Complex Contagions on Configuration Model Graphs with a Power-Law Degree Distribution

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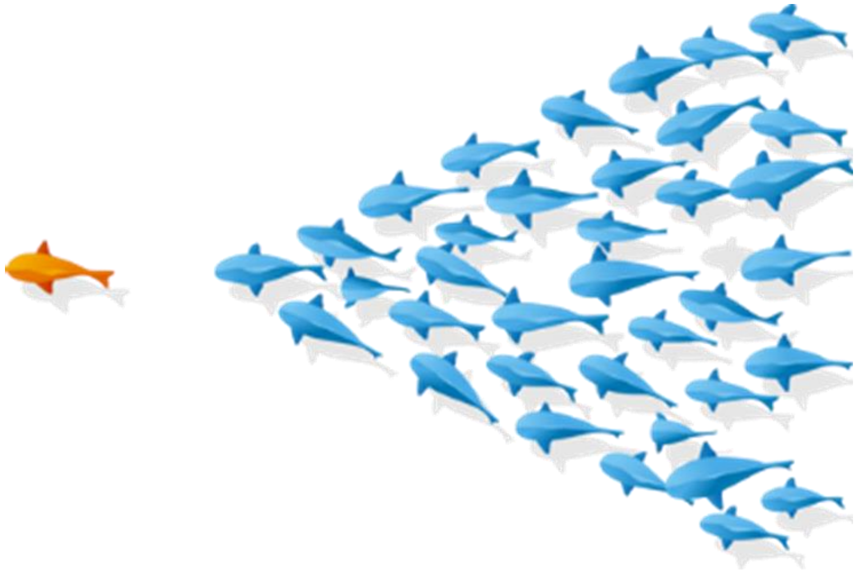
Contagions, diffusion, cascade…

- Ideas, beliefs, behaviors, and technology adoption spread through network
- Why do we need to study this phenomena?
  - Better Understanding
  - Promoting good behaviors/beliefs
  - Stopping bad behavior



Outline

- Models
  - Complex contagions model
  - Power-Law and configuration model graph
- Main result
- Related work
- A happy proof sketch

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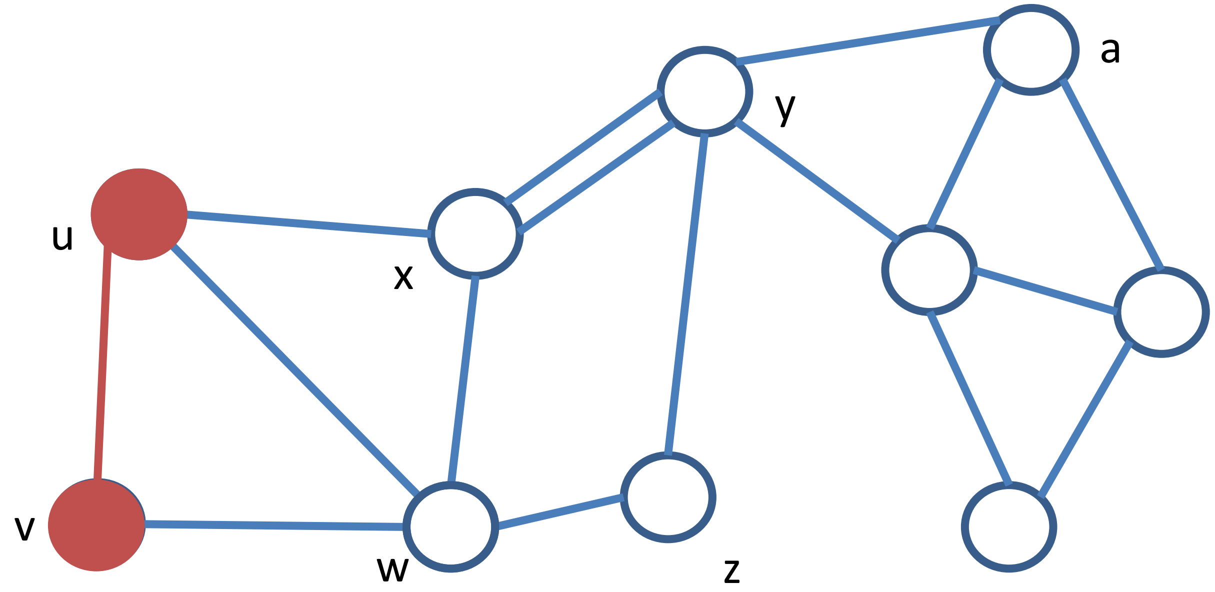
Model of Contagions

- K-Complex Contagions [GEG13; CLR 79]
  - Given an initial infected set $I = \{u, v\}$.



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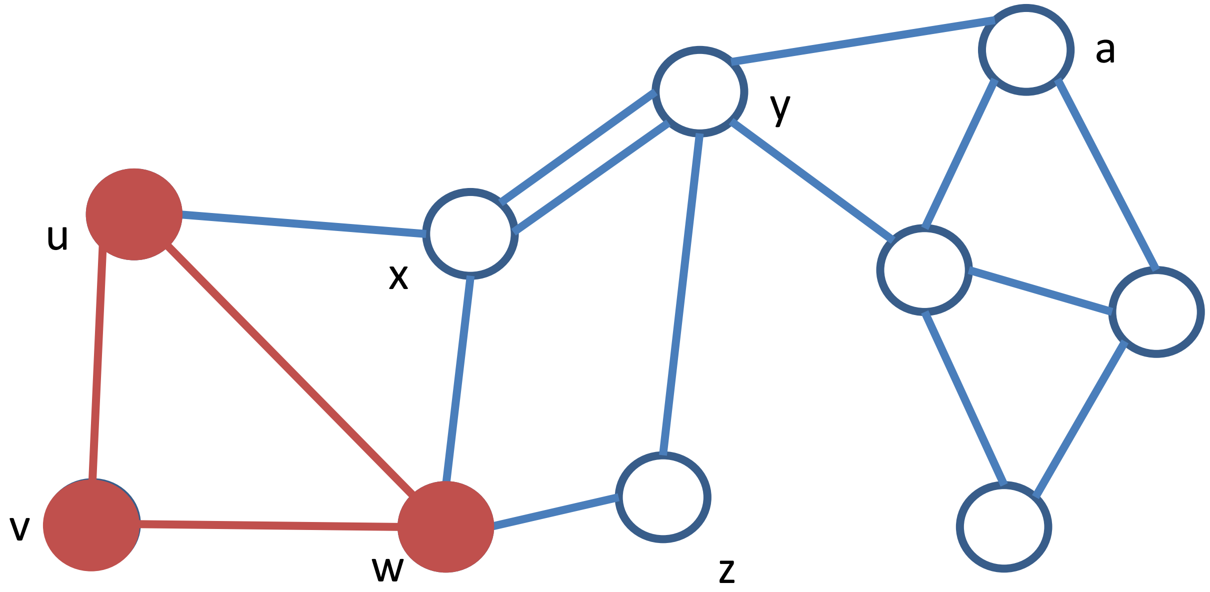
  - Node becomes infected if it has at least k infected neighbor



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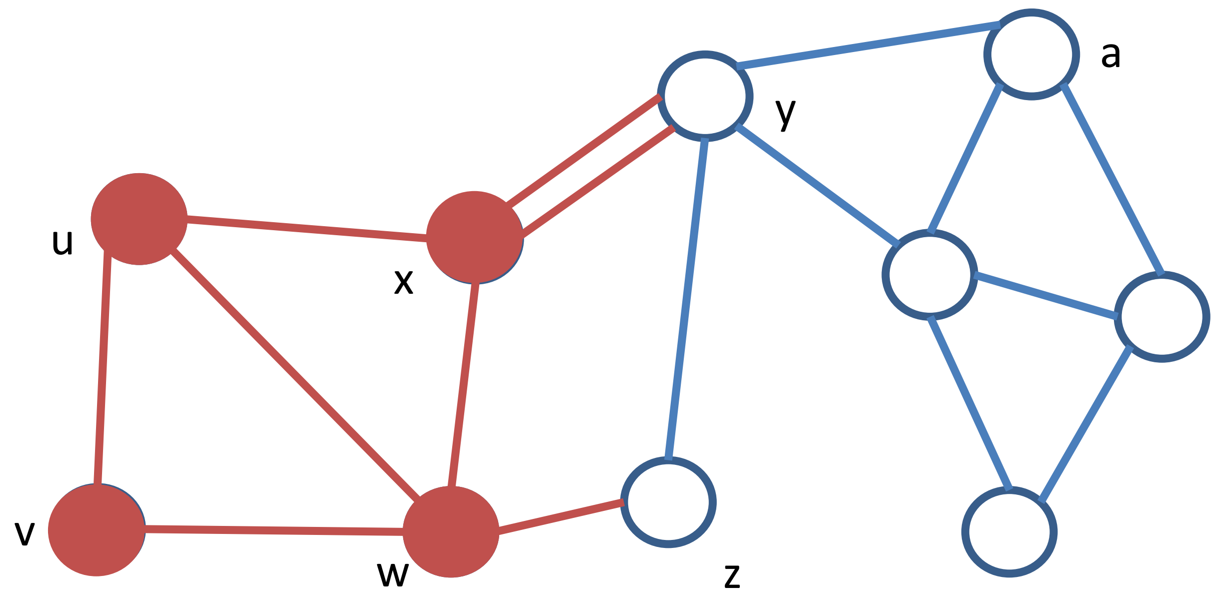

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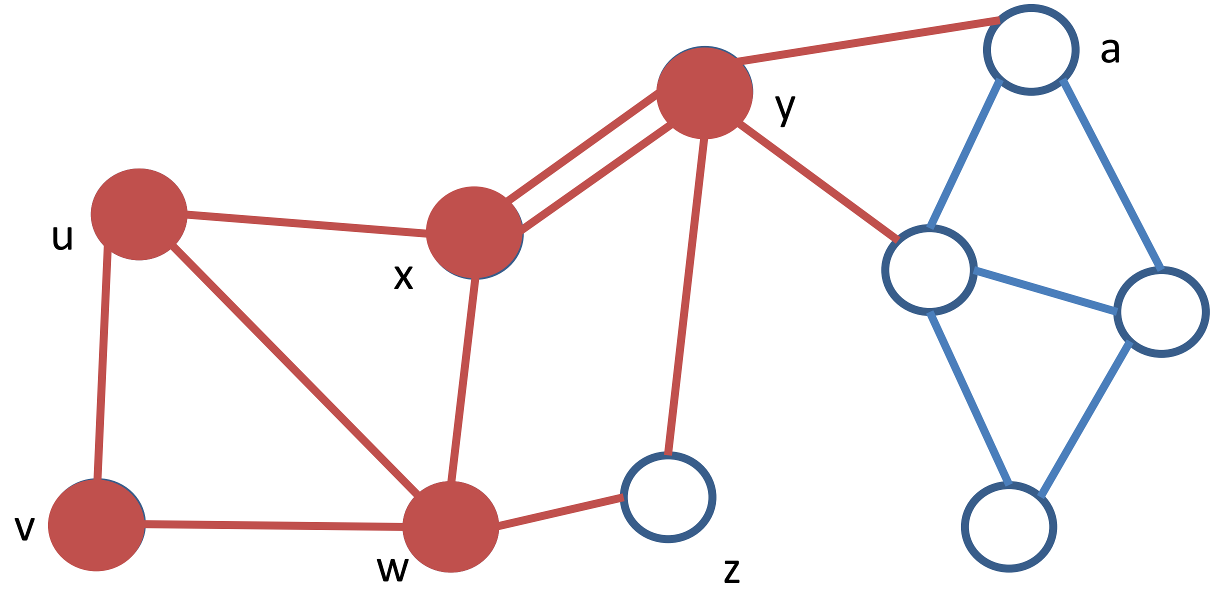

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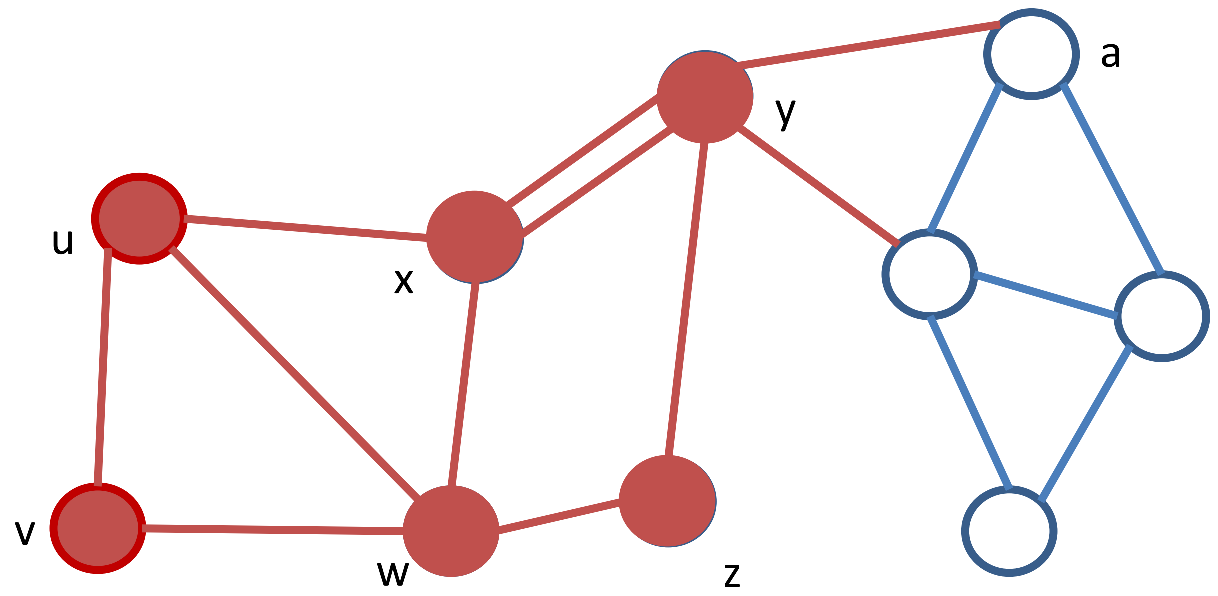

Why k complex contagions?

- One of most classical and simple contagions model
  - Threshold model [Gra 78]
  - Bootstrap percolation [CLR 79]
- Non-submodular

Motivating Question

- Do k complex contagions spread on social networks?

Question

- Do k complex contagions spread on Erdos-Renyi model $G_{n,p}$ where $p = O\left(\frac{1}{n}\right)$?
  - n vertices
  - Each edge (u, v) occurs with probability p
- Need $\Omega(n)$ (random) seeds to infect constant fraction of the graph[JLTV89]?
- Can we categorize all networks which spread slowly/quickly?

What is a social network?

- Qualitatively: special structure
  - Power law degree distribution
  - low-diameter/small-world…
- Quantitatively: generative model?
  - Configuration model graphs
  - Preferential attachment model
  - Kleinberg’s small world model

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Motivating Question

- Do k complex contagions spread on social networks?
  - What properties are shared by social networks?
  - Do these properties alone permit complex contagion spreads?

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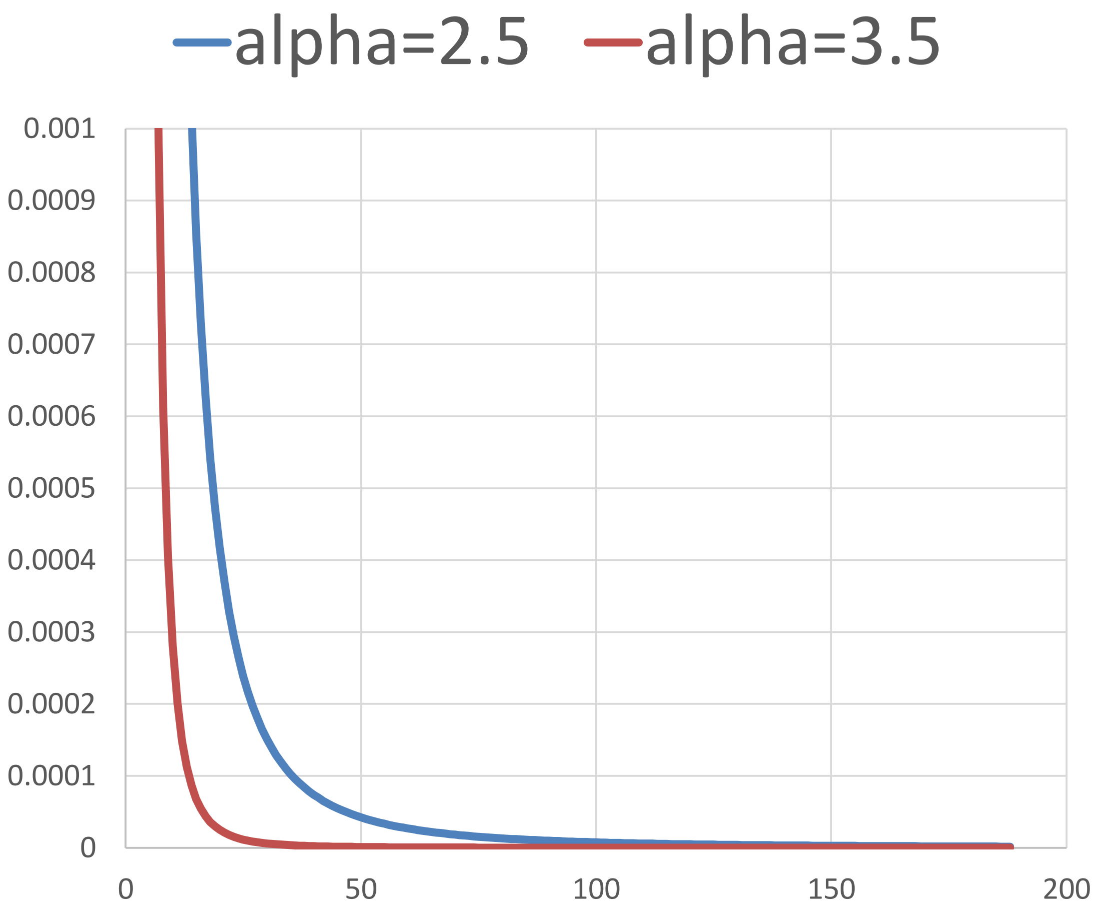
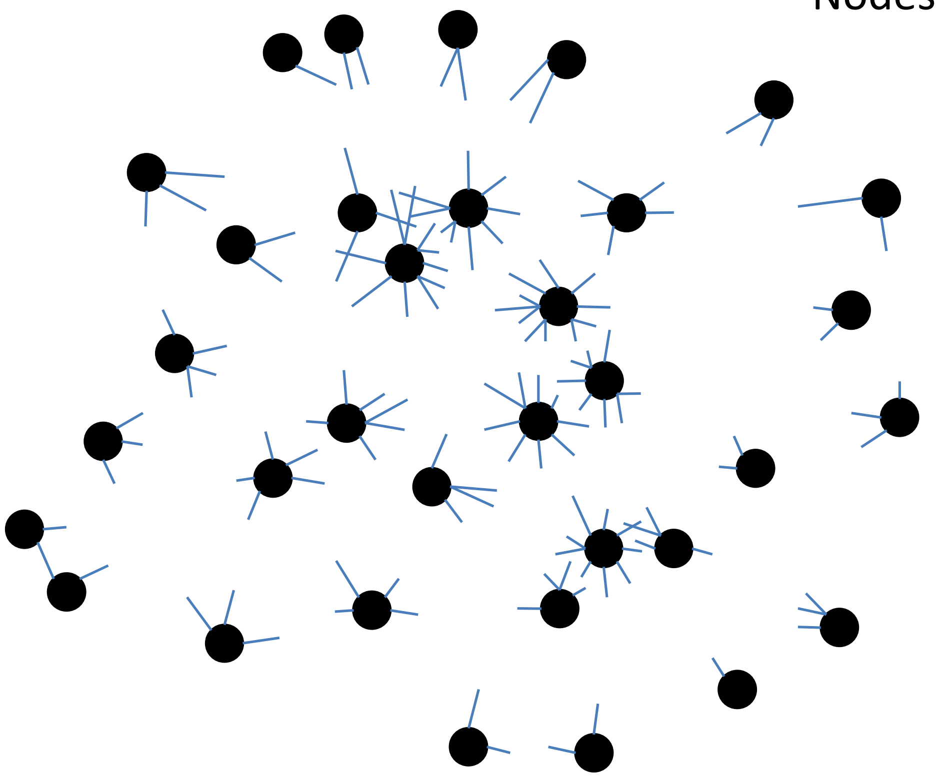
Power-law distribution

- A power-law distribution with α if the $\Pr[X = x] \sim x^{-\alpha}$



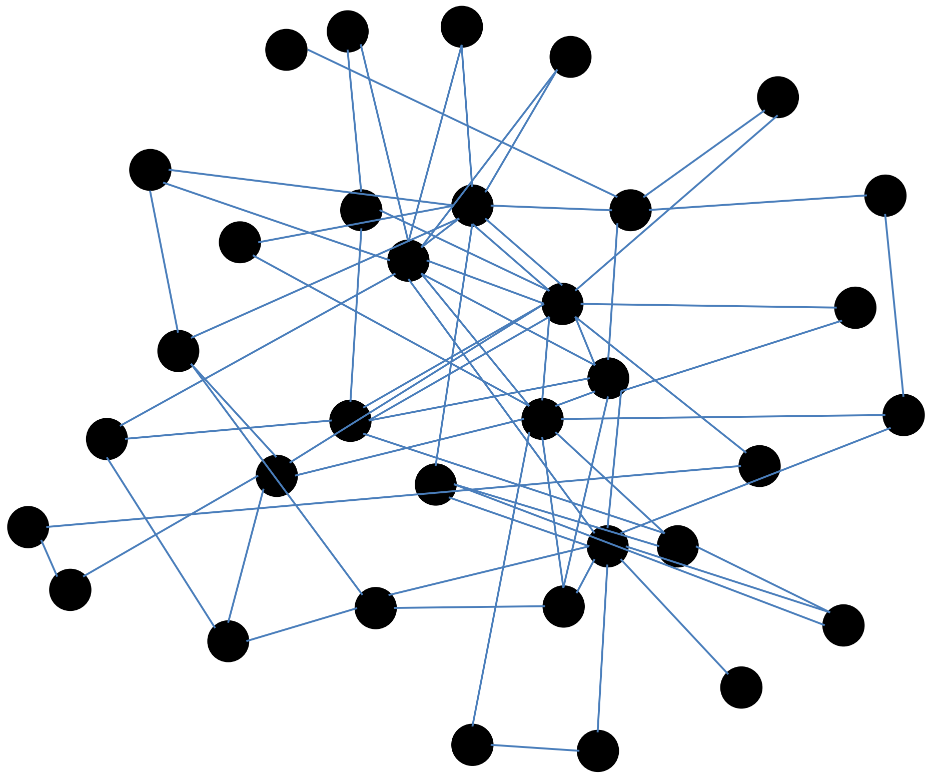
Configuration Model

- Given a degree sequence $\deg(v_1), \deg(v_2), \dots, \deg(v_n)$
- The node v_i has $\deg(v_i)$ stubs



Configuration Model

- Given a degree sequence $\deg(v_1), \deg(v_2), \ldots, \deg(v_n)$
- The node v_i has $\deg(v_i)$ stubs
- Choose a uniformly random matching on the stubs



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Theorems

Main result

- Configuration Model
  - power-law degree distribution $2 < \alpha < 3$
- Initial infected node
  - the highest degree node
- k-complex contagions spreads to $\Omega(1)$ fraction of nodes with high probability.

Corollary from [Amini 10]

- Configuration Model
  - power-law degree distribution $3 < \alpha$
- Initial infected nodes:
  - $o(1)$ fraction of highest degree node
- k-complex contagions spreads to $o(1)$ fraction of nodes with high probability.

The Bottom Line

Main result

- Configuration Model
  - power-law degree distribution $2 < \alpha < 3$
- k-complex contagions
  - Initial infected node: the highest degree node
- Contagions spreads to $\Omega(1)$ fraction of nodes with high probability.

Corollary from [Amini 10]

- Configuration Model
  - power-law degree distribution $3 < \alpha$
- k-complex contagions
  - Initial infected node: $o(1)$ fraction of highest degree node
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Complex contagions spread on most of graphs with power-law degree distribution $2 < \alpha < 3$

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What has been done?

Lattice[98]	Random regular[07]	Configuration model $\alpha>3$[10]	Watts-Strogatz[13] Kleinberg[14]
Tree[79,06]	Erdos-Renyi model[12]	Chung-Lu model[12]	Preferential Attachment[14]

What has been done?

Lattice[98]	Random regular[07]	Configuration model $\alpha>3$ [10] **Configuration model** $2<\alpha<3$[16]	Watts-Strogatz[13] Kleinberg[14]
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Start with $G_{n,p}$

Lattice[98]	Random regular[07]	Configuration model $\alpha>3$ [10] Configuration model $2<\alpha<3$[16]	Watts-Strogatz[13] Kleinberg[14]
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k-complex contagions don't spread

Physics

Lattice[98]

Random regular[07]

Configuration model $\alpha>3$ [10]

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Watts-Strogatz[13]

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k-complex contagions spread

Network Science

Lattice[98]	Random regular[07]	Configuration model α>3 [10] Configuration model 2<α<3[16]	Watts-Strogatz[13] Kleinberg[14]
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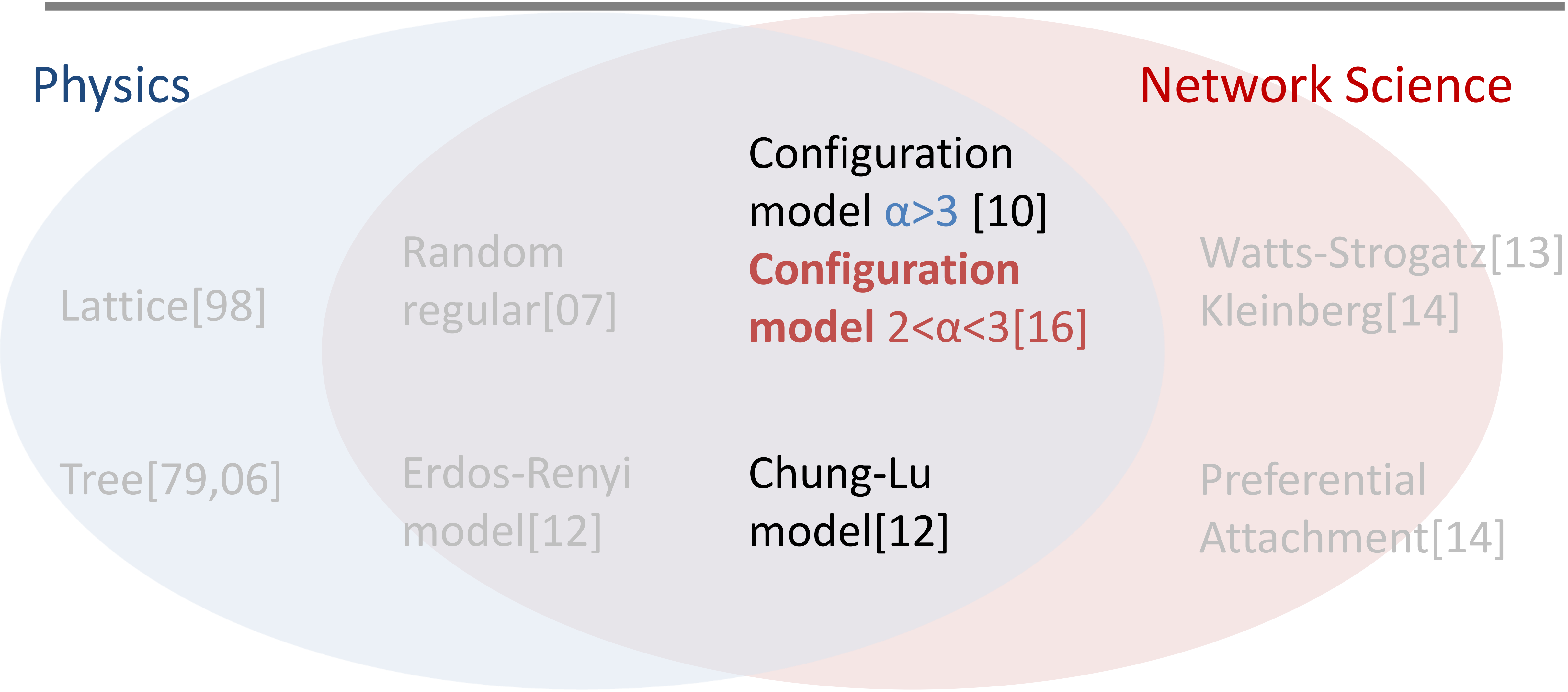
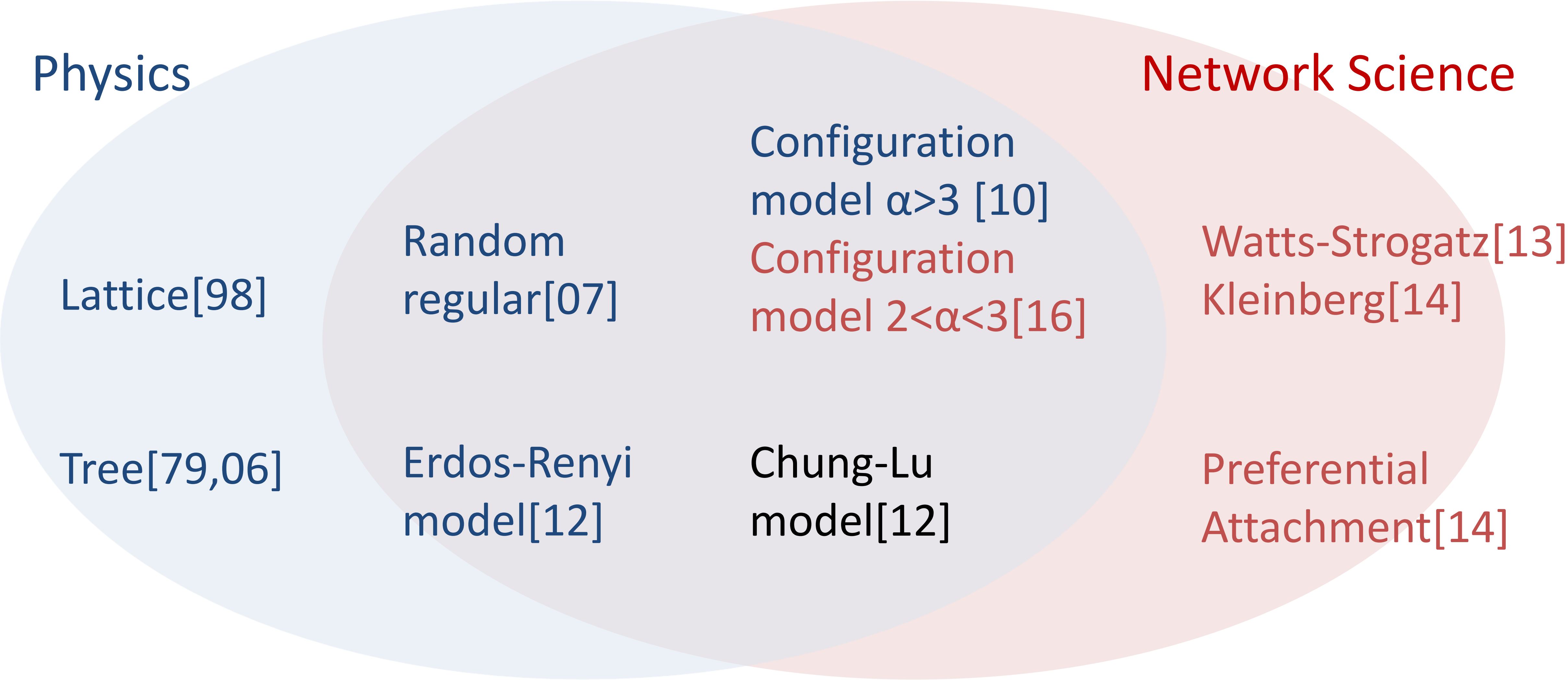
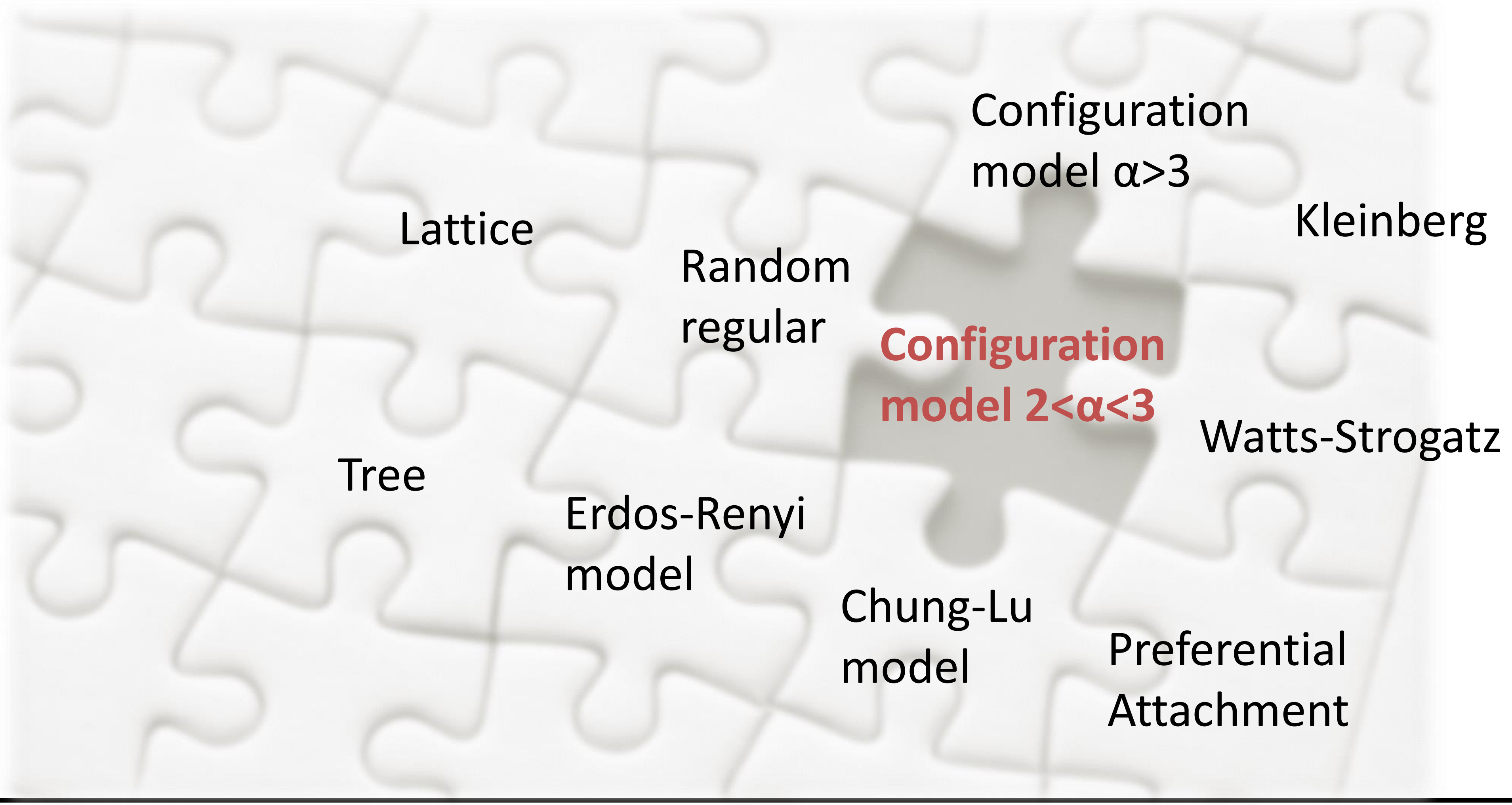
Things get complicated



What have we learned?



Why do we want to solve it?



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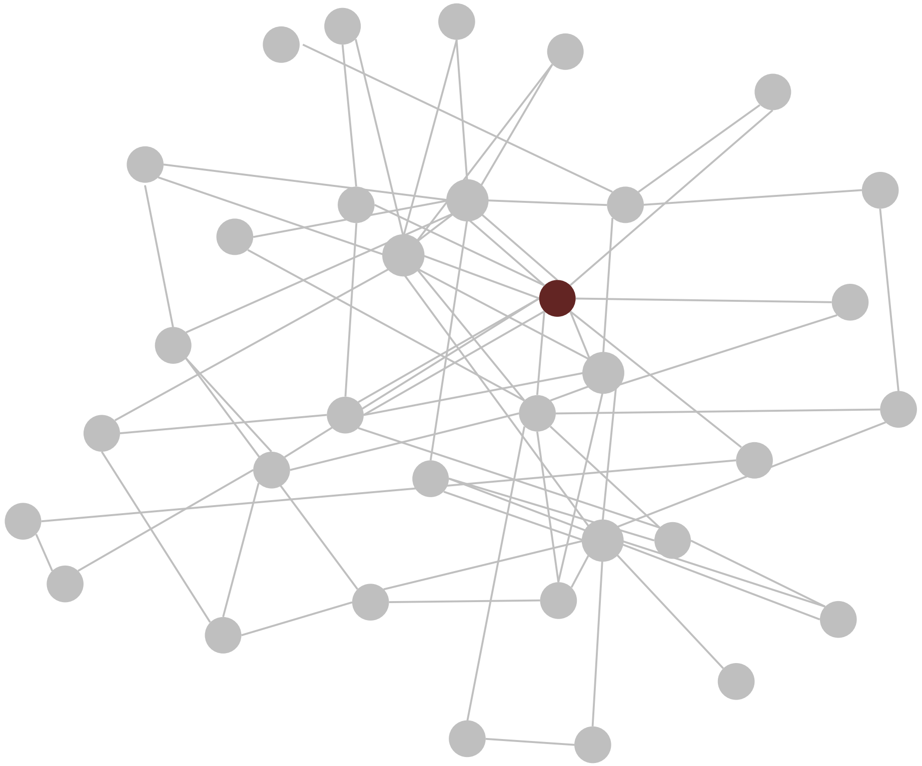
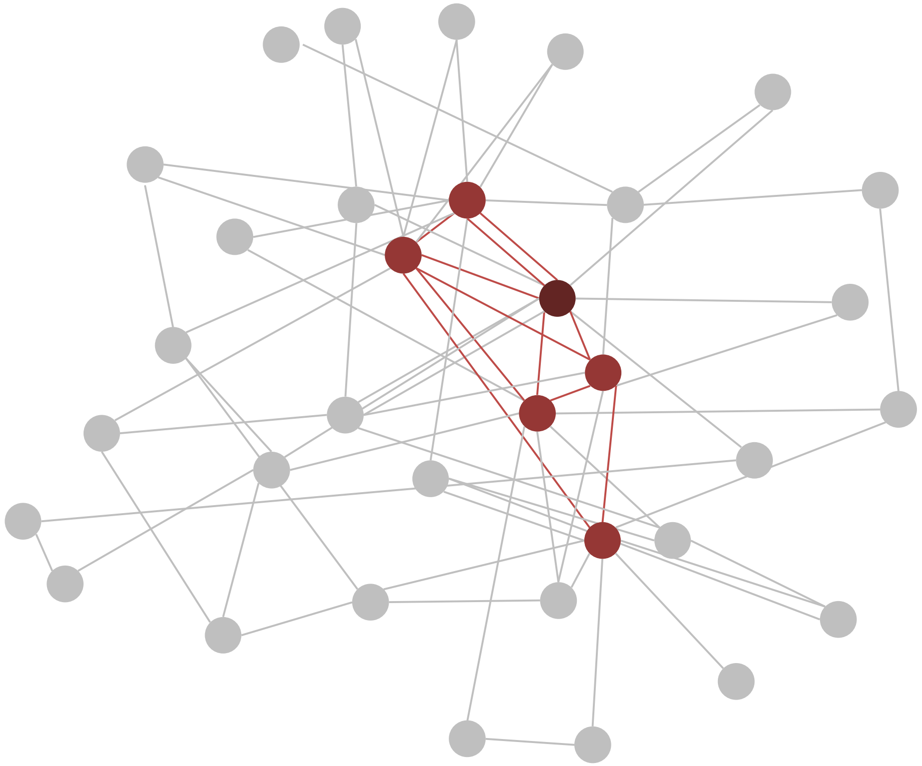
Idea

- Restrict contagions from high degree node to low degree nodes
- Reveal the edges when needed



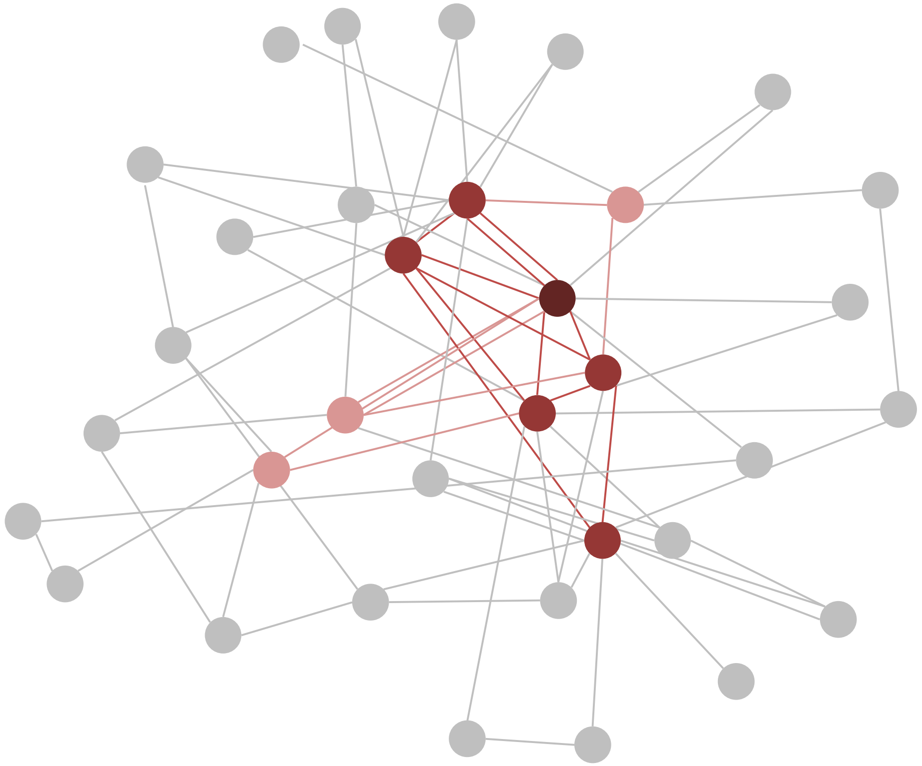
Observations

- The highest degree nodes forms clique



Observations

- The highest degree nodes forms clique
- k degree node has many edges to l degree nodes where $l > k$



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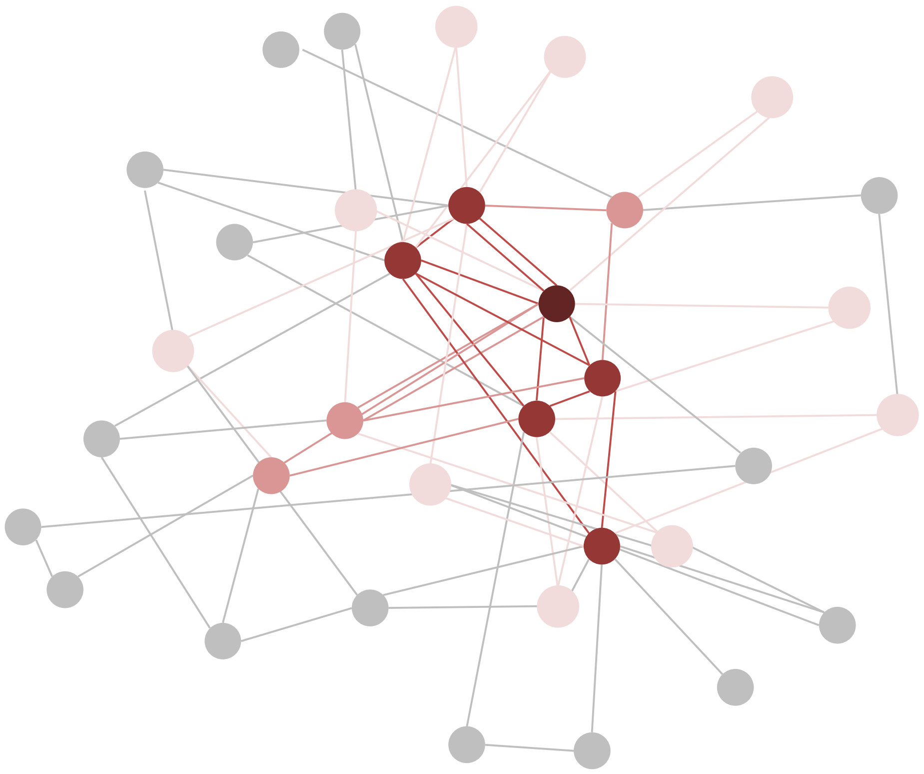

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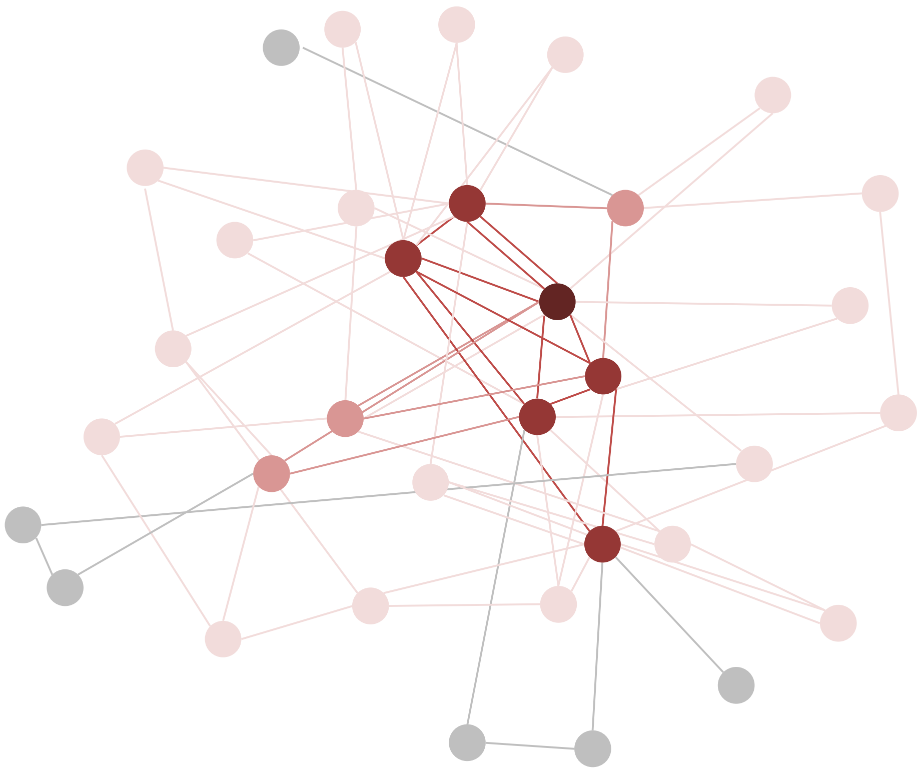

Previous tool and challenges

Physics

Network Science

Lattice[98]

Tree[79,06]

Random regular[07]

Erdos-Renyi model[12]

Configuration model $\alpha>3$ [10]

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Chung-Lu model[12]

Watts-Strogatz[13]

Kleinberg[14]

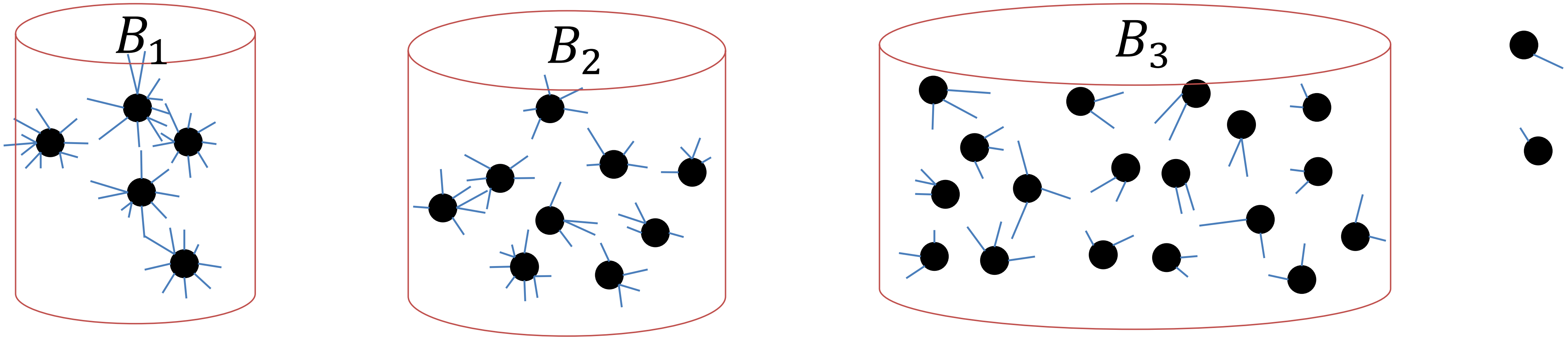
Preferential Attachment[14]

Inductive Structure

- Partition nodes into buckets ordered by degree of nodes

$$B_1, B_2, \dots, B_l$$

- Induction: if infection spreads on previous buckets B_i where $i < k$, the infection also spread on bucket B_k.



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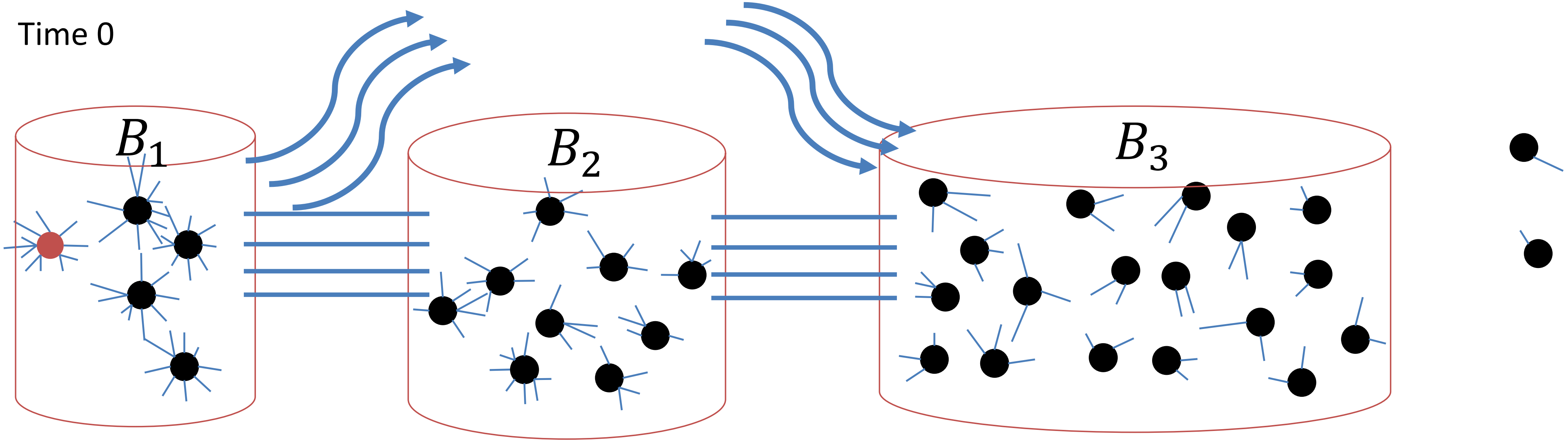

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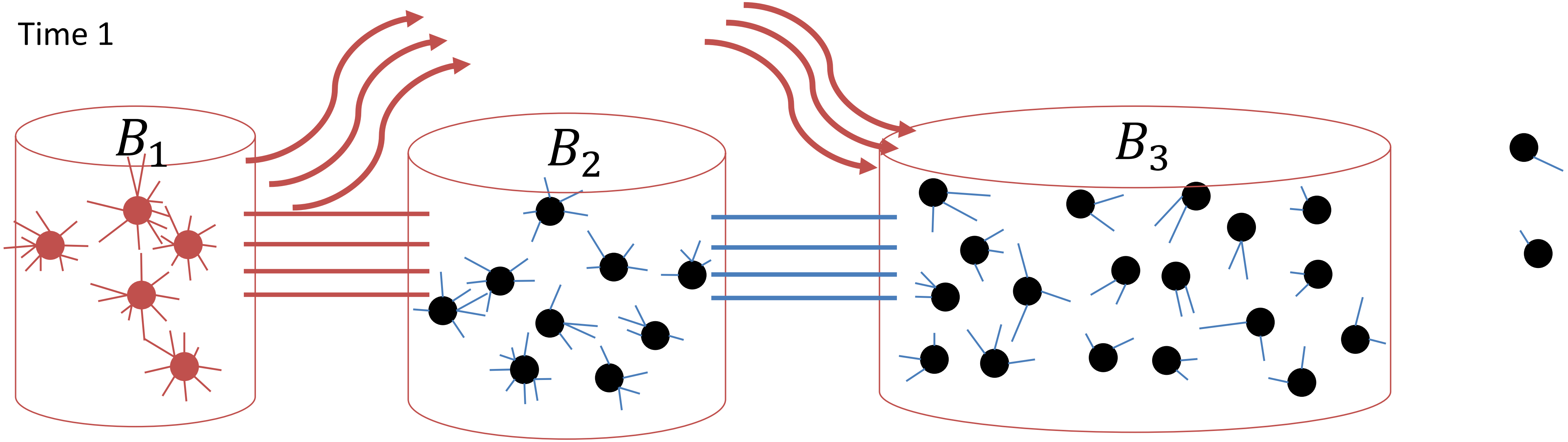

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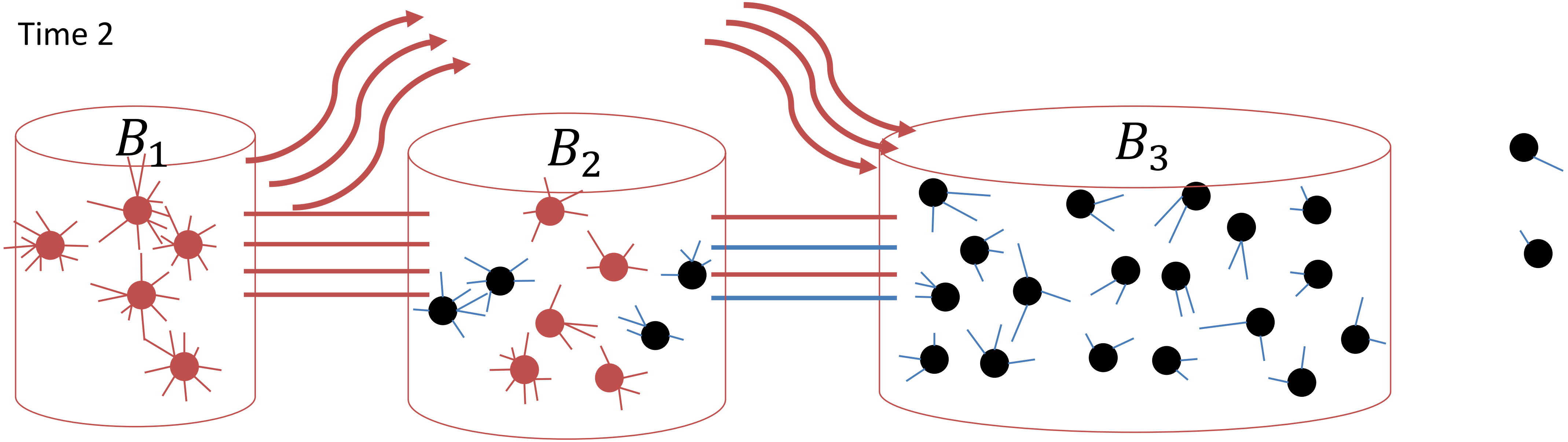

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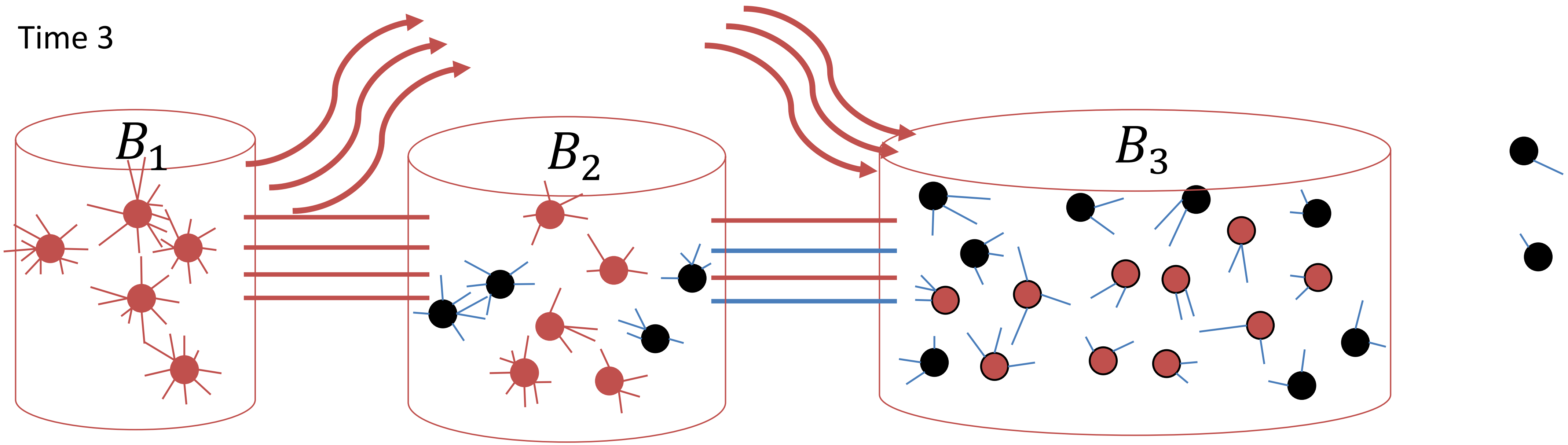

Inductive Structure

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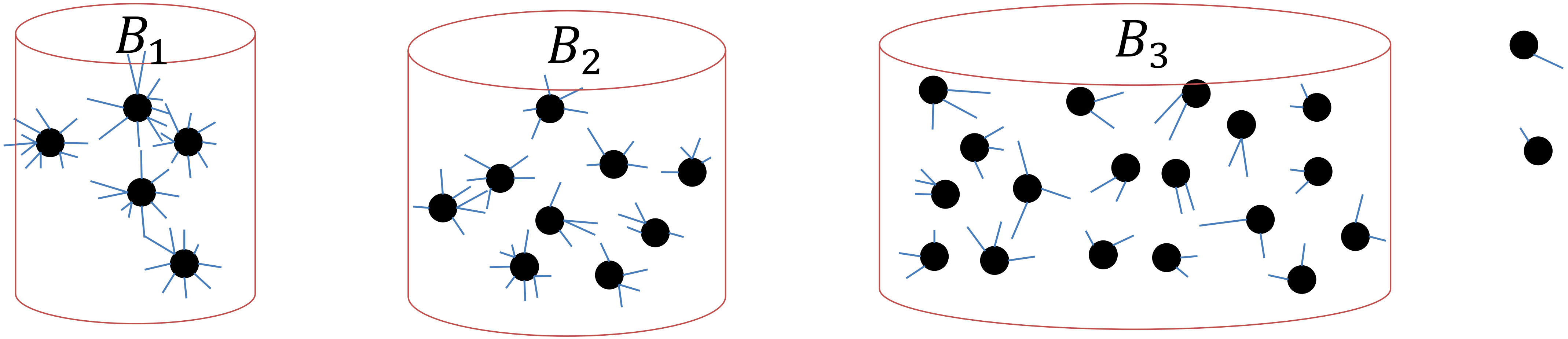
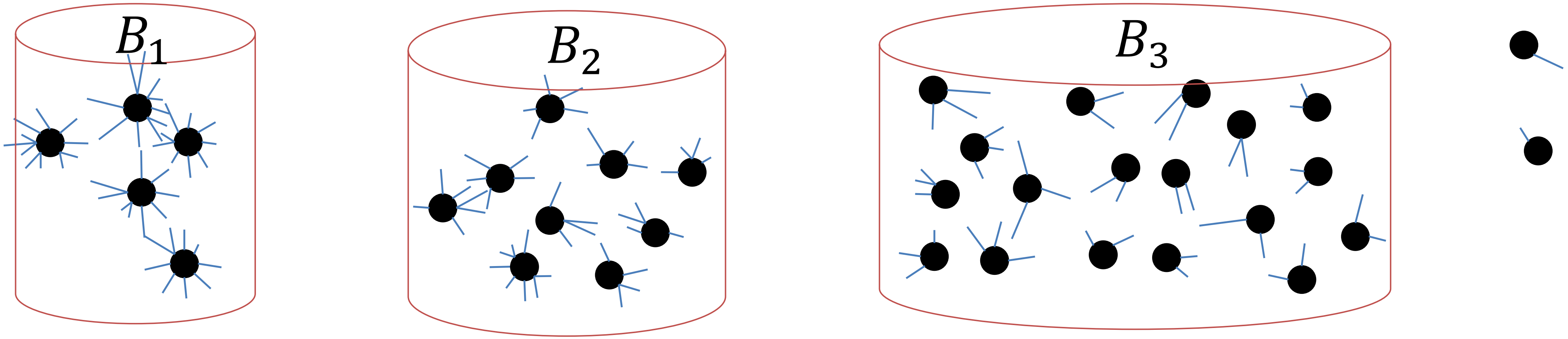
  - – Well connection between buckets
  - – Infection spread in buckets



Inductive Structure

- Induction: if infection spreads on previous buckets B_i where $i < k$, the infection also spread on bucket B_k.
  - Well connection between buckets: Chernoff bound
  - Infection spread in buckets: Chebeshev's inequality



Thanks for your listening

How do we solve it?

- Chebyshev's inequality

$$\Pr[Z > E[Z] + t] \leq \frac{Var[Z]}{t^2}$$

- Chernoff-type bound

$$\Pr[Z_n > Z_0 + t] \leq \exp\left(\frac{-t^2}{c}\right)$$