Oligopolistic Competition and Collaboration in Capacity

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Abstract

We focus on the topic of "***Capacity sharing between firms that compete with partially substitutable goods***". In particular, we examine (i) the conditions under which competing firms engage in capacity sharing, and (ii) how capacity sharing affects the firms' initial and subsequent capacity investment and operational decisions. Using a game-theoretic framework, we analyze the equilibrium, determine the price of shared capacity, and investigate the factors affecting the equilibrium outcome. We specifically present the results of a single period two-stage model where the firms are quantity setters.

1 Introduction

Capacity is defined as a firm's limited ability to produce revenue generating products. In order to respond to changes in demand, the firm tends to adjust its capacity level. For instance, anticipation of consistent increase in demand may trigger a need to build new capacity. These strategic level capacity adjustment decisions substantially alter the firm's assets both in terms of immediate expenditures and the stream of future monetary flow. Building new capacity requires major commitment of capital and human resources over a long span of time. Furthermore it is partially or completely irreversible. Therefore, the firm seeks more efficient ways to increase its processing abilities. In addition to outsourcing and contract manufacturing, a very effective way of achieving this is sharing capacity with the other firms.

Although the idea of sharing capacity with current and potential competitors may sound unrealistic, it is a practice that a significant number of companies have been implementing in increasingly many industries. In semiconductor industry, where building a new fab takes as long as 5 years and extensive initial capital commitment, the firms are forced to engage in strategic partnerships with their possible competitors. These partnerships, allowing the firms to reduce costs, become more important as the margins are pulled down with intense competition. For instance, the world's largest chip manufacturer Taiwan Semiconductor (TSMC) has long term capacity sharing agreements with Hewlett-Packard (HP) and Texas Instruments (TI), under which HP and TI use TSMC facilities to produce chips when orders exceed their own capacity. [15] Another similar agreement is between IBM and Chartered Semiconductor Manufacturing, where Chartered takes excess orders off IBM's hands. [21] Infineon Technologies, who offer semiconductor and system solutions, collaborates with Nanya Technologies in Taiwan and has built a joint venture to manufacture specialized memory chips. [1]

In automotive industry, Toyota Motor Corporation and General Motors Corporation launched a joint venture, New United Motor Manufacturing, Inc. (NUMMI) in Fremont, CA in 1984. NUMMI, very first of its kind, started manufacturing Chevrolet Nova and Toyota Corolla. [2] Currently, the facility produces Toyota Corolla, Toyota Tacoma and Pontiac Vibe, where Toyota Corolla and Pontiac Vibe compete in the same market segment. Another collaboration between Toyota and GM has been in the design stage of Toyota Matrix and Pontiac Vibe, which share the same mechanical foundation and platform. Toyota signed a similar agreement with Subaru on a production capacity sharing deal in late 2005. Under this agreement, Toyota will have the right to utilize Subaru's Indiana facilities to produce new Toyota Camry in case of insufficiency of its own production capacity. [8] Toyota also has another agreement since 2002 with Peugeot Citroen Automobiles SA in France on joint production of compact cars. [22]

[1] www.inotera.com
[2] www.nummi.com

Airline industry extensively use capacity sharing, in the name of code sharing, under which a flight operated by an airline is jointly marketed with one or more other airlines. Most major airlines have code sharing partnerships (such as Star Alliance and Sky Team), which is a way to expand their flight network without a burden of large-scale capital commitment. Through establishing strategic partnerships, airlines can reduce operating costs and expand their coverage to become more competitive in the low margin airline industry. Capacity sharing not only takes place in seating capacity, but also in cargo capacity. While these partnerships allow the participating firms to market seats from the same flights, the individual airlines engage in competition by setting their own prices.

The list of examples where capacity sharing takes place does not end here: The digital satellite television company DirecTV and its long time rival EchoStar have an agreement, which enables them to share satellite bandwidth to compete in the same markets. Gaz de France has signed a contract, regarding plans to cooperate with Suez, another French-based company, on electricity generation and supply, including contracts for reciprocal capacity sharing.

These examples have some common characteristics. First of all, capacity expansion in these examples is both extremely costly and time consuming. Moreover, volatile market conditions, together with competition, bring relatively small margins to the firms. Under these conditions, the firms seek to exploit the advantages of collaboration by sharing capacity.

In this paper we will be focusing on explaining the phenomenon of capacity sharing using mathematical models. After introducing previous work in related fields of literature, we will discuss two models, where we consider a two-stage duopoly with firms producing partially substitutable products. In the first stage the firms independently make their capacity decisions, whereas in the second stage, constrained by these capacity levels, the firms engage in market competition with negotiation over capacity sharing. Our focus will be on explaining the effects of capacity sharing to the firms' initial investment decisions and determining the sharing price, endogenously set by the negotiation process. Particularly we will try to find out what drives the competing firms to practice capacity sharing. On the other hand, we will also concentrate on discovering the potential benefit of sharing capacity over pure competition. We plan to research possible extensions to these models in the future to cover other aspects of capacity sharing.

2 Literature Survey

The study of capacity sharing is closely related to several streams of research in economics and operations management. In economics, it is directly related to the oligopoly literature, which mainly focuses on determining equilibrium conditions under different assumptions and how firms behave in equilibrium by using descriptive models. On the other hand, there is an extensive amount of previous work in strategic capacity management, in which, researchers, unlike the economists, try to establish the optimal capacity investment policies for firms, or aim to specify contracts that enable the coordination of independent agents, possibly in a supply chain setting. These models, however, rarely consider competitive settings. Furthermore, there has been significant amount of work done in inventory management focusing on inventory transshipment within dealer or retailer networks, which is relevant in terms of the attempt to assess the collaboration between possible competitors. Indeed, for some models, inventory transshipment and capacity sharing can be used interchangeably. Finally, we will use results from the Bargaining literature to model the collaboration between the firms. This literature considers situations where the individuals have the opportunity to collaborate for mutual benefit, and tries to predict the equilibrium of such a situation.

2.1 Oligopoly

The study of oligopolistic competition dates back to Cournot [5], who models a duopoly of two quantity-setting firms with linear downward sloping inverse demand function. Provided that the firms cannot engage in collaborative decision making, he shows that the equilibrium will be symmetrical where the firms make positive profits. However, when the firms are assumed to control the prices and have unlimited capacity, Bertrand [2] argues that the firms would name zero prices and make no profit in the noncooperative equilibrium, as they would keep undercutting each other to capture the whole market, reaching a result completely different from Cournot's. By relaxing the sufficient capacity assumption of Bertrand, Edgeworth [12] argues that existence of equilibrium in pure strategies is not immediate and depends on the capacity levels of the firms. He shows that the prices in the market will fluctuate in a specified interval for moderate levels of capacity. Levitan and Shubik [17] give a brief outline of this history and, calculate the range over which the price is expected to fluctuate in Edgeworth's setting. They further show that there exists a mixed strategy equilibrium, whose support may coincide with the range of price fluctuation. Here, it is highly crucial how we determine the residual demand for the high-priced firm when the low-priced firm cannot meet the demand. They assume that consumers all have uniform access to the sellers and further, that, if the demand at the low price exceeds low-priced firm's capacity, then the demand faced by the high priced firm is the maximum of the demand at the high price and the remaining amount at the low price. In other words, the customers, who value the product most, go to the low-priced firm first, whereas the high-priced firm is left with the customers having lower valuation. Using this assumption, Kreps and Scheinkman, in their seminal work [16], show that if the firms compete in two stages, where, in the first stage they commit production capacities and in the second stage they engage in Bertrand-like price competition, the unique equilibrium is the one in which the firms build Cournot capacities and name the corresponding market clearing prices. This result established the robustness of Cournot equilibrium in a duopoly. However, Davidson and Deneckere [6] and others argue that Kreps and Scheinkman's results are very sensitive to the residual demand assumption. There have been several other notable papers focusing on this subject, including Herk [14] who shows that joint Cournot behavior will arise in the equilibrium when the residual demand function for the high-priced firm is determined by customer switching costs. When demand uncertainty is incorporated into the model, however, the existence of pure strategy equilibria is lost, shown by Hviid [11]. Haskel and Martin [10] try to empirically test the two hypotheses using a panel industry data set for the UK, and find that capacity constrained firms become more Cournot-like. We must note that all these models consider homogeneous goods. Friedman [13] extends the model to differentiated products oligopoly, and shows results parallel to the homogenous products case, where the emergence of Cournot equilibrium depends on the assumption made about the residual demand function. As we will be considering differentiated products, this result will be more relevant for our analysis.

2.2 Strategic Capacity Management

The capacity expansion literature is concerned with determining the size, timing, location and type of investment in new capacity. Investment costs, in general, exhibit substantial economies-of-scale. The firm, therefore, must consider the trade-off between the economies-of-scale savings of large expansion sizes versus the cost of installing capacity before it is needed. Manne, in his classical work [19] studies this trade-off with linearly growing deterministic demand and shows that capacity expansion size at a time increases with economies-of-scale and decreases with the time value of money. There have been later attempts to approach the capacity expansion problem in the presence of demand uncertainty. An extensive survey can be found in [26].

When the emphasis is on characterizing the timing of the investment subject to uncertain demand and with the existence of frictions, such as irreversibility, the optimal capacity adjustment policy, usually follows an "*Invest, Stay-put, Disinvest (ISD)*" policy, which is characterized by a continuation region on the state space. When the state, which is the capacity level, falls in this region, it is optimal not to adjust capacity, and otherwise, the optimal adjustment to the capacity is to bring the capacity level to the boundary of the continuation region. Eberly and Van Mieghem [9] consider capacity adjustment of a firm controlling multiple factors when any number of the factors face "kinked" linear adjustment costs, reflecting the costly reversibility. They show that an ISD policy is optimal for a monopolist controlling multiple factors of production which do not compete with each other. Angelus and Porteus [1] study the simultaneous capacity and production management of short-life-cycle, produce-to-stock goods under uncertain demand with partially irreversible capacity investment. Short-life-cycle products, frequently arising in the electronics industry, are characterized by a demand stream that stochastically increase up to a peak and then decrease over a finite time horizon, where leftover inventory cannot be carried to the next period. The optimal capacity adjustment plan, which follows ISD, takes the form of an expansion phase at the beginning of the life cycle, which is followed by a constant phase in the middle and a downsizing phase at the end. As there is no carry-over of inventory from period to period, the optimal capacity levels can be determined before observing the demand.

Note that the capacity adjustment literature reviewed above do not consider independent decision makers in a competitive setting. These solution methods must be augmented with consideration of the interaction between the independent decision makers and their policies in order to solve for Nash equilibrium capacity investment strategies. Complexity quickly mounts and great care must be exerted to specify a tractable multiperson model. [26] Furthermore, capacity sharing has not been a concern for those models. Van Mieghem [25] considers a game-theoretic setting of a manufacturer and a subcontractor and studies three outsourcing contract types, differing in the terms specified by the contracts ex ante, to explore the coordinating power of these contracts. In this setting, the manufacturer and the subcontractor decide separately on their capacity investment levels, then, after demand uncertainty is resolved, the manufacturer has the option to subcontract when he decides on the production and sales. Although the model considers collaboration in capacity, it assumes that the firms are distinguished such that their products do not compete in the market. In another game-theoretic model, with Dada [27], he investigates the benefits of postponement strategies that postpones, one or more of capacity, quantity and pricing decisions to be done after the demand is realized, which is assumed to be linear with additive shocks. They study the implications of price postponement in a competitive setting, without any capacity sharing, and find that the relative value of operational postponement techniques increase as the industry becomes more competitive.

Another approach taken by some researchers to study capacity competition involves queueing. These papers mainly consider the duopolies consisting of make-to-order firms, by assessing their steady state performance measures. Chen and Wan [4], consider a duopoly of two firms, modeled as exponential queues facing a stream of customer arrivals modulated by a Poisson process, where each arrival independently decides from which firm they will get the service, depending on the queue lengths and prices. The emphasis is on the capacity competition, where capacity is defined as the exponential service rate, without considering any capacity sharing. In another paper [3], they study price competition in a duopoly of make-to-order firms delivering different quality levels with different waiting costs, again without capacity sharing. Yu et. al.[28], analyze service capacity pooling, where they examine the impacts of various factors on capacity pooling, with the focus on cost sharing between the participating firms.

2.3 Inventory Transshipment

Models considering inventory transshipment between competitive dealers or retailers in a network show resemblance to possible models to be used to study capacity sharing between competitors. Especially in single period models where capacity does not have strategic long term implications, it is not different from committing in inventory. A related paper is by Rudi et. al. [24] who study a transshipment problem of two sellers, serving customers in distinct regions in a single period newsvendor setting. Although there is no competitive interaction in terms of demands they face, one seller's inventory level decision affects the other through transshipment possibility, thus, a game-theoretic approach is needed to address this setting. With local decision making, a new parameter must be introduced into the model, the transshipment price, which would be neglected in the centrally coordinated case where the transshipment would be intrafirm. Rudi et. al. assume that the transshipment price is exogenously given, rather than obtaining this price as a result of the negotiation process between the sellers. They compare the equilibrium inventory levels with those optimally selected by a centralized decision maker and establish the transshipment price that induce the firms to select the coordinated inventory levels. The extension of this model to more than two sellers is not very straightforward because of the complicated transshipment policies between the sellers. Another similar setting is studied by Zhao et. al. [29], focusing on a continuous review dealer network consisting of two independent dealers, taking a continuous time Markov chain approach. In their setting, in the case of shortage, the dealer who faces the shortage places a sharing request to the other dealer who, then treats that request as a low priority demand and rations his inventory accordingly. Suggesting appropriate inventory rationing and replenishment policies, they construct a tractable model. For special settings of the problem they are able to establish supermodularity of the resulting game and hence show the existence of pure strategy equilibrium, but not for the general case. However, in the numerical study carried out, the structure of the equilibrium can be observed, so that appropriate incentives can be suggested to impose the dealers to select the inventory levels that maximize the overall chain profit. A brief review of inventory transshipment literature can be found in [24].

2.4 Bargaining

In the inventory transshipment context, we have seen that, when we consider collaboration, we need to introduce an exogenously given parameter to the problem to specify the transfer payment for inventory sharing. Then, we can separate and appropriately solve the interrelated problems faced by the firms in order to find the equilibrium. However, one of our main goals is to identify the price of the shared capacity. When we want the transfer payment to be endogenously determined by the model, we can no longer separate the firms' problems. At this point, Bargaining comes into the scene. A classical paper, Nash [20], studies this problem, where the assumptions he states, some of which can be treated as axioms, lead to the results.

Next, we will elaborate his results as they will directly be used in the analysis of the preliminary work. He defines u_1 and u_2 as the utilities to the individuals from entering into negotiation. Assuming that the feasible space of (u_1, u_2) is symmetric along the line $u_1 = u_2$ and convex, Nash shows that the equilibrium of this setting occurs at the allocation where $u_1 u_2$ is maximized. In the next section we will see that this is equivalent to maximizing $u_1 + u_2$ and appropriately allocating the total gain from Bargaining. In this way we can endogenously determine the transfer payment as a result of the negotiation process. Furthermore, although we assume that the firms have equal bargaining power, we can use this approach to study firms with arbitrary bargaining power.

3 The Model: Capacity-Constrained Quantity Competition with Capacity Sharing

In this section we develop our model to study capacity sharing between competing firms. Consider a market of two partially substitutable products, where the price of a product is determined as the market clearing price. In our model, regardless of the market structure, the firms face two decision problems over two stages. In the first stage they build capacity to set their final capacity levels and then in the second stage, constrained by the corresponding capacity levels, they determine the production quantities. We analyze four settings, varying in the degree of collaboration. In the first setting, we assume that both products are produced by a single firm, which we will denote as *Fully Integrated Firm (FIF)*. In the other extreme, we study the setting in which there are two independent firms who cannot collaborate in any stage of the competition, denoted as *Pure Competition(PC)*. Finally, the other two settings include collaboration in capacity. In the first setting there are two firms, who can collaborate in the second stage of the game by setting the production quantities together, as if they formed a cartel. Hence, we will denote this setting as *Cartel Formation (CF)*. The other setting that involves collaboration will be denoted as *Noncooperative Capacity Sharing Negotiation (NC)*, where the firms have the option to share capacity in the quantity setting game, but are only allowed to negotiate over the level of capacity to be shared.

We consider deterministic linear inverse demand functions. The capacity building price will be assumed to be linear and same for both firms, and will be denoted by β. The production cost is not necessarily the same for both products, but it is assumed to be linear. We assume that the production cost does not depend on the facility used, it is product specific, that is, regardless of the facility used to produce product i, it will cost c_i to firm i. Given that the quantities q_1 and q_2 are supplied to the market, the market clearing prices are

$$p_i(q_i, q_j) = a_i' - b_i q_i - \tilde{b}_j q_j \qquad i, j = 1, 2 \quad i \neq j$$

If we further define $a_i = a_i' - c_i$, where c_i is the unit production cost of product i, then the operating profits of the second stage are given by

$$\pi_i^{op}(q_i, q_j) = q_i \left(a_i - b_i q_i - \tilde{b}_j q_j\right) \qquad i,j = 1,2 \quad i \neq j$$

The parameters a_i, $b_i$and $\tilde{b}_i$ are assumed to be positive. Furthermore, we make the following assumptions about the parameters:

$$b_i > \tilde{b}_j \qquad i = 1,2, \quad j = 1,2$$

With this assumption we impose the coefficient of a firm's production quantity in its own inverse demand function to be larger than that in the other firm's inverse demand function. Furthermore, it must also be larger than the coefficient of the other firm's production quantity in its own inverse demand function. In other words, the market clearing price of a product is more sensitive to its own quantity than it is to the other product's quantity. This is a standard assumption in differentiated product settings, known as diagonal dominance.

$$2a_1 b_2 - a_2 \tilde{b}_2 > 0 \quad \text{and} \quad 2a_2 b_1 - a_1 \tilde{b}_1 > 0$$

This assumption assures that, in the case where both firms have positive capacity in the second stage, they will produce positive quantities at the noncooperative equilibrium. Furthermore, we put a lower bound on the capacity building cost: $\beta \geq \max(\frac{\tilde{b}_1}{2}\frac{2a_2b_1+a_1\tilde{b}_1}{4b_1b_2-\tilde{b}_1\tilde{b}_2}, \frac{\tilde{b}_2}{2}\frac{2a_1b_2+a_2\tilde{b}_2}{4b_1b_2-\tilde{b}_1\tilde{b}_2})$, for technical reasons. Moreover, we assume that the market size parameters are greater than the capacity building cost, that is $a_i > \beta$, $i = 1,2$ to avoid trivial results, and, without loss of generality, we assume that product 2 has a larger market size parameter: $a_2 \geq a_1$.

3.1 Benchmark Scenarios

In order to have a benchmark setting to compare the results sharing capaicty, we first study the FIF problem. Consider a monopolistic firm, which first builds a production capacity that is capable of producing both products, and then sets the production quantities, constrained by the generic capacity level. As the demand function is deterministic and there is no competition, the FIF will fully utilize the capacity he builds. Then, denoting the quantities of products that FIF will produce by q_1 and q_2, he solves the following concave maximization problem:

$$\max_{q_1 \geq 0, q_2 \geq 0} q_1(a_1 - b_1 q_1 - \tilde{b}_2 q_2) + q_2(a_2 - b_2 q_2 - \tilde{b}_1 q_1) - \beta(q_1 + q_2) \tag{1}$$

Once this problem is solved and the optimal quantities are determined, the optimal capacity level of FIF will simply be the sum of the production quantities. Next proposition characterizes the solution to this problem.

Proposition 3.1. *Define the following quantities:*

$$Q_1^{FIF} = \frac{2a_1b_2 - a_2(\tilde{b}_1 + \tilde{b}_2) - \beta(2b_2 - \tilde{b}_1 - \tilde{b}_2)}{4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2}$$

$$Q_2^{FIF} = \frac{2a_2b_1 - a_1(\tilde{b}_1 + \tilde{b}_2) - \beta(2b_1 - \tilde{b}_1 - \tilde{b}_2)}{4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2}$$

Then the unique solution to problem (1) will be given by (K_1^{FIF}, K_2^{FIF}), which are:

$$(K_1^{FIF}, K_2^{FIF}) = \begin{cases} \left(0, \frac{a_2-\beta}{2b_2}\right), & \text{if } Q_1^{FIF} \le 0; \\ \left(Q_1^{FIF}, Q_2^{FIF}\right), & \text{otherwise.} \end{cases} \tag{2}$$

At the optimal solution, the FIF will always produce product 2 , which has the larger market parameter. The other product will be produced only if the operating profit from producing that product is higher than its negative effect on the price of the other. In the case where FIF produces both products, the quantities will be given by the unconstrained solution. If it is not worth to produce product 1, then the production quantity of product 2 will simply be the optimal monopoly quantity.

As another benchmark at the other end, we study the equilibrium of the two-stage game where the firms build capacity in the first stage and compete in quantities in the second stage, constrained by the capacity levels, without any capacity sharing. Although we can solve this problem using backward induction, it turns out that there exists an equivalent single stage game, as asserted in the next proposition.

Proposition 3.2. *The two-stage game under the Pure Competition setting reduces to a simple Cournot quantity competition game, where the market clearing prices will be given by*

$$p_1(q_1, q_2) = a_1 - \beta - b_1q_1 - \tilde{b}_2q_2 \quad p_2(q_1, q_2) = a_2 - \beta - b_2q_2 - \tilde{b}_1q_1 \tag{3}$$

where q_1 and q_2 are the quantities supplied to the market. Now define the following quantities:

$$Q_1^{PC} = \frac{2a_1b_2 - a_2\tilde{b}_2 - \beta(b_2 - \tilde{b}_2)}{4b_1b_2 - \tilde{b}_1\tilde{b}_2} \quad Q_2^{PC} = \frac{2a_2b_1 - a_1\tilde{b}_1 - \beta(b_1 - \tilde{b}_1)}{4b_1b_2 - \tilde{b}_1\tilde{b}_2}$$

Then the equilibrium capacity levels will be

$$(K_1^{PC}, K_2^{PC}) = \begin{cases} \left(0, \frac{a_2-\beta}{2b_2}\right), & \text{if } Q_1^{PC} \le 0; \\ \left(Q_1^{PC}, Q_2^{PC}\right), & \text{otherwise.} \end{cases}$$

Proposition 3.2 states that the equilibrium of the two-stage game can be found by solving the simple Cournot quantity setting game by taking the inverse demand functions that are obtained by subtracting the unit capacity building cost from the market clearing prices of the products. Note that, in the equilibrium, there may not be enough incentive for firm 1 to enter the competition, hence the game may result in an equilibrium in which firm 2 is the monopoly of the market. Furthermore, the firms will not build higher capacity than they will need, hence there will be no capacity idling. Note that, the quantities produced and thus the total capacity built will be larger than those of the FIF solution, which is quite intuitive. The competition will force the firms to produce more, thus the market clearing price will decrease, resulting in a higher consumer surplus.

3.2 Capacity Sharing

In this section we present our model of capacity sharing. We employ a two stage setting, in which, firms independently build capacity in the first stage and compete in quantities in the second stage with the option of sharing capacity. Using backward induction, we first characterize the equilibrium of the second stage subgames for any capacity vector realization and then solve for the equilibrium of the first stage capacity building game. Before going into the analysis of the second stage subgames using Nash bargaining solution, we will justify its use by showing that Nash's axioms are satisfied.

Recall that in Nash Bargaining solution, u_1 and u_2 were defined as the utilities to the individuals from entering into negotiation. In our context, these variables correspond to the differences between the before and after bargaining payoffs of the firms, where before bargaining payoffs will be referred as *disagreement payoffs*, which are the payoffs of the firms if they do not share their capacity and play the equilibrium strategies of the capacity constrained competition. The sum of the after bargaining payoffs will be defined as *maximum attainable payoff*, which is the highest total payoff that the firms can reach by sharing capacity.

In addition to the regular assumptions about the anticipations and preferences of the individuals that allow him to formulate the bargaining game with mathematical utility functions representation, Nash assumes that the set of utility pairs is compact, convex and symmetric with respect to the line $u_1 = u_2$. Here the variable u_i is defined as the extra utility that a firm gains by engaging in bargaining negotiations, and it takes a specific value for any anticipation pair the players have. Hence, in our case, for a subgame determined by a capacity pair (K_1, K_2), we let $u_i(K_1, K_2, Q_1, Q_2, t)$ be the utility from engaging in bargaining process, corresponding to an anticipation of the players, which is characterized by the quantities (Q_1, Q_2) and the transfer payment t, similar to the definition by Nash. Then we can write these utilities as

$$u_1(K_1, K_2, Q_1, Q_2, t) = \pi_1^s(K_1, K_2, Q_1, Q_2) - t - d_1(K_1, K_2)$$

$$u_2(K_1, K_2, Q_1, Q_2, t) = \pi_2^s(K_1, K_2, Q_1, Q_2) + t - d_2(K_1, K_2)$$

where $\pi_i^s(K_1, K_2, Q_1, Q_2)$ is the profit from sales in the second period and t is the transfer payment that occurs from firm 1 to firm 2 for the capacity share, and $d_i(K_1, K_2)$ is the disagreement payoff for firm i. If the capacity exchange is from firm 1 to firm 2, then t ideally will take a negative value. From now on we will suppress the dependency of these variables on the capacity levels for brevity. Now, let's restrict our attention to the quantity vectors such that the resulting total operating payoff $\pi_1^s(Q_1, Q_2) + \pi_2^s(Q_1, Q_2)$ is at least as much as the sum of the disagreement payoffs. This is natural as by the definition of disagreement payoffs, the firms will not settle down with a result in which they do not get at least their disagreement payoffs. In the set representation let's define $\mathcal{Q}$ as the set of quantity vectors satisfying the condition above, that is

$$\mathcal{Q} = \{(Q_1, Q_2) : \pi_1^s(Q_1, Q_2) + \pi_2^s(Q_1, Q_2) \geq d_1(K_1, K_2) + d_2(K_1, K_2), (Q_1, Q_2) \in \mathbb{R}^+\}$$

Then the feasible utility pairs set can be defined as:

$$\mathcal{U} = \{(u_1, u_2) : u_1 = u_1(Q_1, Q_2, t), u_2 = u_2(Q_1, Q_2, t), (Q_1, Q_2) \in \mathcal{Q}, t \in [-(\pi_2^s(Q_1, Q_2) - d_2), \pi_1^s(Q_1, Q_2) - d_1]\}$$

Note that we can set the transfer payment t to any value we want, as long as the resulting utilities are nonnegative. By defining t in this way and setting the feasible region for t accordingly, it becomes easy to show the compactness, convexity and symmetricity of the utility pairs feasible set. It essentially depends on the construction in which we freely select the value of the transfer payment. Next, we will show that the feasible utility pairs set satisfies the desired properties.

Convexity

Let (u_1^1, u_2^1) and (u_1^2, u_2^2) be in the set $\mathcal{U}$, i.e.

$$\begin{aligned} u_1^1 &= \pi_1^s(Q_1^1, Q_2^1) - t^1 - d_1 \quad u_2^1 = \pi_2^s(Q_1^1, Q_2^1) + t^1 - d_2 \\ u_1^2 &= \pi_1^s(Q_1^2, Q_2^2) - t^2 - d_1 \quad u_2^2 = \pi_2^s(Q_1^2, Q_2^2) + t^2 - d_2 \end{aligned} \tag{4}$$

for some (Q_1^1, Q_2^1) and (Q_1^2 ,Q_2^2) in $\mathcal{Q}$, and t^i in the interval $[-(\pi_2^s(Q_1^i, Q_2^i) - d_2), \pi_1^s(Q_1^i, Q_2^i) - d_1]$. Now define ($\bar{u}_1$, $\bar{u}_2$) as convex combination of (u_1^1, u_2^1) and (u_1^2, u_2^2):

$$(\bar{u}_1, \bar{u}_2) = (\lambda u_1^1 + (1-\lambda)u_1^2, \lambda u_2^1 + (1-\lambda)u_2^2) \qquad \lambda \in [0, 1]$$

Note that, we can define ($\bar{u}_1$, $\bar{u}_2$) in terms of a quantity vector and a transfer payment, and showing that there exists a feasible quantity vector and a transfer payment will mean that it belongs to the set $\mathcal{U}$, establishing its convexity. Let

$$\bar{u}_1 = \pi_1^s(\bar{Q}_1, \bar{Q}_2) - \bar{t} - d_1 \quad \bar{u}_2 = \pi_2^s(\bar{Q}_1, \bar{Q}_2) - \bar{t} - d_2 \tag{5}$$

Note that $(\bar{Q}_1,\, \bar{Q}_2)$ is feasible since

$$\begin{aligned}\bar{u}_1 + \bar{u}_2 =& \pi_1^s(\bar{Q}_1, \bar{Q}_2) + \pi_2^s(\bar{Q}_1, \bar{Q}_2) - d_1 - d_2 \\ =& \lambda(u_1^1 + u_2^1) + (1-\lambda)(u_1^2 + u_2^2) - d_1 - d_2 \geq 0\end{aligned}$$

The last inequality follows as $(u_1^1,\, u_2^1)$ and $(u_1^2,\, u_2^2)$ are feasible utility pairs. It shows that there exists feasible $(\bar{Q}_1,\, \bar{Q}_2)$ such that $(\bar{u}_1,\, \bar{u}_2)$ can be defined as in (5). Now, by selecting $\bar{t}$ arbitrarily in the range $[-(\pi_2^s(\bar{Q}_1, \bar{Q}_2) - d_2), \pi_1^s(\bar{Q}_1, \bar{Q}_2) - d_1]$, which will be nonempty, we conclude that $\bar{U}$ is convex.

Compactness

Note that, as we restrict the set of feasible pricing vectors to a set in which the resulting total payoff is at least as much as the sum of the disagreement payoffs, the set $\mathcal{U}$ is bounded below in both variables. Furthermore, as the payoffs from sales remain constant with respect to the capacities for large enough capacity levels, without loss of optimality we can restrict our set by imposing upper bounds, hence $\mathcal{U}$ will be bounded above as well, completing the argument of compactness.

Symmetricity

Finally, Nash argued that if the set $\mathcal{U}$ is symmetric, that is if $(a, b) \in \mathcal{U}$ implies that $(b, a) \in \mathcal{U}$, then the equilibrium will be a point on the line $u_1 = u_2$. Now let $(u_1^1,\, u_2^1)$ be an arbitrary utility pair and be defined as in (4). Define $\hat{t}$ as follows

$$\hat{t} = \pi_1^s(Q_1^1, Q_2^1) - \pi_2^s(Q_1^1, Q_2^1) + d_2 - d_1 - t^1$$

It can be shown that $\hat{t}$ is in the desired interval as well. If we plug in $\hat{t}$ instead of t^1 in (4), we get $(u_2^1,\, u_1^1)$, which is defined by $(Q_1^1,\, Q_2^1)$ and $\hat{t}$, and hence feasible. Therefore, $\mathcal{U}$ is symmetric with respect to the line $u_1 = u_2$.

We have shown that the assumptions of Nash bargaining solution are satisfied in our problem. Recall that Nash bargaining solution says that the equilibrium of a bilateral bargaining problem will occur at the point where the product $u_1 u_2$ is maximized. In the next lemma we will see that this point can be found alternatively by maximizing the sum of the utilities, instead of their product.

Lemma 3.3. *Let u_1^* and u_2^* be the equilibrium after bargaining payoffs of the firms, i.e.: $(u_1^*, u_2^*) = \arg\max_{(u_1,u_2)\in\mathcal{U}} u_1 u_2$. Let U^* be defined as*

$$U^* = \{u_1 + u_2 : (u_1, u_2) = \arg\max_{(u_1,u_2)\in\mathcal{U}} u_1 + u_2\}$$

Then,

$$u_1^* = u_2^* = U^*/2$$

If we approach the problem by solving for the maximizer of $u_1 u_2$, as suggested by Nash [20], t term will remain in the expression, and its value cannot be determined endogenously. Using Lemma 3.3 firms' after bargaining payoffs can be determined by maximizing the total payoff they can reach within the feasible allocation of total capacity. The definition of this feasible set constitute the main difference between two different approaches, Cartel Formation and Noncooperative Capacity Sharing Negotiation. As we allow the firms to collaboratively decide not only on the level of capacity share but also on the quantities to supply to the market in the CF setting, the maximum attainable payoff will be defined as the maximum profit of a centralized decision maker, controlling both products. In NC setting, however, the firms are only allowed to negotiate over the capacity share, which must be followed by a noncooperative competition in quantity. Therefore, the maximum attainable payoff is the highest total competitive payoff the firms can attain by reallocating the total capacity. Besides these differences, the disagreement payoffs under both approaches will be the same. For a given capacity vector (K_1, K_2) the disagreement payoffs will be the payoffs the firms will get in the equilibrium of the game in which they compete in quantities, constrained by the capacity levels without any capacity sharing option. This game is identical to the one played in the second stage of the PC setting, hence the disagreement payoffs for both CF and NC settings are simply identical to the payoffs in (17). In the following sections we will determine the maximum attainable payoffs and discuss about the resulting equilibrium of the first stage games under two approaches.

3.2.1 Cartel Formation (CF)

In this approach, after building their individual capacities in the first stage, the firms come together in the second stage and as if they form a hypothetical centralized decision maker that controls the production of both products, i.e. as if they form a cartel, they decide on the quantities of each product to produce. Next we will characterize the MAP in this setting, for which we simply will solve the quantity setting problem of a centralized decision maker, for a given level of capacity. Hence, MAP will be the solution to the following nonlinear program:

$$\max_{y_1\geq 0, y_2\geq 0} y_1(a_1 - b_1 y_1 + \tilde{b}_2 y_2) + y_2(a_2 - b_2 y_2 + \tilde{b}_1 y_1) \tag{6}$$

$$\text{s.t.} \qquad y_1 + y_2 \leq K^t$$

In this problem, the decision variables y_1 and y_2 denote the quantity of the products to be supplied to the market and K^t is the sum of the individual capacities of the firms that were finalized in the first stage. The solution of this problem, includes two cases depending on the parameters and they are characterized in the following two propositions.

Lemma 3.4. *Let*

$$T_1^{CF1} = \frac{a_2 - a_1}{2b_2 - \tilde{b}_1 - \tilde{b}_2} \quad T_2^{CF1} = \frac{2a_1b_2 + 2a_2b_1 - (a_1 + a_2)(\tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2}$$

$$Q_1^{CFU} = \frac{2a_1b_2 - a_2(\tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2} \quad Q_2^{CFU} = \frac{2a_2b_1 - a_1(\tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2}$$

$$Q_1^{CFC}(K^t) = \frac{(2b_2 - \tilde{b}_1 - \tilde{b}_2)K^t + a_1 - a_2}{2(b_1 + b_2 - \tilde{b}_1 - \tilde{b}_2)} \quad Q_2^{CFC}(K^t) = \frac{(2b_1 - \tilde{b}_1 - \tilde{b}_2)K^t - a_1 + a_2}{2(b_1 + b_2 - \tilde{b}_1 - \tilde{b}_2)}$$

If the following inequality is satisfied, then the solution of problem (6) will be given by (Q_1^{CF1}, Q_2^{CF1}).

$$2a_1b_2 - a_2(\tilde{b}_1 + \tilde{b}_2) \geq 0 \tag{7}$$

$$(Q_1^{CF1}, Q_2^{CF1}) = \begin{cases} \left(0, K^t\right), & \text{if } K^t \leq T_1^{IB1}; \\ \left(Q_1^{CFC}(K^t), Q_2^{CFC}(K^t)\right), & \text{if } T_1^{CF1} < K^t \leq T_2^{IB1}; \\ \left(Q_1^{CFU}, Q_2^{CFU}\right), & \text{otherwise.} \end{cases} \tag{8}$$

Here we will say that this proposition corresponds to *Case 1* of Cartel Formation. If the given inequalities are satisfied by the parameters, then the hypothetical centralized decision maker will produce both products, provided that total capacity level is high enough. There exists a threshold value (T_1^{CF1}), so that, if the total capacity is below that threshold, then the optimal action for the centralized decision maker is to fully utilize the total capacity to produce only product 2. There exists another threshold (T_2^{CF1}), so that if the total capacity is higher than that, then the solution will be given by the unconstrained maximizer, (Q_1^{CFU}, Q_2^{CFU}), of the objective function. Among these thresholds, the latter will always take a higher value. For the total capacity levels between these two thresholds the solution will be the one where the capacity constraint will be binding, ($Q_1^{CFC}(K^t)$, $Q_2^{CFC}(K^t)$). The solution for *Case 2* is characterized in the following proposition.

Lemma 3.5. *Let*

$$T^{CF2} = Q_2^{CFM} = \frac{a_2}{2b_2}$$

If inequality (7) is not satisfied, then the solution of problem 6 *will be given by* (Q_1^{CF2}, Q_2^{CF2}).

$$(Q_1^{CF2}, Q_2^{CF2}) = \begin{cases} \left(0, K^t\right), & \text{if } K^t \leq T^{IB2}; \\ \left(0, Q_2^{CFM}\right), & \text{otherwise.} \end{cases} \tag{9}$$

If inequality (7) is not satisfied, then it cannot be optimal for the centralized decision maker to produce product 1 for any level of the total capacity. In this case, the only threshold value characterizing the solution will be the monopoly output level of product 2, so that if the total capacity is lower than that value then the optimal action is to utilize all the capacity, and otherwise it is to produce the monopoly quantity.

Having defined the disagreement payoffs and the maximum attainable payoff, we can obtain the after bargaining payoffs of the firms using the fact that the extra generated payoff will be divided evenly between the firms. Let $d_1(K_1, K_2)$ be the disagreement payoff for firm 1 and $\pi^{MAP}(K_1, K_2)$ be the maximum attainable payoff, then the after bargaining payoff of firm 1 will be

$$\begin{aligned} \pi_1(K_1, K_2) = & \quad d_1(K_1, K_2) + \frac{1}{2}\left(\pi^{MAP}(K_1, K_2) - d_1(K_1, K_2) - d_2(K_1, K_2)\right) \\ = & \quad \frac{1}{2}\left(\pi^{MAP}(K_1, K_2) + d_1(K_1, K_2) - d_2(K_1, K_2)\right) \end{aligned} \tag{10}$$

Note that the after bargaining payoff is a combination of the disagreement and maximum attainable payoffs, that are defined in a case based structure on the capacity space, which is divided into mutually exclusive and collectively exhaustive regions. In Figure 1 these regions are demonstrated for Case 1. Note that depending on the values of the parameters, some of these regions may not exist, whereas the figure exhibits the highest possible number of regions.

This figure shows us some interesting properties of the second stage equilibrium. If the capacity vector falls to the right of the second line that makes 45 degrees with the horizontal axis (Regions 1,2,3 and 4), then in the equilibrium of the second stage game the total quantity supplied to the market will be smaller than the total capacity, where capacity sharing is also possible. If the capacity vector falls in one of the regions 5, 6, and 7, then in the equilibrium total capacity is fully utilized to produce both products, but if it is in one of the regions 8 and 9, then in the equilibrium only product 2 is produced using all the capacity.

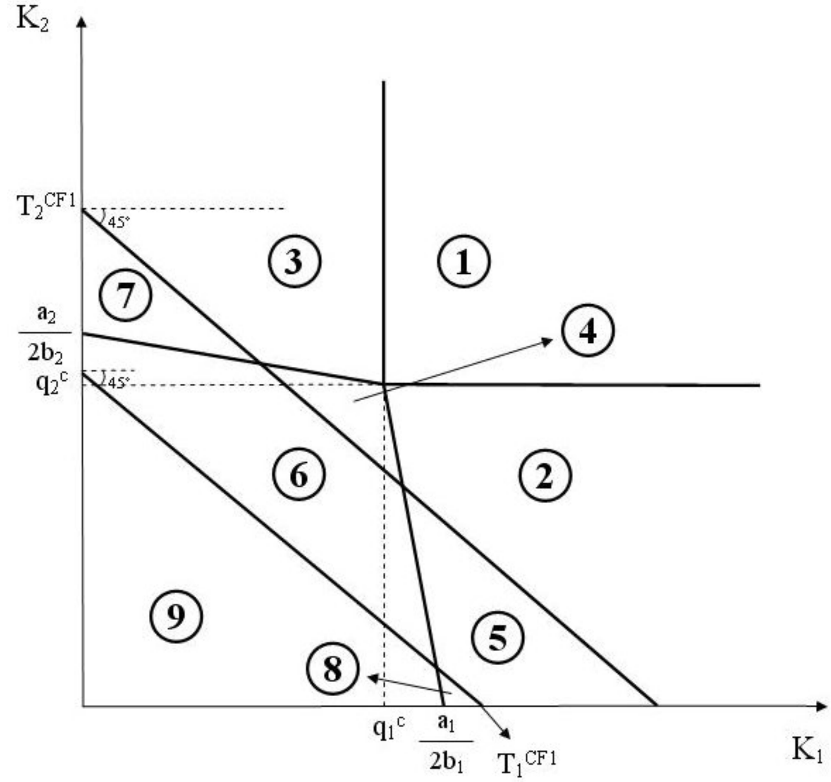

Figure 1: The capacity space and the regions for the after bargaining payoffs

Once we characterize the equilibrium payoffs of the second stage subgames, we apply backward induction to solve for the first stage capacity decisions. Denoting the second stage equilibrium payoffs as functions of the capacity levels, by $\pi_i(K_i, K_{-i})$, $i = 1,\ 2,\ -i \neq i$, the equilibrium capacity levels must satisfy the following.

$$\pi_i(K_i, K_{-i}) - \beta K_i \geq \pi_i(K_i', K_{-i}) - \beta K_i' \quad \forall K_i' \geq 0, \quad i = 1, 2 \quad -i \neq i$$

In the following Proposition we summarize the equilibrium for Case 1 of CF.

Proposition 3.6. *Suppose the following inequality is satisfied.*

$$2a_1 b_2 - a_2(\tilde{b}_1 + \tilde{b}_2) \geq 0 \tag{11}$$

Then the equilibrium under CF will be given by $\left((K_1^{CF1}, K_2^{CF1}) \quad (Q_1^{CF1}, Q_2^{CF1})\right)$*, which is as follows:*

$$
\left((K_1^{CF1}, K_2^{CF1}) \quad (Q_1^{CF1}, Q_2^{CF1})\right) = \begin{cases} \left((L_1^3, L_2^3) \quad (Q_1^{CFU}, Q_2^{CFU})\right), \\ \qquad \textit{if } L_1^3 + L_2^3 \geq T_2^{CF1}; \\ \left((L_1^2, L_2^2) \quad \left(Q_1^{CFC}(L_1^2 + L_2^2), Q_2^{CFC}(L_1^2 + L_2^2)\right)\right), \\ \qquad \textit{else if } L_1^2 + L_2^2 \geq T_1^{CF1}; \\ \left((L_1^1, L_2^1) \quad (0, L_1^1 + L_2^1)\right), \\ \qquad \textit{else if } L_1^1 > 0; \\ \left(\left(0, \frac{a_2 - 2\beta}{2b_2}\right) \quad \left(0, \frac{a_2 - 2\beta}{2b_2}\right)\right), \\ \qquad \textit{otherwise.} \end{cases} \tag{12}
$$

The parameters, denoted by the letter L, stand for the capacity levels satisfying the equilibrium conditions when the corresponding cases occur. For brevity their expressions are given in Appendix.

In (12), the first case occurs when the capacity building price is relatively low, so that in the equilibrium the capacity vector falls in Region 4 of Figure 1, where the firms do not fully utilize their capacities. In other words, firms invest in capacity which will stay idle in the second stage. Osborne and Pitchik [23] find a similar result in their study of cartels. Capacity is costly, but it is worth acquiring, even if not used in production, if it sufficiently improves a firm's bargaining position, which truly summarizes the situation. The second case corresponds to the situation where the capacity building cost takes intermediate values such that the firms will fully utilize their capacities, with the possibility of capacity sharing, but the capacity building cost is low enough such that both firms build positive levels of capacity. And finally the last two cases correspond to the situation where in the equilibrium of the second stage game only product 2, the more profitable product is produced. In the first of these two cases firm 1 builds capacity which will solely be shared with firm 2 to produce product 2 and firm 1 collects the revenue from sharing the capacity only. In this case firm 1 finds it worthwhile to threaten firm 2, so that firm 2 anticipates that move and builds her capacity in such a way that she will utilize the whole capacity to produce her own product in the second stage. On the other hand, the latter case corresponds to the situation where firm 1 has no incentive to build capacity as the expected payoff from the second stage sales together with the revenue from capacity sharing do not cover the costs of building capacity.
One interesting result here is the slopes of the best response functions in region 4 of Figure 1, where one of the best response lines will have a positive slope, whereas in all the other regions the best response lines are downward sloping. A positive sloped best response line means that the firm will have more incentive to increase her capacity if the other firm increases his capacity. Furthermore, in this region, some of the capacity sit idle in the second stage, therefore the firm will be willing to build more in idle capacity. Again, it is solely caused by the relatively

inexpensive capacity building cost.

Inequality (11) is the condition such that if it is not satisfied, then firm 1 will not produce its own product. This will be more clear in the following proposition where the equilibrium for the cases that violate (11) is given.

Proposition 3.7. *If inequality (11) is not satisfied, then the equilibrium capacity levels and quantities of the CF game will be given by* $\left((K_1^{CF2}, K_2^{CF2}) \quad (Q_1^{CF2}, Q_2^{CF2})\right)$*, which is as follows:*

$$\left((K_1^{CF2}, K_2^{CF2}) \quad (Q_1^{CF2}, Q_2^{CF2})\right) = \begin{cases} \left((L_1^3, L_2^3) \quad \left(0, \frac{a_2}{2b_2}\right)\right), \\ \qquad \text{if } L_1^3 + L_2^3 \geq \frac{a_2}{2b_2}; \\ \left((L_1^1, L_2^1) \quad (0, L_1^1 + L_2^1)\right), \\ \qquad \text{else if } L_1^1 > 0; \\ \left(\left(0, \frac{a_2-\beta}{2b_2}\right) \quad \left(0, \frac{a_2-\beta}{2b_2}\right)\right), \\ \qquad \text{otherwise.} \end{cases} \tag{13}$$

where L_1^1*,* L_2^1*,* L_1^3 *and* L_2^3 *are defined in Appendix.*

In this case, it is not worthwhile at all for firm 1 to produce his own product, but when the capacity building price is relatively lower he builds capacity but shares all of it with firm 2. Furthermore, it is possible that in the equilibrium firms may build extra capacity that would stay idle in the second stage.

Next, we examine the outcome of the setting where firms can negotiate capacity sharing but cannot make agreements on the quantities of each other's product, which corresponds to the Noncooperative capacity sharing negotiation (NC) setting.

3.2.2 Noncooperative Capacity Sharing Negotiation (NC)

We have already stated that the difference between CF and NC settings lies in the way to determine the maximum attainable payoff. In NC setting, we do not allow the firms to collaboratively decide on the quantities to be supplied to the market, but they are allowed only to negotiate over the capacity to be exchanged, which will be followed by competition in quantities, where the firms are constrained with their new capacity levels. Therefore, the maximum attainable payoff is the highest payoff the firms can reach in total by sharing capacity but playing the quantity competition game with their capacity levels after they decide on the level of capacity share. Hence, in order to find the maximum attainable payoff, we maximize the sum of the competitive payoffs of the firms subject to the constraint that their capacity levels add up to the sum of the capacity levels. In the following claim we present the resulting maximum attainable payoff as a function of the total capacity.

Lemma 3.8. *Define the following quantities*

$$Q_1^{NCR} = \frac{4a_1b_1b_2 - 2a_2b_1\tilde{b}_2 - a_1\tilde{b}_1\tilde{b}_2}{2b_1(4b_1b_2 - \tilde{b}_2(2\tilde{b}_1 + \tilde{b}_2))} \quad Q_2^{NCR} = \frac{2a_2b_1 - a_1(\tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 - \tilde{b}_2(2\tilde{b}_1 + \tilde{b}_2)}$$

$$Q_1^{NCL} = \frac{2a_1b_2 - a_2(\tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 - \tilde{b}_1(2\tilde{b}_2 + \tilde{b}_1)} \quad Q_2^{NCL} = \frac{4a_2b_1b_2 - 2a_1b_2\tilde{b}_1 - a_2\tilde{b}_1\tilde{b}_2}{2b_2(4b_1b_2 - \tilde{b}_1(2\tilde{b}_2 + \tilde{b}_1))}$$

Then the solution to the maximum attainable payoff problem will be given by (Q_1^{NC}, Q_2^{NC})*.*

$$(Q_1^{NC}, Q_2^{NC}) = \begin{cases} \left(0, K^t\right), & \text{if } K^t \leq T_1^{CF1}; \\ \left(Q_1^{CFC}(K^t), Q_2^{CFC}(K^t)\right), & \text{if } T_1^{CF1} \leq K^t < T^{NC}; \\ \left(Q_1^{NCR}, Q_2^{NCR}\right), & \text{if } \Pi^{in1} > \Pi^{in2}; \\ \left(Q_1^{NCL}, Q_2^{NCL}\right), & \text{otherwise.} \end{cases} \tag{14}$$

where

$$T^{NC} = I_{\{\Pi^{in1} > \Pi^{in2}\}} T_1^{NC} + I_{\{\Pi^{in1} \leq \Pi^{in2}\}} T_2^{NC} \tag{15}$$

and $I_{\{.\}}$ *is the indicator function and the other parameters are defined in Appendix.*

Note the similarity of the expression (14) to (**??**), MAP in case 1 of Cartel Formation. Basically, the maximum attainable payoff for relatively lower values of the total capacity turned out to be same for both approaches. There are, however, two differences, firstly there is no Case 2 in NC setting, hence only in the first case of expression (14) only product 2 is produced, and for high values of the total capacity MAP cannot attain the unconstrained maximum under NC setting. These differences are solely caused by the fact that MAP is determined as a result of a competitive process, which prevents the firms to reach full collaboration.

Having obtained the maximum attainable payoff, we can get the after bargaining payoffs of the firms in the second stage, as functions of the capacity levels. The payoffs will be defined again depending on the region the capacity vector belongs to on the capacity space, Figure (1). Then, similar to the approach taken under CF setting, we can obtain the equilibrium under NC setting, summarized in the following claim.

Proposition 3.9. *The equilibrium capacity levels and quantities of the NC game will be given by*
$\left((K_1^{NC}, K_2^{NC}) \quad (Q_1^{NC}, Q_2^{NC})\right)$*, which is as follows:*

$$\left((K_1^{NC}, K_2^{NC}) \quad (Q_1^{NC}, Q_2^{NC})\right) = \begin{cases} \left((L_1^3, L_2^3) \quad (Q_1^{NCR}, Q_2^{NCR})\right), \\ \qquad \textit{if } L_1^3 + L_2^3 \geq T^{NC} \textit{ and if } \Pi^{in1} > \Pi^{in2}; \\ \left((L_1^3, L_2^3) \quad (Q_1^{NCL}, Q_2^{NCL})\right), \\ \qquad \textit{if } L_1^3 + L_2^3 \geq T^{NC} \textit{ and if } \Pi^{in1} \leq \Pi^{in2}; \\ \left((L_1^2, L_2^2) \quad (Q_1^{CFC}(L_1^2 + L_2^2), Q_2^{CFC}(L_1^2 + L_2^2))\right), \\ \qquad \textit{else if } L_1^2 + L_2^2 \geq T_1^{CF1}; \\ \left((L_1^1, L_2^1) \quad (0, L_1^1 + L_2^1)\right), \\ \qquad \textit{if } L_1^1 > 0; \\ \left(\left(0, \frac{a_2-\beta}{2b_2}\right) \quad \left(0, \frac{a_2-\beta}{2b_2}\right)\right), \\ \qquad \textit{otherwise.} \end{cases} \tag{16}$$

Parallel to the similarities between the maximum attainable payoffs, the equilibrium structures are also similar. In fact, the possible equilibrium capacity levels are same, but the definition of the last case differs. Furthermore, the equilibrium quantities are same when the equilibrium total capacity is relatively lower. Therefore, almost all of the qualitative results for the CF holds for NC as well. The only difference is that, when the capacity building price is relatively low, in CF setting, the firms fully exploit the opportunity to take monopoly behavior, whereas in the NC setting competition pulls the production levels higher than those of CF.

From the equilibrium capacity levels, payoffs and quantities, the value of the transfer payment can be extracted. Let $d_i(K_i^{eq}, K_{-i}^{eq})$ be the equilibrium payoff of firm i if the firms compete without sharing while the equilibrium capacity vector is (K_1^{eq}, K_2^{eq}). Then let $\Pi(K_1^{eq}, K_2^{eq})$ be the maximum attainable payoff of the second stage in the equilibrium, and $\pi_i^s(K_i^{eq}, K_{-i}^{eq})$ be the sales profit of firm i. Then the transfer payment t, can be calculated as follows:

$$t = \pi_1^{op}(K_1^{eq}, K_2^{eq}) - \frac{\Pi(K_1^{eq}, K_2^{eq}) + d_1(K_1^{eq}, K_2^{eq}) - d_2(K_1^{eq}, K_2^{eq})}{2}$$

This simplified model can give us insights about what to expect from the analysis of a more complicated setting, which would better resemble a real-life situation. These insights can be summarized as follows:

- We tackled the problem of capacity sharing in a duopoly with two different approaches. In CF we let the firms behave as if they formed a Cartel and set the quantities collaboratively, whereas under NC we did not. However, in the equilibrium we observed same outcome for most of the cases. The reason is that, even if the firms are only allowed to negotiate over the capacity share, they can anticipate the second stage market competition behavior of each other and they try to make full use of the opportunity to collaborate and get closer to monopoly behavior.

- If the cost of capacity is relatively low, then the firms build excess capacity in the equilibrium, which stays idle in the production stage.
- The sum of the profits of the firms when they can share capacity will be closer to the Fully Integrated Firm profit, than to the Pure Competition.
- Transfer payment for the shared capacity can be obtained from the bargaining process without directly including it in the analysis.

4 Summary and Possible Extensions

Capacity sharing is a widely applied practice in various industries. This may happen in the form of establishing a joint venture or sharing the currently established production or service capacity. Although there has been extensive research on capacity competition, most of the work done is limited to homogenous-product single period models, in which no capacity sharing is considered. On the other hand, another stream of research focuses on firms' optimal capacity adjustment policies, not necessarily considering a competitive setting. Therefore, we believe that, capacity sharing between competitors is a fruitful research field.

In a two-stage quantity competition model of capacity sharing we have seen that the firms will get close to the monopoly behavior by anticipating the possibility to share capacity. Here, as we employed deterministic demand functions, the motive to share capacity mainly depends on the imbalance in the demand functions, price sensitivities and cost differences, rather than variability in demand. We furthermore saw that for most of the values of the parameter set, even if we do not allow the firms to set the quantities to be supplied to the market collaboratively, by anticipating the capacity sharing possibility, the firms end up at the same equilibrium point. Another interesting result was the incentive to build higher capacity even if it will not be needed in the production stage. This mainly stems from the intention of increasing the bargaining power when the capacity building cost is relatively low. We were also able to obtain the equilibrium price of the capacity shared as a result of the negotiation process.

We have studied a one period two-stage model with deterministic and linear demand functions, linear production and capacity building costs, in order to get insights about what drives the firms to share their capacities with their competitors and how the other parameters in the market affect their roles in this relationship. In the business world, however, the real situations are generally more complex. The following extensions on capacity sharing could be rewarding directions to follow:

- The immediate step in the progress of this research will be the study of the single period two stage model with price competition. We already developed the model and obtained some preliminary results, which are similar to the quantity competition model. Again using similar techniques, we obtained the equilibrium structure for special parameter sets. Similar results, such as capacity idling will be observed in price competition version as well.

- Uncertainty plays an important role in business decision making processes. In order to observe how incorporating uncertainty will influence the firms' behavior when they can share capacity, we first plan to extend the results for the single period models we considered above to the settings with stochastic demand functions. Especially, the model where the strategic variable of the second stage is price, may turn out to have interesting insights. We devoted preliminary work on this topic as well, where we study the extension of the quantity setting problem analyzed above to a context where the demand parameters are uncertain at the stage of building capacities, whereas the uncertainty resolves before the firms make their quantity decisions.

- Capacity has a long term strategic importance, and in general its adjustment is costly and time consuming. In fact we claimed that these properties of capacity investment make firms tend to collaborate to provide better service to their customers. A one-shot model cannot capture this strategic importance of capacity. In an ideal infinite horizon model, where the movement of demand from period to period is governed by a Markovian process, firms will be able to adjust their capacity levels less frequently than they do pricing or quantity setting decisions. In such a model, one can study the effects of the changes in market structure to the firms' investment and sharing decisions. Due to the complexity, such a model may lead to analytical intractability, hence it may be wise to first assume simple state transition processes.

- For all the models we have proposed up to now, in both preliminary work and future plan, we assumed symmetric complete information between the players of the game. However, sometimes the firms may have private information about their cost structures, which affects their equilibrium behavior. In fact, in oligopoly literature, the textbook examples studying limit pricing, where firms possibly have excess capacity to deter entry, consider this kind of information asymmetry. We plan to devote research on this subject to see the influence of information asymmetry on the firm's equilibrium behavior.

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A Appendix

Proof of Proposition 3.1 :. If we take the Hessian of the objective function in (1) we get the following:

$$\begin{bmatrix} -2b_1 & -(\tilde{b}_1+\tilde{b}_2) \\ -(\tilde{b}_1+\tilde{b}_2) & -2b_2 \end{bmatrix}$$

which is negative semi definite by the diagonal dominance assumptions, hence solving for the FOC's will give us the unique maximizer, ignoring the nonnegativity constraints. When we solve the first order conditions we get the following:

$$q_1 = \frac{2a_1b_2 - a_2(\tilde{b}_1+\tilde{b}_2) - \beta(2b_2 - \tilde{b}_1 - \tilde{b}_2)}{4b_1b_2 - (\tilde{b}_1+\tilde{b}_2)^2} \qquad q_2 = \frac{2a_2b_1 - a_1(\tilde{b}_1+\tilde{b}_2) - \beta(2b_1 - \tilde{b}_1 - \tilde{b}_2)}{4b_1b_2 - (\tilde{b}_1+\tilde{b}_2)^2}$$

Note that, by the assumption that product 2 is more profitable, q_2 as defined above is always nonnegative, but q_1 may turn out to be negative, in which case, by concavity of the objective function, the optimal action for FIF will be producing the monopoly quantity of product 2, and not producing product 1 at all, as given in (2). □

Proof of Proposition 3.2*:* The natural way to solve a two-stage game is backward induction, in which we would first solve for the second stage equilibrium, taking the first stage's actions as given, and then using the result of the second stage we would solve for the equilibrium actions of the first stage. Therefore, first, for a given capacity vector (K_1, K_2) we solve for the equilibrium of the second stage game, which, after ordinary best response function calculations, turn out to have equilibrium payoffs as functions of K_1 and K_2 as follows:

$$(\pi_1^{PC}, \pi_2^{PC}) = \begin{cases} \left(b_1\left(\frac{2a_1b_2 - a_2\tilde{b}_2}{4b_1b_2 - \tilde{b}_1\tilde{b}_2}\right)^2, \quad b_2\left(\frac{2a_2b_1 - a_1\tilde{b}_1}{4b_1b_2 - \tilde{b}_1\tilde{b}_2}\right)^2\right), & \text{if } K_1 \geq q_1^c \text{ and } K_2 \geq q_2^c; \\ \left(b_1\left(\frac{a_1 - \tilde{b}_2K_2}{2b_1}\right)^2, \quad K_2\left(a_2 - b_2K_2 - \tilde{b}_1\left(\frac{a_1 - \tilde{b}_2K_2}{2b_1}\right)\right)\right), & \text{if } K_1 \geq \frac{a_1 - \tilde{b}_2K_2}{2b_1} \text{ and } K_2 < q_2^c; \\ \left(K_1\left(a_1 - b_1K_1 - \tilde{b}_2\left(\frac{a_2 - \tilde{b}_1K_1}{2b_2}\right)\right), \quad b_2\left(\frac{a_2 - \tilde{b}_1K_1}{2b_2}\right)^2\right), & \text{if } K_1 < q_1^c \text{ and } K_2 \geq \frac{a_2 - \tilde{b}_1K_1}{2b_2}; \\ \left(K_1\left(a_1 - b_1K_1 - \tilde{b}_2K_2\right), \quad K_2\left(a_2 - b_2K_2 - \tilde{b}_1K_1\right)\right), & \text{otherwise.} \end{cases} \tag{17}$$

The cases in expression 17, are demonstrated on Figure 2, where the cases correspond to regions. Here, first case corresponds to the situation where both firms have enough capacity so that they produce the pure Cournot quantities, whereas in the second case firm 2 does not have

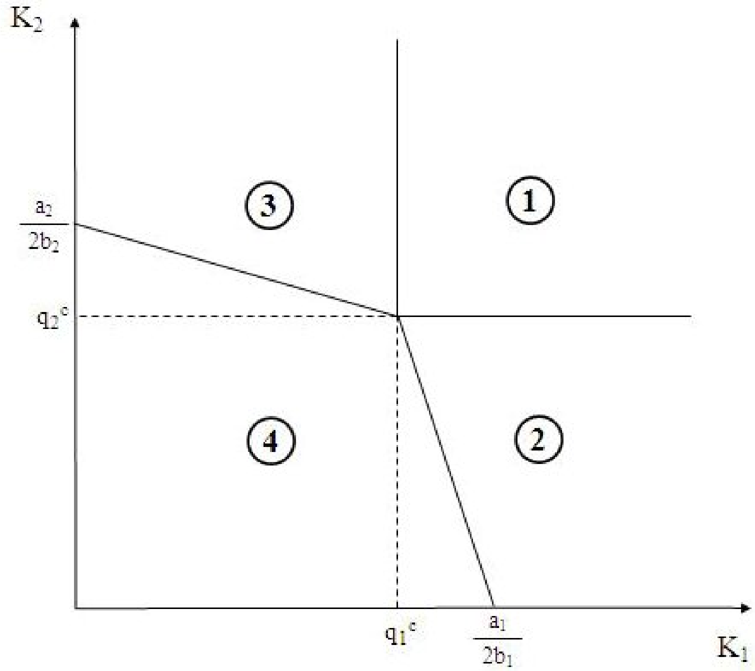

Figure 2: The capacity space and the regions for the payoffs in PC setting, also the Disagreement Payoffs for the Bargaining Games

sufficient capacity to produce the Cournot equilibrium quantity and hence she produces up to her capacity. The third case corresponds to the situation where only firm 1 is constrained and in the last case both firms are constrained and hence produce up to their capacities. Now, using this result we can write the payoff of firm 1 in the first stage as a function of the capacity levels as follows:

$$\Pi_1^{PC} = \begin{cases} K_1\left(a_1 - b_1K_1 - \tilde{b}_2K_2\right) - \beta_1 K_1, & \text{if } K_1 \leq \frac{a_1-\tilde{b}_2K_2}{2b_1}; \\ b_1\left(\frac{a_1-\tilde{b}_2K_2}{2b_1}\right)^2 - \beta_1K_1, & \text{otherwise.} \end{cases}$$

for $K_2 \leq q_2^c$, and

$$\Pi_1^{PC} = \begin{cases} K_1\left(a_1 - b_1K_1 - \tilde{b}_2K_2\right) - \beta_1 K_1, & \text{if } K_1 \leq \frac{a_2-2b_2K_2}{\tilde{b}_1}; \\ K_1\left(a_1 - b_1K_1 - \tilde{b}_2\left(\frac{a_2-\tilde{b}_1K_1}{2b_2}\right)\right) - \beta_1K_1, & \text{if } K_1 \leq q_1^c; \\ b_1\left(\frac{2a_1b_2-a_2\tilde{b}_2}{4b_1b_2-\tilde{b}_1\tilde{b}_2}\right)^2 - \beta_1K_1, & \text{otherwise.} \end{cases}$$

for $K_2 > q_2^c$. Firm 2's payoff function is symmetric to the expressions above. These payoff functions can be shown to be continuous and concave, and hence the first order conditions will give the unique maximizers, the best response functions of the first stage. After some algebra we see that the unique intersection point of the best response functions turns out to be

$$K_1 = \frac{2a_1b_2 - a_2\tilde{b}_2 - \beta(b_2 - \tilde{b}_2)}{4b_1b_2 - \tilde{b}_1\tilde{b}_2} \quad K_2 = \frac{2a_2b_1 - a_1\tilde{b}_1 - \beta(b_1 - \tilde{b}_1)}{4b_1b_2 - \tilde{b}_1\tilde{b}_2}$$

which is the equilibrium point of a quantity competition game , with inverse demand functions defined as in (3). Note that, while K_2, as defined above, is always nonnegative, K_1 may turn out to be negative, which means that in the equilibrium firm 1 does not have enough incentive to engage in the competition and firm 2 will behave as if she is a monopoly in the market. □

Proof of Lemma 3.3*:* We can write the maximization problem to find the equilibrium as follows:

$$\max_{u_1 \geq 0, u_2 \geq 0} u_1 u_2$$

$$\text{s.t.} \quad u_1 + u_2 \leq U^*$$

Note that we converted the constraint enforcing the utility pair to be selected from the feasible set, to nonnegativity constraints and another constraint that restricts the higher total value. At the point where this program's solution occurs, the KKT conditions must hold, hence there must exist $\mu \geq 0$ such that

$$u_2 - \mu = 0$$

$$u_1 - \mu = 0$$

$$(u_1 + u_2 - \mathcal{U}^*)\mu = 0$$

Hence, at the solution, either $\mu = 0$ and the constraint is satisfied with a slack, or $u_1 = u_2 = \mu \geq 0$ and the constraint is satisfied as an equation. In the first case, both variables will take 0, which is the lowest objective function value possible in the feasible set. Therefore, the solution must be at the other point, where $u_1 = u_2 = \frac{\mathcal{U}^*}{2}$. □

Proof of Lemma 3.4*:* The objective function in (6) has the same Hessian matrix as the objective function in the FIF's problem, which was shown to be negative semidefinite, establishing the concavity of the objective function. First, we will find the unconstrained maximizer of the objective function. The FOCs of the unconstrained problem are given by

$$a_1 - 2b_1y_1 - (\tilde{b}_1 + \tilde{b}_2)y_2 = 0 \quad a_2 - 2b_2y_2 - (\tilde{b}_1 + \tilde{b}_2)y_1 = 0$$

whose solution is $(y_1, y_2) = (Q_1^{CFU}, Q_2^{CFU})$. If this unconstrained solution satisfies the constraints, then it will also be the solution of the constrained problem, by concavity of the objective function. As $a_2 \geq a_1$ and by the diagonal dominance assumption, we know that

$Q_2^{CFU} \geq 0$. Furthermore, the condition (7) directly implies that $Q_1^{CFU} \geq 0$ as well. Then if $Q_1^{CFU} + Q_2^{CFU} \leq K^t$, which corresponds to the following condition:

$$K^t \geq \frac{2a_1b_2 + 2a_2b_1 - (a_1 + a_2)(\tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2} = T_2^{CF1} \tag{18}$$

then the unconstrained solution will be feasible and hence optimal for the constrained problem, establishing the last case in expression (8). On the other hand, if the unconstrained solution violates the constraint, which is equivalent to violation of (18), then the unconstrained solution will be infeasible due to insufficient capacity. Hence, as the objective function is concave, at optimality the capacity constraint must be binding. Then, using this fact, we can replace y_2 by $K^t - y_1$ and rewrite the problem in one dimension, whose unique solution will be ($Q_1^{IBC}(K^t)$, $Q_2^{IBC}(K^t)$), establishing the second case. Note that $Q_1^{IBC}(K^t)$ may take negative value, in which case we say that it is not worthwhile to produce product 1 with the current level of capacity. Hence, if it takes a negative value, the problem will reduce to the following problem:

$$\max_{0 \leq y_2 \leq K^t} y_2(a_2 - b_2y_2)$$

The problem above is a simple problem of the capacitated monopolist. As the objective function is concave, the solution is to produce up to capacity if the capacity level is less than the optimal monopoly quantity, and to produce optimal monopoly quantity otherwise. By (11) and the initial assumptions we know that if $Q_1^{IBC}(K^t)$ takes a negative value, then the capacity is smaller than the optimal monopoly quantity, hence the solution will be producing product 2 using all the capacity, establishing the first case, and completing the proof. □

Proof of Lemma 3.5*:* As stated in the proof of Proposition 3.4, when (7) is not satisfied $Q_1^{CFU} < 0$, i.e. the product 1 component of the unconstrained solution takes a negative value, meaning that product 1 will not be produced in the optimal solution. Then the problem reduces to the problem of a capacitated monopolist, producing only product 2. The solution is also discussed above, which is to produce up to capacity if the capacity is lower then the monopoly quantity and otherwise to produce monopoly quantity. □

Proof of Proposition 12*:* Here, we will sketch the main steps of the derivation of equilibrium, and skip the details as they are repetitive and give no further insight. We know that if the given inequality is satisfied, then we can obtain the expressions for the after bargaining payoffs of the firms from Figure 1, where the payoffs will depend the region they belong to on this Figure. Therefore, the payoff of a firm as a function of its own capacity level for a given value of the competitor's capacity level will be a piecewise defined function, whose break points will be at the boundaries between the regions on Figure 1. It can be shown that each piece in the overall payoff function, i.e. the payoff corresponding to each region, is concave with respect to the corresponding decision variable. It can further be shown that the overall payoff functions will be continuous on the breakpoints, and bounded, this, together with the piecewise concavity property, implies that for each value of the competitor's capacity level, there exists a well defined best response capacity level. As we map these best response capacity levels as functions on the capacity space we see that they are downward sloping everywhere, except in region 4 (which does not interfere our results). Furthermore, the slopes of the best response lines will be greater than or equal to -1. This, together with the bounds on the unit capacity building cost implies that the best response functions intersect in one of the regions 4, 6 or 9 on Figure 1. If we further inspect the best response functions, we see that the slopes will increase when we go from region 9 to 6 and from 6 to 4. This leads to expression 12, which exhibits the case based structure of the equilibrium, where the L parameters are the coordinates of the intersection points of the best response lines.

□

Proof of Proposition 3.7*:* Note that, Figure 1 applies when inequality (7) is satisfied. When it is not satisfied, however, the regions on the capacity space determining the after bargaining payoffs of the firms will look like as in Figure 3.

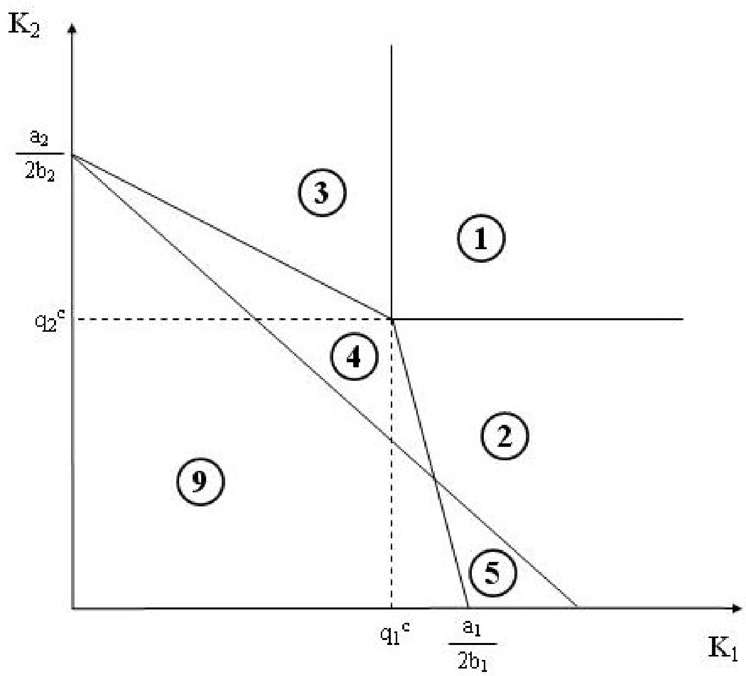

Figure 3: The capacity space and the regions for the after bargaining payoffs

Here, unlike Figure 1, the graph shows exact number of regions, which can be shown using the inequality (7) and the initial assumptions. Again, after straightforward payoff calculations we can get the best response functions of the firms, which turns out to be continuous over the boundary line $K_1 + K_2 = \frac{a_2}{2b_2}$, which together with the initial assumptions, assure the intersection of the best response lines. Case based representation of this intersection point turns out to be the expression in (13). □

Proof of Lemma 3.8*:* Again we will go over the main steps of the derivation, instead of the full proof. For a given capacity vector, we search for the highest total competitive payoff (same as the disagreement payoffs) of the firms on the line $K_1+K_2 = K^t$, where K^t is the total capacity. Note that depending on the regions this line passes on Figure 2, the expression for the total payoff changes. First we will state the following Lemma, as a first step:

Lemma A.1. *There exists a threshold, such that when the total capacity is below that value after the negotiation process, the firms will produce only firm 2's product, and when it is higher both firms' products will be produced. This threshold value is equal to T_1^{CF1}, the analogous threshold in the CF problem.*

To derive this Lemma one can write the total competitive payoff for very low values of total capacity, which will be the sum of the payoffs when both firms produce up to capacity, and can be expressed in one variable. This function will be concave and hence have a unique maximizer. It can further be shown that for total capacity levels lower than T_1^{CF1} the resulting maximum total payoff will be higher than the total payoffs in the other regions, through which it is possible that the line $K_1 + K_2 = K^t$ passes. Then, the threshold value will be obtained from the condition that the first coordinate of the maximizer is nonnegative. Next, we can state another Lemma which establishes the solution for further values of the total capacity.

Lemma A.2. *Depending on the parameter set, there exists another threshold such that for the values below this threshold, but above T_1^{CF1}, the solution is given by ($Q_1^{CFC}(K^t)$, $Q_2^{CFC}(K^t)$), constrained solution for the CF problem. This threshold level is given by T^{NC} as defined in (*15*).*

The derivation of this result is also straightforward. For the values of K^t greater than T_1^{CF1}, the total payoff in Region 4 of Figure 2 is maximized at ($Q_1^{CFC}(K^t)$, $Q_2^{CFC}(K^t)$), which will result in a total payoff higher than the total payoff in other possible regions, until the total capacity hits the threshold value defined above. When the total capacity level is higher than T^{NC} the highest total payoff will be the maximum payoff in one of the regions 2 or 3, depending on the parameter values. Furthermore the total payoff will not depend on the total capacity in these regions. The resulting expression for the MAP is stated in (14). □

Proof of Proposition 3.9*:* The disagreement payoffs are the same for both approaches, CF and NC. Furthermore, in Proposition 3.8 we have shown that in the maximum attainable payoff calculations, for values of K^t lower than T_2^{CF1} and T^{NC} the resulting MAP's are essentially same so are the thresholds used to define the cases. For higher values of total capacity, however, the expressions for MAP become different. Nevertheless, as the payoff above these thresholds do not depend on the capacity levels, they will simply be ignored in the best response function calculations, therefore the best response function will also be same between CF and NC. Therefore, we obtain expression (16) in a same fashion as we obtain the equilibrium capacity levels and quantities for CF. □

Defined Parameters

$$q_1^c = \frac{2a_1b_2 - a_2\tilde{b}_2}{4b_1b_2 - \tilde{b}_1\tilde{b}_2} \quad q_2^c = \frac{2a_2b_1 - a_1\tilde{b}_1}{4b_1b_2 - \tilde{b}_1\tilde{b}_2}$$

$$T_1^{CF1} = \frac{a_2 - a_1}{2b_2 - \tilde{b}_1 - \tilde{b}_2} \quad T_2^{CF1} = \frac{2a_1b_2 + 2a_2b_1 - (a_1 + a_2)(\tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2}$$

$$Q_1^{CFU} = \frac{2a_1b_2 - a_2(\tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2} \quad Q_2^{CFU} = \frac{2a_2b_1 - a_1(\tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2}$$

$$Q_1^{CFC}(K^t) = \frac{(2b_2 - \tilde{b}_1 - \tilde{b}_2)K^t + a_1 - a_2}{2(b_1 + b_2 - \tilde{b}_1 - \tilde{b}_2)} \quad Q_2^{CFC}(K^t) = \frac{(2b_1 - \tilde{b}_1 - \tilde{b}_2)K^t - a_1 + a_2}{2(b_1 + b_2 - \tilde{b}_1 - \tilde{b}_2)}$$

$$Q_1^{NCR} = \frac{4a_1b_1b_2 - 2a_2b_1\tilde{b}_2 - a_1\tilde{b}_1\tilde{b}_2}{2b_1(4b_1b_2 - \tilde{b}_2(2\tilde{b}_1 + \tilde{b}_2))} \quad Q_2^{NCR} = \frac{2a_2b_1 - a_1(\tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 - \tilde{b}_2(2\tilde{b}_1 + \tilde{b}_2)}$$

$$Q_1^{NCL} = \frac{2a_1b_2 - a_2(\tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 - \tilde{b}_1(2\tilde{b}_2 + \tilde{b}_1)} \quad Q_2^{NCL} = \frac{4a_2b_1b_2 - 2a_1b_2\tilde{b}_1 - a_2\tilde{b}_1\tilde{b}_2}{2b_2(4b_1b_2 - \tilde{b}_1(2\tilde{b}_2 + \tilde{b}_1))}$$

$$L_1^1 = \frac{4(a_1 - \beta)b_2 - 2(a_2 - \beta)(\tilde{b}_2 - \tilde{b}_1)}{4b_2(2b_1 + b_2) + (\tilde{b}_1 - \tilde{b}_2)^2} \quad L_2^1 = \frac{4(a_2 - \beta)(b_1 + b_2) - (a_1 + a_2 - 2\beta)(2b_2 + \tilde{b}_1 - \tilde{b}_2)}{4b_2(2b_1 + b_2) + (\tilde{b}_1 - \tilde{b}_2)^2}$$

$$L_1^2 = \frac{2(a_1 - \beta)b_2 - (a_2 - \beta)(\tilde{b}_2 - \tilde{b}_1) - \beta(2b_2 - \tilde{b}_2 + \tilde{b}_1) + 2\hat{C}(a_2 - a_1) + \hat{B}(2b_2 - \tilde{b}_2 + \tilde{b}_1)}{4b_1b_2 + (\tilde{b}_1 - \tilde{b}_2)^2 - 4\hat{C}(b_1 + b_2)}$$

$$L_2^2 = \frac{2(a_2 - \beta)b_1 - (a_1 - \beta)(\tilde{b}_1 - \tilde{b}_2) - \beta(2b_1 - \tilde{b}_1 + \tilde{b}_2) + 2\hat{C}(a_1 - a_2) + \hat{B}(2b_1 - \tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 + (\tilde{b}_1 - \tilde{b}_2)^2 - 4\hat{C}(b_1 + b_2)}$$

$$L_1^3 = \frac{2a_1b_2 - (a_2 - \beta)(\tilde{b}_2 - \tilde{b}_1) - \beta(4b_2 - \tilde{b}_1 + \tilde{b}_2)}{4b_1b_2 + (\tilde{b}_1 - \tilde{b}_2)^2}$$

$$L_2^3 = \frac{2a_2b_1 - (a_1-\beta)(\tilde{b}_1 - \tilde{b}_2) - \beta(4b_1 + \tilde{b}_1 - \tilde{b}_2)}{4b_1b_2 + (\tilde{b}_1 - \tilde{b}_2)^2}$$

$$\hat{A} := \frac{(a_1 - a_2)^2}{4(b_1 + b_2 - \tilde{b}_1 - \tilde{b}_2)}$$

$$\hat{B} := \frac{a_1(2b_2 - \tilde{b}_1 - \tilde{b}_2) + a_2(2b_1 - \tilde{b}_1 - \tilde{b}_2)}{2(b_1 + b_2 - \tilde{b}_1 - \tilde{b}_2)}$$

$$\hat{C} := \frac{-4b_1b_2 + (\tilde{b}_1 + \tilde{b}_2)^2}{4(b_1 + b_2 - \tilde{b}_1 - \tilde{b}_2)}$$

$$\begin{aligned} T_1^{NC1} = \quad & \frac{b_1\left(a_1(2b_2 - \tilde{b}_1 - \tilde{b}_2) + a_2(2b_1 - \tilde{b}_1 - \tilde{b}_2)\right)\left(4b_1b_2 - \tilde{b}_2(2\tilde{b}_1 + \tilde{b}_2)\right)}{b_1\left(4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2\right)\left(4b_1b_2 - \tilde{b}_2(2\tilde{b}_1 + \tilde{b}_2)\right)} \\ + \quad & \frac{\sqrt{(2a_2b_1 - a_1(\tilde{b}_1 + \tilde{b}_2))^2(4b_1b_2 - \tilde{b}_2(2\tilde{b}_1 + \tilde{b}_2))(b_1 + b_2 - \tilde{b}_1 - \tilde{b}_2)b_1(\tilde{b}_1)^2}}{b_1\left(4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2\right)\left(4b_1b_2 - \tilde{b}_2(2\tilde{b}_1 + \tilde{b}_2)\right)} \end{aligned}$$

$$\begin{aligned} T_2^{NC1} = \quad & \frac{b_2\left(a_2(2b_1 - \tilde{b}_1 - \tilde{b}_2) + a_1(2b_2 - \tilde{b}_1 - \tilde{b}_2)\right)\left(4b_1b_2 - \tilde{b}_1(2\tilde{b}_2 + \tilde{b}_1)\right)}{b_2\left(4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2\right)\left(4b_1b_2 - \tilde{b}_1(2\tilde{b}_2 + \tilde{b}_1)\right)} \\ + \quad & \frac{\sqrt{(2a_1b_2 - a_2(\tilde{b}_1 + \tilde{b}_2))^2(4b_1b_2 - \tilde{b}_1(2\tilde{b}_2 + \tilde{b}_1))(b_1 + b_2 - \tilde{b}_1 - \tilde{b}_2)b_2(\tilde{b}_2)^2}}{b_2\left(4b_1b_2 - (\tilde{b}_1 + \tilde{b}_2)^2\right)\left(4b_1b_2 - \tilde{b}_1(2\tilde{b}_2 + \tilde{b}_1)\right)} \end{aligned}$$

$$\Pi^{in1} = \frac{(a_1)^2(4b_1b_2 + (\tilde{b}_1)^2) - 4a_1a_2b_1(\tilde{b}_1 + \tilde{b}_2) + (2a_2b_1)^2}{4b_1(4b_1b_2 - 2\tilde{b}_1\tilde{b}_2 - (\tilde{b}_2)^2)}$$

$$\Pi^{in2} = \frac{(a_2)^2(4b_1b_2 + (\tilde{b}_2)^2) - 4a_1a_2b_2(\tilde{b}_1 + \tilde{b}_2) + (2a_1b_2)^2}{4b_2(4b_1b_2 - 2\tilde{b}_1\tilde{b}_2 - (\tilde{b}_1)^2)}$$

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