Icarus **154**, 449–458 (2001)
doi:10.1006/icar.2001.6692, available online at http://www.idealibrary.com on IDEAL®

The Effect of a Planet in the Asteroid Belt on the Orbital Stability of the Terrestrial Planets

Jack J. Lissauer

Space Sciences Division 245-3, NASA Ames Research Center, Moffett Field, California 94035
E-mail: jlissauer@ringside.arc.nasa.gov

Elisa V. Quintana

Space Sciences Division 245-3, NASA Ames Research Center, Moffett Field, California 94035 and Department of Physics, University of Michigan, Ann Arbor, Michigan 48109

Eugenio J. Rivera

Space Sciences Division 245-3, NASA Ames Research Center, Moffett Field, California 94035 and Department of Astronomy, State University of New York, Stony Brook, New York 10794

and

Martin J. Duncan

Department of Physics, Queen's University, Kingston, Ontario K7L 3N6, Canada

Received October 20, 2000; revised May 14, 2001

If a planetary-mass body were present in the asteroid belt, the orbits of the terrestrial planets and those of the giant planets would be more closely coupled. A greater exchange of angular momentum could affect the stability of the terrestrial planets. Moreover, the planet in the asteroid belt could itself excite terrestrial planet eccentricities. To study these effects, we have simulated several systems consisting of the Solar System planets and a 0.1–10 Earth mass ($M_\oplus$) object on the orbit of a main belt asteroid or on an initially circular orbit at Ceres's semimajor axis. An integration with Ceres at 5 $M_\oplus$ remained stable for a billion years. Ceres at 10 $M_\oplus$, however, caused the system to become unstable at $\sim$25–50 myr. When additional mass was given to both Ceres (bringing it up to 2 $M_\oplus$) and Mars (1 $M_\oplus$), the systems self-destructed in <100 myr. However, if these bodies were grown at an epoch when Mars' eccentricity was small, the resulting system was stable for the entire 500 myr simulated. Analogous results were found when other asteroids were grown to planetary masses. Thus, an "asteroid gap" is not required for a planetary system similar to our own to be stable on geological timescales. © 2001 Elsevier Science (USA)

Key Words: **Solar System dynamics; extrasolar planets.**

1. INTRODUCTION

According to current models, numerous planetary embryos were present during the late stages of terrestrial planet formation, and many of these embryos were on mutually crossing orbits (Chambers and Wetherill 1998, 2001; Agnor *et al.* 1999). Systems such as these, in which bodies experience numerous close approaches to one another, are highly chaotic and extremely sensitive to both small changes in initial conditions and minor perturbations. Indeed, simply displacing one (of over 100) embryo along its orbit by one meter can make the difference between forming three terrestrial planets or five (Chambers *et al.* 2002, in preparation). Even though most simulations are designed to produce configurations similar to those observed in our Solar System, a wide variety of planetary systems result. About one-third of the simulations performed by Chambers and Wetherill (1998, 2001) which started with embryos in the 2–4 AU region had a surviving planet in the asteroid belt when the simulations ended. The strong perturbations of Jupiter prevented substantial growth of this surviving asteroidal planet. However, $\sim$1 $M_\oplus$ embryos could have grown by rapid runaway accretion within the asteroidal region of a smoothly varying disk of material that contained enough material to form a 10–15 $M_\oplus$ jovian core at 5.2 AU (Lissauer 1987). Alternatively, a several $M_\oplus$ embryo formed exterior to Jupiter could be scattered into the asteroidal region (Thommes 2000). Thus, planet formation models suggest that some planetary systems, which otherwise closely resemble our Solar System, may have a planet where we have a nearly empty "gap" known as the asteroid belt. If a massive body orbited between Mars and Jupiter, it could increase the

0019-1035/01 $35.00

dynamical coupling between the terrestrial planets and the giant planets. These interactions might cause the terrestrial planets' orbits to cross, leading to catastrophic consequences.

The potential destabilizing effect of a planet within the asteroid belt can be seen by comparing the angular momentum deficit of the terrestrial planets to that of the giant planets. The angular momentum deficit (AMD), L_d, of a planet is the difference between its actual orbital angular momentum and the angular momentum that it would have on a circular, uninclined orbit with the same semimajor axis. The total AMD of a system of planets is given by

$$L_d = \sum_k \frac{m_k M_\odot}{m_k + M_\odot} \sqrt{G(m_k + M_\odot) a_k} \left(1 - \cos i_k \sqrt{1 - e_k^2}\right). \tag{1}$$

Here, m_k is the mass of the planet, $M_\odot$ is the Sun's mass, G is the gravitational constant, a_k is the semimajor axis of the planet, e_k is the eccentricity, and i_k is the inclination relative to the invariable plane of the system. Because L_d is proportional to the mass of a planet and the square root of its semimajor axis, the combined AMD of the jovian planets is about one thousand times as large as the combined AMD of the terrestrial planets. By numerically integrating the secular equations of planetary motion, Laskar (1997) found a significant exchange of AMD among the inner planets and among the giant planets, but very little exchange in AMD between the inner and outer planets. In Laskar's calculations, which only consider secular perturbations, the total AMD of the terrestrial planets ranges from 6×10^{44} to 10×10^{44} g cm^2/s on a 10^9-year timescale. Full integrations show variations from 5×10^{44} to 12×10^{44} g cm^2/s on 10^6–10^8-year timescales (Rivera 2001). The minimum AMD required for the orbits of a pair of terrestrial planets to cross is 20.3×10^{44} g cm^2/s, which is sufficient for Earth ($e = 0.030$) and Mars ($e = 0.324$) to intersect at 1.030 AU. Venus ($e = 0.090$) and Mercury ($e = 0.700$) can intersect at 0.658 AU with $L_d = 33.7 \times 10^{44}$ g cm^2/s. Terrestrial planets can come within one mutual Hill sphere of one another (at which point mutual perturbations can alter semimajor axes and increase L_d) with somewhat smaller AMD; $L_d = 19.2 \times 10^{44}$ g cm^2/s is sufficient for Earth and Mars to have such a close approach, and $L_d = 32.5 \times 10^{44}$ g cm^2/s is adequate for Venus and Mercury.

A planet orbiting between Mars and Jupiter could more closely couple the dynamics of the terrestrial and giant planets, allowing a greater exchange in AMD between the two systems. This could subsequently affect the stability of the inner planets. If the extra planet began on an orbit with significant eccentricity and/or inclination, its AMD could also destabilize the system by being transferred to the orbits of the terrestrial planets.

Can a planetary system similar to our own be stable if a planet-sized body exists in the asteroid belt? The answer to this question has implications for the diversity of expected planetary systems and the likely abundance of habitable planets. Dynamical models for the origin of large near-Earth asteroids (Migliorini *et al.* 1998) imply that a large asteroid placed at the "wrong" location would be perturbed into a Mars-crossing orbit by giant planets and destabilize our terrestrial planet system. We wish to know whether or not there is also a substantial range of "right" parameters allowing a planetary system similar to the Solar System, but lacking a "gap" between the terrestrial and giant planets, to be stable for geological times.

To study the above-mentioned effects, we have performed multiple simulations by numerically integrating the orbits of the major Solar System planets along with a planet which begins on an asteroid's orbit between Mars and Jupiter. Our intent is to analyze the stability of systems which are similar to our Solar System apart from the addition of a planetary mass body on an asteroidal orbit; we do not model planetary formation herein. We discuss the techniques used and the initial conditions in the following section. Results of our integrations are presented in Section 3, and Section 4 contains a discussion of the implications of these results.

2. METHOD

Our study consisted of numerical integrations of numerous synthetic systems comprising the Sun, the terrestrial and giant planets, and a planetary-mass object in the asteroidal region. The orbital elements of one of the four most massive main belt asteroids (1 Ceres, 2 Pallas, 4 Vesta, 10 Hygeia), or asteroid 6115 Martinduncan, were chosen for the extra planet in our simulations. The initial mass and orbital elements of these asteroids are listed in Table I, where ω is the argument of pericenter, Ω is the longitude of the ascending node, and M is the mean anomaly. If

TABLE I
Asteroid Initial Conditions

Asteroid	Mass ($M_\oplus$)	L_d/M (cm^2/s)[a]	a (AU)	e	i (degrees)	ω (degrees)	Ω (degrees)	M (degrees)
1 Ceres	1.78653×10^{-4}	1.55016×10^{18}	2.76618	0.07789	10.58293	73.79917	80.50163	249.60022
2 Pallas	4.18719×10^{-5}	1.55113×10^{19}	2.77236	0.22965	34.84603	310.27260	173.19797	4.80621
4 Vesta	4.85713×10^{-5}	9.21376×10^{17}	2.36076	0.09030	7.13487	149.79980	103.96142	192.91051
10 Hygeia	1.50739×10^{-5}	1.12697×10^{18}	3.13687	0.11966	3.84391	314.61601	283.68850	242.41937
6115 Martinduncan	1.08867×10^{-23}	6.06796×10^{17}	2.22568	0.15489	4.89382	66.37720	316.40076	37.32239

[a] Specific AMD of asteroids are averaged over 10^5 years.

a massive asteroid were suddenly added to the Solar System, the extra mass would alter planetary orbits in a manner very sensitive to the time and longitude at which the body was inserted. To avoid shocking the systems in this manner, each integration was begun by first linearly growing the asteroid from its original mass to a desired final mass of 0.1 to 10 $M_\oplus$ during the first 100,000 years (cf. Duncan and Lissauer 1998). The asteroid then remained at this final mass for the duration of the integration, which ranged from 100 myr to 1 Gyr, or until a planet was either ejected from the system or collided with the Sun (the radius of the Sun was defined to be 0.005 AU).

Each planetary system was simulated twice using codes based on the N-body mapping technique of Wisdom and Holman (1991, which is referred to as the mixed-variable symplectic (MVS) method by Saha and Tremaine 1994). In one case we used the SWIFT MVS integrator (Levison and Duncan 1994) and in the other we used the Mercury5 MVS hybrid integrator (Chambers 1999). For the SWIFT simulations, the initial orbital elements and masses for the asteroids and planets were provided by Ted Bowell (personal communication, 1997) at the epoch JD2450400.5 (November 13, 1996). The planetary parameters for the Mercury5 runs were taken from JPL's DE200 ephemeris data at the epoch 2451000.5 (July 6, 1998). The initial parameters for the asteroids in these runs were taken from the IAU Minor Planet Center (http://cfa-www.harvard.edu/cfa/ps/mpc.html) at the epoch JD2451560.5 (January 17, 2000) and were integrated backward to the epoch of the planetary data prior to each simulation. The small differences in the initial conditions for each given pair of simulations, and variations resulting from slightly different time steps chosen for the two sets of integrations, are magnified by the chaotic nature of the systems being studied (even without considering the asteroids, the Lyapunov time of the Solar System is only $\sim 5 \times 10^6$ years, Laskar 1989). The SWIFT MVS integrator does not accurately simulate close approaches among planets, so it only provides an estimate of the time to orbit crossing. The integrations done with the Mercury5 MVS hybrid scheme yield both instability time and the consequences of the instability. Both integration packages were slightly modified to grow a selected asteroid in the main belt into a planet-sized body according to the prescription described above. To examine whether or not the small mass of Mars substantially increases the stability of the terrestrial planets, the codes were further modified for several simulations to grow both an asteroid and Mars into (at least) 1 $M_\oplus$ bodies.

The runs were integrated with time steps of 5.5 days (using SWIFT) or 6 days (using Mercury5); these time steps are approximately 1/16 the orbital period of the planet Mercury. Thus, planets which had been perturbed into orbits with very small perihelia were not followed accurately and may have been artificially absorbed by the Sun (Rauch and Holman 1999). Consequently, we are not confident in the "planet hit Sun" instability mode. However, systems with such an eccentric planet are already highly chaotic, and thus the times to instability (our primary concern) is not likely to be affected substantially.

3. RESULTS

Table II summarizes the results of our simulations. The first column lists the planetary systems integrated, denoted by the first letter of the asteroid (or MD for Martinduncan) involved in each run, followed by the asteroid's final mass (in units of the Earth's mass, $M_\oplus$). The systems which are listed near the bottom of Table II involved growing both Ceres and Mars to the specified mass (in $M_\oplus$) and/or starting Ceres on a circular orbit. The next two columns give for each integrator the first crossing time (t_c) if the orbits of any two terrestrial planets overlap in heliocentric distance. The fourth column lists the time a planet is lost (t_l), either by being ejected (beyond 100 AU) from the system or impacting the Sun, in the Mercury5 integrations. The final column lists the consequence of the instability for the Mercury5 runs. The integrations with Mercury5 would also have terminated if two planets had collided, but this did not happen in any of our simulations. Note that the first planet lost in each of our simulations is always either Mercury or Mars, the lowest mass (and smallest AMD) terrestrial planets.

3.1. *Individual Asteroid Grown Using Current Orbit*

Figure 1 shows the evolution of the orbits and the AMD of the terrestrial planets and a 10 $M_\oplus$ Ceres in the Mercury5 run. Note that throughout the simulation, Ceres has dozens of times

TABLE II
Results of Simulations

	SWIFT	Mercury5		
Run (asteroid/mass)	t_c	t_c	t_l	Fate of lost planet
C10	14.09	5.79	45.42	Mercury ejected
C5	>1000	>200	—	
H10	3.59	>500	—	
H5	>100	>200	—	
MD5	0.47[c]	0.47[c]	32.51	Mars ejected
MD1	67.63[c]	30.38[c]	159.79	Mars "hit Sun"
MD0.1	>100	>200	—	
P5	165.52[c]	144.14[c]	151.72	Mars ejected
P2	>200[c]	>200[c]	—	
V5	2.73	1.34	65.40	Mars "hit Sun"
V2	>100	>200	—	
V1	>100	>200	—	
C5M1	58.16	16.96	36.52	Mercury ejected
C2M1	40.65	13.8	98.62	Mercury ejected
C1M1	139.79	284.4	437.70	Mercury ejected
C5M1[a]	57.45	96.94	112.27	Mercury "hit Sun"
C2M1[a]	>500	>500	—	
C10[b]	>500	>500	—	
C10[b]M1[a]	202.29	271.36	326.315	Mercury ejected

Note. Instability times are in myr.
[a] System began at an epoch where Mars (M) had a minimum eccentricity.
[b] Ceres (C) started on circular orbit with $i = 0$ with respect to the invariable plane.
[c] Asteroid crossed Mars's orbit before any pair of terrestrial planets' orbits crossed.

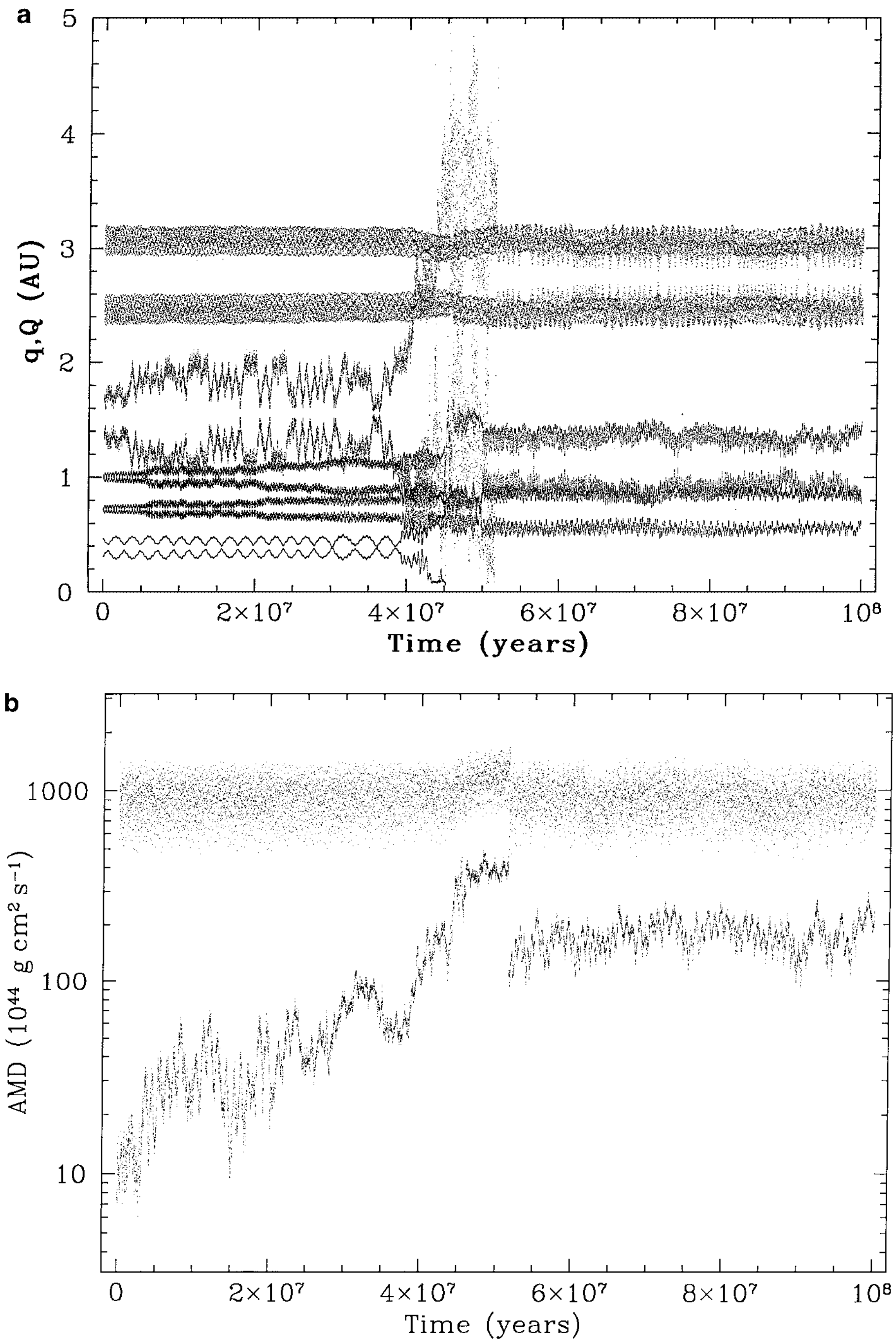

FIG. 1. (a) The perihelia (q) and aphelia (Q) of the terrestrial planets and Ceres (grown to 10 $M_{\oplus}$) are shown as functions of time for the Mercury5 C10 run. Note that Mars's orbit is excited to high eccentricity and it becomes Earth-crossing within 10 myr. Close approaches between Mars and Earth excite the eccentricities of both bodies, and Venus and Mercury also become highly excited. Mercury is ejected from the system after 45 myr, and Mars is ejected at ~7 myr thereafter. (b) The combined AMD of the four terrestrial planets is shown as a function of time in the lower curve, beginning 10^5 years after the start of the integration (when the asteroid reached its final mass). The upper curve shows the combined AMD of the terrestrial planets and Ceres at 10 $M_{\oplus}$.

as much AMD as would be required to destabilize the terrestrial planets if it were all transferred to Mercury and Venus or to Earth and Mars. However, it takes a few million years for the AMD of the terrestrial planets to reach the minimum value for orbit crossing. At ~5 myr, the orbit of Mars crosses Earth's orbit, and within 40 myr Mars has close approaches with Earth and Venus, perturbing Mercury enough to be ejected from the Solar System at 45 myr. The Solar System with Ceres grown to

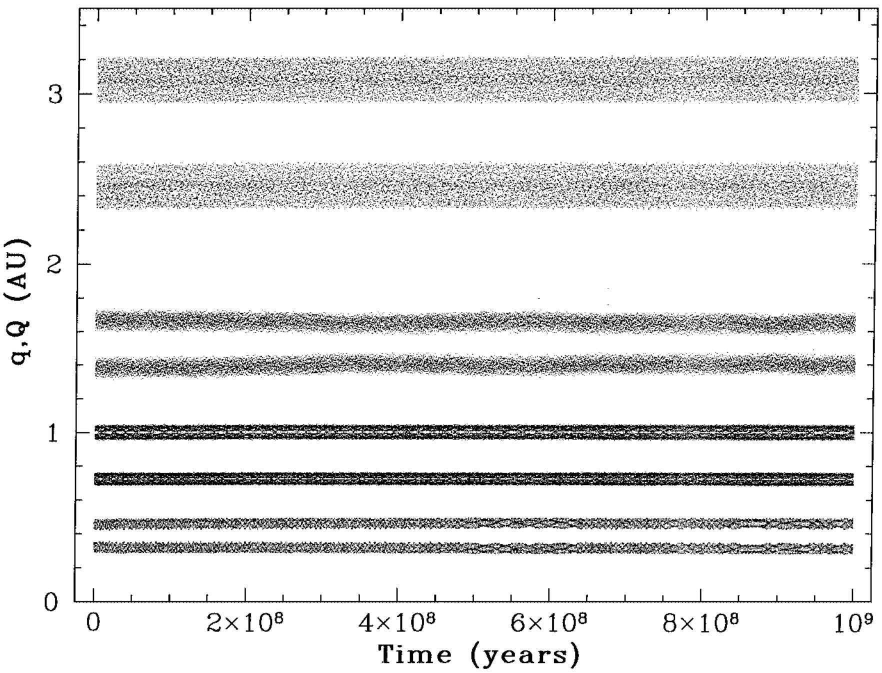

FIG. 2. The perihelia (q) and aphelia (Q) of the terrestrial planets and a 5 $M_\oplus$ Ceres are shown as functions of time for the SWIFT C5 run. All bodies in this system remained in stable orbits for the entire one billion years simulated.

10 $M_\oplus$ became unstable after 14 myr in the SWIFT run, when Mars became Earth-crossing. Variations of this magnitude in instability time are typical of chaotic systems, where small differences in initial conditions, rounding errors, etc., grow exponentially with time, and thus they are not indicative of any problems with our integration techniques (e.g., Duncan and Lissauer 1997). When Ceres was grown to 5 $M_\oplus$, however, all bodies remained in quiescent orbits for a billion years (Fig. 2), despite the fact that Ceres had more than an order of magnitude more AMD than would be needed to destabilize the terrestrial planets were it all transferred inward.

In both the SWIFT and Mercury5 simulations with Hygeia grown to 10 $M_\oplus$, Mercury's eccentricity was excited to high values during the first few million years. In the SWIFT run, Mercury's eccentricity was large enough for it to become Venus-crossing at $t_c \sim 3.6 \times 10^6$ years. In the Mercury5 run, Mercury's eccentricity almost but not quite reached Venus-crossing values many times during the 500 myr simulated (Fig. 3a). This discrepancy may be due to the highly chaotic nature of the inner planets when they are near instability. Alternatively, even though their orbits had not yet crossed, Mercury and Venus might have approached one another closely enough that a small error in their trajectories occurred in the SWIFT run and that this error allowed the planets' orbits to subsequently cross. Note that in the (stable) 10 $M_\oplus$ Hygeia run with Mercury5, the AMD of the terrestrial planets is frequently above the threshold required for orbital crossing (Fig. 3b), but orbits never cross as Mars always retains a significant amount of this AMD and the eccentricities of the three innermost planets are positively correlated. When Hygeia was grown to 5 $M_\oplus$, it did not destabilize the inner planets in either simulation.

When asteroid Martinduncan was grown to 5 $M_\oplus$, Mars's eccentricity was highly excited and that planet had close approaches to Earth in less than half a million years in each simulation. Mars continued to cross both Earth and Venus in the Mercury5 run, persisting for $\sim$32 myr before being ejected. Systems with Martinduncan at 1 $M_\oplus$ became orbit-crossing after tens of millions of years, and those with Martinduncan at 0.1 $M_\oplus$ were stable for the entire 100 or 200 myr simulated. In all three cases, Martinduncan efficiently transfers AMD to Mars, but at a mass of 0.1 $M_\oplus$ there is barely enough AMD to lead to orbit-crossing with an optimal allocation of this AMD among the planets, and the AMD does not all get concentrated in a neighboring pair of objects during either simulation of this system.

In the systems with Vesta grown to 5 $M_\oplus$, Mars became Earth-crossing at $t_c \sim 2.7 \times 10^6$ years in the SWIFT simulation. In the Mercury5 run, Mars's eccentricity increased substantially during the first few hundred thousand years and it became Earth-crossing at $t_c \sim 1.34 \times 10^6$ years, but the system persisted for over 65 myr (Fig. 4). In this integration, Mars first crossed Vesta's orbit after 60 myr, and began crossing the orbits of Jupiter and Saturn at 63.4 myr. Ultimately, the periapse of Mars dropped so low that we stopped the integration. Vesta grown to 1 or 2 $M_\oplus$, however, did not cause the orbits of the inner planets to cross.

The Solar System with Pallas grown to 5 $M_\oplus$ remained stable for $\sim$150–170 myr. In each of our simulations involving Pallas, the aphelion of Mars crosses the perihelion of Pallas for

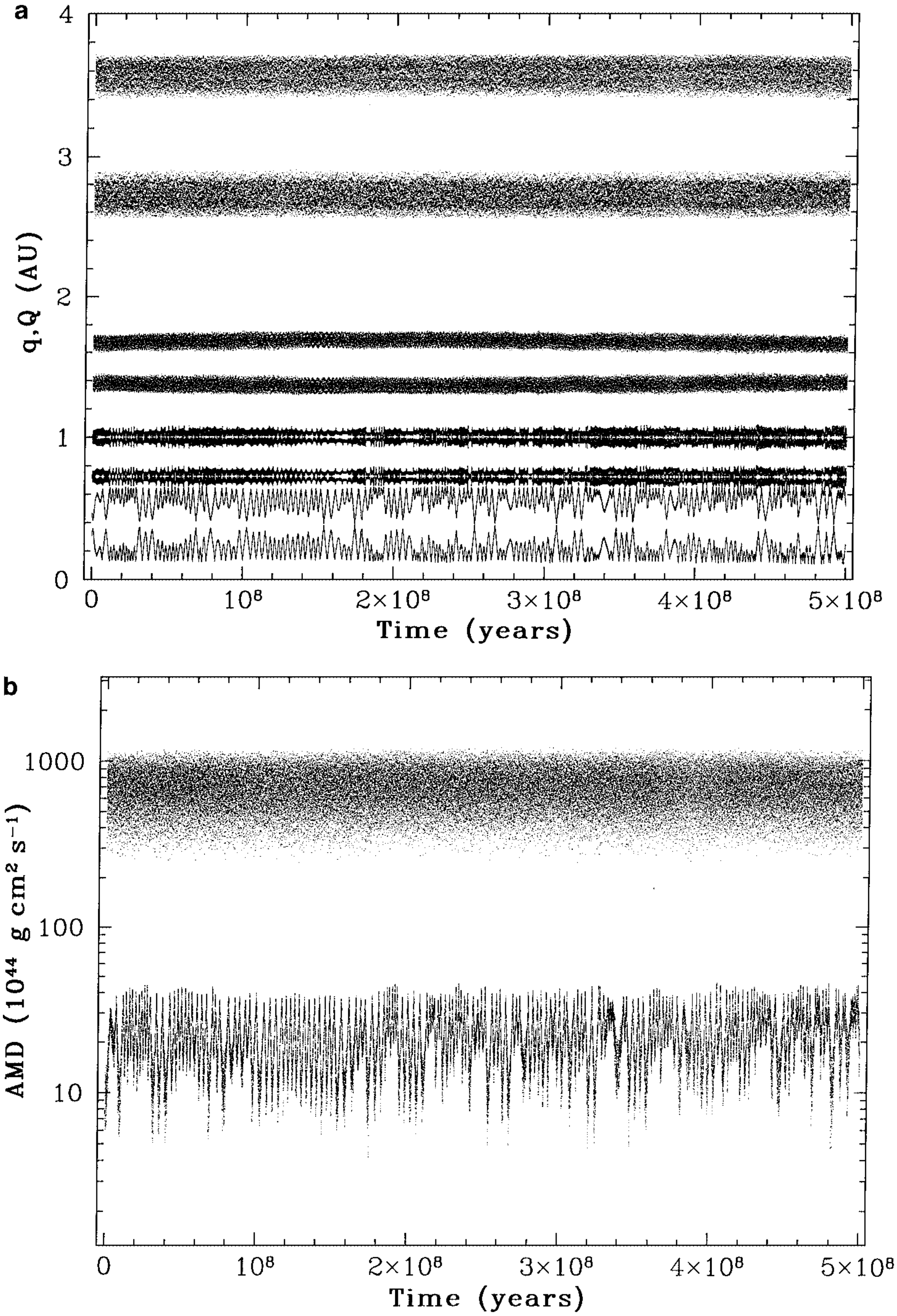

FIG. 3. (a) The perihelia (q) and aphelia (Q) of the terrestrial planets and Hygeia for the Mercury5 H10 run. Mercury and Venus approach but never cross, allowing the system to survive for the entire 500 myr simulated. Note the positive correlations among the eccentricities of the three innermost planets in this system. (b) The combined AMD of the four terrestrial planets in the Mercury5 H10 run is shown as a function of time in the lower curve, while the combined AMD of the terrestrial planets and Hygeia at 10 $M_\oplus$ is shown in the upper curve. Here, although the AMD of the terrestrial planets reaches high enough values to destabilize the system, Mars retains enough AMD itself such that orbits never cross in this simulation.

the duration of the integrations (Fig. 5). This situation also occurs with the real Mars and Pallas. The two bodies are prevented from collision, however, by a coupling of Pallas's eccentricity and argument of perihelion (Fig. 6). To study the stability of this protection mechanism in the actual Solar System, we performed two simulations in which 25 test particles were placed near Pallas's orbit; one of these integrations was performed using SWIFT and the other using Mercury5. For the SWIFT integration, the

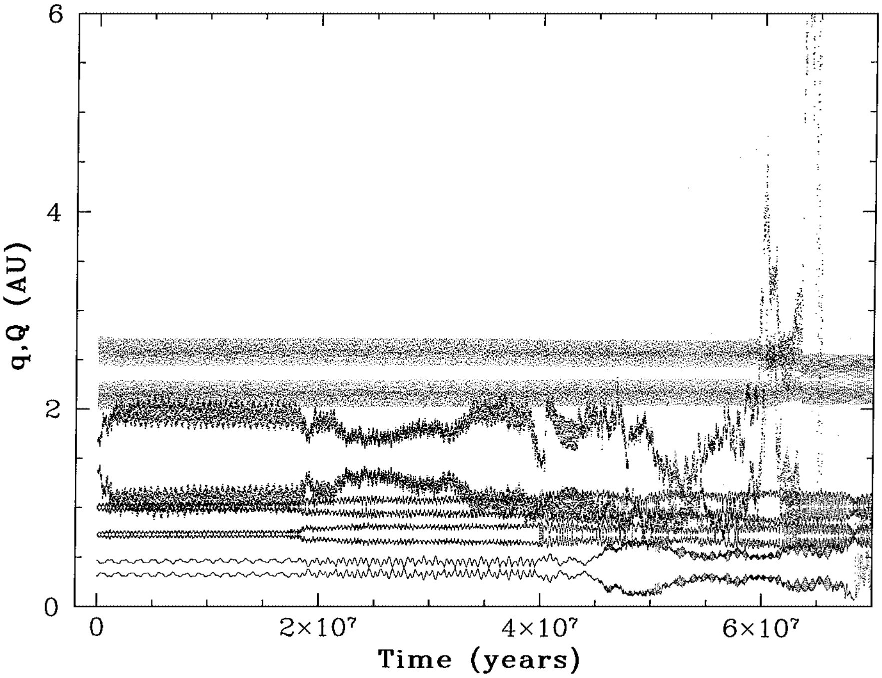

FIG. 4. The perihelia (q) and aphelia (Q) of the terrestrial planets and Vesta for the Mercury5 V5 run. Note that Mars made excursions into the giant planetary region prior to its demise.

z components of the position and velocity of the 25 test particles were varied from the nominal initial coordinates of Pallas in increments of 2×10^{-12} AU in position and 2×10^{-13} AU/year in velocity. For the Mercury5 run, the semimajor axis was increased from its nominal value by up to 2.4×10^{-12} AU in increments of 1×10^{-13} AU. In each case, the systems were stable (all 25 test particles remained in orbit) for a billion years.

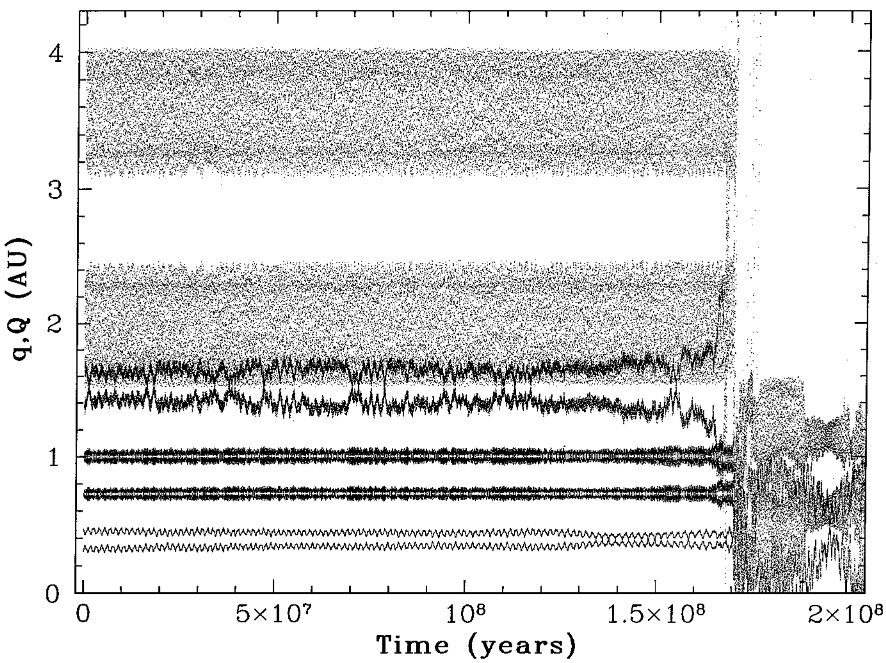

FIG. 5. The perihelia (q) and aphelia (Q) of the terrestrial planets and Pallas at 5 $M_\oplus$ are shown as functions of time for the SWIFT P5 run. Although all bodies remained in well-behaved orbits for the first 150 myr, Mars's aphelion was exterior to Pallas's perihelion during much of the integration interval.

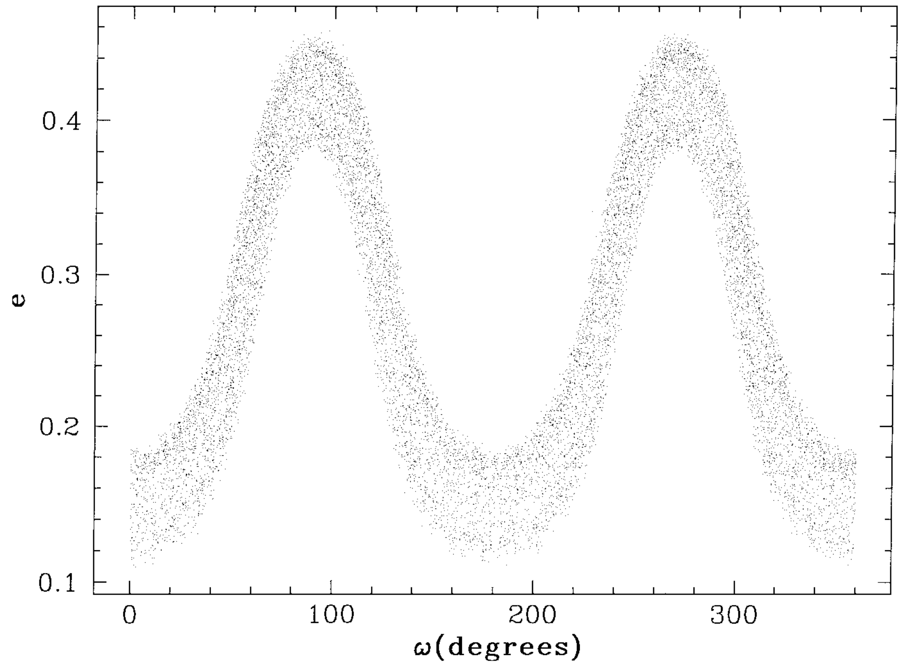

FIG. 6. This plot shows the relationship between Pallas's eccentricity (e) and argument of perihelion (ω) during the first 100 myr of the P5 SWIFT run.

3.2. Ceres Begins on a Circular Orbit

In the simulations discussed above, the planet was grown on one of the orbits of the four most massive asteroids or asteroid Martinduncan. Each of these five asteroids has a moderately large eccentricity; the specific AMD of each of the asteroids in the real Solar System averaged over 10^5 years beginning at the epoch 2451000.5 (July 6, 1998) is given in Table I. Thus when each is grown to a planetary mass, its AMD is larger than that of the terrestrial planets. To determine the relative importance of this "initial" AMD of the asteroids and the AMD transported from the jovian planets via the asteroids, we performed simulations in which the asteroid Ceres was initially on a circular orbit with zero inclination with respect to the invariable plane and grown to 10 $M_\oplus$. Even in this case, the giant planets rapidly excited the eccentricity of Ceres up to 0.086, giving it several times the amount of AMD needed to destabilize the terrestrial planets. Nonetheless, in both systems, the terrestrial planets remained on stable orbits for the entire 500 myr simulated. The initial AMD of a 10 $M_\oplus$ Ceres is thus a critical factor in destabilizing the C10 systems in $\sim 10^7$ years.

3.3. Ceres and Mars Both Enlarged

Because a 5 $M_\oplus$ Ceres had little effect on the dynamics of the inner and outer planets, we performed several simulations in which both Ceres and Mars were grown to (at least) 1 $M_\oplus$ bodies. When Ceres was grown to 5 $M_\oplus$ and Mars to 1 $M_\oplus$, Earth was excited and had close approaches with Venus at $\sim$16 myr in the Mercury5 run; ultimately Mercury was ejected at 36 myr. In the SWIFT run, Venus's eccentricity became excited at $\sim$40 myr and the orbits of Venus and Mercury crossed at $t_c \sim 58 \times 10^6$ years. In both systems with Ceres grown to 2 $M_\oplus$ and Mars to 1 $M_\oplus$, Venus again had close approaches with Mercury between 10 and 40 myr. In the Mercury5 case, however, the system persisted for nearly 100 myr until Mercury was ejected. Systems with both Ceres and Mars grown to 1 $M_\oplus$ became orbit crossing in $\sim$200 myr. Thus, adding some mass to Ceres and some to Mars can destabilize systems more rapidly than would adding the same total amount of mass to Ceres alone.

Mars currently has a high orbital eccentricity, so by enlarging Mars to 1 $M_\oplus$ we are substantially increasing the total AMD of the terrestrial planets. Thus, the increase in the AMD of Mars may play a larger role in destabilizing the system than does the improved coupling with the asteroid and giant planets. Mars's AMD varies substantially on timescales longer than the 10^5 year interval over which we increased the selected planets' masses (Fig. 7), so growing Mars at a different epoch could add far less AMD to the terrestrial planets than does increasing its mass at present. We thus performed several integrations in which Ceres and Mars were both grown starting at an epoch 5.78 myr after the present, when Mars's AMD will be much smaller than it is at present (Fig. 7). (Note that the mean AMD of Ceres, shown in Fig. 8, does not vary substantially on these timescales, so the epoch at which Ceres is grown makes little difference.) A Mercury5 integration with Ceres at 5 $M_\oplus$ and Mars at 1 $M_\oplus$ became planet-crossing in just under 100 myr. In a SWIFT simulation of this system, Venus became excited at $\sim$57 myr before having a close encounter with Mercury. Systems with Ceres grown to 2 $M_\oplus$ and Mars to 1 $M_\oplus$ remained stable for the entire 500 myr simulated. Adding mass to Ceres and Mars when

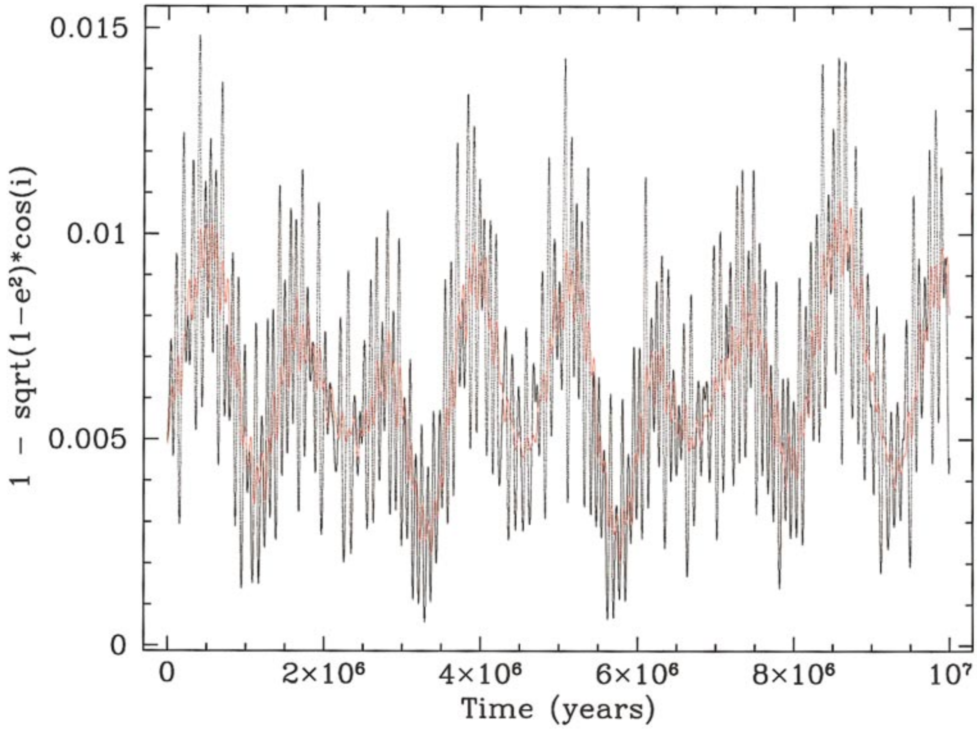

FIG. 7. Mars's nondimensional angular momentum deficit (the last term in Eq. (1)) and average nondimensional AMD over the preceeding 100,000 years are shown as functions of time in the actual Solar System, with Ceres included at its actual mass. The black curve represents the instantaneous value of AMD, whereas the red curve represents a running average over an interval of 10^5 years.

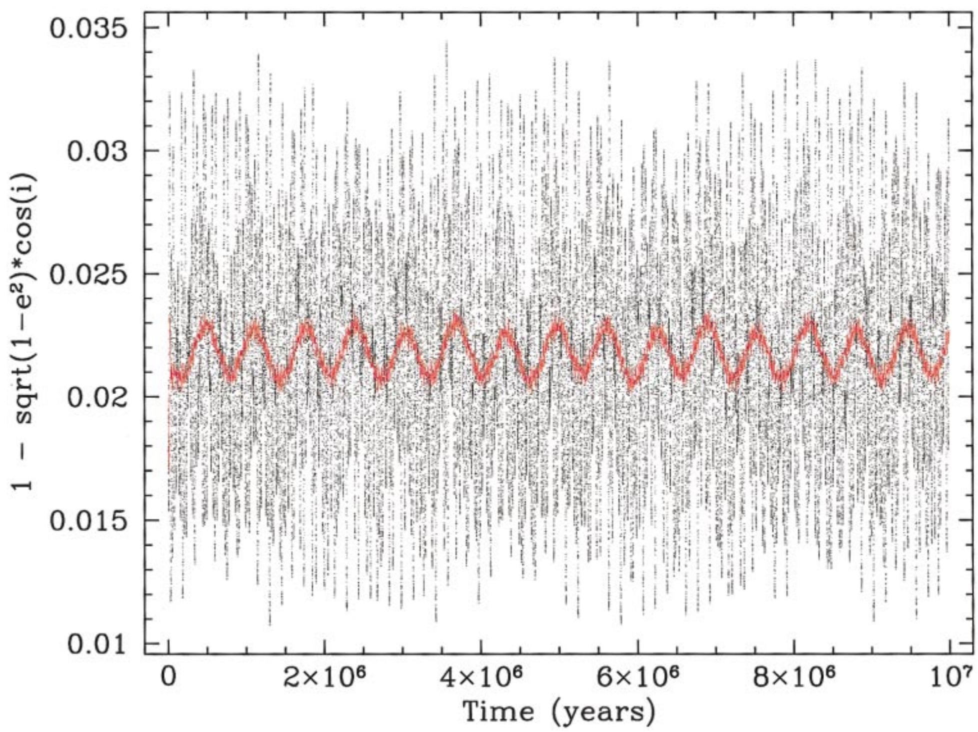

FIG. 8. The nondimensional angular momentum deficit of Ceres at its actual mass (black points) and average nondimensional AMD over the preceeding 100,000 years (red curve) are shown as functions of time.

Mars is on a less eccentric orbit thus appears to be less effective at destabilizing the system than similar additions when Mars is more eccentric (presumably because less AMD is being added to the terrestrial planets), but still more destabilizing than adding mass to Ceres alone, presumably because the larger mass of Mars facilitates the transport of AMD from the giant planets and Ceres to the terrestrial planets.

In an additional pair of simulations, we integrated Ceres at its actual mass up to the epoch in which Mars was at a minimum eccentricity; we then circularized Ceres's orbit and grew Ceres to 10 $M_\oplus$ and Mars to 1 $M_\oplus$. Orbits began to cross after 200–300 myr in these simulations.

4. DISCUSSION

It is possible to have a planetary-sized body in the asteroid belt region without destroying the orbital stability of the terrestrial planets. Systems with Ceres or Hygeia at 5 $M_\oplus$, or Vesta or Pallas at 2 $M_\oplus$, were stable for the entire 10^8–10^9 years simulated. A gap between Mars and Jupiter, therefore, is not necessary to have a stable Solar System. Furthermore, the terrestrial planets can remain stable when additional mass is given to both Mars and an asteroid, although less additional mass is accomodated if it is split between Mars and the asteroid than would be allowed if all of the additional mass is placed in the asteroid alone. This is especially true if the extra mass is added to Mars at an epoch when Mars's eccentricity is large, as is the case at present.

Outer belt asteroids Ceres, Pallas, and Hygeia are closely coupled with the jovian planets but generally do not transfer a substantial fraction of their AMD to the terrestrial planets. In contrast, inner belt object Martinduncan is excited less by the outer planets, but it is more tightly coupled to the terrestrial planets and can efficiently transfer a large fraction of its AMD inward.

ACKNOWLEDGMENTS

We thank John Chambers for many informative discussions and comments. Tony Dobrovolskis and two anonymous reviewers provided comments which helped us improve the manuscript. This research was supported in part by NASA's Origins of Solar Systems Research Program through Grant NAG5-9680 and a NASA Graduate Student Researchers Program Fellowship awarded to E.V.Q. M.J.D. is also grateful for the continuing financial support of the National Science and Engineering Research Council of Canada.

REFERENCES

Agnor, C. B., R. M. Canup, and H. F. Levison 1999. On the character and consequences of large impacts in the late stage of terrestrial planet formation. *Icarus* **142**, 219–237.

Chambers, J. E. 1999. A hybrid symplectic integrator that permits close encounters between massive bodies. *Mon. Not. R. Astron. Soc.* **304**, 793–799.

Chambers, J. E., and G. W. Wetherill 1998. Making the terrestrial planets: N-body integrations of planetary embryos in three dimensions. *Icarus* **136**, 304–327.

Chambers, J. E., and G. W. Wetherill 2001. Planets in the asteroid belt. *Meteorites Planet. Sci.* **36**, 381–399.

Duncan, M. J., and J. J. Lissauer 1997. Orbital stability of the uranian satellite system. *Icarus* **125**, 1–12.

Duncan, M. J., and J. J. Lissauer 1998. The effects of post-main sequence solar mass loss on the stability of the Solar System. *Icarus* **134**, 303–310.

Laskar, J. 1989. A numerical experiment on the chaotic behaviour of the Solar System. *Nature* **338**, 237–238.

Laskar, J. 1997. Large scale chaos and the spacing of the inner planets. *Astron. Astrophys.* **317**, L75–L78.

Levison, H. F., and M. J. Duncan 1994. The long-term dynamical behavior of short-period comets. *Icarus* **108**, 18–36.

Lissauer, J. J. 1987. Timescales for planetary accretion and the structure of the protoplanetary disk. *Icarus* **69**, 249–265.

Migliorini, F., P. Michel, A. Morbidelli, D. Nesvorny, and V. Zappala 1998. Origin of Earth-crossing asteroids: A quantitative simulation. *Science* **281**, 2022–2024.

Rauch, K. P., and M. Holman 1999. Dynamical chaos in the Wisdom–Holman integrator: Origins and solutions. *Astron. J.* **117**, 1087–1102.

Rivera, E. J. 2001. *Dynamical Evolution of the Earth–Moon Progenitors*. Ph.D. thesis. State University of New York, Stony Brook, New York.

Saha, P., and S. Tremaine 1994. Long-term planetary integration with individual time steps. *Astron. J.* **108**, 1962–1969.

Thommes, E. W. 2000. *On the formation of Uranus and Neptune*. Ph.D. thesis. Queen's University.

Wisdom J., and M. Holman 1991. Symplectic maps for the N-body problem. *Astron. J.* **102**, 1528–1538.