1. **a.** The population of a country is currently 52 million and is growing at a rate of 2.3% per year. Write a formula for the population P in millions t years from now.

Answer: $P =$ $52 \cdot 1.023^t$

b. The population of a different country is currently 5.8 million and is decreasing at a rate of 1.9% per year. Write a formula for the population P in millions t years from now.

Answer: $P =$ $5.8 \cdot 0.981^t$

c. The amount of money in Red's savings account is currently ₱80000 and is growing by 1.99% per year. Write a formula for the amount S of money in the account t <u>months</u> from now.

Answer: $S =$ $8000 \cdot 1.0199^{t/12}$

d. The mass of a sample of a radioactive substance is currently 0.7 grams. The substance has a half-life of 7.54 hours, which means that half of the substance decays every 7.54 hours. Write a formula for the mass m of the sample in grams t hours from now.

Answer: $m =$ $0.7 \cdot \left(\frac{1}{2}\right)^{t/7.54}$

e. The mass of a sample of a radioactive substance is currently 25 mg, and the mass is decaying at a *continuous* yearly rate of 8.1%. Write a formula for the mass m of the sample (in mg) t years from now.

Answer: $m =$ $25e^{-0.081t}$

f. The population of a country was 32 million in 2005, and the population increases by 12% every 10 years. Write a formula for the population P of the country (in millions) t years after the year 2000.

Answer: $P =$ $32 \cdot 1.12^{(t-5)/12}$

g. The population of a country was 12 million in 2008, and the population doubles every 30 years. Write a formula for the population P of the country in millions t years after the year <u>2000</u>.

Answer: $P =$ $12 \cdot 2^{(t-8)/30}$

2. Let $S(t)$ be the amount of money in Blue's savings account (in thousands of ₱) t years after January 1, 2005, and suppose that $S(t)$ is given by

$$S(t) = 70 \cdot 1.023^t.$$

a. How much money was in the account at the beginning of 2005?

Answer: ₱70,000

b. By what percent does the amount of money in the account increase each year?

Answer: 2.3%

3. Let $S(t)$ be the amount of money in Green's savings account (in thousands of ₱) t years after January 1, 2005, and suppose that $S(t)$ is given by

$$S(t) = 50 \cdot 1.0045^{12t}.$$

a. How much money was in the account at the beginning of 2005?

Answer: ₱50,000

b. By what percent does the amount of money in the account increase each year?

Answer: 5.54%

4. Let $S(t)$ be the amount of money in Gold's savings account (in thousands of ₱) t years after January 1, 2005, and suppose that $S(t)$ is given by

$$S(t) = 60 \cdot 2^{t/55}.$$

a. How much money was in the account at the beginning of 2005?

Answer: ₱60,000

b. By what percent does the amount of money in the account increase each year?

Answer: 1.27%

5. The population of Grubbin on Melemele Island is growing exponentially, doubling in size every 8.5 years.

a. Suppose that at the start of 2010, there were 25,000 Grubbin on Melemele island. Let $G(y)$ be the number of Grubbin on the island (in thousands) y years after the start of 2010. Write a formula for $G(y)$. Leave your answer in exact form (no decimal approximations).

Answer: $G(y) =$ $25 \cdot 2^{y/8.5}$ or $25 \cdot \left(2^{1/8.5}\right)^y$

b. By what percent does the Grubbin population grow each year? Give your answer as a percentage rounded to the nearest hundredth of a percent (like 0.12% or 34.56%).

Solution: Rewrite the formula for $G(y)$ as

$$G(y) = 25 \cdot \left(2^{1/8.5}\right)^y \approx 25 \cdot (1.08496)^y$$

So each time y increases by 1, $G(y)$ is multiplied by 1.08496, which increases $G(y)$ by 8.496%.

Answer: the population grows by 8.50 % each year

6. Suppose that the number of cups of coffee sold nationwide each day by a large coffee chain is a function of the price charged per cup. Below is some data about the number of cups sold at different prices.

price	\$1.25	\$1.60
cups sold	550,000	450,000

Let $C(p)$ be the number of thousands of cups of coffee sold each day if the company charges a price of p dollars per cup, assuming the number of cups sold is an exponential function of the price charged. Find a formula for $C(p)$.

Solution: When the price increases from \$1.25 to \$1.60, the number of cups sold is multiplied by a factor of $450000/550000 = 9/11$. If the number of cups sold $C(p)$ is an *exponential* function of the price p, then *every* time the price increases by \$0.35, the number of cups sold will be multiplied by $9/11$. This means that it will be convenient to write $C(p)$ in the form

$$\boxed{?}\left(\frac{9}{11}\right)^{p/0.35}.$$

We don't know $C(0)$, but we do know that $C(1.25) = 550$; similarly to how we got point-slope form for a line, we can write

$$C(p) = 550\left(\frac{9}{11}\right)^{(p-1.25)/0.35}.$$

You could also find a formula by saying that $C(p) = ab^p$ for some numbers a and b, then solving the system of equations

$$550 = ab^{1.25}$$
$$450 = ab^{1.60}$$

for a and b. Dividing the second equation by the first gives

$$\frac{450}{550} = \frac{ab^{1.60}}{ab^{1.25}}$$

which simplifies to

$$\frac{11}{9} = b^{0.35}$$

so that

$$b = \left(\frac{11}{9}\right)^{1/0.35}.$$

Plug this back into the first equation to get

$$550 = a\left(\left(\frac{11}{9}\right)^{1/0.35}\right)^{1.25}$$

which gives

$$a = \frac{550}{\left(\frac{11}{9}\right)^{1.25/0.35}}.$$

Then, plugging these values for a and b back into $C(p) = ab^p$, the formula is

$$C(p) = \frac{550}{\left(\frac{11}{9}\right)^{1.25/0.35}}\left(\left(\frac{11}{9}\right)^{1/0.35}\right)^{p}.$$

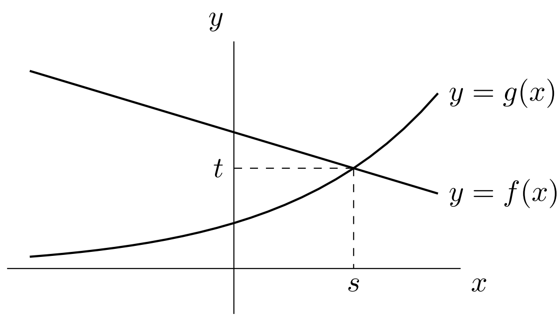
7. Suppose that the amount of money in Silver's savings account grows at a *continuous* annual rate of 4.9%. By what percent does Silver's money grow each year?

Solution: If the initial amount in the account is A_0, then the amount A after t years is given by $A = A_0e^{0.049t}$. This can be rewritten as $A = A_0(e^{0.049})^t \approx A_0(1.05022)^t$.

8. If the half-life of a substance is 95 days, what percentage of a sample of this substance remains after 320 days?

Solution: If the initial mass of the substance is m_0, then the mass m after t days is given by $m = m_0(0.5)^{t/95}$. Plugging in $t = 320$, we get $m \approx m_0 \cdot 0.09683$, so about 9.683% of the original mass remains.

9. The graphs of two functions $y = f(x)$ and $y = g(x)$ are shown below. The functions are given by the formulas $f(x) = ax + b$ and $g(x) = cd^x$. The graphs intersect at the point (s, t).



Circle your answers. You do not need to explain your answers.

a. What can be said about the constant a?

$a > 0$ $\boxed{a < 0}$ NOT ENOUGH INFO

b. What can be said about the constants b and c?

$\boxed{b > c}$ $b < c$ NOT ENOUGH INFO

c. What can be said about the constant d? (Circle the best answer.)

$d > 0$ $d < 0$ $\boxed{d > 1}$ $d < 1$

d. If the constant b increases, then which of the following must happen? (Circle all correct answers.)

$\boxed{s \text{ increases}}$ s decreases $\boxed{t \text{ increases}}$ t decreases

e. If the constant d increases, then which of the following must happen? (Circle all correct answers.)

s increases $\boxed{s \text{ decreases}}$ $\boxed{t \text{ increases}}$ t decreases

10. A textbook publisher finds that the number of math textbooks it sells each semester depends on the price they charge for each book. Below is some data about the number of books the publisher sells.

price	\$120	\$135
books sold	20,500	19,125

Let $B(p)$ be the number of books the publisher sells each semester if they charge p dollars per book, assuming the number of books sold is an exponential function of the price charged.

a. By what percent do sales decrease when the publisher raises the price of their book by \$15?

Solution: Note that the table above illustrates a price increase of 15%.

Answer: sales decrease by about 6.707 %

b. Find a formula for $B(p)$.

Answer: $B(p) = 20500\left(\frac{19125}{2050}\right)^{(p-120)/15}$

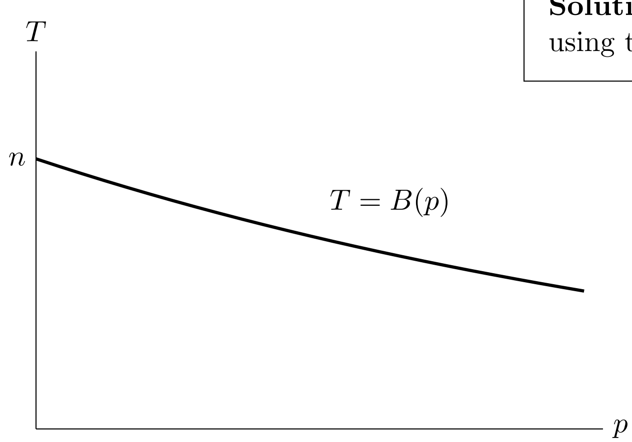
c. By what percent do sales increase when the publisher lowers the price of their book by \$20?

Solution: You could compute $B(100)$ using your formula from **b** and then compute $B(100)/B(120)$.

Answer: sales increase by about 9.699 %

d. Below is a graph of $T = B(p)$. What is the value of the constant n?

Solution: n is equal to $B(0)$. Compute $B(0)$ using the formula from part **b** to get ...



Answer: $n =$ 35725.1