Constraint Semantics and the Language of Subjective Uncertainty

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The doxastic hypothesis:

Propositions do not suffice to characterize a typical agent's doxastic state. A complete inventory of the propositions to which I lend high credence would omit my belief that it might rain today.

The assertion hypothesis:

Propositions suffice to characterize the contents of assertions. For example, the content of my assertion that it might rain today is a proposition about information available to me or to my community.

Kratzer's hypothesis:

A given modal has a "common kernel of meaning" whether it is used to target epistemic modality, deontic modality, circumstantial modality, or some other flavor of modality (1977, 338–342). That common kernel pertains to the relationship between the modal's prejacent and a contextually supplied body of information or set of premises.

Section 1 reconciles compositional semantics and the doxastic hypothesis.

Section 2 refines Kratzer's hypothesis, and uses it to help explain the evidential features of epistemic modals.

Section 3 argues that epistemic modals demand a hybrid of 'pure' probabilistic semantics, à la section 1, and premise semantics, à la section 2.

Here I model doxastic states using *probability spaces*; $\langle W, \mathcal{F}, \mu \rangle$ such that

1. $\mathcal{F}$ is an algebra over W (i.e., $\mathcal{F}$ is a set of subsets of W, $W \in \mathcal{F}$, and $\mathcal{F}$ is closed under complementation and union);
2. μ (the *measure function* of the triple) is a function from $\mathcal{F} \to [0, 1]$;
3. $\mu(W) = 1$;
4. If M and N are disjoint elements of $\mathcal{F}$, then $\mu(M \cup N) = \mu(M) + \mu(N)$.

A constraint on doxastic states is a set of probability spaces that are admissible by the lights of that constraint.

(1) There's a 50% chance that it's raining now.

(2) There's a 25% chance that the next ball drawn will be white, and a 25% chance that the next ball drawn will be red.

Types:

e is a type ($D_{\langle e \rangle}$ = {Al, Betty});

t is a type ($D_{\langle t \rangle}$ = {*true*, *false*});

a is a type ($D_{\langle a \rangle}$ = {the set of $\langle W, \mathcal{F}, \mu \rangle$ triples such that W is the set of all possible worlds, and $\langle W, \mathcal{F}, \mu \rangle$ is a probability space};

if α and β are types, then $\langle \alpha, \beta \rangle$ (sometimes abbreviated '$\alpha\beta$') is a type;

nothing else is a type.

$$[\![\textbf{is/are tall}]\!]_{\langle e,\langle a,t\rangle\rangle} = \lambda e.\lambda a. \begin{cases} \textit{true} \text{ if the measure function of } a \\ \text{takes the proposition that } e \text{ is tall to 1;} \\ \textit{false} \text{ otherwise.} \end{cases}$$

$$[\![\textbf{is/are nice}]\!]_{\langle e,\langle a,t\rangle\rangle} = \lambda e.\lambda a. \begin{cases} \textit{true} \text{ if the measure function of } a \\ \text{takes the proposition that } e \text{ is nice to 1;} \\ \textit{false} \text{ otherwise.} \end{cases}$$

$$[\![\textbf{there is an } x\textbf{\% chance that}]\!]_{\langle\langle a,t\rangle,\langle a,t\rangle\rangle} = \lambda C \in D_{\langle a,t\rangle}.\lambda a. \begin{cases} \textit{true} \text{ if } a \text{ takes } p \text{ to } \frac{x}{100}, \\ \text{ (where } p \text{ is a proposition that every} \\ \text{measure function of } C \text{ takes to 1);} \\ \textit{false} \text{ otherwise.} \end{cases}$$

$$[\![\textbf{and}]\!] = \lambda F \in D_{\langle a,t \rangle}.\lambda G \in D_{\langle a,t \rangle}.\lambda a.$$
$$\begin{cases} \textit{true} \text{ if } F(a) = \textit{true} \text{ and } G(a) = \textit{true}; \\ \textit{false} \text{ otherwise.} \end{cases}$$

We might be tempted to give the following constraint semantic entry for 'it is not the case that':

$$[\![\textbf{it is not the case that}]\!] = \lambda F \in D_{\langle a,t \rangle}. \lambda a. \begin{cases} \textit{true} \text{ if, if } F(b) = \textit{true} \text{ and } b \text{ gives } x \text{ to a proposition,} \\ \text{then } a \text{ gives } 1 - x \text{ to that proposition;} \\ \textit{false} \text{ otherwise.} \end{cases}$$

(3) It is not the case that Al is tall.

(4) There is a 5% chance that Al is tall.

(5) It is not the case that there is a 95% chance that Al is tall.

(6) There is a 50% chance that Betty is nice.

(7) It is not the case that there is a 50% chance that Betty is nice.

(8) It is not the case that Al is tall.

(9) It is not the case that there is a 100% chance that Al is tall.

This semantic entry works well:

$$[\![\textbf{it is not the case that}]\!] = \lambda F \in D_{\langle a,t\rangle}.\lambda a. \begin{cases} \textit{true} \text{ if } F(a) = \textit{false}; \\ \textit{false} \text{ otherwise.} \end{cases}$$

(10) We’re either about as likely as not to hire John, or we’re about as likely as not to hire James—you know how bad I am with names.

$$[\![\textbf{or}]\!] = \lambda F \in D_{\langle a,t \rangle}.\lambda G \in D_{\langle a,t \rangle}.\lambda a. \begin{cases} \textit{true} \text{ if } F(a) = \textit{true} \text{ or } G(a) = \textit{true}; \\ \textit{false} \text{ otherwise.} \end{cases}$$

(*S* satisfies $\mathcal{C}$ if *S* treats the elements of $\mathcal{C}$ as 'possible end states,' in the sense that if *S*'s state were to *rule out* all of the elements of $\mathcal{C}$ but one, then *S*'s state would be the element of $\mathcal{C}$ that *S* does not rule out.)

(11) "Every moment you spend with your child could be the one that really matters" (Russert 2006, xv–xvi).

(12) "Každyj priëm kokaina možet stat' poslednim."
Every dose of cocaine could become the last.
'Every time you take cocaine could be your last.'

(13) Given only what we can be certain of, no one here has to be the thief.

(14) Almost every square inch of the floor might have paint on it.

The semantic value of ‘almost every square inch of the floor’ is type $\langle\langle e, \langle a, t\rangle\rangle, \langle a, t\rangle\rangle$. It takes an open sentence like ‘λx. [might [x has paint on x]]’ to the characteristic function of the set of admissibles each element of which has the following property: for each square inch of the floor that is in some set of square inches on the floor consisting of almost every such square inch, the proposition that that square inch of the floor has paint on it gets at least ‘might’ level credence.

Authority reflects range. The *authority* that a speaker claims in asserting that φ decreases with increases in the size of the *range* of credences such that '*S* believes that φ' is true (holding fixed context, content of the prejacent, vagueness of expression, intonation, stakes, background conditions, …)

The White spies are spying on the Red spies, who are spying on the gun for hire. The gun for hire has left evidence suggesting that he is in Zurich, but one clever White spy knows that he is in London. After finding the planted evidence, one Red spy says to the others, "The gun for hire might be in Zurich," and the others respond "That's true." The clever White spy says "That's false—he's in London" to the other White spies, and explains how he knows this. (cf. Egan et al. 2005)

Authority reflects range helps explain why we have relativist-friendly judgments here: the less authority we claim when making an assertion, the more lenient the norms that govern the assertion.

G. E. Moore (foreshadowing Karttunen 1972):

> 'You *must* have omitted to turn the light off' means: 'There's conclusive evidence that you didn't.' The evidence is: It wouldn't have been on now, if you had turned it off, for (a) nobody else has been in the room & (b) switches can't turn on by themselves. But 'you certainly didn't' doesn't = 'You *must* have omitted': we shouldn't say the latter if we *saw* you come out without turning it off: we then shouldn't have *inferred* that you didn't. (1962, 188, dating to 1941–1942)

(17) John must be here by now.

(18) John has to be here by now.

(19) John should be here by now.

(20) John ought to be here by now.

(21) John couldn't be here by now.

(22) I don't think John could be here by now.

(23) I doubt that John could be here by now.

Following KRATZER 1976 (cf. VAN FRAASSEN 1973 and VELTMAN 1976), I hold that all readings of 'must,' 'have to,' 'should,' 'ought,' 'can,' 'could,' 'might,' and the like pertain to the relation between the prejacent and a set of premises. This is how and why epistemic modals carry an 'evidential' signal.

To a first approximation, on the Kratzer/Veltman semantics (24) means that 'φ' follows from all the strongest arguments available.

(24) It must be/has to be that φ.

(25) means (to a first approximation) that some strongest argument available does not falsify 'φ':

(25) It might be that φ.

Definition 1. *A relation is a preorder iff it is conditionally reflexive and transitive.*

Definition 2. *A preorder* $\lesssim$ *totally preorders a set* S *iff* $\forall x \forall y \big((x \in S \wedge y \in S) \rightarrow (x \lesssim y \vee y \lesssim x) \big)$.

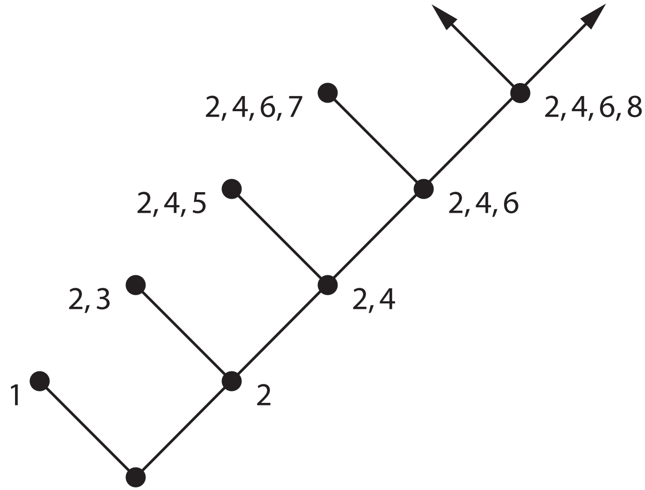
Definition 3. $\lesssim_i$ *(read 'is at least as good as at world i') is a partial preorder of a set* $S_{\lesssim_i}$ *of worlds such that* $S_{\lesssim_i} = \{ w : w \lesssim_i i \vee i \lesssim_i w \}$.

Definition 4. $<_i$ *(read 'is better than at i') is a strict partial order such that* $\forall x \forall y \big(x <_i y \leftrightarrow (x \lesssim_i y \wedge y \not\lesssim_i x) \big)$.

PM (Partial 'Must'):

'Must C' is true at i (relative to $\lesssim_i$) iff for every world $h \in S_{\lesssim_i}$ there is some world j such that $j \lesssim_i h$ and every world k such that $k \lesssim_i j$ is a C-world. (Kratzer 1981, 298; 1991, 644)

A problem case. Suppose that John and Karen deontically value proper supersets of children strictly increasingly, and that, because they think every life is uniquely precious, they think that sets of children neither of which is a subset of the other are deontically incomparable. John and Karen know that they have an unusual condition: they will have only boys unless they have an operation that will allow them to conceive one girl but will also make them infertile. They believe that they must have at most finitely many children.



Cheaper by the dozen

(26) It must be that a girl is conceived.

(27) It must be that the last child conceived is a girl.

(28) It is permissible that only boys are conceived.

(29) It must be that only boys are conceived.

(30) If John and Karen were obligated to conceive at most *n* children, they wouldn't have to conceive a girl.

Definition 5. *A set S is a $\lesssim$ antichain iff*

$$\forall x\Big(x \in S \to \big(\exists y(x \lesssim y \vee y \lesssim x) \wedge \neg\exists z(z \in S \wedge x \lesssim z \vee z \lesssim x)\big)\Big).$$

Definition 6. *A $\lesssim$ antichain S is a maximal $\lesssim$ antichain iff no $\lesssim$ antichain properly includes S.*

Definition 7. *A set S is a $\lesssim$ chain iff $\lesssim$ totally preorders S.*

Definition 8. *A $\lesssim$ chain S is a maximal $\lesssim$ chain iff no $\lesssim$ chain properly includes S.*

AM (Antichain 'Must'):

'Must $\mathcal{C}$' is true at i (relative to $\lesssim_i$) iff there is some maximal $\lesssim_i$ antichain, B, such that $\forall h \forall j \big((h \in B \wedge j \lesssim_i h) \rightarrow j \in C \big)$.

Theorem 1. *'Must $\mathcal{C}$' is true at i (relative to $\lesssim_i$) according to PM iff it is true at i (relative to $\lesssim_i$) according to AM.*

But it's possible for a maximal $\lesssim$ antichain to be disjoint from some maximal $\lesssim$ chain.

Definition 9. *A set S is a $\lesssim$ cutset iff S contains an element of each maximal $\lesssim$ chain.*

CM (Cutset 'Must'):

'Must C' is true at i (relative to $\lesssim_i$) iff there is some $\lesssim_i$ cutset, B, such that $\forall h \forall j \big((h \in B \land j \lesssim_i h) \rightarrow j \in C \big)$.

Lemma 1. *For each $\lesssim_i$ cutset B there is some maximal $\lesssim_i$ antichain A such that $A \subseteq B$.*

Theorem 2. *'Must C' is true at i (relative to $\lesssim_i$) according to CM only if it is true at i (relative to $\lesssim_i$) according to AM.*

Lemma 2. *Let* $t \in T$ *iff* $\forall k(k \lesssim_i t \rightarrow k \in C)$. *If 'Must* $\mathcal{C}$*' is not true at i (relative to* $\lesssim_i$*) according to CM, then some maximal* $\lesssim_i$ *chain* M *is such that* $M \cap T = \varnothing$.

Theorem 3. *As before, let* $t \in T$ *iff* $\forall k(k \lesssim_i t \rightarrow k \in C)$. *If 'Must* $\mathcal{C}$*' is true at i (relative to* $\lesssim_i$*) according to AM and not according to CM, then there is some maximal* $\lesssim_i$ *chain* M *and some maximal* $\lesssim_i$ *antichain* A *such that* $A \subseteq T$ *and every element of* M *is* $\lesssim_i$ *bettered by some element of* A.

Bas van Fraassen foreshadowed premise semantics in his 1973. To a first approximation, on van Fraassen's semantics (31) means that there is *some* strongest argument available such that 'φ' follows from it.

(31) It ought to be/should be that φ.

OSO (Ordering Semantics 'Ought'):

'Ought C' is true at i (relative to $\lesssim_i$) iff there is some world j such that $j \lesssim_i i$ and every world k such that $k \lesssim_i j$ is a C-world.

(32) It ought to be that the last child conceived is a girl.

(33) It ought to be that the last child conceived is a boy.

MCO (Maximal Chain 'Ought'):

'Ought $\mathcal{C}$' is true at i (relative to $\lesssim_i$) iff 'Must $\mathcal{C}$' is true at i relative to some maximal $\lesssim_i$ chain.

The intuitive thought is that weak necessity modals like 'ought' and 'should' abstract away from incomparability: 'Ought C' is true iff there is some way of bracketing moral dilemmas on which 'Must C' is true. So weak necessity modals decompose partial preorders into their constituent maximal chains and test those maximal chains against the standards associated with strong necessity modals like 'must' and 'have to.'

(32) It ought to be that the last child conceived is a girl.

(33) It ought to be that the last child conceived is a boy.

(27) It must be that the last child conceived is a girl.

(34) It must be that the last child conceived is a boy.

The quantitative aspects of the language of subjective uncertainty make pure constraint semantics look attractive. The evidential aspects of epistemic modals make premise semantics look attractive. I advocate a hybrid.

(17) John must be here by now.

(18) John has to be here by now.

(19) John should be here by now.

(20) John ought to be here by now.

(35) But he’s not here yet.

- epistemic strong necessity modals = CM plus a doxastic constraint wrt the prejacent,
- epistemic weak necessity modals = MCO without *any* doxastic constraint wrt the prejacent.

Surprising fact: possibility modals are stronger than weak necessity modals in an analogous way.

(36) #He left an hour ago, and there isn't any traffic. So John might be here by now, but he's not here yet.

(37) They should be here by now, but they're not.

(38) #They might be here by now, but they're not.

So (epistemic) ‘ought’ does not imply (epistemic) ‘can.’ To explain this we need a hybrid—the above plus

- epistemic possibility modals are dual to epistemic strong necessity modals, in both their premise semantic and constraint semantic aspects.

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