STABLE VECTORS IN THE MOY-PRASAD FILTRATION

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LARGE GOAL / MOTIVATION
Understand and classify all complex representations of p-adic groups, i.e., Local Langlands correspondence.

WHAT ARE P-ADIC NUMBERS?
In number theory the study of congruences plays an important role, and hence we want to define a norm on the integers so that two numbers \( n \) and \( m \) are close iff \( n \equiv m \mod p^k \) for a large \( N \). This is achieved by \( |p^k \cdot r| = p^{-k} \) with \( r \) and \( p \) coprime.

The p-adic integers \( \mathbb{Z}_p \) are the completion of the integers by this norm \( | \cdot | \), i.e., a p-adic integer is of the form \( a_0 + a_1 \cdot p + a_2 \cdot p^2 + a_3 \cdot p^3 + \ldots \) for some integers \( a_i \).

The p-adic numbers \( \mathbb{Q}_p \) are the fraction field of the p-adic integers. They are a completion of the rational numbers \( \mathbb{Q} \).

We call a field \( \mathbb{F} \) that is a finite extension of the p-adic \( \mathbb{Q}_p \) a p-adic field.

WHAT ARE P-ADIC GROUPS?
P-adic groups, or more precisely, reductive groups over p-adic fields, are certain subgroups of the group of invertible \( n \times n \) matrices whose entries are elements of a p-adic field \( \mathbb{F} \), e.g., \( \text{GL}_n(\mathbb{F}), \text{SL}_n(\mathbb{F}), \text{SO}_n(\mathbb{F}), \text{Sp}_{2n}(\mathbb{F}). \) On this poster, we will restrict our attention to the simple factors of these groups, and we make the assumption that our group is split - a technical assumption that is always satisfied for reductive groups over algebraically closed fields. These split simple groups are classified up to a finite center in terms of a combinatorial object, the Dynkin diagram.

Moy-Prasad filtration

The Bruhat-Tits building \( \mathcal{B}(G,F) \)
- is a building associated to a given p-adic group \( G \) by Bruhat and Tits
- for \( \text{SL}_2(\mathbb{Q}_p) \) it is an infinite tree in which each vertex has \( p+1 \) neighbors, see Figure 1
- for every point \( x \) in the building, Bruhat and Tits define a compact subgroup \( G_x \) in \( G \), called the parahoric subgroup, which has finite index in the stabilizer \( \text{Stab}_x \).

\[ \mathcal{B}(G,F) \]

Figure 1: The Bruhat-Tits building of \( \text{SL}_2(\mathbb{Q}_p) \); source: [Rab05].

Stable vectors and epipelagic representations

Supercuspidal representations
- are the building blocks for all representation of p-adic groups
- very mysterious, only few constructions known, see [Adl98] (special case) and [Yu01] (for large \( p \))

Epipelagic representations are supercuspidal representations of smallest positive depth.

Main theorem

Theorem 1 (in words). The existence of stable vectors in the Moy-Prasad filtration quotient does not depend on the prime \( p \).

Theorem 1 (details for experts). Let \( x \in \mathcal{B}(G,F) \) be a rational point of order \( m \). Then there exist stable vectors in \( G_x \) under the action of \( G_0/G_0 \) if and only if there exists an elliptic, \( \mathbb{Z} \)-regular element of order \( m \) in the Weyl group of \( G \) and \( x \) is conjugate to \( x_0 \) under the affine Weyl group for some hyperspecial point \( x_0 \). Here \( p \) is half of the sum of the positive co-roots.

This theorem was known for large primes \( p \) thanks to [RY14].

Applications
- We obtain supercuspidal (epipelagic) representations uniformly for all primes \( p \).
- As a corollary of the proof we obtain a different description of the Moy-Prasad filtration quotient as a representation of the reductive quotient for all primes \( p \) without restriction.
- The proof involves a construction of the filtration quotient representations over the integers. As a consequence we can compare the occurring representations of the reductive quotients at different primes.

References