

# “Token” Equilibria in Sensor Networks with Multiple Sponsors\*

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## Abstract

*When two sponsoring organizations, working towards separate goals, can employ wireless sensor networks for a finite period of time, it can be efficiency-enhancing for the sponsors to program their sensors to cooperate. But if each sensor privately knows whether it can provide a favor in any particular period, and the sponsors cannot contract on ex post payments, then no favors are performed in any Nash equilibrium. Allowing the sponsors to contract on ex post payments, we construct equilibria based on the exchange of “tokens” that yield significant cooperation and increase expected sponsor payoffs. Increasing the sponsors’ liability is beneficial because it enables them to use more tokens.*

## 1 Introduction

First generation Wireless Sensor Networks (WSNs) are primarily application-specific in terms of their infrastructure, protocol/architecture, and user set, and typically all sensors within a given network are assumed to be deployed and controlled by a single administrative entity, or (equivalently) by a mutually altruistic consortium. The first generation networks are self configuring—able to configure themselves to collaborate with other sensors on their own network—but they are not generally able to interoperate

with other sensor networks, or to make their resources available to different clients.

The success and increasing deployment of WSNs is motivating a second generation that allows a diverse set of clients to learn about and access resources spanning multiple networks. Clients compose end-to-end services using the services provided by individual networks. If sensors that belong to different organizations can cooperate with each other to provide various services like sensing, routing, data processing, and storage, overall utility to a diverse set of end users can be significantly improved. Since different networks may be deployed and managed by independent organizations, they may have different protocols, architectures, security policies, and pricing mechanisms. When the WSNs of multiple sponsoring organizations interact, strategic considerations become salient.

Resource management in such large scale, multi-sponsor distributed systems is a daunting task. Game theory brings two important benefits to bear on such problems. First, it provides a set of tools for constructing and evaluating equilibria in WSNs, where each sensor uses locally available information to inform its behavior. Second, it provides a method for analyzing the human and institutional motives that underly the use of WSNs.

This second benefit is the main focus of this study, where we consider an network made up of sensors belonging to two different sponsoring organizations. Each sponsor programs its own sensors to further its own objectives, but both sponsors can gain if their sensors share information and resources in the field. In a simple model of this type of situation, we construct an equilibrium in the WSN that yields a

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significantly higher payoff to the sponsors than they could attain separately. Further, this equilibrium in the WSN corresponds to a Nash equilibrium in the game between the sponsors; i.e., neither sponsor can gain by programming its sensors differently.

Once the sensors are in the field, they can request favors from one another. (We use the words “favors” and “services” interchangeably.) Canonical examples of services are routing, data storage, and data aggregation. The benefit of receiving a favor is greater than the cost of providing one, so for efficiency favors should always be provided. However, whether or not each sensor has the ability to provide a favor varies stochastically over time, in a way that is undetectable to other sensors. Hence a sensor from one sponsor could “claim” to be unable to provide a favor to a sensor from another sponsor, and by doing so save on the costs of providing the favor. Thus a sponsor could potentially gain by deviantly programming its sensors to request favors, but not provide them. This is similar to the classic prisoners’ dilemma problem, and in a finite horizon setting without contracts, the unique Nash equilibrium is that neither sponsor ever allows its sensors to provide favors to the other sponsor.

The problem, then, is to construct an equilibrium in which both sponsors are willing to program their sensors to cooperate. In the equilibrium we construct, each sponsor starts with an agreed-upon number of tokens, which it distributes among its sensors. When a sensor from one sponsor requests a favor from another sponsor’s sensor, it offers a token in exchange for the favor. If the requestee is able to provide the favor, it does so in return for the token.

If a sensor runs out of tokens, it can no longer request favors. So the more tokens are available to the sensors, the more favors will be performed and the more efficient will be the equilibrium. We consider a game with a fixed, finite horizon, but we allow the sponsors to write a contract at the outset of the game that can obligate them to make payments to each other that depend on the distribution of tokens in the final period. Each sponsor has limited liability, so its ex post payment is uniformly bounded across all realizations of the final distribution of tokens. Since the equilibrium value of a token is determined endogenously, this exogenous bound yields an endogenous limit on the number of tokens available.

**Related literature** There is a substantial literature applying game theoretic models and tools to network problems, particularly relating to the internet, for example applying incentive compatible mechanism design to distributed computing [4] and community resource sharing [7, 10], or considering more abstract issues [11]. Specifically for the setting of WSNs, recent work has applied game theoretic tools to models involving a single sponsor, such as load-balanced

tree construction for data gathering [12] and sensor role assignment [3]. Other work has investigated network architecture when each node is controlled by a different strategic agent [13]. We depart from the existing literature by considering a higher level problem: that of coordinating groups of sensors sponsored by multiple organizations. Rather than evaluating tradeoffs between specific favors (e.g., routing vs. sensing), we consider abstract favors.

We make use of two classic results from the game theory literature. First, as a benchmark we employ a theorem from [2], which implies that, when the sponsors cannot write a contract before programming their sensors, the unique Nash equilibrium outcome is that the sponsors do not program their sensors to cooperate.

Second, our result is in the spirit of the classic “folk theorem,” which states that any level of cooperation can be supported in an equilibrium of an infinitely repeated game if the players are sufficiently patient. Recent work has extended the folk theorem to games with private information (in particular, [5] and [8]). Still, no folk theorem applies to our setting, since costly communication is required to implement cooperation. But the equilibria we construct are similar to the types of equilibria that are used to prove the folk theorem.

Several authors have recently considered games in which strategic agents trade favors when their ability to perform favors is private information. Our token equilibria are based on [9], which constructs a token equilibrium in a two-player favors game; [9] also constructs simpler equilibria that induce cooperation among a network of players. [6] constructs optimal equilibria in a continuous-time two-player favors game, and [1] constructs cooperative equilibria in a more complex game with both favors and stochastic investments. Although in contrast to these authors we consider a finite horizon environment, we allow the sponsors to write a contract compelling them to pay monetary transfers based on what occurs during the game. This means that the strategic considerations in our setting are quite similar to those of infinite horizon environments.

## 2 The model

Two sponsoring organizations,  $i \in \{A, B\}$ , each employ  $K$  sensors,  $s_{i1}, \dots, s_{iK}$ , on a rectangular grid with  $2K$  nodes. Each sensor can communicate only with its immediate neighbors (vertically and horizontally, not diagonally). The game operates in continuous time from  $t = 0$  up to a fixed end point  $T < \infty$ . At certain times during the game, each sensor will need a favor from a particular one of its neighbors. For simplicity we assume that each sensor’s need for favors arrives according to a Poisson process with Poisson parameter  $\lambda$ . Any sensor of which a favor is requested will instantaneously learn whether it is able to

provide the favor: with probability  $\pi$  it can provide the favor, and with probability  $1 - \pi$  it cannot. The process of requesting and receiving a favor will be described in the subsequent section.

The “payoff” for sensor  $s_{ik}$  for the period  $[0, t]$  is

$$u_{ik}(t) = \alpha R_t - \beta P_t - \gamma C_t,$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are positive parameters;  $R_t$  is the number of favors received by  $s_{ik}$  during the period;  $P_t$  is the number of favors provided by  $s_{ik}$  during the period; and  $C_t$  is the number of communication signals sent by  $s_{ik}$  during the period. The utility for a sponsor in the entire game is

$$U_i = \sum_{k=1}^K u_{ik}(T) + \tau_i,$$

where  $\tau_i$  is the monetary transfer received by sponsor  $i$  at the end of the game. The monetary transfers must be zero sum ( $\tau_A = -\tau_B$ ) and are bounded above by  $\bar{\tau} < \infty$ . Throughout, we assume  $\gamma/\pi < \alpha - \beta$ , so the sponsors can gain by cooperating.

The equilibrium concept we employ is Nash equilibrium between the sponsors, even though the events of the game play out dynamically. In dynamic games, theorists usually employ subgame perfect equilibrium or one of its refinements, to rule out the possibility that a strategic agent could threaten to take actions that are not sequentially optimal. In our setting, however, all strategic activity takes place simultaneously at the outset of the game, when the sponsors write a contract (see below) and program their sensors. Since the sensors themselves are automata rather than strategic agents, it is reasonable in this context for them to be able to commit to threats that may not be sequentially optimal.

**Token equilibria** At the outset of a token equilibrium, the sponsors jointly agree on a number of tokens,  $n_i$  for each sponsor  $i$  to distribute among its sensors. They can also write a legally binding contract that obliges them to make payments at the end of period  $T$  that depend on the data stored in the sensors. For simplicity, we assume that this data is complete and verifiable. (This assumption could be weakened by requiring tokens to be encrypted and requiring sensors to confirm each transaction, at the cost of additional communication.) When the sponsors program their sensors according to the equilibrium, the payments need depend only on the number of tokens held by each sponsor at the end of period  $T$ .

When (for example) sensor  $s_{A1}$  has a token and needs a favor from sensor  $s_{B1}$ ,  $s_{A1}$  sends a request to  $s_{B1}$ , and tentatively forwards a token along with the request. If  $s_{B1}$  provides the requested favor, then  $s_{B1}$  obtains the token; otherwise  $s_{A1}$  retains the token. The number of communication signals sent by a particular sensor is the total number

of requests it made. (If  $s_{B1}$  is unwilling or unable to provide a favor, it does not incur a communication cost when  $s_{A1}$  sends its request. If it does provide a favor, the communication cost of doing so is incorporated into  $\beta$ .)

### 3 Analytical results

**Proposition 1.** *Suppose that the sponsors cannot write a contract. Then the unique Nash equilibrium outcome is that, with probability 1, no sensor requests a favor. Equilibrium expected utilities are  $U_A = U_B = 0$ .*

*Proof.* If all sensors are programmed never to provide favors, then it must also be that all sensors are programmed never to request favors, since requesting favors is costly.<sup>1</sup> So suppose that some sensors are programmed to provide favors after a set of histories that arise with positive probability in equilibrium. Consider a sensor that receives a request for a favor at time  $t$ , following a history along the equilibrium path. It is a best response to grant the favor only if the cost of doing so outweighs the benefit, and the only possible benefit is the prospect that it or one of its siblings might subsequently receive a favor in turn. (Two sensors are called *siblings* if they belong to the same sponsor.) Given the time remaining before  $T$ , the probability that any of its siblings will subsequently have a need for a favor is  $K(T - t)\lambda$ . Thus the benefit of performing a favor is less than the cost if  $t \geq t^* \equiv T - \beta/K\alpha\lambda$ , and so along any Nash equilibrium path each sensor must be programmed to ignore any favor request that arrives after  $t^*$ . Since no sensor can be programmed to grant a favor request after  $t^*$ , it cannot be a best response for any sensor to request a favor after  $t^*$ . Now apply the same reasoning as if  $T = t^*$ , and continue inductively to obtain the result.  $\square$

**Proposition 2.** *Now suppose that the sponsors **can** write a contract as described in Section 2. If  $\bar{\tau} \geq \beta/2$ , then there exists a token equilibrium in which the expected sum of equilibrium utilities is positive.*

*Proof.* Suppose sponsor  $A$  begins with 1 token, sponsor  $B$  begins with none, and the contract binds the sponsor without the token at the end the game to pay an amount  $x$  to the other sponsor. Suppose  $A$  programs each of its sensors to request a favor if it has the token, but ignore any favor requests, while  $B$  programs its sensors to provide favors when a token is offered, if able to do so, but never to request favors. Let  $s_{Ai}$  be the sensor that starts out with the token, and  $q = (T - t)\lambda\pi$  be the probability that  $s_{Ai}$  needs a favor during the game from some sensor  $s_{Bj}$ , and that  $s_{Bj}$  is

<sup>1</sup>Since there is a continuum of possible histories, sensors could request favors after a set of histories that arise with zero probability without affecting the sponsors’ expected payoffs. In the remainder of the proof, we ignore possible deviations after sets of histories that arise with zero probability.

able to provide the favor. Then  $B$  receives expected utility of  $q(x - \beta) - (1 - q)x$ , compared to  $-x$  if it programmed its sensors to ignore such requests. Hence  $B$ 's strategy is a best response to  $A$ 's strategy if  $x \geq \beta/2$ . In turn,  $A$  receives expected utility of  $q(\alpha - x) + (1 - q)x - \gamma$ , compared to  $x$  if it programmed its sensors not to make such requests. Hence the sponsors' strategies are mutual best responses if  $\alpha - \frac{\gamma}{q} \geq 2x \geq \beta$ ; e.g.,  $x = \beta/2 \leq \bar{\tau}$ . Finally, neither sponsor, acting unilaterally, can gain by programming its sensors to make or respond to requests in earlier periods. The expected sum of equilibrium utilities is  $q(\alpha - \beta) - \gamma > 0$ .  $\square$

**Proposition 3.** *Suppose that  $T < \infty$  and the sponsors can write a contract as described in Section 2. Then there exists a token equilibrium in which, along the equilibrium path, each sensor requests a favor if and only if it holds a token when its need for a favor arises, and each sensor grants a favor if it is able to do so when it receives a request. No more than  $2\bar{\tau}/\beta$  tokens that can be accommodated in such an equilibrium.*

*Proof.* Suppose the contract binds sponsor  $A$  to pay  $(p_B - p_A)x$  to sponsor  $B$ , where  $p_i$  is the number of tokens held by  $i$ 's sensors at the end of period  $T$ , and  $x = \beta/2$ . Suppose that  $A$  programs its sensors as described. Consider sensor  $s_{B_j}$  at time  $t$ , and suppose it receives a request for a favor. As in Proposition 2, the cost of providing the favor is outweighed by the benefit, which is at least  $x$  (since the token can at least be held until the end of period  $T$ ),  $B$ 's best response is to program its sensors to provide favors upon request, if able.

Now suppose that sensor  $s_{B_j}$  holds one or more tokens at time  $t$ , when its need for a favor from sensor  $s_{A_i}$  arises. Let  $z_t$  be the value of  $s_{B_k}$ 's marginal token at  $t$ , when  $s_{B_k}$  employs the token optimally by either requesting a favor or holding it until time  $t + dt$ . Then

$$z_t = \max\{\pi\alpha + (1 - \pi)z_{t+dt} - \gamma, z_{t+dt}\} \geq z_{t+dt}.$$

Hence if  $s_{B_j}$  ever finds it optimal to request a favor, then it should request a favor whenever it holds a token. Finally, as computed in Proposition 2, sponsors will optimally program each sensor to request a favor with its last token if  $\alpha - \gamma/p \geq 2x$ . The number of tokens is maximized subject to these constraints by setting  $x = \beta/2$ .  $\square$

## 4 Simulation results

## 5 Discussion

Our model features one-shot contractual interaction between the sponsors with exogenously limited liability. However, it is well known that results in such environments are qualitatively similar, and can be made quantitatively

similar by appropriate choice of the liability limit, to settings of infinitely repeated interaction without contracts. As in the classic full information folk theorem (so called due to the lack of a definitive originator), in an infinitely repeated setting the non-contractual monetary transfers can be enforced by the threat of cutting off cooperation permanently. The effective liability limit can then be endogenously derived from the rate of time preference.

Furthermore, in the infinitely repeated setting, there are additional complicated, but well-understood modifications by which monetary transfers can be dispensed with entirely, and their incentive effects replicated by changing the number of tokens allocated to each sponsor in each successive project as a function of the outcomes in previous projects (see [5, 8]).

Also by standard arguments, our qualitative results extend further to infinitely repeated, non-contractual environments with many sponsors, in which pairs of sponsors match randomly each period, and in which each sponsor's history of cooperation is publicly observed. Such an environment may come closest to describing the potential use of multi-sponsor WSNs within a relatively small scientific community. But as the use of WSNs diffuses to wider audiences of users, such communal reputation mechanisms may begin to break down, making explicit contracts more useful.

## 6 Future work

We are now simulating the operation of small scale multiple-sponsor networks using the ns-2 network simulator to estimate the fraction of efficient payoffs attained in token equilibria as a function of the sponsors' liability. We are developing methods to evaluate the performance of token equilibria for a large-scale sensor networks under various realistic scenarios, including a range of probability distributions for requesting and providing favors.

It has been shown by [6] in a related model that there can be efficiency gains to gradually forgiving debts of favors. In our context this suggests that it may be helpful to depreciate disparities in token holdings over time. Ultimately, under any given liability limit the optimal value of each sensor's marginal token ought to vary with the number of tokens held by it and its siblings. The fact that sensors do not know how many tokens their siblings hold may make computing an optimal equilibrium an intractable problem, but token equilibria are certainly amenable to tractable improvements, which we intend to pursue.

Existing middlewares for sensor networks typically focus on rapid development and integration, but within a network for a single application and administrative domains. To that end, we would like to propose an *agoric* approach to middleware design. We have started the development of its initial prototype. We then plan to deploy this middle-

ware on the real sensors for performance evaluation. For scalability, we plan to leverage this middleware substrate towards development of *autonomous agents* to automate a broad range of activities including resource discovery and price negotiation.

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