Marginal Jobs, Heterogeneous Firms, & Unemployment Flows*

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Abstract

Much recent research has sought to explain the cyclical amplitude of unemployment fluctuations in the US. This paper shows that amplification of the cyclical variation of unemployment can be obtained from adding two very simple features to an otherwise standard model of the aggregate labor market, namely downward sloped short run labor demand and endogenous job destruction. This generalized model is able to match more closely the cyclicality of both job finding and employment to unemployment flows observed in US data. Contrary to standard models, the model can generate amplification while maintaining realistic surplus to employment relationships. In addition, we uncover a novel source of amplification of cyclical shocks that is generated by the interaction of countercyclical unemployment inflows and job creation.

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Much recent research has sought to explain the cyclical amplitude of unemployment fluctuations in the US. Shimer (2005) has shown that a leading model of the aggregate labor market, the Mortensen-Pissarides (MP) model, cannot explain this cyclical volatility. A common solution to this problem proposed in subsequent literature has been to invoke rigidity in the wages of newly hired workers (see among others Shimer, 2004; and Hall, 2005). However, the empirical validity of such an assumption has been questioned by Haefke, Sonntag, & van Rens (2007), and by Pissarides (2007).[1] This paper takes a different approach. We show that amplification of the cyclical volatility of unemployment can instead be obtained simply by adding two very conventional features to the standard search model, namely downward sloped short run demand for labor and endogenous job destruction.

The motivation for these additional features is simple. First, downward sloped labor demand is motivated by the fact that other production inputs, notably capital, are not fully flexible at cyclical frequencies.[2] Second, the inclusion of endogenous job destruction is informed by empirical evidence that part of the cyclical upswing in unemployment in times of recession is accounted for by increased flows from employment to unemployment.[3]

However, incorporating these two conventional features simultaneously is not a trivial exercise. We show that it is also not a daunting one. In particular, downward sloped labor demand implies that firms face a non-linear production technology which poses a number of theoretical challenges. First, this complicates wage setting because the surplus generated by each of the employment relationships within a firm is not the same (e.g. "the" marginal worker generates less surplus than infra-marginal workers). In section 1, we derive a very intuitive and explicit wage bargaining solution for this environment, something that has been considered challenging in recent research (see Cooper, Haltiwanger & Willis, 2007; and Hobijn & Sahin, 2007). In particular, the solution is a very natural generalization of the wage bargaining solution in the standard MP model. The simplicity of our solution is therefore a useful addition to the literature.[4]

[1] This echoes the earlier results of Baker, Gibbs, & Holmstrom (1994) who observe substantial wage flexibility among new hires in data for a particular firm (see especially their Figure II).

[2] Another motivation for downward sloped labor demand is the existence of imperfect product market competition. For a model with this feature (but with exogenous job destruction) see Rotemberg (2006).

[3] See Perry (1972); Marston (1976); Blanchard & Diamond (1990); Elsby, Michaels, & Solon (2007); Fujita & Ramey (2007); Pissarides (2007); Shimer (2007); and Yashiv (2006).

[4] Related models with endogenous separations such as Cooper et al. (2007) and Hobijn & Sahin (2007) have set worker bargaining power to zero in order to derive wages. Acemoglu & Hawkins (2006) characterize wages in a model with exogenous separations, but they focus on a time to hire aspect to job creation, which leads to a more challenging bargaining problem. Our solution is analogous to the wage bargaining solutions derived by Smith (1999), Cahuc & Wasmer (2001), and Krause & Lubik (2007) for models with exogenous job destruction.

The wage bargaining solution then enables us to characterize the properties of the optimal labor demand policy of an individual firm in the presence of idiosyncratic firm heterogeneity. An interesting by-product of this exercise is that the optimal labor demand solution in the generalized model is analogous to that of a model of kinked hiring costs in the spirit of Bentolila & Bertola (1990), but where the hiring cost is endogenously determined by frictions in the labor market. Thus, the correspondence between the two major approaches to the economics of aggregate labor markets – search and matching models and employment adjustment cost models – sharpens in the process of generalizing the standard search model.

A second analytical challenge in models with a non-linear production technology and idiosyncratic heterogeneity is that aggregation of microeconomic behavior is not straightforward, because a representative firm interpretation of the model doesn't exist. To address this, in section 2 we develop a method for aggregating the behavior of individual firms that holds for a wide class of optimal labor demand policies at the microeconomic level. In particular, we are able to solve for the equilibrium distribution of employment across firms, which in turn allows us to determine the level of the aggregate (un)employment stock. In addition, we also provide a related method that allows us to solve for aggregate unemployment flows (hires and separations) implied by microeconomic behavior. Together, these characterize the aggregate steady state equilibrium of the model economy.

In section 3 we introduce aggregate shocks to the model. We show that this is not a trivial exercise. Out of steady state, individual firms must forecast future wages, which involves forecasting the future path of the distribution of employment across firms, an infinite order state variable. We provide an analytical approximation to a firm's optimal labor demand policy in the presence of aggregate shocks. Using this, we employ an approach that mirrors the method proposed by Krusell & Smith (1998) to solve for the transition paths for the unemployment stock and flows in the presence of aggregate shocks.

These results allow us to quantitatively evaluate our model, which we turn to in section 4. An attractive feature of the model is that, by incorporating both a notion of firm size as well as idiosyncratic heterogeneity, it delivers important cross sectional implications. We show that the model can be used to match key features of the distribution of employment growth across establishments that have been documented since the work of Davis & Haltiwanger (1992). We use this calibration strategy to discipline the process of idiosyncratic shocks facing firms. Applying this to an otherwise standard calibration reveals that our generalized model can more closely match the observed cyclical variation of both the job finding rate and the employment to unemployment transition rate in the US, and is a substantial improvement on the basic MP model.

A potential concern in models, such as this, that incorporate countercyclical job destruction has been that they often cannot generate the observed procyclicality of vacancies (Shimer, 2005; Mortensen & Nagypal, 2007b). Importantly, we find that our model makes considerable progress in this regard: Our calibration of the model generates approximately half of the observed comovement between vacancies and output per worker. Moreover, we suggest that the remaining procyclicality is likely due to procyclical job-to-job flows that are observed in the data (Fallick & Fleischman, 2004) but are abstracted from in the present paper.

The common factor that generates both the procyclicality of the job finding rate and of vacancies in the model is the procyclicality of desired job creation. To uncover the processes underlying this, we derive a simple approximation to the decline in job creation following an adverse aggregate shock in the generalized model, analogous to the method employed by Shimer (2005) and Mortensen & Nagypal (2007a,b). This exercise reveals two sources of amplification. The first generalizes a well-known result that the standard MP model is consistent with observed unemployment cyclicality if the average flow surplus to employment relationships is sufficiently small.[5] We show that an analogous result occurs in the generalized model if a weighted average of the average and *marginal* flow surplus is sufficiently small. However, because downward sloped labor demand implies that the marginal surplus will be smaller than the average surplus, the generalized model can deliver amplification of the job creation response to aggregate shocks at the same time as preserving a sizeable average surplus from employment relationships.[6]

This result is important because recent research has suggested that the average surplus required for the standard model to match the observed cyclicality of the job finding rate is implausibly small (Mortensen & Nagypal, 2007a). An appealing feature or our calibration strategy is that it provides a formalization of this intuition. By calibrating to the observed cross sectional distribution of employment growth, we obtain a sense of the plausible size of idiosyncratic shocks facing firms. We then show that the standard MP model faces a tension in the face of plausible shocks. To match the volatility of the job finding rate, the

[5]Intuitively, a small surplus to employment relationships implies that small reductions in the productivity of labor (as in a recession) quickly exhaust the surplus and lead firms to cut back substantially on hiring. See Mortensen & Pissarides (1994), Hagedorn & Manovskii (2005), and Costain & Reiter (2005).

[6]One might imagine that a symmetric logic holds on the supply side of the labor market if there is heterogeneity in workers' valuations of leisure so that "the" marginal worker obtains a low surplus from employment. Interestingly, Mortensen & Nagypal (2007a) argue that this is not the case. They show that if firms cannot differentiate workers' types when making hiring decisions, they will base their decision on the average, rather than the marginal, valuation of leisure among the unemployed. The same is unlikely to be true of the model studied here, since firms presumably know their production technology when making hiring decisions.

model requires a small surplus. But this small surplus in turn implies a counterfactually high separation rate.[7]

Our results also suggest a second, more novel source of amplification that is generated by the interaction of heterogeneous firms and downward sloped labor demand. Following a reduction in aggregate labor demand, low productivity firms wish to shed more workers, and high productivity firms wish to hire fewer workers. Thus inflows into the unemployment pool rise, and outflows from the unemployment pool fall, *ceteris paribus*, causing unemployment to rise. To restore equilibrium in the model, hiring firms must be convinced to hire enough workers to equate inflows to outflows once more. The model achieves this by allowing the labor market to slacken, so that unemployed workers become more abundant, and hiring (suitable) workers becomes less costly for firms. With downward sloped labor demand, increased hiring retards the productivity of additional employment relationships, and so the labor market must slacken further, and unemployment must rise more, in order to return the economy back to equilibrium once again.

Section 5 of the paper discusses the broader implications of our analysis. We argue that the model developed in the paper provides a rich, yet analytically tractable model of the aggregate labor market in the short run. As such, we believe that this model will provide a useful laboratory for the cyclical analysis of aggregate labor markets in future empirical and theoretical research. In addition, we suggest that, by developing a model with a well-defined concept of a firm, the analytical results derived here are a natural complement to recent research that has investigated the empirical implications of search frictions using establishment level data (Cooper, Haltiwanger & Willis, 2007; Davis, Faberman, & Haltiwanger, 2007). Finally, we emphasize the wider applicability of our framework. The model lends itself to a number of natural extensions including: estimation using establishment level data; a treatment of the steady state and dynamics of the distribution of firm size; implications with respect to the establishment size wage effect; the inclusion of other popular forms of labor adjustment costs; and a deeper analysis of the implications of the multilateral dimension of wage setting between an individual firm and its many workers.

[7] A small average surplus also jars with widespread evidence for the prevalence of long term employment relationships in the US economy, which researchers have taken to imply substantial rents to ongoing matches (Hall, 1982; Stevens, 2005).

1 The Firm's Problem

In what follows we consider a model in which there is a mass of firms, normalized to one, and a mass of potential workers equal to the labor force, L.[8] In order to hire unemployed workers, firms must post vacancies. However, frictions in the labor market limit the rate at which unemployed workers and hiring firms can meet. As is conventional in the search and matching literature, these frictions are embodied in a matching function, $M = M(U, V)$, that regulates the number of hires, M, that the economy can sustain given that there are V vacancies and U unemployed workers. We assume that $M(U, V)$ exhibits constant returns to scale.[9] Vacancies posted by firms are therefore filled with probability $q = M/V = M(U/V, 1)$ each period. Likewise, unemployed workers find jobs with probability $f = M/U = M(1, V/U)$. Thus, the ratio of aggregate vacancies to aggregate unemployment, $V/U \equiv \theta$, is a sufficient statistic for the job filling (q) and job finding (f) probabilities in the model. Taking these flow probabilities as given, firms choose their optimal level of employment, to which we now turn.

1.1 Labor Demand

We consider a discrete time, infinite horizon model in which firms use labor, n, to produce output according to the production function, $y = pxF(n)$ where $F' > 0$ and $F'' \leq 0$. The latter is a key generalization of the standard MP model that we consider: When $F'' < 0$, the marginal product of labor will decline with firm employment, and thereby will generate a downward sloped demand for labor at the firm level. p represents the state of aggregate labor demand, whereas x represents shocks that are idiosyncratic to an individual firm. We assume that the evolution of the latter idiosyncratic shocks is described by the c.d.f. $G(x'|x)$.

A typical firm's decision problem is completely analogous to that in Mortensen & Pissarides (1994), and is as follows. Firms observe the realization of their idiosyncratic shock, x, at the beginning of a period. Given this, they then make their employment decision. Specifically, they may choose to separate from part or all of their workforce, which we assume

[8] Assuming a fixed number of firms is important for the model to depart from the standard MP model. Free entry would yield an economy of infinitesimal firms that converges to the MP limit. In principle, one could allow for costly firm entry as a middle ground. We abstract from this in part for simplicity. But our choice is also informed by evidence in Davis and Haltiwanger (1992). They find that, in manufacturing, establishment births account for 20% of job creation on average each year. While this is not a small number, the majority of job creation is clearly accounted for by incumbent establishments. For a model that explores the impact of firm entry on job creation, see Garibaldi (2006).

[9] See Petrongolo & Pissarides (2001) for a summary of empirical evidence that suggests this is reasonable.

may be done at zero cost. Any such separated workers then join the unemployment pool in the subsequent period. Alternatively, firms may hire workers by posting vacancies, $v \geq 0$, at a flow cost of c per vacancy. If a firm posts vacancies, the matching process then matches these up with unemployed workers inherited from the previous period. After the matching process is complete, production and wage setting are performed simultaneously.

It follows that we can characterize the expected present discounted value of a firm's profits, $\Pi(n_{-1}, x)$, recursively as:[10]

$$\Pi(n_{-1}, x) = \max_{n,v} \left\{ pxF(n) - w(n,x)\, n - cv + \beta \int \Pi(n, x')\, dG(x'|x) \right\} \tag{1}$$

where $w(n,x)$ is the bargained wage in a firm of size n and productivity x. A typical firm seeks a level of employment that maximizes its profits subject to a dynamic constraint on the evolution of a firm's employment level. Specifically, firms face frictions that limit the rate at which vacancies may be filled: A vacancy posted in a given period will be filled with probability $q < 1$ prior to production. Thus, the number of hires an individual firm achieves is given by:

$$\Delta n \mathbf{1}^{+} = qv \tag{2}$$

where Δn is the change in employment, and $\mathbf{1}^{+}$ is an indicator that equals one when the firm is hiring, and zero otherwise. Substituting the constraint, (2), into the firm's value function, we obtain:

$$\Pi(n_{-1}, x) = \max_{n} \left\{ pxF(n) - w(n,x)\, n - \frac{c}{q}\Delta n \mathbf{1}^{+} + \beta \int \Pi(n, x')\, dG(x'|x) \right\} \tag{3}$$

Note that the value function is not fully differentiable in n: There is a kink in the value function around $n = n_{-1}$. This reflects the (partial) irreversibility of separation decisions in the model. While firms can shed workers costlessly, it is costly to reverse such a decision because hiring (posting vacancies) is costly. In this sense, the labor demand side is formally analogous to the kinked employment adjustment cost model of the form analyzed in Bentolila & Bertola (1990), except that the per–worker hiring cost, $c/q(\theta)$, is endogenously determined.

In order to determine the firm's optimal employment policy, we take the first-order conditions for hires and separations (i.e. conditional on $\Delta n \neq 0$):

$$pxF'(n) - w(n,x) - w_n(n,x)\, n - \frac{c}{q}\mathbf{1}^{+} + \beta D(n,x) = 0, \text{ if } \Delta n \neq 0 \tag{4}$$

[10] We adopt the convention of denoting lagged values with a subscript, $_{-1}$, and forward values with a prime, $'$.

where $D(n,x) \equiv \int \Pi_n(n,x') \, dG(x'|x)$ reflects the marginal effect of current employment decisions on the future value of the firm. Equation (4) is quite intuitive. It states that the marginal product of labor $(pxF'(n))$ net of any hiring costs $(\frac{c}{q}\mathbf{1}^+)$, plus the discounted expected future marginal benefits from an additional unit of labor $(\beta D(n,x))$ must equal the marginal cost of labor $(w(n,x) + w_n(n,x)n)$. To provide a full characterization of the firm's optimal employment policy, it remains to characterize the future marginal benefits from current employment decisions, $D(n,x)$, and the wage bargaining solution, $w(n,x)$, to which we now turn.

1.2 Wage Setting

The existence of frictions in the labor market implies that it is costly for firms and workers to find alternative employment relationships. As a result, there exist quasi–rents over which the firm and its workers must bargain. The assumption of constant marginal product in the standard MP model has the tractable implication that these rents are the same for all workers within a given firm. It follows that firms can bargain with each of their workers independently, because the rents of each individual employment relationship are independent of the rents of all other employment relationships.

However, because we allow for the possibility of downward–sloped labor demand $(F'' < 0)$, these rents will depend on the number of workers within a firm. Intuitively, the rent that a firm obtains from "the" marginal worker will be lower than the rent obtained on all infra–marginal hires due to diminishing marginal product. An implication of the latter is that the multilateral dimension of the firm's bargain with its many workers becomes important: The rents of each individual employment relationship within a firm are no longer independent.

To take this into account, we adopt the bargaining solution of Stole & Zwiebel (1996) which generalizes the Nash solution to a setting with downward–sloped labor demand.[11] Stole & Zwiebel present a game where the bargained wage is the same as the outcome of simple Nash bargaining over the *marginal* surplus. The game that supports this simple result is one in which a firm negotiates with each of its workers in turn, and where the breakdown of a negotiation with any individual worker leads to the renegotiation of wages with all remaining workers.[12]

[11] This approach was first used by Cahuc & Wasmer (2001) to generate a wage equation for the exogenous job destruction case.

[12] The intuition for the Stole & Zwiebel result is as follows. If the firm has only one worker, the firm and worker simply strike a Nash bargain. If a second worker is added, the firm and the additional worker know that, if their negotiations break down, the firm will agree to a Nash bargain with the remaining worker. By induction, then, the firm approaches negotiations with the nth worker as if that worker were marginal.

In accordance with timing of decisions each period, wages are set after employment has been determined. Thus, hiring costs are sunk at the time of wage setting, and the marginal surplus, which we denote as $J(n,x)$, is equal to the marginal value of labor gross of the costs of hiring:

$$J(n,x) = pxF'(n) - w(n,x) - w_n(n,x)\,n + \beta D(n,x) \tag{5}$$

The surplus from an employment relationship for a worker is the additional utility a worker obtains from working in her current firm over and above the utility she obtains from unemployment. The value of employment in a firm of size n and productivity x, $W(n,x)$, is given by:

$$W(n,x) = w(n,x) + \beta\mathbb{E}\left[sU' + (1-s)\,W(n',x')\,|n,x\right] \tag{6}$$

While employed, a worker receives a flow payoff equal to the bargained wage, $w(n,x)$. She loses her job with (endogenous) probability s next period, upon which she flows into the unemployment pool and obtains the value of unemployment, U'. With probability $(1-s)$, she retains her job and obtains the expected payoff of continued employment in her current firm, $W(n',x')$. Likewise, the value of unemployment to a worker is given by:

$$U = b + \beta\mathbb{E}\left[(1-f)\,U' + fW(n',x')\right] \tag{7}$$

Unemployed workers receive flow payoff b, which represents unemployment benefits and/or the value of leisure to a worker. They find a job next period with probability f, upon which they obtain the expected payoff from employment, $W(n',x')$.

Wages are then the outcome of a Nash bargain between a firm and its workers over the marginal surplus, with worker bargaining power denoted as η:

$$(1-\eta)\left[W(n,x) - U\right] = \eta J(n,x) \tag{8}$$

Given this, we are able to derive a wage bargaining solution with the following simple structure:

Proposition 1 *The bargained wage, $w(n,x)$, solves the differential equation*[13]

$$w(n,x) = \eta\left[pxF'(n) - w_n(n,x)\,n + \beta f\frac{c}{q}\right] + (1-\eta)\,b. \tag{9}$$

Therefore, the wage that solves the bargaining problem is that which maximizes the marginal surplus.

[13] An attractive feature of this solution is its similarity to the solution obtained by Cahuc & Wasmer (2001) for the exogenous job destruction model. It is also consistent with Acemoglu & Hawkins' (2006) Lemma 2, except that it holds both in and out of steady state.

The intuition for (9) is quite straightforward. As in the MP model, wages are increasing in the worker's bargaining power, η, the marginal product of labor, $pxF'(n)$, workers' job finding probability, f, the marginal costs of hiring for a firm, c/q, and workers' flow value of leisure, b. There is an additional term, however, in $w_n(n,x)\,n$. To understand the intuition for this term, consider a firm's negotiations with a given worker. If these negotiations break down, the firm will have to pay its remaining workers a higher wage. The reason is that fewer workers imply that the marginal product of labor will be higher in the firm, which will partially spillover into higher wages ($w_n n < 0$). The more powerful this effect is (the more negative is $w_n n$), the more the firm loses from a given breakdown of negotiations with a worker, and the more workers can extract a higher wage from the bargain.

In what follows, we will adopt the simple assumption that the production function is of the Cobb-Douglas form, $F(n) = n^{\alpha}$ with $\alpha \leq 1$. Given this, the differential equation for the wage function, (9), has the following simple solution:

$$w(n,x) = \eta \left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} + \beta f \frac{c}{q} \right] + (1-\eta)\, b \tag{10}$$

Setting $\alpha = 1$ yields the discrete time analogue to the familiar wage bargaining solution for the MP model.

1.3 The Firm's Optimal Employment Policy

Now that we have obtained a solution for the bargained wage at a given firm, we can combine this with the firm's first–order condition for employment and thereby characterize the firms optimal employment policy, which specifies the firm's optimal employment as a function of its state, $n(n_{-1},x)$. Thus, combining (4) and (9) we obtain:

$$(1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b \right] - \eta\beta f \frac{c}{q} - \frac{c}{q}\mathbf{1}^{+} + \beta D(n,x) = 0 \tag{11}$$

Given (11) we are able to characterize the firm's optimal employment policy as follows:

Proposition 2 *The optimal employment policy of a firm is of the form*

$$n(n_{-1},x) = \begin{cases} R_v^{-1}(x) & \textit{if} \quad x > R_v(n_{-1}) \\ n_{-1} & \textit{if} \quad x \in [R(n_{-1}), R_v(n_{-1})] \\ R^{-1}(x) & \textit{if} \quad x < R(n_{-1}) \end{cases} \tag{12}$$

where the functions $R_v(\cdot)$ and $R(\cdot)$ satisfy

$$(1-\eta)\left[\frac{pR_v(n)\,\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}-b\right]-\eta\beta f\frac{c}{q}+\beta D(n,R_v(n))\equiv\frac{c}{q} \tag{13}$$

$$(1-\eta)\left[\frac{pR(n)\,\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}-b\right]-\eta\beta f\frac{c}{q}+\beta D(n,R(n))\equiv 0. \tag{14}$$

The firm's optimal employment policy will be similar to that depicted in Figure 1. It is characterized by two reservation values for the firm's idiosyncratic shock, $R(n_{-1})$ and $R_v(n_{-1})$. Specifically, for sufficiently bad idiosyncratic shocks ($x < R(n_{-1})$ in the figure), firms will shed workers until the first-order condition in the separation regime, (14), is satisfied. Moreover, for sufficiently good idiosyncratic realizations ($x > R_v(n_{-1})$ in the figure), firms will post vacancies and hire workers until the first-order condition in the hiring regime, (13), is satisfied. Finally, for intermediate values of x, firms freeze employment so that $n = n_{-1}$. This occurs as a result of the kink in the firm's profits at $n = n_{-1}$, which arises because hiring is costly to firms, while separations are costless.

To complete our characterization of the firm's optimal employment policy, it remains to determine the marginal effect of current employment decisions on future profits of the firm, $D(n,x)$. It turns out that we can show that $D(n,x)$ has the following recursive structure:

Proposition 3 *The marginal effect of current employment on future profits, $D(n,x)$, is given by*

$$D(n,x)=d(n,x)+\beta\int_{R(n)}^{R_v(n)}D(n,x')\,dG(x'|x) \tag{15}$$

where

$$d(n,x)\equiv\int_{R(n)}^{R_v(n)}\left\{(1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}-b\right]-\eta\beta f\frac{c}{q}\right\}dG(x'|x)+\int_{R_v(n)}^{\infty}\frac{c}{q}dG(x'|x). \tag{16}$$

Equation (15) is a contraction mapping in $D(n,\cdot)$, and therefore has a unique fixed point.

The intuition for this result is as follows. Because of the existence of kinked adjustment costs (costly hiring and costless separations) the firm's employment will be frozen next period with positive probability. In the event that the firm freezes employment next period ($x' \in [R(n), R_v(n)]$), the current employment level persists into the next period and so do the marginal effects of the firm's current employment choice. Proposition 3 shows that these

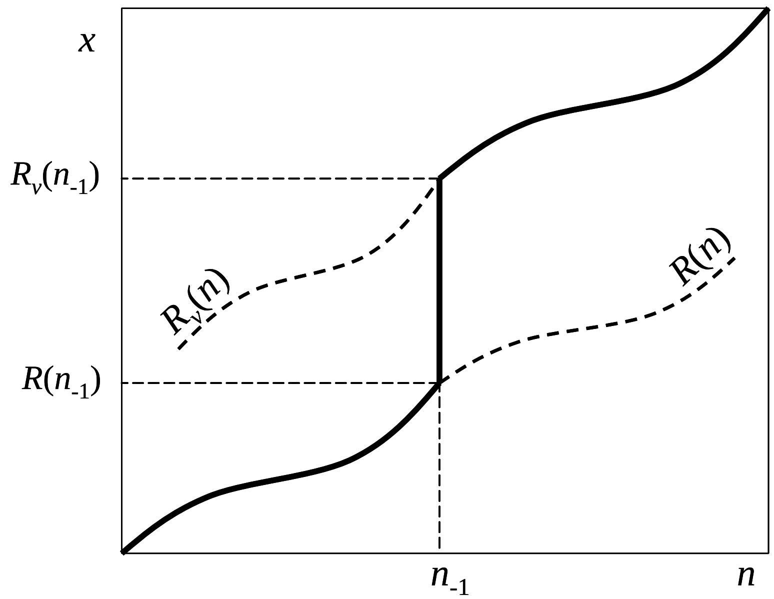

Figure 1: Optimal Employment Policy of a Firm

marginal effects persist into the future in a *recursive* fashion. Propositions 2 and 3 thus summarize the microeconomic behavior of firms in the model.[14]

To get a sense for how the microeconomic behavior of the model works, we next derive the response of an individual firm's employment policy function to changes in (exogenous) aggregate productivity, p, and the (endogenous) aggregate vacancy–unemployment ratio, θ. To do this, we assume that the evolution of idiosyncratic shocks is described by:

$$x' = \begin{cases} x & \text{with probability } 1-\lambda \\ \tilde{x} \overset{c.d.f.}{\sim} \tilde{G}(x) & \text{with probability } \lambda \end{cases} \tag{17}$$

Thus, idiosyncratic shocks display some persistence ($\lambda < 1$) with innovations drawn from the distribution function $\tilde{G}$. Given this, we can establish the following result:

Proposition 4 *If idiosyncratic shocks, x, evolve according to (17), then the effects of the aggregate state variables p and θ on a firm's optimal employment policy are*

$$\frac{\partial R_v}{\partial p} < 0; \frac{\partial R}{\partial p} < 0; \frac{\partial R_v}{\partial \theta} > 0; \text{ and } \frac{\partial R}{\partial \theta} > 0 \iff n \text{ is sufficiently large.} \tag{18}$$

[14] It is straightforward to show that equations (10) to (16) reduce down to the discrete time analogue to the Mortensen & Pissarides (1994) model when $\alpha = 1$.

The intuition behind these marginal effects is quite simple. First, note that increases in aggregate productivity, p, shift a firm's employment policy function downwards in Figure 1. Thus, unsurprisingly, when labor is more productive, a firm of a given idiosyncratic productivity, x, is more likely to hire workers, and less likely to shed workers. Second, increases in the vacancy–unemployment ratio, θ, unambiguously reduce the likelihood that a firm of a given idiosyncratic productivity will hire workers (R_v increases for all n). The reason is that higher θ implies a lower job–filling probability, q, and thereby raises the marginal cost of hiring a worker, c/q. Moreover, higher θ implies a tighter labor market and therefore higher wages (from (9)) so that the marginal cost of labor rises as well. Both of these effects cause firms to cut back on hiring. Finally, increases in the vacancy–unemployment ratio, θ, will reduce the likelihood of shedding workers for small firms, but will raise it for large firms. This occurs because higher θ has countervailing effects on the separation decision of firms. On the one hand, higher θ reduces the job–filling probability, q, rendering separation decisions less reversible (since future hiring becomes more costly), so that firms become less likely to destroy jobs. On the other hand, higher θ implies a tighter labor market, higher wages, and thereby a higher marginal cost of labor, rendering firms more likely to shed workers. The former effect is dominant in small firms because the likelihood of their hiring in the future is high.

2 Aggregation and Steady State Equilibrium

2.1 Aggregation

Since we are ultimately interested in the equilibrium behavior of the aggregate unemployment rate, in this section we take on the task of aggregating up the microeconomic behavior of section 1 to the macroeconomic level. This exercise is non–trivial because each firm's employment is a non–linear function of the firm's lagged employment, n_{-1}, and its idiosyncratic shock realization, x. As a result, there is no representative firm interpretation that will aid aggregation of the model.

To this end, we are able to derive the following result which characterizes the steady state aggregate employment stock and flows in the model:

Proposition 5 *If idiosyncratic shocks, x, evolve according to (17), the steady state c.d.f. of* *employment across firms is given by*

$$H(n) = \frac{\tilde{G}[R(n)]}{1 - \tilde{G}[R_v(n)] + \tilde{G}[R(n)]}. \tag{19}$$

Thus, the steady state aggregate employment stock is given by

$$N = \int n dH(n) \tag{20}$$

and the steady state aggregate number of separations, S, and hires, M, is equal to

$$S = \lambda \int [1 - H(n)] \tilde{G}[R(n)] dn = \lambda \int H(n) \left(1 - \tilde{G}[R_v(n)]\right) dn = M. \tag{21}$$

Proposition 5 is useful because it provides a tight link between the solution for the microeconomic behavior of an individual firm and the macroeconomic outcomes of that behavior. Specifically, it shows that once we know the optimal employment policy function of an individual firm (that is, the functions $R(n)$ and $R_v(n)$) then we can directly obtain solutions for the aggregate employment stock and flows. An important feature to note about Proposition 5 is its generality. Specifically, it allows one to generate analytically the steady state aggregate employment stock and labor flows for any given employment policy function at the microeconomic level, not just that derived above. In addition, the expressions for aggregate employment and flows are straightforward to compute numerically.

The three components of Proposition 5 are also quite intuitive. The steady state distribution of employment across firms, (19), is obtained by setting the flows into and out of the mass $H(n)$ equal to each other. The inflow into the mass comes from firms who reduce their employment from above n to below n. There are $[1 - H(n)]$ such firms, and since they are reducing their employment, it follows from (12) that each firm will reduce its employment below n with probability equal to $\Pr[x < R(n)] = \lambda\tilde{G}[R(n)]$. Thus, the inflow into $H(n)$ is equal to $\lambda[1 - H(n)] \tilde{G}[R(n)]$. Similarly, one can show that the outflow from the mass is equal to $\lambda H(n) \left(1 - \tilde{G}[R_v(n)]\right)$. Setting inflows equal to outflows yields the expression for $H(n)$ in (19).[15] Given this, the expression for aggregate employment, (20), follows directly.

The intuition for the final expression for aggregate flows in Proposition 5, (21), is as follows. Recall that the mass of firms whose employment switches from above some number n to below n is equal to $\lambda[1 - H(n)] \tilde{G}[R(n)]$. Equation (21) states that the aggregate

[15] This mirrors the mass-balance approach used in Burdett & Mortensen (1998) to derive the equilibrium wage distribution in a search model with wage posting.

number of separations in the economy is equal to the *cumulative* sum of these downward switches in employment over n. To get a sense for this, consider the following simple discrete example. Imagine an economy with two separating firms: one that switches from three employees to one, and another that switches from two employees to one. It follows that two firms have switched from > 2 employees to ≤ 2 employees, and one firm switched from > 1 to ≤ 1 employee. Thus, the cumulative sum of downward employment switches is three, which is also equal to the total number of separations in the economy.

2.2 Steady State Equilibrium

Given (19), (20), and (21), the conditions for aggregate steady state equilibrium can be obtained as follows. First note that each firm's optimal policy function, summarized by the functions $R(n)$ and $R_v(n)$ in Proposition 2, depends on two aggregate variables: The (exogenous) state of aggregate productivity, p; and the (endogenous) ratio of aggregate vacancies to aggregate unemployment, $V/U \equiv \theta$, which uniquely determines the flow probabilities q and f.

In the light of Proposition 5, we can characterize the aggregate steady state of the economy for a given p in terms of two relationships. The first, the *job creation condition*, is simply equation (20), which we re–state here in terms of unemployment, making explicit its dependence on the aggregate vacancy–unemployment ratio, θ:

$$U(\theta)_{JC} = L - \int n dH(n; \theta) \tag{22}$$

(22) simply specifies the level of aggregate employment that is consistent with the inflows to (hires) and outflows from (separations) aggregate employment being equal as a function of θ. The second steady state condition is the *Beveridge Curve* relation. This is derived from the difference equation that governs the evolution of unemployment over time:

$$\Delta U' = S(\theta) - f(\theta) U \tag{23}$$

(23) simply states that the change in the unemployment stock over time, $\Delta U'$, is equal to the inflow into the unemployment pool – the number of separations, S – less the outflow from the unemployment pool – the job finding probability, f, times the stock of unemployed workers, U. In steady state, aggregate unemployment will be stationary, so that we obtain

the steady state unemployment relation:

$$U(\theta)_{BC} = \frac{S(\theta)}{f(\theta)} \tag{24}$$

The steady state value of the vacancy–unemployment ratio, θ, is co–determined by (22) and (24).

3 Introducing Aggregate Shocks

The previous section characterized the determination of steady state equilibrium in the model. However, in what follows, we are interested in the dynamic response of unemployment, vacancies and worker flows to aggregate shocks. To address this, we need to characterize the dynamics of the model out of steady state. The latter is not a trivial exercise in the context of the present model. Out of steady state, firms in the model need to forecast future wages and therefore, from equation (9), future labor market tightness. Inspection of the steady state equilibrium conditions (22) and (24) reveals that, in order to forecast future labor market tightness, firms must predict the evolution of the entire distribution of employment across firms, $H(n)$, an infinite order state variable.

Our approach to this problem mirrors the method proposed by Krusell & Smith (1998). We consider shocks to aggregate labor productivity that evolve according to the simple random walk:

$$p' = \begin{cases} p + \sigma_p & \text{w.p.} \quad 1/2, \\ p - \sigma_p & \text{w.p.} \quad 1/2. \end{cases} \tag{25}$$

Following Krusell & Smith, we conjecture that a forecast of the *mean* of the distribution of employment across firms, $N = \int n dH(n)$, provides an accurate forecast of future labor market tightness. We then exploit the fact that shocks to aggregate labor productivity, denoted by σ_p in equation (25), are small in U.S. data.[16] This allows us to approximate the evolutions of aggregate employment, N, and labor market tightness, θ, around their steady state values N^* and θ^* as follows:

$$\begin{aligned} N' &\approx N^* + \nu_N (N - N^*) + \nu_p (p' - p), \\ \theta' &\approx \theta^* + \theta_N (N' - N^*) + \theta_p (p' - p), \end{aligned} \tag{26}$$

[16] Examples of other studies that have exploited the fact that aggregate shocks are small include Mortensen & Nagypal (2007) and Gertler & Leahy (2008).

for $\sigma_p \approx 0$. Under these conditions, we can approximate the optimal employment policy of an individual firm out of steady state. To see how this might be done, note from the first order conditions (13) and (14) that to derive optimal employment in the presence of aggregate shocks, one must characterize the marginal effect of current employment decisions on future profits, $D(\cdot)$, out of steady state.

Proposition 6 *If a) aggregate shocks evolve according to (25); b) a forecast of N provides an accurate forecast of future θ; c) aggregate shocks are small ($\sigma_p \approx 0$); and d) idiosyncratic shocks evolve according to (17), then the marginal effect of current employment on future profits is given by*

$$D(n, x; N, p, \sigma_p) \approx D(n, x; N^*, p, 0) + D_N^* (N - N^*), \tag{27}$$

where D_N^* is a known function of the parameters of the forecast equation (26) and the steady state employment policy defined in (13) and (14).

Proposition 6 shows that, in the presence of aggregate shocks, the forward looking component to the firm's decision, $D(n, x; N, p, \sigma_p)$, is approximately equal to its value in the absence of aggregate shocks, $D(n, x; N^*, p, 0)$, plus a known function of the deviation of aggregate employment from steady state, $D_N^* (N - N^*)$. Practically, Proposition 6 allows us to derive analytically an approximate solution for the optimal policy function in the presence of aggregate shocks, for given values of the parameters of the forecast equation (26).

To complete our description of the dynamics of the model, we need to aggregate the microeconomic behavior summarized in the employment policies of individual firms. A simple extension of the result of Proposition 5 implies that the aggregate number of separations and hires in the economy at a point in time are respectively given by:

$$\begin{aligned} S(N,p) &= \lambda \int [1 - H_{-1}(n_{-1})] \tilde{G}[R(n_{-1}; N, p)] \, dn_{-1}, \\ M(N,p) &= \lambda \int H_{-1}(n_{-1}) \Big(1 - \tilde{G}[R_v(n_{-1}; N, p)]\Big) \, dn_{-1}, \end{aligned} \tag{28}$$

where $H_{-1}(n_{-1})$ is the distribution of lagged employment across firms. Notice that the timing is emphasized in the out of steady state case.

A number of observations arise from this. First, the aggregate flows depend on the level of aggregate employment, N. Recalling the accumulation equation for N yields:

$$N = N_{-1} + M(N,p) - S(N,p). \tag{29}$$

It follows that, to compute aggregate employment, all one need do is find the fixed point value of N that satisfies equation (29). This allows us to compute equilibrium labor market tightness by noting that

$$f(\theta) = M/(L-N). \tag{30}$$

A second observation from equation (28) is that, in order to compute the path of aggregate unemployment flows, and hence employment, we need to describe the evolution of the distribution of employment across firms, $H(n)$. It turns out that the evolution of $H(n)$ can be inferred by a simple extension of the discussion following Proposition 5. Recall that the change in the mass $H(n)$ over time is simply equal to the inflows less the outflows from that mass. Following the logic of Proposition 5 provides a difference equation for the evolution of $H(n)$:

$$H(n) = H_{-1}(n) + \lambda\tilde{G}[R(n;N,p)][1-H_{-1}(n)] - \lambda\left(1-\tilde{G}[R_v(n;N,p)]\right)H_{-1}(n). \tag{31}$$

This allows us to update the aggregate flows $S(N,p)$ and $M(N,p)$ over time, and hence derive the evolution of equilibrium employment.

The previous results allow us to compute the evolution of aggregate employment and labor market tightness for a given configuration of the parameters of the forecast equations (26). This of course does not guarantee that those parameters are consistent with the behavior that they induce. To complete our characterization of equilibrium in the presence of aggregate shocks, we follow Krusell & Smith and iterate numerically over the parameters $\{\nu_N, \nu_p, \theta_N, \theta_p\}$ to find the fixed point. In the simulations of the model that follow, the fixed point of the conjectured forecast equations in (26) provides a very accurate forecast in the sense that the R^2s of regressions based on (26) exceed 0.999.

4 Quantitative Applications

Shimer (2005) demonstrated that the standard MP model cannot generate enough cyclical amplitude in the job finding rate to match that observed in US data. A natural question is whether the generalized model analyzed here can alleviate this problem. To this end, we perform the following illustrative numerical exercise. We normalize aggregate productivity, p, to 1 and calculate the dynamic response of the job finding rate, f, and the unemployment inflow rate, $s \equiv S/N$, to fluctuations in p.

4.1 Calibration

The first part of our calibration strategy is very conventional. We take a time period to be equal to one week, which in practice acts as a good approximation to the continuous time nature of unemployment flows. We set the dispersion of the innovation to aggregate labor productivity, σ_p, in order to match the empirical dispersion of output per worker in the US economy of 0.02 (see Shimer, 2005). We then assume that the matching function is of the conventional Cobb-Douglas form, $M = \mu U^{\phi} V^{1-\phi}$.[17] We set $\phi = 0.6$ based on the estimates reported in Petrongolo & Pissarides (2001). We target a weekly unemployment outflow probability of $f = 0.1125$, to be approximately consistent with a monthly outflow hazard of 0.45. In addition, we follow Pissarides (2007) and target a mean value of the vacancy–unemployment ratio of $\theta = 0.72$. Noting from the matching function that $f = \mu\theta^{1-\phi}$, the latter implies that $\mu = 0.129$ on a weekly basis.

A more distinctive feature of our strategy is the calibration of idiosyncratic productivity shocks, x. These are assumed to arrive with probability λ each period, as in (17). Innovations to idiosyncratic productivity are lognormally distributed according to $\ln \tilde{x} \sim N\left(-\frac{1}{2}\sigma_x^2, \sigma_x^2\right)$, so that the mean innovation is normalized to one. Given this, we solve for firms' optimal employment policy (see Appendix A for details). An important outcome is that we can derive the steady state distribution of employment growth:

Proposition 7 *The steady state density of employment growth, $\delta = \Delta \ln n$, across firms is given by:*

$$h_{\Delta}(\delta) = \begin{cases} \lambda \int e^{\delta} n \tilde{G}' \left[R'\left(e^{\delta} n\right)\right] dH(n) & \text{if} \quad \delta < 0, \\ \lambda \int \left(\tilde{G}\left[R_v(n)\right] - \tilde{G}\left[R(n)\right]\right) dH(n) & \text{if} \quad \delta = 0, \\ \lambda \int e^{\delta} n \tilde{G}' \left[R_v'\left(e^{\delta} n\right)\right] dH(n) & \text{if} \quad \delta > 0. \end{cases} \quad (32)$$

Proposition 7 provides us with a novel approach to calibrating the parameters of the process of idiosyncratic shocks, λ and σ_x. Specifically, there is abundant evidence on the properties of the cross sectional distribution of employment growth since the seminal work of Davis & Haltiwanger (1992). Empirically, this distribution is characterized by a dominant spike at zero employment growth, with relatively symmetric tails corresponding to job creation and job destruction (see Davis & Haltiwanger, 1992, Figure 1.A). Note that this is exactly the form of the employment growth distribution implied by the model in Proposition 7.

[17] An issue that can arise when using a Cobb–Douglas matching function in a discrete time setting is that the flow probabilities f and q are not necessarily bounded above by 1. This issue does not arise here due to the short time period of one week.

In practice, we choose λ to match the spike at zero in this distribution, and σ_x to match the dispersion of the tails of employment growth. Intuitively, the cross sectional distribution of employment growth is a manifestation of idiosyncratic shocks across firms. The more often these shocks arrive (the higher is λ in the model), the more likely a firm is to alter its employment, and the smaller is the implied spike at zero employment growth. Likewise, the greater the dispersion of innovations, σ_x, the larger the implied adjustment that firms will make, hence determining the tails of the distribution. In practice, we target an annual spike of 30 percent, based on Figure 1.A of Davis & Haltiwanger (1992),[18] and an annual standard deviation of employment growth of 0.37, based on estimates reported in Davis et al. (2006).

We then use the results of Proposition 5 and sections 2 and 3 to derive numerically the evolution of the unemployment stocks and flows implied by the model. Given these, we target the vacancy cost c to generate per worker hiring costs c/q approximately equal to 14% of quarterly worker compensation. This is in accordance with the results of Silva & Toledo (2007) who used the Saratoga Institute's (2004) estimate of the labor costs of posting vacancies. In the context of the model, this implies a value of c approximately equal to 0.27 of the average worker's wage.[19]

To pin down workers' bargaining power, η, we target the elasticity of average wages of newly hired workers with respect to output per worker to be equal to 0.94, based on the results of Haefke et al. (2007).[20] We target the elasticity of the wages of newly hired workers rather than the elasticity of wages of all workers for two reasons. First, it is well known empirically that the wages of workers in ongoing relationships are rigid (see among others Card & Hyslop, 1997), which is at odds with the assumption of Nash wage setting that we employ here.[21] Second, it is also well known that it is the flexibility of wages of new hires, rather than of ongoing workers, that is relevant to the cyclicality of the job finding rate implied by search and matching models of the labor market such as the one studied here (Shimer, 2004; Hall, 2005).

[18] Since we abstract from firm entry and exit, we reweight the spike in Davis & Haltiwanger (1992) to obtain a spike conditional on no entry or exit of approximately 30 percent.

[19] We want to equate the per worker hiring cost c/q to 14% of quarterly wages, $0.14 \cdot [12 \cdot \mathbb{E}(w)]$, since there are 12 weeks per quarter. Then note that the implied weekly job filling probability is given by $q = \mu\theta^{-\phi} = 0.129 \cdot 0.72^{-0.6} = 0.16$. Piecing this together yields $c/\mathbb{E}(w) = 0.16 \cdot 0.14 \cdot 12 = 0.27$.

[20] We target an elasticity of 0.94 based on Haefke et al.'s baseline results. It is worth bearing in mind that this is at the upper end of the range of estimates presented in their paper.

[21] Indeed, in the calibration that follows, the Nash wage setting assumption implies an elasticity of average worker wages with respect to output per worker of approximately 1. This overstates the cyclicality of ongoing wages observed in the data, which display an elasticity with respect to output per worker of approximately 0.5 (see Solon, Barsky, & Parker, 1994; Pissarides, 2007).

Table 1: Calibrated Parameters (Weekly)

Parameter	Meaning	Value	Reason
ϕ	Matching elasticity	0.600	Petrongolo & Pissarides (2001)
μ	Matching efficiency	0.129	Pissarides (2007)
α	$F(n) = n^{\alpha}$	0.590	Labor share = 0.72
β	Discount factor	0.999	Quarterly interest rate = 0.012
b	Value of leisure	0.385	Mean inflow rate = 0.0078
c	Flow vacancy cost	0.120	Hiring cost = 14% qtly. wages
η	Worker bargaining power	0.443	Elasticity of new hires' wage
L	Labor force	3.331	Mean job finding prob. = 0.1125
λ	x: arrival rate	0.063	Spike at $\Delta \ln n = 0$
σ_x	x: standard deviation	0.217	Dispersion of $\Delta \ln n$

Table 1: Calibrated Parameters. Flow parameters are reported at a weekly frequency, consistent with the timing of the model.

This leaves three remaining parameters: the production function parameter, α; workers' opportunity cost of employment, b; and the potential labor force, L.[22] To determine α, we target an aggregate labor share based on the estimates reported in Gomme & Rupert (2007). Their estimates suggest a labor share for market production of 0.72.

We calibrate the flow payoff from unemployment b by targeting a mean weekly unemployment inflow probability of $s = 0.0078$, consistent with estimates of the unemployment inflow rate in Shimer (2007). Intuitively, the higher is the worker's payoff from unemployment, the lower are the implied rents to employment relationships in the model. For a given level of dispersion in idiosyncratic shocks implied by our calibration of λ and σ_x above, a lower surplus implies that jobs will be destroyed more frequently, raising the inflow rate into unemployment, s. Thus, we set the flow payoff from unemployment b in such a way to yield employment rents that match observed levels of the unemployment inflow rate.

Finally, we choose the size of the labor force, L, to match a mean unemployment rate of 6.5 percent. Since we have chosen b to match the observed mean inflow rate, this is equivalent to choosing L to match an average weekly job finding rate of 0.1125, as observed in U.S. data.

[22] Taken literally, L represents the labor force as a fraction of the number of the number of firms in the model economy. In reality, however, L is more accurately described as the labor force as a fraction of the number of production units in the economy. The latter may correspond to a small firm, a small division within a large firm etc. For this reason, we do not calibrate L directly.

4.2 Implications for Unemployment Volatility

The parameter results of this numerical exercise are reported in Table 1, and the implied model outcomes are in Table 2. Note that the model is calibrated to match the mean levels of the job finding rate and the unemployment inflow rate. The model is *not* calibrated to match the elasticities of these flow hazard rates with respect to output per worker. The results are remarkably encouraging: The model implies an elasticity of the job finding rate that is surprisingly close to that observed in the data. In addition, the model almost exactly replicates the elasticity of employment to unemployment transition rate observed in the data.

These results make substantial progress relative to the standard MP model. To see this, we provide two comparison exercises on the standard model.[23] First, taking as given the process for idiosyncratic shocks implied by the distribution of employment growth derived above, we calibrate the MP model to match the mean levels of the job finding rate f, the unemployment inflow rate s, and the elasticity of s with respect to output per worker. This allows the model to speak to the implied elasticity of the job finding rate, and thereby the elasticities of vacancies and labor market tightness. The outcomes of this first exercise are in column (1) of Table 3. This confirms the result emphasized in previous literature that the standard MP model is unable to generate enough cyclicality in job creation. Shimer's (2005a) calibration of the standard MP model yields an elasticity of f equal to 0.48. Mortensen & Nagypal (2007a) favor a different calibration of the standard MP model that yields an elasticity of f equal to 1.56 (see their section 3.2). Pissarides' (2007) calibration of the standard model with endogenous job destruction obtains an elasticity of f equal to 1.54. Thus, the standard MP model, with or without endogenous job destruction, appears to be able explain approximately one half of the observed elasticity of the job finding rate. The results of Table 3 suggest that the generalized model studied in the present paper can plausibly account for almost all of the observed cyclical comovement between f and output per worker.

Our second exercise provides a new perspective on the MP model's inability to match the cyclicality of unemployment flows. In this case, we again take as given the process for idiosyncratic shocks implied by the empirical distribution of employment growth. However, instead of targeting the mean level of the inflow rate into unemployment s, we now allow the MP model to match the empirical elasticity of the job finding rate f. The results of this exercise are reported in column (2) of Table 3. This reveals that, for plausible idiosyncratic

[23] In practice, we use a variant of the Mortensen & Pissarides (1994) model suggested by Mortensen & Nagypal (2007b). This allows for a distribution of idiosyncratic shocks with unbounded upper support, such as the lognormal shocks we use here.

Table 2: Model Outcomes (Weekly)

Outcome	Mean Level		Elasticity w.r.t. $\frac{Y}{N}$	
	Data	Model	Data	Model
f	0.1125	0.1126	2.95	2.73
s	0.0078	0.0078	–2.48	–2.54
V	–	–	3.68	1.91
θ	0.72	0.72	7.56	6.82

Parameter values as in Table 2.

Table 2: Model Outcomes. Flow outcomes are reported at a weekly frequency. Following Mortensen and Nagypal (2007a), elasticities are obtained by regressing the log deviation from trend of f and s on the log deviation from trend of non-farm business output per worker obtained from the Bureau of Labor Statistics. Following Shimer (2005), series are detrended using a Hodrick–Prescott filter with smoothing parameter 100000. The series for f and s are respectively the job-finding rate and the employment to unemployment transition rate derived in Shimer (2007).

shocks, the MP model must dramatically overstate the magnitude of unemployment inflows in order to match the cyclical comovement of f. In order to match the variation in the job finding rate, the model requires a small surplus to employment relationships, as emphasized by Mortensen & Nagypal (2007a).[24] Mortensen & Nagypal go on to argue that the required surplus is unrealistically small. The results of Table 3 formalize this intuition: For realistic variation in idiosyncratic shocks to firms, a surplus small enough to match the cyclicality of f implies an employment to unemployment transition rate that is more than double what is observed empirically. Intuitively, a small surplus implies that small idiosyncratic shocks to employment relationships are enough to exhaust the surplus and lead to destruction of a match. Consequently, realistic dispersion in idiosyncratic shocks generates excessive worker turnover.

Thus, the standard MP model faces a tension: To match plausible levels of unemployment inflows, the model must generate a sufficiently large surplus at the expense of matching the comovement of the job finding rate. Conversely, to generate the cyclical variation in the job finding rate, the the surplus must be small, which in turn yields excessive employment

[24] A common diagnostic for the size of the flow surplus is the ratio between worker's payoff from unemployment, b, and the average product of labor. For Column 1 of Table 3, this ratio equals 0.6520; for Column 2, it equals 0.7273. Thus, the MP model demands a smaller surplus to match the volatility of the job finding rate.

Table 3: MP Model Outcomes (Weekly)

Outcome	**(1)**		**(2)**	
	Mean	Elasticity	Mean	Elasticity
f	0.1125	1.66	0.1125	2.95
s	0.0078	–2.48	0.0183	–2.48
V	–	–0.58	–	0.21
θ	0.72	3.29	0.72	4.86

Table 3: Model Outcomes from Calibration of Mortensen and Pissarides model.

to unemployment transitions.

4.3 Cyclicality of Vacancies

Until now we have ignored the cyclicality of vacancies generated by our generalized model. Readers of Shimer (2005), however, will recall that the standard MP search and matching model also fails to match the observed cyclical volatility in the vacancy rate in the U.S. Specifically, as shown in Table 2, the empirical elasticity of the vacancy rate with respect to output per worker in the U.S. derived by Shimer (2005) is equal to 3.68. The implied elasticity from Shimer's calibration of the standard MP model is $0.995 \times 0.027/0.020 = 1.34$ (see Shimer, 2005, Table 3). Moreover, the calibration of the MP model with $\alpha = 1$ in column (1) of Table 3 reveals that the standard model with endogenous job destruction performs even worse on this dimension, yielding a *counter*cyclical vacancy elasticity of -0.58. This arises because countercyclical job destruction leads to an offsetting increase in hires in times of recession to maintain balance between unemployment inflows and outflows, and thereby stymies the procyclicality of vacancies (Shimer, 2005; Mortensen & Nagypal, 2007b).

The analogous elasticity generated by our simulation of the generalized model studied here is 1.91. This is clearly a substantial improvement over the standard model, especially given that the generalized model incorporates countercyclical job destruction. However, there remains a question of why the generalized model, which matches the cyclicality of job finding and employment to unemployment transition rates so well, cannot fully explain the cyclicality of vacancies. We believe that the answer is that a complete understanding of the cyclical behavior of vacancies requires an understanding of the processes underlying job-to-job employment flows, a phenomenon that we abstract from in our analysis of unemployment flows. To see why, note the following identity that relates the job-filling rate, q, vacancies (or job openings), V, and the *numbers* (not the hazard rates) flowing from unemployment to employment, UE, and from job to job, EE:

$$qV = UE + EE \tag{33}$$

Log differentiation of this identity yields:

$$d\log q + d\log V = \varphi d\log UE + (1-\varphi)\, d\log EE \tag{34}$$

where φ is the share of total hires that originates from unemployment. Recent research has shown that job-to-job flows (EE) are substantially procyclical and account for approximately 60% of total hires using Current Population Survey data from 1994 onwards (Fallick & Fleischman, 2004). Equation (34) shows that the procyclicality of EE flows must therefore contribute substantially to the procyclicality of vacancies observed in the data. For this reason, we feel that the elasticity of vacancies obtained from the generalized model without on-the-job search may in fact be quite reasonable: If we were able to match the empirical elasticity, we would implicitly be *over*-explaining the procyclicality of vacancies. For the same reason, however, we also feel that extending the model to account for job-to-job flows is an important task for future research.

4.4 Understanding Amplification

Figure 2 plots the response of the Job Creation, (22), and Beveridge Curve, (24), conditions to a decline in p for the simulation detailed in Table 2. The figure reveals that allowing for downward sloped labor demand amplifies the response of the vacancy-unemployment ratio to aggregate disturbances primarily through movements in the Job Creation condition. This naturally raises the question of why the JC condition moves so much. The following result provides a sense for where this amplification comes from by taking a log–linear approximation to a firm's marginal surplus around mean employment:

Proposition 8 *For small λ, the horizontal shift in the JC condition induced by a change in aggregate productivity, p, is given approximately by*

$$\left.\frac{d\ln\theta}{d\ln p}\right|_{JC} \approx \frac{(1-\eta)\,\tilde{p}}{\omega\phi\left[(1-\eta)(\tilde{p}-b)-\eta\beta c\theta\right]+\eta\beta c\theta}, \tag{35}$$

where ω is the steady state employment share of hiring firms, and $\tilde{p} \equiv \rho\overline{apl} + (1-\rho)\,\overline{mpl}$

where $\overline{apl}$ and $\overline{mpl}$ are respectively the average and marginal product of labor of the average–sized firm, and $\rho \equiv \frac{\alpha\eta}{1-\eta(1-\alpha)}$.

Corollary 1 *The elasticity of the vacancy–unemployment ratio to aggregate productivity in the $\alpha = 1$ case (Mortensen & Pissarides, 1994) is approximately equal to*

$$\frac{d\ln\theta}{d\ln p} \approx \frac{(1-\eta)\,p}{\phi\left[(1-\eta)\,(p-b)-\eta\beta c\theta\right]+\eta\beta c\theta}. \tag{36}$$

Equation (36) extends results presented in Mortensen & Nagypal (2007a) for the standard MP model with exogenous job destruction to the endogenous job destruction case. It echoes Mortensen & Nagypal's results in that it shows that the cyclical response of the vacancy-unemployment ratio, θ, is amplified in the endogenous job destruction case when the average flow surplus to employment relationships, $p-b$, is small. Intuitively, when the flow surplus is small, small reductions in aggregate productivity, p, can easily exhaust that surplus and lead firms to cut back substantially on hiring. Thus, to incentivize firms to hire once more and thereby restore equilibrium, the model must allow the labor market to slacken, and labor market tightness to fall, substantially.

Equation (35) generalizes this result to the case of downward sloped labor demand and endogenous job destruction. Inspection of (36) and (35) reveals that there are two ways that the addition of downward sloped labor demand can potentially yield amplification of the response of labor market tightness (and thereby of the job finding rate, $f = \mu\theta^{1-\phi}$) to changes in aggregate productivity. The first is that the effective surplus that matters for amplification is now given by by $\tilde{p}-b$, and this is smaller than the average flow surplus. The reason is that the effective flow surplus, $\tilde{p}-b$, is now a weighted average of the average and marginal flow surplus. When the demand for labor slopes downward, the marginal surplus will be smaller than the average surplus, because infra–marginal employment relationships are more productive. This provides a sense for why the numerical exercise above is able to generate greater volatility in θ even when the average flow surplus is relatively large: It is because the *marginal* flow surplus is relatively small in the simulation.

It's important to stress that while the model implies a small marginal surplus, it does allow for a substantial average flow surplus to employment relationships, in contrast to the standard MP model. For instance, Mortensen and Nagypal (2007a) report that their preferred estimate of the average flow match surplus is $\frac{1}{b/(Y/N)} - 1 = \frac{1}{0.73} - 1 = 37$ percent. The corresponding value implied by our calibration in Table 2 is an average flow surplus

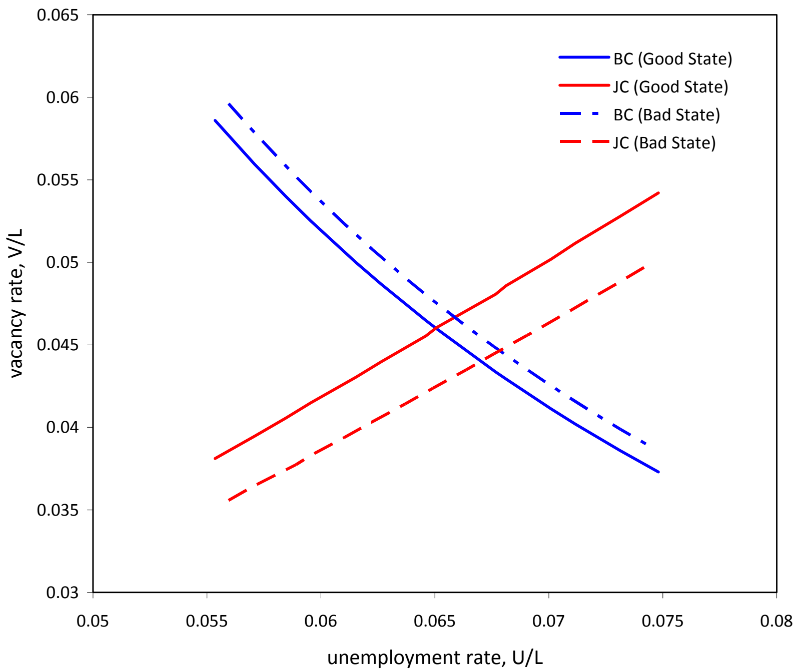

Figure 2: Job Creation and Beveridge Curve in the Simulated Model

equal to $\frac{1}{0.61} - 1 = 64$ percent.[25]

Equation (35) also suggests that there is an additional effect at work in the form of the variable ω, the steady state employment share of hiring firms. To understand the significance of this term, note that in the standard MP model with flat labor demand (the special case where $\alpha = 1$), ω is equal to 1. When $\alpha = 1$, a firm that reduces its employment will shed all of its workers since, if one worker is unprofitable at a firm, all workers are unprofitable. Similarly, when $\alpha = 1$, if it is profitable to hire one worker, it is profitable to hire any number of workers. Thus shedding firms have zero employment, and all of steady state employment is accounted for by hiring firms in the standard MP model.

The latter is a useful point of contrast with the model with downward sloped labor demand and endogenous job destruction. Because of downward sloped labor demand, shedding firms do not reduce their employment to zero because reducing employment replenishes the marginal product of labor in those firms. Likewise, hiring firms' desired employment level is bounded because additional hiring depletes the marginal product of labor. Hence ω will

[25] The worker's surplus in our simulation is also substantial compared to previous calibrations using the standard MP model. Our calibration implies workers obtain a $(\mathbb{E}[w] - b)/b = 18$ percent flow surplus from employment over unemployment.

be less than unity, and inspection of (35) and (36) reveals that this will lead to greater amplification relative to the standard MP model.[26]

The intuition for this effect is related to the interaction of downward sloped labor demand and heterogeneous firms. Following a reduction in aggregate productivity, shedding firms wish to shed more workers, and hiring firms wish to hire fewer workers. Thus, inflows into the unemployment pool rise, and outflows from the unemployment pool fall, *ceteris paribus*, and unemployment rises. To return the model to steady state, hiring firms must be convinced to hire enough workers to equate inflows to outflows once more. The model achieves this by allowing the job filling probability, $q(\theta)$, to rise (and labor market tightness, θ, to fall) so that hiring becomes less costly for firms. However, when the demand for labor slopes downward, additional hiring reduces the marginal product of labor, making additional employment relationships less attractive to hiring firms. As a result, the job filling probability, $q(\theta)$ must rise (and hence θ must fall) more to convince these firms to increase hiring and return the economy back to steady state once more.

4.5 Propagation

A less well–documented limitation of the standard MP model relates to the propagation of the response of equilibrium labor market tightness to aggregate shocks. In the MP model, tightness is a jump variable and therefore moves contemporaneously with changes in output per worker. Empirically, however, the vacancy–unemployment ratio displays more persistence than output per worker (see Shimer, 2005).

In contrast, in the model presented in sections 2 and 3, labor market tightness does not move contemporaneously with vibrations in aggregate labor productivity. Rather, the determination of θ over time depends on the evolution of the distribution of employment across establishments, $H(n)$. Inspection of equation (31), the law of motion for the distribution of employment, reveals that $H(n)$ is not a jump variable but is instead a slow moving state variable in the model. It follows that the model admits a natural channel for the propagation of aggregate shocks on labor market tightness.

To provide a sense for this, Figure 3 plots the dynamic response of the job finding rate to a one percent decline in output per worker in both the model of sections 2 and 3, as well as the calibration of the standard MP model underlying column (1) of Table 3. Figure 3 confirms that the generalized model yields some propagation of the response of labor market

[26] The reader may worry whether $(1-\eta)(\tilde{p}-b)-\eta\beta c\theta$ is positive or not. To see that it is, note that we can rewrite it as $(1-\eta)\left(\frac{p\alpha x n^{\alpha-1}}{1-\eta(1-\alpha)}-b\right)-\eta\beta c\theta$, and observe from equations (13) and (14) that it is, in fact, the marginal flow surplus of a firm, and therefore must be positive.

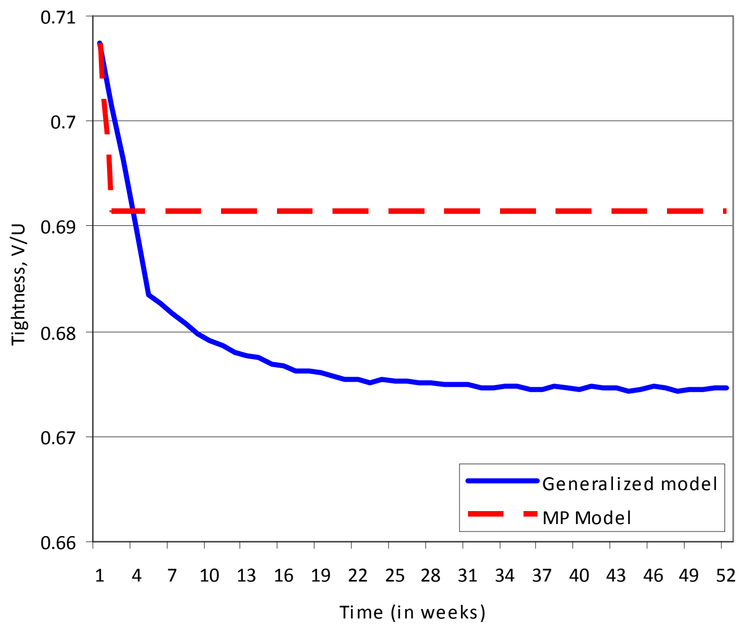

Figure 3: Dynamic response to a permanent decline in output per worker: MP model (dashed line) vs. Generalized Model (solid line).

tightness, in addition to amplification, relative to the MP model. It now takes one to two quarters for full adjustment to the shock to be achieved. .However, while the model has the ability to generate propagation, quantitatively the effects are modest relative to the persistence observed in the data.

It is worth emphasizing that, while the generalized model appears unable to match the degree of propagation observed in the data, it does provide an important source of intuition for what factors are likely to enhance propagation: i.e. whatever it is that slows down the evolution of the distribution of employment across firms, $H(n)$. In particular, rewriting the law of motion for $H(n)$, (31), yields

$$H(n) - H_{-1}(n) = -\lambda \left(1 - \tilde{G}\left[R_v(n;N,p)\right] + \tilde{G}\left[R(n;N,p)\right]\right)\left[H_{-1}(n) - H^*(n)\right], \qquad (37)$$

where $H^*(n)$ is the steady state distribution that sets $H(n) - H_{-1}(n) = 0$. Equation (37) suggests two important factors that slow the evolution of $H(n)$. First, less frequent idiosyncratic shocks, as implied by a lower value of λ, will cause fewer firms to adjust their employment, and thereby slow the reallocation of employment across firms. Second, the size of adjustment costs will determine the gap between $R_v(n;N,p)$ and $R(n;N,p)$ in a firm's optimal employment policy function in Figure 1. Larger adjustment costs will widen this gap, reducing the term $\left(1 - \tilde{G}\left[R_v\left(n; N, p\right)\right] + \tilde{G}\left[R\left(n; N, p\right)\right]\right)$ in equation (37), and slowing the dynamics of $H\left(n\right)$. This suggests that the nature of labor adjustment costs has important implications for the propagation of the response of unemployment to aggregate shocks.

5 Summary and Discussion

This paper has shown that the addition of two very simple features – downward sloped short run labor demand and endogenous job destruction – to an otherwise standard model of the aggregate labor market can help explain the observed cyclical variation in the job finding rate, the employment to unemployment transition rate, and vacancies observed in US data. We show that this is driven by two effects. First, cyclicality in job creation is generated by the fact that marginal employment relationships generate a low surplus in the short run. Small aggregate disturbances quickly exhaust this surplus and lead to substantial reductions in hiring. Importantly, however, due to downward sloped labor demand, low marginal surplus is nevertheless consistent with a sizeable surplus to the average employment relationship, contrary to the standard search model. Second, increased job destruction in recessions must be soaked up by increased hiring in equilibrium. With downward sloped labor demand, increased hiring diminishes the value of additional employment relationships to firms. As a result, hiring firms are less willing to soak up the separations of shedding firms, and unemployment rises more in the wake of a recession. Calibration of the model to available moments suggests that, while the majority of the cyclicality in job creation is attributable to small marginal surplus, both of these mechanisms appear to be quantitatively significant.

In the course of establishing these results, we also provide a rich, yet analytically tractable model of the aggregate labor market in the short run. As such, we believe that this model will provide a useful laboratory for the cyclical analysis of aggregate labor markets in future empirical and theoretical research. A number of avenues arise naturally in the light of this. First, the model has a well-defined concept of a firm and so lends itself to estimation using establishment level data. As a result, the analytical framework developed here will complement recent research efforts that have sought to solve and estimate search models using numerical methods (e.g. Cooper, Haltiwanger & Willis, 2007).

Second, the model provides a very simple set of analytical results that characterize the steady state and associated dynamics of the distribution of employment across firms—the firm size distribution. Consequently, the model can speak to the wealth of literature that has attempted to model the equilibrium properties of the firm size distribution (see Luttmer, 2007, and the references therein). In addition, the implications of the model for variation in the distribution of firm size over the business cycle can be related to recent research by Moscarini & Postel–Vinay (2008) that has emphasized the differential effects of business cycles between small and large firms.

Third, the addition of a notion of firm size in the model provides a natural motivation for the firm size wage effect that has been emphasized in empirical labor economics (Brown & Medoff, 1989). In the model presented here, firms with idiosyncratically higher productivity pay higher wages as workers capture part of the additional rents embodied in these more productive employment relationships. An open question is the extent to which our model can account for the magnitude of the firm size wage effect observed in the data.

Fourth, our interpretation of the standard search and matching model as a model of kinked adjustment costs raises the question of the aggregate implications of other forms of adjustment costs in the labor market. Recent research has emphasized the importance of fixed adjustment costs in explaining the empirical properties of labor demand at the micro level (see for example Caballero, Engel, & Haltiwanger, 1997, and Cooper, Haltiwanger, & Willis, 2004). Incorporating these adjustment costs into the model will therefore provide a unification of the joint insights of the two dominant approaches to the modelling of aggregate labor markets—the search and matching framework, and models of adjustment costs.

A final extension relates to the nature of wage setting. An attractive feature of incorporating firm size into models of the labor market is that an assessment of the multilateral dimension to wage bargaining between a firm and its many workers becomes feasible. This has been of especial interest in recent literature that has emphasized the importance of rigidities in the structure of wages within a firm, as well as of individual wages over time, for determining the volatility of unemployment (Bewley, 1999; Hall, 2005). While the wage bargaining solution derived in the present paper seeks to improve upon approaches in previous work, it is in many ways an idealized environment in which the wages of all workers can be renegotiated costlessly. This idealized setting, however, provides a fruitful benchmark for analyzing the implications of rigidities in renegotiation of wages within a firm, as well as across time.

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7 Appendix

A Solution of the Simulated Model

Steady State Optimal Employment Policy. Idiosyncratic shocks evolve according to (17) with $\ln \tilde{x} \sim N\left(-\frac{1}{2}\sigma_x^2, \sigma_x^2\right)$. In this case, we can rewrite the recursion for the function $D(n,x)$ in Proposition 3 as:

$$\begin{aligned} D(n,x) &= (1-\lambda)\chi(x) + \lambda \int_{R(n)}^{R_v(n)} \chi(x')\, d\tilde{G}(x') + \lambda \int_{R_v(n)}^{\infty} \frac{c}{q} d\tilde{G}(x') \\ &\quad + \beta(1-\lambda) D(n,x) + \beta\lambda \int_{R(n)}^{R_v(n)} D(n,x')\, d\tilde{G}(x') \end{aligned} \tag{38}$$

where $\chi(x) \equiv (1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f \frac{c}{q}$. We then conjecture that the function $D(n,x)$ is of the form $D(n,x) = d_0 + d_1\chi(x)$. Substituting this into the latter, and equating coefficients, we obtain the following solution for $D(n,x)$:

$$\begin{aligned} D(n,x) &= \frac{1-\lambda}{1-\beta(1-\lambda)}\chi(x) \\ &\quad + \frac{\lambda}{1-\beta(1-\lambda)} \frac{\tilde{G}[R_v(n)] - \tilde{G}[R(n)]}{1-\beta(1-\lambda) - \beta\lambda\left(\tilde{G}[R_v(n)] - \tilde{G}[R(n)]\right)} \mathcal{Q}(n) \\ &\quad + \frac{1-\tilde{G}[R_v(n)]}{1-\beta(1-\lambda) - \beta\lambda\left(\tilde{G}[R_v(n)] - \tilde{G}[R(n)]\right)} \lambda \frac{c}{q}, \end{aligned} \tag{39}$$

where $\mathcal{Q}(n) \equiv \mathbb{E}(\chi(x') \mid x' \in [R(n), R_v(n)])$. Substituting into the first order conditions for hires and separations (13) and (14) yields two nonlinear equations in the optimal employment policy $R(n)$ and $R_v(n)$ that are straightforward to solve numerically. The aggregate employment stock and flows are then obtained directly from applying the results of Proposition 5.

Average Product and Average Marginal Product. The average product of labor implied by the model is given by $APL = \mathbb{E}[pxn^{\alpha-1}]$. Note that:

$$\mathbb{E}\left[xn^{\alpha-1}\right] = \int \left[\int x dG(x|n)\right] n^{\alpha-1} dH(n)$$

Moreover, the optimal employment policy implies that, given n, x must lie in the interval $[R(n), R_v(n)]$, but is otherwise independently distributed. Thus:

$$\int x dG(x|n) = \frac{\int_{R(n)}^{R_v(n)} x dG(x)}{G[R_v(n)] - G[R(n)]} = \frac{1}{2}[R(n) + R_v(n)] \tag{40}$$

where the last equality follows from the assumption of uniform idiosyncratic shocks in the simulation. Thus:

$$APL = \mathbb{E}\left[pxn^{\alpha-1}\right] = p\int \frac{1}{2}\left[R(n) + R_v(n)\right] n^{\alpha-1} dH(n) \tag{41}$$

Moreover, the average marginal product of labor is simply given by $\mathbb{E}[MPL] = \mathbb{E}[px\alpha n^{\alpha-1}] = \alpha APL$.

Average Wages. It follows from equation (9) that the average wage across firms is given by:

$$\bar{w}_f = \frac{\eta}{1-\eta(1-\alpha)}\mathbb{E}[MPL] + \eta\beta f\frac{c}{q} + (1-\eta)b \tag{42}$$

To obtain the average wage across workers, which we denote $\bar{w}_w$, note that $\bar{w}_w = \mathbb{E}\left[\frac{n}{\mathbb{E}(n)}w(n,x)\right]$ where $w(n,x)$ is the wage in a given firm defined in (9). That is, it is the employment-weighted average of wages across firms. Thus:

$$\bar{w}_w = \frac{\eta}{1-\eta(1-\alpha)}\frac{1}{\mathbb{E}(n)}\mathbb{E}[px\alpha n^{\alpha}] + \eta\beta f\frac{c}{q} + (1-\eta)b \tag{43}$$

This has a very similar structure to the average wage across firms. It follows that:

$$\bar{w}_w = \frac{\eta p\alpha}{1-\eta(1-\alpha)}\frac{1}{\mathbb{E}(n)}\int\frac{1}{2}\left[R(n) + R_v(n)\right]n^{\alpha}dH(n) + \eta\beta f\frac{c}{q} + (1-\eta)b \tag{44}$$

Finally, the average wage of new hires, which we denote $\bar{w}_m$, is equal to a hiring–weighted average of wages across hiring firms. Noting from (12) that idiosyncratic productivity of hiring firms is given by $x = R_v(n)$, we have that:

$$\bar{w}_m = \mathbb{E}\left[\mathbb{E}(w(n,x)\,|n > n_{-1}, n_{-1})\right] = \int\int_{n_{-1}} w(n, R_v(n))\frac{dG[R_v(n)]}{1-G[R_v(n_{-1})]}dH(n_{-1}) \tag{45}$$

B Proofs

Conjecture 1 *The optimal employment policy function is of the form specified in (12).*

We will later verify in the proof of Proposition 2 that the Conjecture is consistent with the solution for the wage equation obtained in Proposition 1.

Proof of Proposition 1. Note first that, under the Conjecture, we can write the marginal surplus to a firm recursively as:

$$J(n,x) = pxF'(n) - w(n,x) - w_n(n,x)n + \beta\int_{R_v(n)}^{\infty}\frac{c}{q}dG(x') + \beta\int_{R(n)}^{R_v(n)} J(n,x')\,dG(x') \tag{46}$$

In addition, we can write the value to a worker of unemployment as:

$$U = b + \beta \left\{ (1-f) U' + f \int_0^\infty \int_{R_v(n)}^\infty W\left(R_v^{-1}(x'), x'\right) \frac{dG(x')}{1 - G(R_v(n))} dH(n) \right\} \tag{47}$$

Upon finding a job, which occurs with probability f, the new job must be in a firm which is posting vacancies. This implies that the idiosyncratic productivity of the firm, $x' > R_v(n)$, and that the level of employment in the hiring firm, $n = R_v^{-1}(x')$. Moreover, since firms differ in size, there is a distribution of employment levels, $H(n)$, over which an unemployed worker will take expectations when evaluating the expected future benefits of being hired. It is useful to rewrite the worker's value of unemployment as:

$$U = b + \beta \left\{ U' + f \int_0^\infty \int_{R_v(n)}^\infty \left[W\left(R_v^{-1}(x'), x'\right) - U'\right] \frac{dG(x')}{1 - G(R_v(n))} dH(n) \right\} \tag{48}$$

Then note that, due to Nash sharing, the worker's surplus in an expanding firm, $W\left(R_v^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} J\left(R_v^{-1}(x'), x'\right)$, and moreover that, by the first-order condition for a hiring firm (see (4)), $J\left(R_v^{-1}(x'), x\right) = c/q$. Thus, we obtain the simple result:

$$U = b + \beta U' + \beta f \frac{\eta}{1-\eta} \frac{c}{q} \tag{49}$$

The value of employment to a worker can be written as:

$$\begin{aligned} W(n,x) &= w(n,x) + \beta \left\{ \int_0^{R(n)} \left[\tilde{s} U' + (1-\tilde{s}) W\left(R^{-1}(x'), x'\right)\right] dG(x') \right. \\ &\quad \left. + \int_{R(n)}^{R_v(n)} W(n, x') dG(x') + \int_{R_v(n)}^\infty W\left(R_v^{-1}(x'), x'\right) dG(x') \right\} \end{aligned} \tag{50}$$

An employed worker's expected future payoff can be split into three regimes. If the firm sheds workers next period ($x' < R(n)$) then the worker may separate from the firm. We denote by $\tilde{s}$ the probability that a worker separates from a firm conditional on the firm shedding workers. If the worker separates, she transitions into unemployment and receives a payoff U'. Otherwise she continues to be employed in a firm of size $n' = R^{-1}(x')$. Note that Nash sharing implies that $W\left(R^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} J\left(R^{-1}(x'), x'\right)$, and that, by the first-order condition, $J\left(R^{-1}(x'), x'\right) = 0$. Thus, $W\left(R^{-1}(x'), x'\right) = U'$. In the event that a firm freezes employment next period ($x' \in [R(n), R_v(n)]$) then Nash sharing implies that $W(n, x') - U' = \frac{\eta}{1-\eta} J(n, x')$. Finally, in the event that the firm hires next period, $W\left(R_v^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} \frac{c}{q}$. Thus, we have that:

$$W(n,x) = w(n,x) + \beta U' + \beta \frac{\eta}{1-\eta} \int_{R_v(n)}^\infty \frac{c}{q} dG(x') + \beta \frac{\eta}{1-\eta} \int_{R(n)}^{R_v(n)} J(n, x') dG(x') \tag{51}$$

Subtracting the value of unemployment to a worker from the latter, we obtain the following description of the worker's surplus:

$$W(n,x)-U = w(n,x)-b+\beta\frac{\eta}{1-\eta}\int_{R_v(n)}^{\infty}\frac{c}{q}dG(x')+\beta\frac{\eta}{1-\eta}\int_{R(n)}^{R_v(n)}J(n,x')\,dG(x')-\beta f\frac{\eta}{1-\eta}\frac{c}{q} \tag{52}$$

Under Nash, this must be equal to $\frac{\eta}{1-\eta}J(n,x)$, where $J(n,x)$ is as derived in (46) so that we have:

$$w(n,x)=\eta\left[pxF'(n)-w_n(n,x)\,n+\beta f\frac{c}{q}\right]+(1-\eta)\,b \tag{53}$$

as required. ■

Proof of Proposition 2. Given the wage function in (9), it follows that the firm's objective, (3), is continuous in (n_{-1},x) and concave in n. Thus, it follows from the Theorem of the Maximum that the firm's optimal employment policy function is continuous in (n_{-1},x). Given this, it follows that the employment policy function must be of the form stated in Proposition 2. This verifies that the Conjecture stated at the beginning of the appendix holds. ■

Proof of Proposition 3. First, note that one can re-write the continuation value conditional on each of the three possible continuation regimes:

$$\Pi(n,x')=\begin{cases}\Pi^{-}(n,x') & \text{if} \quad x'<R(n)\\ \Pi^{0}(n,x') & \text{if} \quad x'\in[R(n),R_v(n)]\\ \Pi^{+}(n,x') & \text{if} \quad x'>R_v(n)\end{cases} \tag{54}$$

where superscripts $^{-/0/+}$ refer to whether their are separations, a hiring freeze, or hires tomorrow. Thus we can write[27]:

$$\int\Pi(n,x')\,dG(x'|x)=\int_0^{R(n)}\Pi^{-}(n,x')\,dG+\int_{R(n)}^{R_v(n)}\Pi^{0}(n,x')\,dG+\int_{R_v(n)}^{\infty}\Pi^{+}(n,x')\,dG \tag{55}$$

Taking derivatives with respect to n, recalling the definition of $D(\cdot)$, and noting that, since $\Pi(n,x')$ is continuous, it must be that $\Pi^{-}(n,R(n))=\Pi^{0}(n,R(n))$ and $\Pi^{0}(n,R_v(n))=\Pi^{+}(n,R_v(n))$, yields:

$$D(n,x)=\int_0^{R(n)}\Pi_n^{-}(n,x')\,dG+\int_{R(n)}^{R_v(n)}\Pi_n^{0}(n,x')\,dG+\int_{R_v(n)}^{\infty}\Pi_n^{+}(n,x')\,dG \tag{56}$$

Finally, using the Envelope conditions in Lemma 1 below, and substituting into (56) we

[27] Henceforth, "dG" without further elaboration is to be taken as "$dG(x'|x)$".

obtain (15) and (16) in the main text:

$$\begin{aligned} D(n,x) &= \int_{R(n)}^{R_v(n)} \left\{ (1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b \right] - \eta\beta f \frac{c}{q} \right\} dG(x'|x) \\ &\quad + \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x'|x) + \beta \int_{R(n)}^{R_v(n)} D(n,x')\, dG(x'|x) \\ &\equiv (\mathbf{C}D)(n,x) \end{aligned} \tag{57}$$

To verify that $\mathbf{C}$ is a contraction mapping, we confirm that Blackwell's sufficient conditions for a contraction hold here (see Stokey & Lucas, 1989, p.54). To verify monotonicity, fix $(n,x) = (\bar{n},\bar{x})$, and take $\hat{D} \geq D$. Then note that:

$$\int_{R(\bar{n})}^{R_v(\bar{n})} \hat{D}(\bar{n},x')\, dG(x'|\bar{x}) - \int_{R(\bar{n})}^{R_v(\bar{n})} D(\bar{n},x')\, dG(x'|\bar{x}) = \int_{R(\bar{n})}^{R_v(\bar{n})} \left[\hat{D}(\bar{n},x') - D(\bar{n},x') \right] dG(x'|\bar{x}) \geq 0 \tag{58}$$

Since $(\bar{n},\bar{x})$ were arbitrary, it thus follows that $\mathbf{C}$ is monotonic in D. To verify discounting, note that:

$$[\mathbf{C}(D+a)](n,x) = (\mathbf{C}D)(n,x) + \beta a \left[G(R_v(n)|x) - G(R(n)|x) \right] \leq (\mathbf{C}D)(n,x) + \beta a \tag{59}$$

Since $\beta < 1$ it follows that $\mathbf{C}$ is a contraction. It therefore follows from the Contraction Mapping Theorem that $\mathbf{C}$ has a unique fixed point. ■

Lemma 1 *The value function defined in (3) has the following properties:*

$$\begin{aligned} \Pi_n^-(n,x') &= 0 \\ \Pi_n^0(n,x') &= (1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b \right] - \eta\beta f \frac{c}{q} + \beta D(n,x') \\ \Pi_n^+(n,x') &= c/q \end{aligned} \tag{60}$$

Proof of Lemma 1. First, note that standard application of the Envelope Theorem implies that $\Pi_n^-(n,x') = 0$ and $\Pi_n^+(n,x') = c/q$. It is only slightly less obvious what happens when $\Delta n' = 0$, i.e. when the employment is frozen next period. In this case, $n' = n$ and this implies that:

$$\Pi^0(n,x') = px'F(n) - w(n,x')\,n + \beta \int \Pi(n,x'')\, dG(x''|x') \tag{61}$$

It therefore follows that:

$$\Pi_n^0(n,x') = px'F'(n) - w(n,x') - w_n(n,x')\,n + \beta \int \Pi_n(n,x'')\, dG(x''|x') \tag{62}$$

Since, by definition $D(n,x') \equiv \int \Pi_n(n,x'')\, dG(x''|x')$, the statement holds as required. ■

Proof of Proposition 4. First note that if x evolves according to (17), then we can rewrite the recursion for $D(n,x)$ as:

$$\begin{aligned} D(n,x) &= \frac{1-\lambda}{1-\beta(1-\lambda)}\chi(x) + \frac{\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} \chi(x')\, d\tilde{G}(x') \\ &+ \frac{\lambda}{1-\beta(1-\lambda)}\int_{R_v(n)}^{\infty} \frac{c}{q} d\tilde{G}(x') + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} D(n,x')\, d\tilde{G}(x') \end{aligned} \quad (63)$$

where $\chi(x) \equiv (1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta c\theta$. It follows that the LHS of the first–order conditions, (13) and (14) are increasing in x, because $\chi(x)$ is increasing in x. Thus, to establish that $\partial R_v/\partial p < 0$ and $\partial R/\partial p < 0$, simply note that the function $D(n,x)$ is also increasing in p and thus the LHS of (13) and (14) are increasing in p.

To ascertain the marginal effects of θ we first need to establish the marginal effect of θ on the function $D(n,x)$. Rewriting $f/q = \theta$ and $q = q(\theta)$ in (63), differentiating with respect to θ, and using the first–order conditions, (13) and (14), to eliminate terms we obtain:

$$D_\theta = -\eta\beta c\frac{1-\lambda(1-p^0)}{1-\beta[1-\lambda(1-p^0)]} - \frac{c}{q}\frac{q'(\theta)}{q}\frac{\lambda p^+}{1-\beta[1-\lambda(1-p^0)]} \quad (64)$$

where $p^0 \equiv \tilde{G}(R_v(n)) - \tilde{G}(R(n))$, $p^+ \equiv 1-\tilde{G}(R_v(n))$, and $p^- \equiv \tilde{G}[R(n)]$. Note that D_θ is independent of x. Differentiating the first–order condition for a hiring firm, (13), with respect to θ we obtain:

$$-\eta\beta c + \frac{c}{q}\frac{q'(\theta)}{q} + \beta D_\theta = -\frac{\eta\beta c}{1-\beta[1-\lambda(1-p^0)]} + \frac{c}{q}\frac{q'(\theta)}{q}\frac{1-\beta(1-\lambda p^-)}{1-\beta[1-\lambda(1-p^0)]} < 0 \quad (65)$$

since $q'(\theta) < 0$. Thus it follows that $\partial R_v/\partial\theta > 0$. Likewise, differentiating the first–order condition for a shedding firm, (14), with respect to θ we obtain:

$$-\eta\beta c + \beta D_\theta = -\frac{\eta\beta c}{1-\beta[1-\lambda(1-p^0)]} - \beta\frac{c}{q}\frac{q'(\theta)}{q}\frac{\lambda p^+}{1-\beta[1-\lambda(1-p^0)]} \quad (66)$$

Thus, $\partial R/\partial\theta > 0 \iff n > R_v^{-1}\tilde{G}^{-1}\left(1+\frac{\eta}{\varepsilon_{q\theta}}\frac{f}{\lambda}\right)$ where $\varepsilon_{q\theta} \equiv \frac{d\ln q}{d\ln\theta}$. ■

Proof of Proposition 5. *Proof of (19) and (20):* See main text.

Proof of (21): First note that a necessary condition for a firm to shed workers is that it receives an idiosyncratic shock, which occurs with probability λ. In this event, the number of separations in a firm that is shedding workers is equal to $[n_{-1} - R^{-1}(x)]$, since separating firms set employment, $n = R^{-1}(x)$. Now imagine, counterfactually, that all firms shared the same lagged employment level, n_{-1}. Then, the aggregate number of separations in the economy would equal:

$$\Lambda(n_{-1}) = \lambda\int_{n_{\min}}^{R(n_{-1})}\left[n_{-1} - R^{-1}(x)\right] d\tilde{G}(x) \quad (67)$$

where $n_{\min}$ is the lower support of employment. Using the change of variables, $x = R(n)$, and integrating by parts:

$$\Lambda(n_{-1}) = \lambda \int_{n_{\min}}^{n_{-1}} (n_{-1} - n) \frac{d\tilde{G}[R(n)]}{dn} dn = \lambda \int_{n_{\min}}^{n_{-1}} \tilde{G}[R(n)]\, dn \tag{68}$$

Now, of course, the true aggregate number of separations is equal to $S = \int \Lambda(n_{-1})\, dH(n_{-1})$, where $H(\cdot)$ is the c.d.f. of employment. Denoting $n_{\max}$ as the upper support of $H(\cdot)$, further integration by parts reveals that:

$$S = \Lambda(n_{\max}) - \lambda \int \tilde{G}[R(n_{-1})]\, H(n_{-1})\, dn_{-1} = \lambda \int [1 - H(n)]\, \tilde{G}[R(n)]\, dn \tag{69}$$

as required. A similar method reveals that the aggregate number of hires in the economy, $M = \lambda \int H(n) \left(1 - \tilde{G}[R_v(n)]\right) dn$. It follows from the steady state condition for the distribution for employment, (19), that separations, S, are equal to hires, M. ■

Proof of Proposition 6. Given that aggregate shocks evolve according to (25), and denoting the forecast equations for N and θ in (26) as $N'(N,p)$ and $\theta'(N',p)$ respectively, we can write the marginal effect of current employment on future profits as

$$D(n,x,N,p;\sigma_p) = \frac{1}{2} d(n,x,N'(N,p+\sigma_p), p+\sigma_p) + \frac{1}{2} d(n,x,N'(N,p-\sigma_p), p-\sigma_p), \tag{70}$$

where

$$\begin{aligned} d(n,x,N'(N,p'),p') \;=\; & \int_{R(n,N',p')}^{R_v(n,N',p')} \chi(n,x',N',p')\, dG(x'|x) + \int_{R_v(n,N',p')}^{\infty} c\,[\theta'(N',p')]^{\phi}\, dG(x'|x) \\ & + \beta \int_{R(n,N',p')}^{R_v(n,N',p')} D(n,x',N'(N,p\prime),p')\, dG(x'|x), \end{aligned} \tag{71}$$

and $\chi(n,x,N,p) = (1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta c E[\theta'(N',p')\,|p] \equiv \chi_0 + \chi_1 px + \chi_2 E[\theta'(N',p')\,|p]$. Taking a Taylor series approximation to $D(n,x,N,p;\sigma_p)$ around $\sigma_p = 0$ we obtain

$$D(n,x,N,p;\sigma_p) \approx D(n,x,N^*,p;0) + D_{\sigma_p}(n,x,N^*,p;0)\,\sigma_p + D_N^*(N - N^*), \tag{72}$$

where $D_N^* \equiv D_N(n,x,N^*,p;0)$. It is straightforward to show that $D_{\sigma_p}(n,x,N^*,p;0) = 0$, and that $D_N^* = d_{N'}(n,x,N^*,p)\,\nu_N$. Under the conjectured forecast equations in (26), we can write[28]

$$
\begin{aligned}
d_{N'}\left(n,x,N'\left(N,p'\right),p'\right) &= \int_{R(n,N',p')}^{R_v(n,N',p')} \chi_{N'}\left(n,x',N',p'\right)dG\left(x'|x\right) \\
&+ \int_{R_v(n,N',p')}^{\infty} c\phi\left[\theta'\left(N',p'\right)\right]^{\phi-1}\theta'_{N'}\left(N',p'\right)dG\left(x'|x\right) \\
&+\beta\int_{R(n,N',p')}^{R_v(n,N',p')} D_N\left(n,x',N'\left(N,p'\right),p'\right)dG\left(x'|x\right). \qquad (73)
\end{aligned}
$$

Evaluating at $N = N^*$ and $p' = p$, and noting that $\chi_{N'}\left(n,x,N',p\right) = \chi_2\theta_N\nu_N$, and $\theta'_{N'}\left(N^*,p\right) = \theta_N$, we obtain

$$
\begin{aligned}
d_{N'}\left(n,x,N^*,p'\right) &= \chi_2\theta_N\nu_N\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} dG\left(x'|x\right) + c\phi\theta_N{\theta^*}^{\phi-1}\int_{R_v(n,N^*,p)}^{\infty} dG\left(x'|x\right) \\
&+\beta\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} D_N\left(n,x',N^*,p\right)dG\left(x'|x\right). \qquad (74)
\end{aligned}
$$

Recall from above that $D_N^* = d_{N'}\left(n,x,N^*,p\right)\nu_N$. Putting this together yields

$$
\begin{aligned}
D_N^* &= \chi_2\theta_N\nu_N^2\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} dG\left(x'|x\right) + c\phi\theta_N\nu_N{\theta^*}^{\phi-1}\int_{R_v(n,N^*,p)}^{\infty} dG\left(x'|x\right) \\
&+\beta\nu_N\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} D_{N'}\left(n,x',N^*,p;0\right)dG\left(x'|x\right). \qquad (75)
\end{aligned}
$$

Under the form of idiosyncratic shocks in (idiosync. eq.) we obtain:

$$
\begin{aligned}
D_N^* &= \theta_N\nu_N^2\chi_2\frac{(1-\lambda)+\lambda\left(\tilde{G}\left[R_v^*(n)\right]-\tilde{G}\left[R^*(n)\right]\right)}{1-\beta\nu_N\left[(1-\lambda)+\left(\tilde{G}\left[R_v^*(n)\right]-\tilde{G}\left[R^*(n)\right]\right)\right]} \\
&+\theta_N\nu_N\lambda c\phi{\theta^*}^{\phi-1}\frac{1-\tilde{G}\left[R_v^*(n)\right]}{1-\beta\nu_N\left[(1-\lambda)+\left(\tilde{G}\left[R_v^*(n)\right]-\tilde{G}\left[R^*(n)\right]\right)\right]}, \qquad (76)
\end{aligned}
$$

where $R_v\left(n,N^*,p\right) \equiv R_v^*(n)$ and $R\left(n,N^*,p\right) \equiv R^*(n)$ summarize the steady state employment policy function. ■

Proof of Proposition 7. Consider the c.d.f. of employment growth for a given lagged

[28] Note that the effects of N' on the limits of integration will cancel by virtue of the first order conditions for optimal hiring and firing.

employment level, n_{-1}, and for the case where employment growth is negative:

$$\begin{aligned}\Pr\left(\Delta \ln n < \delta | n_{-1}, \delta < 0\right) &= \Pr\left(\ln R^{-1}(x) - \ln n_{-1} < \delta | n_{-1}\right) \\ &= \Pr\left(x < R\left(e^{\delta} n_{-1}\right) | n_{-1}\right) \\ &= \lambda \tilde{G}\left[R\left(e^{\delta} n_{-1}\right)\right]. \end{aligned} \tag{77}$$

It follows that the unconditional c.d.f. of employment growth, given that $\Delta \ln n < 0$ is equal to:

$$H_{\Delta}(\delta) \equiv \Pr\left(\Delta \ln n < \delta\right) = \lambda \int \tilde{G}\left[R\left(e^{\delta} n_{-1}\right)\right] dH\left(n_{-1}\right), \tag{78}$$

It follows that the density of employment growth is given by $h_{\Delta}(\delta) = H'_{\Delta}(\delta) = \lambda \int \tilde{G}'\left[R'\left(e^{\delta} n_{-1}\right)\right] e^{\delta} n_{-1} dH$ as stated in the Proposition. A similar method reveals that, in the case where $\Delta \ln n > 0$:

$$H_{\Delta}(\delta) = \lambda \int \tilde{G}\left[R_v\left(e^{\delta} n_{-1}\right)\right] dH\left(n_{-1}\right), \text{ and } h_{\Delta}(\delta) = \lambda \int \tilde{G}'\left[R'_v\left(e^{\delta} n_{-1}\right)\right] e^{\delta} n_{-1} dH\left(n_{-1}\right). \tag{79}$$

Finally there is a mass point at zero employment growth. Clearly that is given by:

$$h_{\Delta}(0) = H_{\Delta}\left(0^{+}\right) - H_{\Delta}\left(0^{-}\right) = \lambda \int \left(\tilde{G}\left[R_v\left(n_{-1}\right)\right] - \tilde{G}\left[R\left(n_{-1}\right)\right]\right) dH\left(n_{-1}\right). \tag{80}$$

■

Lemma 2 *If idiosyncratic shocks evolve according to (17), and the matching function is of the form* $M(U,V) = \mu U^{\phi} V^{1-\phi}$*, then the marginal firm surplus defined in (46) is given by*

$$\begin{aligned} J &= \frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)}\left[x + \frac{\beta\lambda p^0}{1-\beta(1-\lambda)-\beta\lambda p^0}\mathcal{E}(n)\right] \\ &\quad - \frac{(1-\eta) b}{1-\beta(1-\lambda)-\beta\lambda p^0} - \beta\frac{c}{q}\frac{\eta f - \lambda p^{+}}{1-\beta(1-\lambda)-\beta\lambda p^0}, \end{aligned} \tag{81}$$

and the marginal effects of n*,* p *and* θ *on* J *are given by*

$$\begin{aligned} J_n &= -\frac{1-\alpha}{n}\frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)}\left[x + \frac{\beta\lambda p^0}{1-\beta(1-\lambda)-\beta\lambda p^0}\mathcal{E}(n)\right] \\ J_p &= \frac{1}{p}\frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)}\left[x + \frac{\beta\lambda p^0}{1-\beta(1-\lambda)-\beta\lambda p^0}\mathcal{E}(n)\right] \\ J_{\theta} &= -\beta\frac{c}{q}\frac{1}{\theta}\frac{\eta f - \phi\lambda p^{+}}{1-\beta(1-\lambda)-\beta\lambda p^0}, \end{aligned} \tag{82}$$

where $\psi \equiv \frac{1-\eta}{1-\eta(1-\alpha)}$, $\mathcal{E}(n) \equiv \mathbb{E}\left(x' | x' \in \left[R(n), R_v(n)\right]\right)$*, and* p^0*,* p^{+} *are as defined in the Proof to Proposition 4.*

Proof. Since firms only receive an idiosyncratic shock with probability λ each period, we can use the recursion for $J(n,x)$, (46), to write:

$$\begin{aligned} J(n,x) &= \frac{1}{1-\beta(1-\lambda)}\left[\psi p x \alpha n^{\alpha-1} - (1-\eta) b - \eta\beta c\theta\right] \\ &\quad + \frac{\beta\lambda}{1-\beta(1-\lambda)}\frac{c}{q}\int_{R_v(n)} d\tilde{G} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J(n,x')\, d\tilde{G} \end{aligned} \tag{83}$$

We then conjecture that $J(n,x)$ is of the form $j_0 + j_1 x$. Substituting this assumption into the latter, and equating coefficients yields:

$$\begin{aligned} j_0 &= -\frac{(1-\eta) b}{1-\beta(1-\lambda)} - \beta\frac{c}{q}\frac{\eta f - \lambda p^+}{1-\beta(1-\lambda)} + \frac{\beta\lambda p^0}{1-\beta(1-\lambda)}\left[j_0 + j_1 \mathcal{E}(n)\right] \\ j_1 &= \frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)} \end{aligned} \tag{84}$$

Solving for j_0 we obtain the required solution for $J(n,x)$. Likewise, we can obtain recursions for the marginal effects of n and θ:

$$\begin{aligned} J_n(n,x) &= -\frac{1}{1-\beta(1-\lambda)}\frac{1-\alpha}{n}\psi p x \alpha n^{\alpha-1} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_n(n,x')\, dG \\ J_p(n,x) &= \frac{1}{1-\beta(1-\lambda)}\psi x \alpha n^{\alpha-1} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_p(n,x')\, d\tilde{G} \\ J_\theta(n,x) &= -\frac{\eta\beta c + \beta\lambda\frac{c}{q^2}q'(\theta)\int_{R_v(n)} dG}{1-\beta(1-\lambda)} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_\theta(n,x')\, dG \end{aligned} \tag{85}$$

Again using the method of undetermined coefficients, and noting that the Cobb Douglas matching function implies $q = \mu\theta^{-\phi} \implies \frac{c}{q^2}q'(\theta) = -\frac{c}{q}\frac{\phi}{\theta}$, yields the required solutions for J_n, J_p and J_θ. ■

Proof of Proposition 8. Total differentiation of the JC condition, $U(\theta) = L - \mathbb{E}(n)$, yields $\frac{d\theta}{dp} = -\frac{\mathbb{E}(\partial n/\partial p)}{\mathbb{E}(\partial n/\partial\theta)}$. In steady state, the probabilities of raising, freezing, and cutting employment will all be constants. Denoting these probabilities as $\mathbf{p}^+, \mathbf{p}^0$, and $\mathbf{p}^-$ respectively, it follows that we can write:

$$\mathbb{E}\left(\frac{\partial n}{\partial \xi}\right) = \mathbf{p}^+\mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n > 0\right) + \mathbf{p}^0\mathbb{E}\left(\frac{\partial n_{-1}}{\partial \xi}\right) + \mathbf{p}^-\mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n < 0\right) \tag{86}$$

for any variable ξ. Note further that in steady state $\mathbb{E}(\partial n/\partial\xi) = \mathbb{E}(\partial n_{-1}/\partial\xi)$ so that we obtain the result that:

$$\mathbb{E}\left(\frac{\partial n}{\partial \xi}\right) = \pi\mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n > 0\right) + (1-\pi)\,\mathbb{E}\left(\frac{\partial n}{\partial \xi}|\Delta n < 0\right) \tag{87}$$

where $\pi \equiv \frac{\mathbf{p}^+}{1-\mathbf{p}^0}$. Thus, we can rewrite the marginal effect of a change in p on θ as:

$$\frac{d\theta}{dp} = -\frac{\pi \mathbb{E}\left(\frac{\partial n}{\partial p}|\Delta n > 0\right) + (1-\pi)\mathbb{E}\left(\frac{\partial n}{\partial p}|\Delta n < 0\right)}{\pi \mathbb{E}\left(\frac{\partial n}{\partial \theta}|\Delta n > 0\right) + (1-\pi)\mathbb{E}\left(\frac{\partial n}{\partial \theta}|\Delta n < 0\right)} \tag{88}$$

Then note that the first–order conditions for optimal labor demand set the marginal firm surplus, $J(n,x)$ as follows:

$$J(n,x) = \begin{cases} c/q(\theta) & \text{if} \quad \Delta n > 0 \\ 0 & \text{if} \quad \Delta n < 0 \end{cases} \tag{89}$$

It is immediate from Lemma 2 that $\frac{\partial n}{\partial p} = -\frac{J_p}{J_n} = \frac{1}{1-\alpha}\frac{n}{p}$ regardless of whether $\Delta n > 0$ or $\Delta n < 0$. Thus it remains to derive $\frac{\partial n}{\partial \theta}$ in each case. Log–linearizing the function J around n, p, x, and θ, we obtain:

$$\log J \approx \varepsilon_{Jn} \log n + \varepsilon_{Jp}(\log p + \log x) + \varepsilon_{J\theta} \log \theta + const. \tag{90}$$

Using this and totally differentiating the first–order conditions for optimal labor demand with respect to n and θ, we obtain:

$$\varepsilon_{Jn} d\log n + \varepsilon_{J\theta} d\log\theta \approx \begin{cases} -d\log q(\theta) & \text{if} \quad \Delta n > 0 \\ 0 & \text{if} \quad \Delta n < 0 \end{cases} \tag{91}$$

Given the Cobb Douglas matching function assumption, $q(\theta) = \mu\theta^{-\phi}$, and it follows that $d\log q(\theta) = -\phi d\log\theta$. Thus:

$$\frac{\partial n}{\partial \theta} = \frac{\partial \log n}{\partial \log \theta}\frac{n}{\theta} \approx \begin{cases} \frac{\phi - \varepsilon_{J\theta}}{\varepsilon_{Jn}}\frac{n}{\theta} & \text{if} \quad \Delta n > 0 \\ -\frac{\varepsilon_{J\theta}}{\varepsilon_{Jn}}\frac{n}{\theta} & \text{if} \quad \Delta n < 0 \end{cases} \tag{92}$$

Substituting this into (88), we obtain:

$$\frac{d\log\theta}{d\log p}|_{JC} \approx -\frac{1}{1-\alpha}\frac{\varepsilon_{Jn}}{\omega\phi - \varepsilon_{J\theta}} \tag{93}$$

where $\omega \equiv \frac{\pi \mathbb{E}(n|\Delta n>0)}{\mathbb{E}(n)}$ is the steady state share of employment in hiring firms. In what follows, we evaluate the approximation (90) to the marginal surplus around mean employment, $\bar{n} \equiv \mathbb{E}(n)$, and mean productivity conditional on mean employment, $x = \mathcal{E}(\bar{n}) \equiv \mathbb{E}(x'|x' \in [R(\bar{n}), R_v(\bar{n})])$. Thus, using the results of Lemma 2 it follows that we can write:

$$J_n = -\frac{1}{\bar{n}}\frac{(1-\alpha)\psi p \alpha \bar{n}^{\alpha-1}}{1-\beta(1-\lambda)-\beta\lambda p^0}\mathcal{E}(\bar{n})$$

and:

$$J\left[1-\beta(1-\lambda)-\beta\lambda p^0\right] = \psi p \mathcal{E}(\bar{n})\alpha\bar{n}^{\alpha-1} - (1-\eta)b - \beta\frac{c}{q}\left[\eta f - \lambda p^+\right] \tag{94}$$

where $\psi \equiv \frac{1-\eta}{1-\eta(1-\alpha)}$. Substituting back into the aggregate elasticity of θ with respect to p, we obtain:

$$\frac{d\log\theta}{d\log p}|_{JC} \approx \frac{\psi p\mathcal{E}\left(\bar{n}\right)\alpha\bar{n}^{\alpha-1}}{\omega\phi\left[\psi p\mathcal{E}\left(\bar{n}\right)\alpha\bar{n}^{\alpha-1}-\left(1-\eta\right)b-\eta\beta c\theta\right]+\eta\beta c\theta-\left(1-\omega\right)\phi\beta\frac{c}{q}\lambda p^{+}} \tag{95}$$

Noting that the marginal product of labor in the average–sized firm is equal to $p\mathcal{E}\left(\bar{n}\right)\alpha n^{\alpha-1}$, and assuming λ is sufficiently small, we obtain:

$$\frac{d\log\theta}{d\log p}|_{JC} \approx \frac{\left(1-\eta\right)\tilde{p}}{\omega\phi\left[\left(1-\eta\right)\left(\tilde{p}-b\right)-\eta\beta c\theta\right]+\eta\beta c\theta} \tag{96}$$

where $\tilde{p} \equiv \rho p\mathcal{E}\left(\bar{n}\right)\bar{n}^{\alpha-1}+\left(1-\rho\right)p\mathcal{E}\left(\bar{n}\right)\alpha\bar{n}^{\alpha-1}$ and $\rho \equiv \frac{\alpha\eta}{1-\eta(1-\alpha)}$, as required. ■