d\boldsymbol{\varepsilon}^{**}$$

Calculating the joint likelihood requires us to integrate over an N-dimensional multivariate normal distribution.