THE LANDAU-GINZBURG/CALABI-YAU CORRESPONDENCE

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Given a nondegenerate quasihomogeneous polynomial $W(x_1, \ldots, x_N)$ with weights $c_1, \ldots, c_N$, two theories can be defined:

- the Gromov-Witten theory of the hypersurface
$$X_W = \{W = 0\} \subset \mathbb{P}(c_1, \ldots, c_N);$$
- the FJRW theory of the polynomial W with the group $\langle J \rangle \subset G$.

The (genus-zero) Landau-Ginzburg/Calabi-Yau correspondence is the assertion that, when X_W is Calabi-Yau, these two theories should be "equivalent" in genus zero. Specifically, this means that:

(1) There should be an isomorphism on the level of state spaces:
$$H^*(X_W) \cong \mathcal{H}_{W,\langle J \rangle};$$

(2) Both sets of genus-zero correlators should be encoded by state-space-valued J-functions, and these should be related to one another by a change of variables, a linear transformation on the state space parameters, and an analytic continuation of the generating function variables.

More generally, the correspondence relates the FJRW theory of (W, G) to the Gromov-Witten theory of $[X_W/(G/\langle J \rangle)]$ whenever a Calabi-Yau condition is satisfied.[1]

In this lecture, we will explain why such a correspondence might be expected, at least in the case where $W = x_1^5 + \cdots + x_5^5$ is the Fermat quintic. The perspective we will describe, which is given in terms of variation of stability conditions, is useful both in motivating the correspondence and in pointing towards a number of generalizations [7] [10]. Moreover, it naturally leads to a definition of the virtual cycle on narrow components of the FJRW moduli space.

[1] The reason we consider $G/\langle J \rangle$ instead of G on the Gromov-Witten side is that J acts trivially on the ambient projective space.

1. The state space correspondence for the quintic

Let $W = x_1^5 + \cdots + x_5^5$. To understand why the state spaces of FJRW theory and Gromov-Witten theory might be related to one another in this case, consider an action of $\mathbb{C}^*$ on $\mathbb{C}^5 \times \mathbb{C}$ by

$$\lambda(x_1, \ldots, x_5) = (\lambda x_1, \ldots, \lambda x_5, \lambda^{-5} p).$$

The quotient $(\mathbb{C}^5 \times \mathbb{C})/\mathbb{C}^*$ is non-separated, but it admits two maximal separated subquotients:[2]

- Restricting to the locus where $(x_1, \ldots, x_5) \neq 0$ yields the quotient
$$\frac{(\mathbb{C}^5 \setminus \{0\}) \times \mathbb{C}}{\mathbb{C}^*},$$
which is the total space of the bundle $\mathcal{O}_{\mathbb{P}^4}(-5)$.
- Restricting to the locus where $p \neq 0$ yields the quotient
$$\frac{\mathbb{C}^5 \times (\mathbb{C} \setminus \{0\})}{\mathbb{C}^*}.$$
Rescaling the p-coordinate to 1 shows this to be the orbifold $[\mathbb{C}^5/\mathbb{Z}_5]$.

The polynomial

$$\overline{W}(x_1, \ldots, x_5, p) = pW(x_1, \ldots, x_5)$$

gives a well-defined map out of either of these quotients. By definition,

$$\mathcal{H}_{W,J} = H^*_{CR}([\mathbb{C}^5/\mathbb{Z}_5], W^{+\infty}),$$

since $J \cong \mathbb{Z}_5$. On the other hand, deformation-retracting the fibers of $\mathcal{O}_{\mathbb{P}^4}(-5)$ shows that

$$H^*(\mathcal{O}_{\mathbb{P}^4}(-5), W^{+\infty}) \cong H^*(\mathbb{P}^4, \mathbb{P}^4 \setminus X_W) \cong H^*(X_W),$$

where the second step follows from the Thom isomorphism.

Thus, the state spaces of FJRW theory and Gromov-Witten theory arise in completely analogous ways. In fact, this observation can be leveraged, with the help of a number of exact sequences, to prove that the two state spaces are isomorphic [6] [4].

[2] In the language of geometric invariant theory, these are the GIT quotients associated to characters $\theta : \mathbb{C}^* \to \mathbb{C}^*$ given by $\theta(\lambda) = \lambda^d$, and the two possibilities correspond to $d > 0$ or $d < 0$.

2. The virtual cycles for the quintic

The moduli space for the FJRW theory of the quintic (with the group $\langle J\rangle = \mathbb{Z}_5$) is

$$\mathcal{W}_{g,n} = \{(C, x_1, \ldots, x_n; L; \varphi) \mid \varphi : L^{\otimes 5} \cong \omega_{C,\log}\}.$$

(The restriction to the group $\langle J\rangle$ forces the five line bundles associated to the variables of W to be isomorphic.) We would like to mimic the ideas of the previous section to explain why this might be related to the moduli space of stable maps to the quintic $X_W = \{W = 0\} \subset \mathbb{P}^4$. Thus, we will begin with a larger but badly-behaved space, in which the two moduli spaces can be seen to sit as substacks by imposing two different stability conditions.

Let us first explain the procedure in the case where $n = 0$. In this case, the larger moduli space is

$$\mathfrak{S}_{g,0,\beta} = \{(C, L, s_1, \ldots, s_5, p) \mid s_i \in H^0(L),\ p \in H^0(P)\},$$

where $\beta = \deg(L)$ and the line bundle P is defined by

$$P := L^{\otimes -5} \otimes \omega_C.$$

This is a non-separated Artin stack, but inside of it we can find two separated Deligne-Mumford substacks by imposing a stability and non-degeneracy condition either on $s_1, \ldots, s_5$ or on p:

- Let $\mathcal{D}^{CY}_{g,0,\beta} \subset \mathfrak{S}_{g,0,\beta}$ be the locus where $s_1, \ldots, s_5$ define a stable map $f : C \to \mathbb{P}^4$. That is, we require that these sections have no common vanishing, and that $L^{\otimes 3} \otimes \omega_{C,\log}$ is ample. Then

  $$\mathcal{D}^{CY}_{g,0,\beta} = \{(C, f : C \to \mathbb{P}^4, p) \mid p \in H^0(f^*\mathcal{O}(-5) \otimes \omega_{C,\log})\}.$$

- Let $\mathcal{D}^{LG}_{g,0,\beta} \subset \mathfrak{S}_{g,0,\beta}$ be the locus where p defines a stable map $C \to \mathbb{P}^0 = \{\bullet\}$. This means that p is nowhere vanishing, so it gives an isomporphism $P \cong \mathcal{O}_C$. Hence:

  $$\mathcal{D}^{LG}_{g,0,\beta} = \{(C, L, s_1, \ldots, s_5, \varphi) \mid s_i \in H^0(L),\ \varphi : L^{\otimes 5} \cong \omega_{C,\log}\}.$$

The two stacks $\mathcal{D}^{CY/LG}_{g,0,\beta}$ admit analogous perfect obstruction theories

$$\mathbb{E}^\bullet = R^\bullet\pi_*(\mathcal{L}^{\oplus 5} \oplus \mathcal{P})$$

relative to the moduli space of stable curves equipped with a line bundle L of degree β. Here, $\mathcal{L}$ is the universal line bundle on the universal curve $\pi : \mathcal{C} \to \mathcal{D}^{CY/LG}_{g,0,\beta}$, and $\mathcal{P} = \mathcal{L}^{\otimes -5} \otimes \omega_\pi$. We will not prove that this indeed gives a perfect obstruction theory in each case; however, the intuition is that deformations of $(C, L, s_1, \ldots, s_5, p)$ relative to (C, L) are given by sections of $L^{\oplus 5} \oplus P$.

Define a homomorphism

$$\sigma : R^1\pi_*(\mathcal{L}^{\oplus 5} \oplus \mathcal{P}) \to \mathcal{O}_{\mathcal{D}^{CY/LG}_{g,0,\beta}},$$

over either of the moduli spaces $\mathcal{D}^{CY/LG}_{g,0,\beta}$ as follows: The fiber of σ over a point $(C, L, s_1, \ldots, s_5, p)$ is the homomorphism

$$\sigma|_{(C,L,s_1,\ldots,s_5,p)} : H^1(L)^{\oplus 5} \oplus H^1(P) \to \mathbb{C}$$

$$(t_1, \ldots, t_5, u) \mapsto \sum_{i=1}^{5} \frac{\partial \overline{W}}{\partial x_i}(s_1, \ldots, s_5, p) \cdot t_i + \frac{\partial \overline{W}}{\partial p}(s_1, \ldots, s_5, p) \cdot u.$$

Here, as before, $\overline{W}(x_1, \ldots, x_5, p) = p \cdot (x_1^5 + \cdots + x_5^5)$.

It is worth checking that σ is well-defined. For example,

$$\frac{\partial \overline{W}}{\partial p}(s_1, \ldots, s_5, p) \in H^0(L^{\otimes 5}) = H^1(L^{\otimes -5} \otimes \omega_C)^\vee = H^1(P)^\vee, \tag{1}$$

so pairing this with u indeed gives an element of $\mathbb{C}$.

Homomorphisms like σ, from the obstruction sheaf to the structure sheaf of a moduli stack, are referred to as **cosections**.[3] The crucial fact about cosections is the following result of Kiem-Li [8]:

Theorem 2.0.1. *Let $\overline{\mathcal{M}}$ be a Deligne-Mumford stack endowed with a perfect obstruction theory. Suppose that there exists a cosection*

$$\sigma : \mathcal{O}b_{\overline{\mathcal{M}}} \to \mathcal{O}_{\overline{\mathcal{M}}}$$

that is surjective on an open set $U \subset \overline{\mathcal{M}}$. Then $\overline{\mathcal{M}}$ has a "localized virtual cycle"

$$[\overline{\mathcal{M}}]^{vir}_{\sigma,loc} \in A_*(\overline{\mathcal{M}} \setminus U).$$

This cycle enjoys the usual properties of virtual cycles, and pushes forward to the usual virtual cycle of $\overline{\mathcal{M}}$ under the inclusion.

The locus $\overline{\mathcal{M}} \setminus U$ on which σ is not surjective is known as the **degeneracy locus** of the cosection. In our case, the degeneracy locus depends on which of the two moduli stacks we are working with:

- An element $(C, L, s_1, \ldots, s_5, p) \in \mathcal{D}^{CY}_{g,0,\beta}$ lies in the degeneracy locus of σ if

$$\sum_{i=1}^{5} \frac{\partial \overline{W}}{\partial x_i}(s_1, \ldots, s_5, p) \cdot t_i + \frac{\partial \overline{W}}{\partial p}(s_1, \ldots, s_5, p) \cdot u = 0.$$

[3]To be precise, the homomorphism σ defined above is not yet a cosection, since its domain is a relative rather than an absolute obstruction sheaf. However, the absolute obstruction sheaf is a quotient of the relative obstruction sheaf, and one can check that σ descends to this quotient.

for all $t_1, \dots, t_5, u$. This forces that

$$\frac{\partial \overline{W}}{\partial p}(s_1, \dots, s_5, p) = s_1^5 + \dots + s_5^5 = 0,$$

so in other words, the map $f : C \to \mathbb{P}^4$ induced by $s_1, \dots, s_5$ lands in the quintic hypersurface X_W. Furthermore, we must have

$$\frac{\partial \overline{W}}{\partial x_i}(s_1, \dots, s_5, p) = 5p \cdot s_i^4 = 0$$

for each i. Since the sections s_i have no common vanishing, this implies that $p = 0$. We conclude that the degeneracy locus of σ is precisely $\overline{\mathcal{M}}_{g,0}(X_W, \beta)$.

- The degeneracy locus of σ on $\mathcal{D}^{LG}_{g,0,\beta}$ is defined by the same condition. In this case, though, the fact that

$$\frac{\partial W}{\partial x_i}(s_1, \dots, s_5, p) = 5p \cdot s_i^4 = 0$$

for all i implies that $s_1 = \dots = s_5 = 0$, since p is nowhere-vanishing. Thus, this degeneracy locus is the moduli space $\mathcal{W}_{g,0}$ of FJRW theory.

We have shown, then, that the moduli spaces of Gromov-Witten theory and FJRW theory arise out of entirely analogous constructions. Furthermore, Kiem-Li's theorem yields a virtual cycle on each of the moduli spaces. Chang-Li have proven [1] that, on the Gromov-Witten side, this cosection-localized virtual cycle yields invariants that agree (up to a sign) with the usual Gromov-Witten invariants of the quintic X_W; analogously, Chang-Li-Li have shown [2] that the cosection-localized virtual cycle agrees with Fan-Jarvis-Ruan's original definition of the virtual cycle when both are defined.[4]

It is somewhat subtle to extend this argument to the case where $n > 0$. The issue is that we now want to define

$$P = L^{\otimes -5} \otimes \omega_{C,\log},$$

but then the appeal to Serre duality in (1), which is necessary in order to define the cosection, fails.

The solution is to work on a single component $\mathcal{W}_{g,n}(\gamma_1, \dots, \gamma_n)$, where

$$\gamma_i = e^{2\pi i \frac{m_i}{5}} \in \mathbb{Z}_5$$

[4] In the case where $W = x^r$, the agreement of the cosection construction of the virtual class in FJRW theory with all other existing constructions also follows from [9].

gives the multiplicity of L at the ith marked point. Then, we set

$$\mathfrak{S}_{g,n,\beta} = \{(C, x_1, \ldots, x_n, L, s_1, \ldots, s_5, p) \mid s_i \in H^0(L),\ p \in H^0(P)\},$$

where C is a stable curve with $\mathbb{Z}_5$ orbifold structure *only* at nodes and

$$P := L^{\otimes -5} \otimes \omega_{C,\log} \otimes \mathcal{O}\left(-\sum_{i=1}^{n} m_i[x_i]\right).$$

Now, *under the assumption that* $m_i \geq 1$ for all i, this maps into $L^{\otimes -5} \otimes \omega_C$, so the application of Serre duality in the definition of σ still goes through. From that point, the argument proceeds exactly as previously, and the resulting moduli space on the Gromov-Witten side is still $\overline{\mathcal{M}}_{g,n}(X_W, \beta)$. On the Landau-Ginzburg side, one obtains a moduli space parameterizing

$$(C, x_1, \ldots, x_n, L, \varphi),$$

in which

$$\varphi : L^{\otimes 5} \cong \omega_{C,\log} \otimes \mathcal{O}\left(-\sum_{i=1}^{n} m_i[x_i]\right).$$

Recall, however, that this is precisely the equation that $|\tilde{L}|$ satisfies on $|\tilde{C}|$ when $\tilde{L}$ is a fifth root of $\omega_{\tilde{C},\log}$ on an orbifold curve $\tilde{C}$. Indeed, forgetting the orbifold structure at the marked points shows the above moduli space to be isomorphic to the moduli space $\mathcal{W}_{g,n}(\gamma_1, \ldots, \gamma_n)$ of FJRW theory.

Remark 2.0.2. The cosection construction yields a virtual class *only* on components $\mathcal{W}_{g,n}(\gamma_1, \ldots, \gamma_n)$ of the moduli space on which every γ_i is narrow. Thus, it can be applied only in situations where the FJRW invariants are known to vanish— or, as explained in the previous lecture, can be defined to vanish while preserving the properties of a CohFT— on the remaining components.

3. The Landau-Ginzburg/Calabi-Yau correspondence for the quintic

Equipped with a definition of the virtual cycle for FJRW theory, we are ready to state the genus-zero correspondence for the quintic:

Theorem 3.0.3 (Chiodo-Ruan [5]). *The genus-zero FJRW theory of* $(W, \langle J \rangle)$ *can be encoded in a generating function* $J^{FJRW}(t, z)$ *taking values in* $\mathcal{H}_{W,\langle J \rangle}$*, which is analogous to the* $H^*(X_W)$*-valued Gromov-Witten J-function* $J^{GW}(q, z)$ *for the quintic threefold* $X_W \subset \mathbb{P}^4$*. After identifying* $q = t^{-5}$*, these two J-functions are related by a change of* *variables in q and t, an analytic continuation, and a linear transformation in the state space coordinate.*

More generally, Chiodo-Iritani-Ruan [3] have proven the LG/CY correspondence for any hypersurface in weighted projective space with the group $G = \langle J \rangle$.

While the cosection construction is very useful in motivating the connection between the FJRW theory of $(W, \langle J \rangle)$ and the Gromov-Witten theory of X_W, it does not directly lead to any proof of this result in the way that the argument of Section 1 led to a proof of the state space isomorphism.

Rather, Chiodo-Ruan prove the Landau-Ginzburg/Calabi-Yau correspondence in roughly the following steps:

(1) Realize the FJRW theory of $(W, \langle J \rangle)$ as a twisted theory over the moduli space of curves.
(2) By relating the twisted to the untwisted theory, construct an explicit function $I^{FJRW}(t, z)$ as a hypergeometric modification of the untwisted J-function. This differs from $J^{FRW}(t, z)$ via a change of variables, essentially by construction.
(3) Observe that, when expanded in a basis for the state space, $I^{FJRW}(t, z)$ encodes a basis of solutions to the same Picard-Fuchs equation as does $I^{GW}(q, z)$. Thus, after an analytic continuation from $q = 0$ to $t = 0$ and a linear transformation matching the two bases of solutions, the two I-functions coincide.

Since Chiodo and Ruan's work, a simpler proof of the correspondence has been given by Ross-Ruan [10] using the notion of quasimaps developed by Ciocan-Fontanine and Kim.

4. Generalizing to complete intersections

The Landau-Ginzburg/Calabi-Yau correspondence, as stated above, relates the Gromov-Witten theory of a Calabi-Yau hypersurface in weighted projective space to the FJRW theory of a polynomial W. We would like to generalize this statement to Calabi-Yau complete intersections, but in order to do so, it is necessary to define a Landau-Ginzburg theory associated to a collection of polynomials rather than just one.

Let $W_1, \ldots, W_r \in \mathbb{C}[x_1, \ldots, x_N]$ be a collection of homogeneous polynomials of the same degree d defining a nonsingular Calabi-Yau complete intersection

$$X_{\overline{W}} = \{W_1 = \cdots = W_r = 0\} \subset \mathbb{P}^{N-1}.$$

(More generally, one can allow $W_1, \ldots, W_r$ to be quasihomogeneous of the same weights, but we will restrict to the homogeneous case for ease of exposition.)

Then the polynomial

$$\overline{W}(x_1, \ldots, x_N, p_1, \ldots, p_r) = p_1 W_1(\mathbf{x}) + \cdots + p_r W_r(\mathbf{x})$$

gives a well-defined map out of the (non-separated) quotient

$$\frac{\mathbb{C}^N \times \mathbb{C}^r}{\mathbb{C}^*},$$

where $\mathbb{C}^*$ acts by

$$\lambda(x_1, \ldots, x_N, p_1, \ldots, p_r) = (\lambda x_1, \ldots, \lambda x_N, \lambda^{-d} p_1, \ldots, \lambda^{-d} p_r).$$

The two maximal separated subquotients given by imposing $\mathbf{x} \neq 0$ or $\mathbf{p} \neq 0$ are:

$$\frac{(\mathbb{C}^N \setminus \{0\}) \times \mathbb{C}^r}{\mathbb{C}^*} = \mathcal{O}_{\mathbb{P}^{N-1}}(-d)^{\oplus r}$$

and

$$\frac{\mathbb{C}^N \times (\mathbb{C}^r \setminus \{0\})}{\mathbb{C}^*} = \mathcal{O}_{\mathbb{P}(d,\ldots,d)}(-1)^{\oplus N}.$$

The relative cohomologies of these two orbifolds give the state spaces of the two theories:

$$H^*(\mathcal{O}_{\mathbb{P}^{N-1}}(-d)^{\oplus r}, \overline{W}^{+\infty}) \cong H^*(\mathbb{P}^{N-1}, \mathbb{P}^{N-1} \setminus X_{\overline{W}}) \cong H^*(X_{\overline{W}}),$$

while on the Landau-Ginzburg side, we *define*

$$\begin{aligned} \mathcal{H}_{\overline{W}} &= H^*_{CR}(\mathcal{O}_{\mathbb{P}(d,\ldots,d)}(-1)^{\oplus N}, \overline{W}^{+\infty}) \\ &= H^*(\mathcal{O}_{\mathbb{P}^{r-1}}, \overline{W}^{+\infty}) \oplus H^*(\mathbb{P}^{r-1})^{\oplus d-1}. \end{aligned}$$

One can show, by an argument analogous to the hypersurface case, that

$$H^*(X_{\overline{W}}) \cong \mathcal{H}_{\overline{W}}.$$

The proof of this fact appears in unpublished work of Chiodo-Nagel [4]. In the special cases where $X_{\overline{W}}$ is a Calabi-Yau threefold complete intersection, it can also be found in [7].

From here, the route to a moduli space and virtual cycle on the Landau-Ginzburg side is reasonably clear by analogy to the hypersurface case. For example, when $n = 0$, one defines

$$\mathfrak{S}_{g,0,\beta} = \{(C, L, s_1, \ldots, s_N, p_1, \ldots, p_r) \mid s_i \in H^0(L),\ p_j \in H^0(P)\},$$

where

$$P = L^{\otimes -d} \otimes \omega_C.$$

This has two Deligne-Mumford substacks:

- The locus on which $s_1, \ldots, s_N$ define a stable map to P^{N-1} yields

$$\mathcal{D}^{CY}_{g,0,\beta} = \{(C, f : C \to \mathbb{P}^{N-1}, p_1, \ldots, p_r) \mid p_j \in H^0(f^*\mathcal{O}(-d) \otimes \omega_C)\}.$$

- The locus on which $p_1, \ldots, p_r$ define a stable map to $\mathbb{P}^{r-1}$ yields

$$\mathcal{D}^{LG}_{g,0,\beta} = \{(C, L, f : C \to \mathbb{P}^{r-1}, s_1, \ldots, s_N) \mid {}^{L^{\otimes d} \cong f^*\mathcal{O}(-1)\otimes\omega_C,}_{s_i \in H^0(L)}\}.$$

The derivatives of $\overline{W}$ can again be used to define a cosection on either of these two moduli spaces. It is straightforward to check that the degeneracy locus of this cosection is

$$\overline{\mathcal{M}}_{g,0}(X_{\overline{W}}, \beta) = \{(C, f : C \to X_{\overline{W}} \subset \mathbb{P}^{N-1}, p_1 = \cdots = p_r = 0\} \subset \mathcal{D}^{CY}_{g,0,\beta}$$

on the Calabi-Yau side and is

$$\{(C, L, f : C \to \mathbb{P}^{r-1}) \mid L^{\otimes d} \cong f^*\mathcal{O}(-1) \otimes \omega_C\} \subset \mathcal{D}^{LG}_{g,0,\beta}.$$

This latter is *defined* to be the moduli space on the Landau-Ginzburg side, and the virtual cycle is defined to be the result of cosection localization.

More generally, the moduli space of the Landau-Ginzburg theory is defined to have components $\widetilde{M}_{g,(m_1,\ldots,m_n)}(\mathbb{P}^{r-1}, \beta)$ for each choice of $m_1, \ldots, m_n \in \{0, 1, \ldots, d-1\}$. Such a component parameterizes

$$\{(C; x_1, \ldots, x_n; L, f : C \to \mathbb{P}^{r-1}) \mid {}^{L^{\otimes d} \cong f^*\mathcal{O}(-1)\otimes\omega_{C,\log},}_{\text{mult}_{x_i}(L) = m_i}\}.$$

As long as $m_i \geq 1$ for each i, a virtual cycle can be obtained via the cosection construction.

Correlators, then, must be defined as integrals against these virtual cycles. Recall that

$$\mathcal{H}_{\overline{W}} = H^*(\mathcal{O}_{\mathbb{P}^{r-1}}, \overline{W}^{+\infty}) \oplus H^*(\mathbb{P}^{r-1})^{\oplus d-1},$$

with all but the first being narrow sectors. For narrow insertions, we define the correlator $\langle \tau_{a_1}(\phi_1^{(m_1)}) \cdots \tau_{a_n}(\phi_n^{(m_n)})\rangle_{g,n,\beta}$ to be

$$\int_{[\widetilde{\mathcal{M}}_{g,(m_1,\ldots,m_n)}(\mathbb{P}^{r-1},\beta)]^{\text{vir}}} \psi_1^{a_1} \text{ev}_1^*(\phi_1) \cdots \psi_n^{a_n} \text{ev}_n^*(\phi_n),$$

where $\phi_i^{(m_i)}$ denotes the element $\phi_i \in H^*(\mathbb{P}^{r-1})$ in the sector indexed by $m_i \in \{1, \ldots, d-1\}$. Any correlator with an insertion from the broad sector $H^*(\mathcal{O}_{\mathbb{P}^{r-1}}, \overline{W}^{+\infty})$ is defined to be zero.

Having made this definition, the theorem is exactly analogous to Chiodo-Ruan's result:

Theorem 4.0.4 (Chiodo-Ruan [5], Clader [7]). *Let $W_1, \ldots, W_r$ be homogeneous polynomials of the same degree d defining a nonsingular Calabi-Yau threefold complete intersection in $\mathbb{P}^{N-1}$. Then the genus-zero Landau-Ginzburg (or "hybrid") theory can be encoded in a generating function $J^{hyb}(t,z)$ taking values in $\mathcal{H}_{\overline{W}}$, which is analogous to the $H^*(X_{\overline{W}})$-valued Gromov-Witten J-function $J^{GW}(q,z)$ for the complete intersection $X_{\overline{W}}$. After an identification between q and t, these two J-functions are related by a change of variables in q and t, an analytic continuation, and a linear transformation in the state space coordinate.*

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