Model Reference Neuroadaptive Control Revisited: How to Keep the System Trajectories on a Given Compact Set

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We revisit the design of model reference neuroadaptive control laws. This class of control laws have the capability to approximate any system uncertainty with an unknown structure and parameters on a compact set using neural networks. Yet, a challenge in their design is to keep the controlled system trajectories on this compact set for satisfying the universal function approximation property. Motivated by this challenge, a new model reference neuroadaptive control architecture is proposed to keep the controlled system trajectories within a-priori, user-defined compact set while addressing disturbance rejection and system uncertainty suppression. The presented architecture is illustrated by a numerical example.

I. Introduction

Neuroadaptive control laws (see, for example, Refs. 1–10 and references therein) are preferred over classical adaptive control laws for applications involving system uncertainties with unknown structures and parameters. This is due to the fact that they are based on the universal function approximation property, and hence, handle larger class of system uncertainties that may not be tolerated by classical adaptive control laws. Specifically, neuroadaptive control laws have the capability to closely approximate any system uncertainty with an unknown structure and parameters on a compact set using neural networks. Yet, a challenge in their design is to keep the controlled system trajectories on this compact set for satisfying the universal function approximation property.

To this end, notable contributions include Refs. 11–16. In particular, an error transformation approach

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is developed in Ref. 11 to achieve constrained performance guarantees under the assumption that every element of the state vector can be accessed by the control signals. This limitation is avoided in Ref. 12 by considering a backstepping approach under assumption that a desired trajectory and its derivatives are available and all bounded, where it is also extended to a generalized class of uncertain dynamical systems in Ref. 13. Restricted potential functions (barrier Lyapunov functions) are employed in the neuroadaptive control schemes of Refs. 14 and 15 in the context of backstepping approach to construct a closed-loop dynamical system with strict performance guarantees. Additionally, a model reference neuroadaptive control methodology is considered in Ref. 16 under the assumption that the approximation tolerance resulting from the neural network approximation of the system uncertainty is known inside a compact set and upper bounded by a known function outside.

In this paper, we propose a new model reference neuroadaptive control architecture to keep the controlled system trajectories within a-priori, user-defined compact set while addressing disturbance rejection and system uncertainty suppression. This is accomplished by utilizing a set-theoretic approach based on restricted potential functions, where the approximation tolerance resulting from the neural network approximation of the system uncertainty is treated as unknown. The presented architecture is illustrated by a numerical example. Finally, it should be noted that the proposed architecture can be viewed as a generalization of authors’ previous work documented in Ref. 17 that utilize parameterized system uncertainty models with a known structure but unknown parameters.

II. Preliminaries

We begin with the definition of the projection operator. Let \( \Omega = \{ \theta \in \mathbb{R}^n : (\theta^\text{min}_i \leq \theta_i \leq \theta^\text{max}_i)_{i=1,2,\ldots,n} \} \) be a convex hypercube in \( \mathbb{R}^n \) where \( (\theta^\text{min}_i, \theta^\text{max}_i) \) represent the minimum and maximum bounds for the \( i^{th} \) component of the \( n \)-dimensional parameter vector \( \theta \). Additionally, for a sufficiently small positive constant \( \nu \), a second hypercube is defined by \( \Omega_\nu = \{ \theta \in \mathbb{R}^n : (\theta^\text{min}_i + \nu \leq \theta_i \leq \theta^\text{max}_i - \nu)_{i=1,2,\ldots,n} \} \) where \( \Omega_\nu \subset \Omega \).

Then, the projection operator \( \text{Proj} : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n \) is defined component-wise by

\[
\text{Proj}(\theta, y) \triangleq \begin{cases} 
(\theta^\text{max}_i - \theta_i) \frac{\nu}{\nu} y_i, & \text{if } \theta_i > \theta^\text{max}_i - \nu \text{ and } y_i > 0 \\
(\theta^\text{min}_i - \theta_i) \frac{\nu}{\nu} y_i, & \text{if } \theta_i < \theta^\text{min}_i + \nu \text{ and } y_i < 0 \\
y_i, & \text{otherwise}
\end{cases}
\]

(1)

where \( y \in \mathbb{R}^n \) (see Ref. 16). As a consequence of this definition, it follows that

\[
\text{Proj} (\theta - \theta^*)^T (\text{Proj} (\theta, y) - y) \leq 0, \quad \theta^* \in \mathbb{R}^n,
\]

(2)

holds\textsuperscript{16,18}. Additionally, it can be generalized to matrices as

\[
\text{Proj}_m(\Theta, Y) = (\text{Proj}(\text{col}_1(\Theta), \text{col}_1(Y)), \ldots, \text{Proj}(\text{col}_m(\Theta), \text{col}_m(Y)))
\]

(3)
where $\Theta \in \mathbb{R}^{n \times m}$, $Y \in \mathbb{R}^{n \times m}$, and $\text{col}_i(\cdot)$ denotes $i$th column operator. In this case, for a given $\Theta^* \in \mathbb{R}^{n \times m}$, it follows from (2) that

$$\text{tr} \left[ ((\Theta - \Theta^*)^T(\text{Proj}_m(\Theta, Y) - Y) \right] = \sum_{i=1}^{m} \left[ \text{col}_i(\Theta - \Theta^*)^T(\text{Proj}(\text{col}_i(\Theta), \text{col}_i(Y)) - \text{col}_i(Y)) \right] \leq 0. \quad (4)$$

We now overview the key results from Ref. 17. We start with the definition of the generalized restricted potential function (generalized barrier Lyapunov function). In particular, let $\|z\|_H = \sqrt{z^T H z}$ be a weighted Euclidean norm, where $z \in \mathbb{R}^p$ is a real column vector and $H \in \mathbb{R}^{p \times p}$. We define $\phi(\|z\|_H), \phi : \mathbb{R}^p \to \mathbb{R}$, to be a generalized restricted potential function (generalized barrier Lyapunov function) on the set

$$D_\epsilon \triangleq \{\|z\|_H : \|z\|_H \in [0, \epsilon)\}, \quad (5)$$

with $\epsilon \in \mathbb{R}_+$ being a-priori, user-defined constant, if the following statements hold:

i) If $\|z\|_H = 0$, then $\phi(\|z\|_H) = 0$.

ii) If $\|z\|_H \in D_\epsilon$ and $\|z\|_H \neq 0$, then $\phi(\|z\|_H) > 0$.

iii) If $\|z\|_H \to \epsilon$, then $\phi(\|z\|_H) \to \infty$.

iv) $\phi(\|z\|_H)$ is continuously differentiable on $D_\epsilon$.

v) If $\|z\|_H \in D_\epsilon$, then $\phi_d(\|z\|_H) > 0$, where

$$\phi_d(\|z\|_H) \triangleq \frac{d\phi(\|z\|_H)}{d\|z\|_H^2}. \quad (6)$$

vi) If $\|z\|_H \in D_\epsilon$, then

$$2\phi_d(\|z\|_H)\|z\|_H^2 - \phi(\|z\|_H) > 0. \quad (7)$$

As noted in Ref. 17, a candidate generalized restricted potential function satisfying the above conditions has the form $\phi(\|z\|_H) = \|z\|_H^2 / (\epsilon - \|z\|_H)$, $\|z\|_H \in D_\epsilon$. Furthermore, the above definition can be viewed as a generalized version of the restricted potential function (barrier Lyapunov function) definitions used by the authors of Refs. 14, 15, 19–22.

Next, consider the uncertain dynamical system given by

$$\dot{x}(t) = Ax(t) + B\Lambda(u(t) + \delta(t, x(t))), \quad x(0) = x_0, \quad t \geq 0, \quad (8)$$

where $x(t) \in \mathbb{R}^n$, $t \geq 0$, is the measurable state vector, $u(t) \in \mathbb{R}^m$, $t \geq 0$, is the control input, $A \in \mathbb{R}^{n \times n}$ is a known system matrix, $B \in \mathbb{R}^{n \times m}$ is a known input matrix, $\delta : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^m$ is a system uncertainty, $\Lambda \in \mathbb{R}_+^{m \times m} \cap \mathbb{D}^{m \times m}$ is an unknown control effectiveness matrix, and the pair $(A, B)$ is controllable. The
Assumption 1. The system uncertainty in (8) is parameterized as

$$\delta(t, x(t)) = W_s^T(t)\sigma_s(x(t)), \quad x(t) \in \mathbb{R}^n$$

where $W_s(t) \in \mathbb{R}^{s \times m}$, $t \geq 0$, is a bounded unknown weight matrix (i.e., $\|W_s(t)\|_2 \leq w_s, t \geq 0$) with a bounded time rate of change (i.e., $\|\dot{W}_s(t)\|_2 \leq \dot{w}_s, t \geq 0$), $\sigma : \mathbb{R}^n \to \mathbb{R}^s$ is a known basis function.

Note that the above assumption is not valid for applications involving unstructured system uncertainties and it is significantly relaxed in the next section motivated by this fact. However, to continue overviewing the results in Ref. 17, we make this assumption only in this section. Now, we consider the feedback control law given by

$$u(t) = u_n(t) + u_a(t), \quad t \geq 0,$$

where $u_n(t) \in \mathbb{R}^m$, $t \geq 0$, and $u_a(t) \in \mathbb{R}^m$, $t \geq 0$, are the nominal and adaptive control laws, respectively. Furthermore, let the nominal control law be

$$u_n(t) = -K_1x(t) + K_2c(t), \quad t \geq 0,$$

where $c(t) \in \mathbb{R}^{n_c}$ is a bounded reference command such that $A_r \triangleq A - BK_1$, $K_1 \in \mathbb{R}^{m \times n}$, is Hurwitz and $B_r \triangleq BK_2$, $K_2 \in \mathbb{R}^{m \times n_c}$. Consider next the reference model given by

$$\dot{x}_r(t) = A_r x_r(t) + B_r c(t), \quad x_r(t) \in \mathbb{D}_r, \quad x_r(0) = x_{r0}, \quad t \geq 0,$$

where $x_r(t) \in \mathbb{R}^n$ is the reference model state vector, $A_r \in \mathbb{R}^{n \times n}$ is the desired Hurwitz system matrix, $B_r \in \mathbb{R}^{n \times n_c}$ is the command input matrix, and $\mathbb{D}_r$ is a user-defined set. Using (8), (9), (10), (11) and (12), the system error dynamics is given by

$$\dot{e}(t) = A_r e(t) + BA(W_0^T(t)\sigma_0(x(t)) + u_a(t)), \quad e(0) = e_0, \quad t \geq 0,$$

where $W_0(t) \triangleq [W_s^T(t), (\Lambda^{-1} - I_{m \times m})K_1, -(\Lambda^{-1} - I_{m \times m})K_2]^T \in \mathbb{R}^{(s+n+n_c) \times m}$, $t \geq 0$, is an unknown aggregated weight matrix and $\sigma_0(x(t), c(t)) \triangleq [\sigma^T(x(t)), x^T(t), c^T(t)]^T \in \mathbb{R}^{s+n+n_c}$, $t \geq 0$, is a known basis function.

Finally, let the adaptive control law be given by

$$u_a(t) = -\hat{W}_0^T(t)\sigma_0(x(t)), \quad t \geq 0,$$
where $\hat{W}_0(t) \in \mathbb{R}^{(s+n+\nu) \times m}$, $t \geq 0$, is the estimate of $W_0(t)$, $t \geq 0$, satisfying the update law

$$\dot{\hat{W}}_0(t) = \gamma \text{Proj}_m (\hat{W}_0(t), \phi_d(\|e(t)\|_P)\sigma_0(x(t))e^T(t)PB), \quad \hat{W}_0(0) = \hat{W}_{00}, \quad t \geq 0,$$

with $\hat{W}_{0\max}$ and $-\hat{W}_{0\min}$ being the maximum and minimum element-wise projection bounds respectively, $\gamma \in \mathbb{R}_+$ is the learning rate (i.e., adaptation gain), and $P \in \mathbb{R}_+^{n \times n}$ is a solution of the Lyapunov equation given by

$$0 = A_1^T P + PA_1 + R,$$

with $R \in \mathbb{R}_+^{n \times n}$. Using (13), (14), and (15) the system error dynamics and the weight estimation error dynamics are given by

$$\dot{e}(t) = A_e e(t) - BA\hat{W}_0^T(t)\sigma_0(x(t)), \quad e(0) = e_0, \quad t \geq 0,$$

$$\dot{\hat{W}}_0(t) = \gamma \text{Proj}_m (\hat{W}_0(t), \phi_d(\|e(t)\|_P)\sigma_0(x(t))e^T(t)PB) - \hat{W}_0(t), \quad \hat{W}_0(0) = \hat{W}_{00}, \quad t \geq 0,$$

where $\hat{W}_0(t) \triangleq \hat{W}_0(t) - W_0(t) \in \mathbb{R}^{(s+n+\nu) \times m}$, $t \geq 0$, is the weight estimation error and $e_0 \triangleq x_0 - x_{r0}$. Note that $\|W_0(t)\|_2 \leq w_0, t \geq 0$, and $\|\hat{W}_0(t)\|_2 \leq \tilde{w}_0, t \geq 0$, automatically holds.

By considering the Lyapunov function $V(e, \hat{W}_0) = \phi(\|e\|_P) + \gamma^{-1}\text{tr}[(\hat{W}_0\Lambda^{1/2}T)(\hat{W}_0\Lambda^{1/2})],$ one can calculate its time derivative along the closed-loop system trajectories (17) and (18) as $\dot{V}(e(t), \hat{W}_0(t)) \leq -\frac{1}{2}\alpha V(e, \hat{W}_0) + \mu_0$, where $\alpha \triangleq \frac{\lambda_{\text{min}}(R)}{\lambda_{\text{max}}(P)}$ and $\mu_0 \triangleq \gamma^{-1}\|\Lambda\|_2 \tilde{w}_0(\frac{1}{2}\alpha \tilde{w}_0 + \tilde{w}_0)$ where $\tilde{w}_0 = \hat{W}_{0\max} + w_0$. Following the results in Ref. 17, the boundedness of the closed-loop dynamical system given by (17) and (18) as well as the strict performance bound on the system error given by $\|e(t)\|_P < \epsilon$ is now immediate.

### III. A New Model Reference Neuroadaptive Control Law

First, we define

$$\text{tanh}(x) \triangleq [\text{tanh}(x(1)), \text{tanh}(x(2)), \ldots, \text{tanh}(x(s))]^T \in \mathbb{R}^s;$$

for any vector $x \in \mathbb{R}^s$. Furthermore, it follows from Ref. 23 that

$$|\eta| - \eta \text{tanh}(\eta) \leq \mathcal{L}, \quad \mathcal{L} = 0.2785,$$

holds for any scalar $\eta \in \mathbb{R}$. Using (19), this can be readily generalized to

$$\|\eta\|_2 - \eta^T \text{tanh}(\eta) \leq \mathcal{L},$$

for any vector $\eta \in \mathbb{R}^s$. We now state the key assumption, which relaxes the assumption stated in the previous section.
**Assumption 2.** The system uncertainty in (8) is parameterized as

$$
\delta(t, x(t)) = W_N^T(t) \sigma_N(x(t)) + \varepsilon_N(x(t)), \; x(t) \in \mathcal{D}
$$

(22)
on a compact set \(\mathcal{D}\), where \(W_N(t) \in \mathbb{R}^{s \times m}, t \geq 0\), is a bounded unknown weight matrix (i.e., \(\|W_N(t)\|_2 \leq w_N, t \geq 0\)) with a bounded time rate of change (i.e., \(\|\dot{W}_N(t)\|_2 \leq \dot{w}_N, t \geq 0\)), \(\sigma_N : \mathbb{R}^n \rightarrow \mathbb{R}^s\) is a basis function constructed using neural networks of the form \(\sigma_N(x(t)) = [\sigma_{N1}(x(t)), \sigma_{N2}(x(t)), \ldots, \sigma_{Nn}(x(t))]^T\), and \(\varepsilon_N : \mathbb{R}^n \rightarrow \mathbb{R}^m\) is the approximation tolerance upper bounded by an unknown constant \(\varepsilon^*\) such that

$$
\|\varepsilon_N(x(t))\|_2 \leq \varepsilon^*, \; \forall x(t) \in \mathcal{D}.
$$

(23)

Next, we consider the feedback control law given by (10). Using (8), (10), (11) and (12), the system error dynamics is given by

$$
\dot{e}(t) = A_r e(t) + B \Lambda (W_N^T(t) \sigma(x(t)) + u_a(t) + \varepsilon_N(x(t))), \; e(0) = e_0, \; t \geq 0,
$$

(24)

where \(e(t) \triangleq x(t) - x_r(t), t \geq 0\), is the system error, \(W(t) \triangleq [W_N^T(t), (\Lambda^{-1} - I_{m \times m}) K_1, -(\Lambda^{-1} - I_{m \times m}) K_2]^T \in \mathbb{R}^{s+n+n_e \times m}, t \geq 0\), is an unknown weight matrix, and \(\sigma(x(t), c(t)) \triangleq [\sigma_N^T(x(t)), x^T(t), c^T(t)]^T \in \mathbb{R}^{s+n+n_e}, t \geq 0\), is a known function involving neural network-based basis function, system state vector, and the reference command. Based on (24), let the adaptive control law be given by

$$
u_a(t) = -\dot{W}^T(t) \sigma(x(t)) - v(t), \; t \geq 0,
$$

(25)

where \(v(t) \in \mathbb{R}^m\) is a corrective signal, and \(\dot{W}(t) \in \mathbb{R}^{s+n+n_e \times m}, t \geq 0\), is the estimate of \(W(t), t \geq 0\), satisfying the update law

$$
\dot{\dot{W}}(t) = \gamma_2 \text{Proj}_{\mathcal{I}} \left( \dot{W}(t), \phi_d(\|e(t)\|_P) \sigma(x(t)) e^T(t) P B \right), \; \dot{W}(0) = W_0, \; t \geq 0,
$$

(26)

with \(\dot{W}_{\text{max}}\) and \(-\dot{W}_{\text{max}}\) being the maximum and minimum element-wise projection bounds respectively, \(\gamma_2 \in \mathbb{R}_+\) is the learning rate (i.e., adaptation gain), and \(P \in \mathbb{R}^{n \times n}\) is a solution of the Lyapunov equation given by (16).

Now, we design the corrective signal \(v(t)\) as

$$
v(t) = \tanh \left( \phi_d(\|e(t)\|_P) B^T P e(t) \right) \hat{q}(t), \; t \geq 0,
$$

(27)

where \(\hat{q}(t) \in \mathbb{R}_+, t \geq 0\), is the estimate of \(q \triangleq \frac{\lambda_{\text{max}}(\Lambda)}{\lambda_{\text{min}}(\Lambda)} \varepsilon^*\) satisfying the corrective update law

$$
\dot{\hat{q}}(t) = \gamma_2 \text{Proj} \left( \hat{q}(t), \phi_d(\|e(t)\|_P) e^T(t) P B \tanh \left( \phi_d(\|e(t)\|_P) B^T P e(t) \right) - \xi \hat{q}(t) \right), \; \hat{q}(0) = \hat{q}_0 \in \mathbb{R}_+, \; t \geq 0,
$$

(28)
with $\hat{q}_{\text{max}}$ and 0 being the maximum and minimum projection bounds respectively, $\gamma_2 \in R_+$ is the learning rate, and $\xi \in R_+$ is the $\sigma$-modification gain.

Now, one can write the system error dynamics, the weight estimation error dynamics, and the residual bound estimation error dynamics respectively as

$$
\dot{e}(t) = A_t e(t) - B \hat{W}^T(t) \sigma(x(t)) + B \Delta(x(t)) - v(t), \quad e(0) = e_0, \quad t \geq 0, \\
\hat{W}(t) = \gamma_1 \text{Proj}_m \left( \hat{W}(t), \phi_d ||e(t)||_P \sigma(x(t)) \right) \sigma^T(t) PB - \hat{W}(t), \quad \hat{W}(0) = \hat{W}_0, \quad t \geq 0, \\
\tilde{q}(t) = \gamma_2 \text{Proj} \left( \tilde{q}(t), \phi_d ||e(t)||_P \right) \sigma^T(t) PB \tanh \left( \phi_d ||e(t)||_P \right) B^T Pe(t) - \xi \hat{q}(t), \quad \tilde{q}(0) = \tilde{q}_0, \\
$$

$t \geq 0$, (31)

where $\|W(t)\|_2 \leq w, t \geq 0$, and $\|\hat{W}(t)\|_2 \leq \hat{w}, t \geq 0$, automatically holds, $\hat{W}(t) \triangleq \hat{W}(t) - W(t), t \geq 0$, is the weight estimation error, and $\tilde{q}(t) \triangleq \tilde{q}(t) - q, t \geq 0$, is the residual bound estimation error.

The main result of this paper can now be stated as follows. Consider the uncertain dynamical system given by (8) subject to Assumption 2, the reference model given by (12), and the feedback control law given by (10) along with the update laws (11), (25), (26), (27), and (28). If $||e_0||_P < \epsilon$, then the closed-loop dynamical system given by (29), (30) and (31) can be shown to be bounded, where the bound on the system error strictly satisfies a-priori given, user-defined worst-case performance

$$
||e(t)||_P < \epsilon, \quad t \geq 0. \\
$$

The above key result significantly generalizes the results presented in the previous work of authors in Ref. 17 by considering an unstructured uncertainty for the dynamical system and using the universal function approximation property of neural networks. Specifically, it shows that the proposed neuroadaptive control framework has the capability to keep the controlled system trajectories within a-priori, user-defined compact set such that the universal function approximation property is not violated. To elucidate this point, let $\mathcal{D}$ be a compact set such that the neural network approximation is valid. Additionally, let $\mathcal{D}_r \subset \mathcal{D}$ be the set such that $x_r(t) \in \mathcal{D}_r$. Then, since the size of $\mathcal{D}_r$ is user-defined by the above key result, one can a-priori set $\mathcal{D}_t \cup \mathcal{D}_r \subset \mathcal{D}$ to make the system uncertainty parameterization in (22) always valid. This answers the question stated in the title of our paper (i.e., how to keep the system trajectories on a given compact set).

We now present a numerical example in the following section.

## IV. Numerical Example

Consider the scalar uncertain dynamical system given by

$$
\dot{x}(t) = x(t) + Au(t) + \delta(x(t)), \quad x(0) = 0, \quad t \geq 0, \\
$$

(33)
with $\Lambda = 0.75$ and $\delta(x(t)) = x^3(t)$. For command tracking, we choose linear nominal controller gain matrices in (11) as $K_1 = 2$ and $K_2 = 1$. In addition, we choose the command signal $c(t)$ as a square wave with unity amplitude.

For approximating the system uncertainty $\delta(x(t)) = x^3(t)$, we utilize a neural network with the basis vector as

$$\sigma(x) = [\sigma_1(x), \sigma_2(x), \sigma_3(x), \sigma_4(x), \sigma_5(x), 1]^T, \quad t \geq 0,$$

with $\sigma_i(x) = \exp(-5(x - x_i)^2), i = 1, \ldots, 5$, for uniform distribution of radial basis functions (RBF) on the interval $(-2, 2)$ as depicted in Figure 1. For the proposed set-theoretic model reference adaptive control architecture highlighted in Section III, we use the generalized restricted potential function given in Section II with $\epsilon = 0.2$ to strictly guarantee $\|x(t) - x_r(t)\|_P < 0.1, t \geq 0$. Finally, we set the projection norm bound imposed on the parameter estimate to 5 and use $R = I$ to calculate $P$ from (16) for the resulting $A_r$.

The closed-loop dynamical system performance with the nominal controller is shown in Figures 2 and 3, where the nominal controller results in an unstable closed-loop dynamical system due to the presence of system uncertainty. We now apply the proposed set-theoretic adaptive controller with $\gamma_1 = 1, \gamma_2 = 5$ and $\xi = 0.01$ as seen in Figures 4 and 5, where satisfactory tracking performance is obtained. Figure 6 then clearly shows that this controller strictly guarantees $\|x(t) - x_r(t)\|_P < 0.2$, and the evolution of the effective learning rate $\gamma_1\phi_d(\cdot)$ is shown in Figure 7. Finally, the evolution of the weight estimation $\hat{W}(t)$ and the residual bound estimation $\hat{q}(t)$ are depicted in Figures 8 and 9, respectively.

V. Conclusion

A new model reference neuroadaptive control law was presented to keep the controlled system trajectories within a-priori, user-defined compact set while addressing disturbance rejection and system uncertainty suppression. This was accomplished by utilizing a set-theoretic approach based on restricted potential functions. The efficacy of the proposed law was illustrated by a numerical example.

References

Figure 1. Distribution of the RBFs in (34).

Figure 2. System performance with the nominal controller.
Figure 3. System control input with the nominal controller.

Figure 4. System performance with the proposed set-theoretic adaptive controller.
Figure 5. System control input with the proposed set-theoretic adaptive controller.

Figure 6. Norm of the system error trajectories with the proposed set-theoretic adaptive controller.
Figure 7. The evolution of the effective learning rate $\gamma_1 \phi_d(\cdot)$ with the proposed set-theoretic adaptive controller.

Figure 8. The evolution of the weight estimation $\hat{W}(t)$. 
Figure 9. The evolution of the residual bound estimation $\hat{q}(t)$.


