MAT 412 HOMEWORK 2

DUE: JANUARY 20, 2016 (BEGINNING OF CLASS)

This is the first regular homework set and covers section 1.1–1.3. References are to Hungerford, 3rd. edition.

Problem 1. Use the division algorithm to show that every odd integer is either of the form $4k+1$ or of the form $4k+3$ for some integer k. (Exercise 1.1.B.8)

Problem 2.

(a) Prove that a positive integer is divisible by 3 if and only if the sum of its digits is divisible by 3 (Exercise 1.2.B.28).
(b) Prove that a positive integer is divisible by 11 if and only if the alternating sums of its digits is a multiple of 11 (e.g., for 1234 the alternating sum of its digits is $1-2+3-4$).

Problem 3. Define the *least common multiple* $\mathrm{lcm}(m,n)$ of two positive integers m and n to be the smallest positive integer v such that both $m|v$ and $n|v$.

(a) Compute $\mathrm{lcm}(85, 65)$, $\mathrm{lcm}(2017, 2019)$.
(b) Show that $v = \frac{mn}{(m,n)}$. (Exercise 1.2.C.33)

Problem 4.

Show that if b and c are both relatively prime to a, then bc is also relatively prime to a.

Problem 5.

(a) Express the following numbers as a product of primes: $2002, 5040, 40320$.
(b) Which of the following are prime: $2^5-1, 2^7-1, 2^{11}-1$? Justify your answer! (Exercise 1.3.A.3)
(c) Let p be a prime and $p>0$. Show that $p|\binom{p}{k}$ for any $1 \leq k < p$. [Note that: $\binom{p}{k} = \frac{p!}{k!(p-k)!}$ is an integer ≥ 0.]

Problem 6.

(a) If p is a prime, prove that one cannot find nonzero integers a and b such that $a^2 = pb^2$. [Hint: Fundamental theorem of arithmetic]
(b) Show that $\sqrt{2}$ is irrational. [Hint: You may want to use part (a)] What can you conclude for $\sqrt{p}$, p prime?

Problem 7. Read ahead sections 2.1–2.3 in the book.