Engineering Agreement: The Naming Game with Asymmetric and Heterogeneous Agents

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Social Convention

- Conventions are universally adopted from two or more alternatives.
- Language, etiquette, or custom.
Engineering Agreement

• What can help or harm convergence?
  – Homogeneity or heterogeneity
  – Community structure

• How robust are the dynamics to possible manipulations?
Naming Game [Baronchelli 06]

- A agent-based process on a network
Naming Game

• A agent-based process on a network
  – Each agent has inventory of names
Naming Game

- A agent-based process on a network
  - Each agent has inventory of names
Naming Game

• A agent-based process on a network
  – Each agent has *inventory* of names
  – At each time an edge is selected at random
Naming Game

• A agent-based process on a network
  – Each agent has inventory of names
  – At each time an edge is selected at random, and one is speaker and the other is listener.
Naming Game

- A agent-based process on a network
  - Each agent has inventory of names
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Naming Game

• A agent-based process on a network
  – Each agent has inventory of names
  – At each time an edge is selected at random, and one is speaker and the other is listener.
• Failure
Naming Game

- A agent-based process on a network
  - Each agent has inventory of names
  - At each time an edge is selected at random, and one is speaker and the other is listener.
  - Failure: listener adds the new name

Example names:
- pop
- coke
- Soft drink
- Soda
- pop
- coke
- Soft drink
- Soda
Naming Game

• A agent-based process on a network
  – Each agent has inventory of names
  – At each time an edge is selected at random, and one is speaker and the other is listener.
    • Failure: listener adds the new name

Soft drink
coke
Soda
pop
coke
Soft drink

coke
Soda
coke
pop
Soft drink
coke
Soda
Naming Game

- A agent-based process on a network
  - Each agent has inventory of names
  - At each time an edge is selected at random, and one is speaker and the other is listener.
    - Failure: listener adds the new name
Naming Game

- A agent-based process on a network
  - Each agent has inventory of names
  - At each time an edge is selected at random, and one is speaker and the other is listener.
    - Failure: listener adds the new name
    - Success
Naming Game

• A agent-based process on a network
  – Each agent has inventory of names
  – At each time an edge is selected at random, and one is speaker and the other is listener.
    • Failure: listener adds the new name
    • Success: both remove all other names
Naming Game

- A agent-based process on a network
  - Each agent has inventory of names
  - At each time an edge is selected at random, and one is speaker and the other is listener.
    - Failure: listener adds the new name
    - Success: both remove all other names
    - Empty
Naming Game

• A agent-based process on a network
  – Each agent has inventory of names
  – At each time an edge is selected at random, and one is speaker and the other is listener.
    • Failure: listener adds the new name
    • Success: both remove all other names
    • Empty: speaker invent a new word
Naming Game

• A agent-based process on a network
  – Each agent has inventory of names
  – At each time an edge is selected at random, and one is speaker and the other is listener.
    • Failure: listener adds the new name
    • Success: both remove all other names
    • Empty: speaker invent a new word
  – Convergence
Different initial states

Empty initial states

Segregated initial states
Motivating Questions

• What can help or harm convergence?
  – Homogeneity or heterogeneity
  – Community structure

• How robust are the dynamics to possible manipulations?
Different graphs

d-ary tree

star

grid

disjoint cliques

complete

regular

Kleinberg

Watts-Strogatz
Fast and Slow Convergence

![Graph showing consensus time vs. number of nodes for different graph structures: grid, complete graph, regular graph, and Star. The graph highlights local structure with a red arrow indicating fast convergence for local structures compared to other graph types.]
Fast and Slow Convergence

Consensus time vs. # nodes for different network structures:
- Grid
- Complete graph
- Regular graph
- Star

Homogeneous structure compared to local structure.
Fast and Slow Convergence

Consensus time vs. # nodes for different network structures:
- **Homogeneous** network structures:
  - Grid
  - Complete graph
  - Regular graph
- **Heterogeneous** network structures:
  - Local structure
  - Star

Legend:
- **grid**
- **complete graph**
- **regular graph**
- **Star**
L = 7

R = 1

\[ \frac{R}{R+L} = \frac{1}{8} \]

Normalized consensus time
Heterogeneous

L = 6
R = 2

R/(R+L) = 2/8

Diagram showing a network with labeled nodes and edges, and a graph plotting normalized consensus time against R/(R+L) with two lines representing different values.
Heterogeneous

\[ \frac{R}{R+L} = \frac{3}{8} \]

L = 5
R = 3

Normalized consensus time

\[ R/(R+L) \]

Graph showing normalized consensus time for different values of \( R/(R+L) \) with data points for 10000 and 5000.
Motivating Questions

• What can help or harm convergence?
  – Homogeneity or heterogeneity
  – Community structure

• How robust are the dynamics to possible manipulations?
Community Structure

Few edges between groups

Many edges within groups
Disjoint cliques

Many edges within groups

Few edges between groups
Tree Structure

Few edges between groups

Many edges within groups
Tree Structure

Few edges between groups

Many edges within groups
Adding Homogeneity

\[ p \]

\[ 1 - p \]
Simulation on Disjoint Cliques

Empty initial states

Segregated initial states

Fraction of non-consensus

$p$

emp-1000: emp-5000: emp-10000:

seg-1000: seg-5000: seg-10000:
Simulation on Disjoint Cliques

Empty initial states

Segregated initial states
Theoretical Analysis

• Segregated start: for $p < p_0 \approx 0.110$, consensus time $= \exp(\Omega(n))$
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• Segregated start: for $p < p_0 \approx 0.110$, consensus time $= \exp(\Omega(n))$
  – Mean field approximation
Theoretical Analysis

- Segregated start: for $p < p_0 \approx 0.110$, consensus time $= \exp(\Omega(n))$
  - Mean field approximation
  - Stability of autonomous system
    - Local stability
    - Global stability
Motivating Questions

• What can help or harm convergence?
  – Homogeneity or heterogeneity
  – Community structure

• How robust are the dynamics to possible manipulations?
Robustness
Stubborn nodes

• How and when can such nodes affect the name to which the dynamics converge?
Stubborn nodes

- How and when can such nodes affect the name to which the dynamics converge?
  - The network topology
  - The time when the stubborn nodes are activated
Stubborn nodes and network

Graph size = 1000

Graph size = 10000
Adding stubborn nodes after consensus

• After consensus: with $p < p_0 \approx 0.108$ fraction of stubborn nodes, the consensus time $= \exp(\Omega(n))$. 
Engineering Agreement

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• How robust are the dynamics to possible manipulations?
QUESTIONS?