Tripartite Entanglement in Two-Photon Cascades

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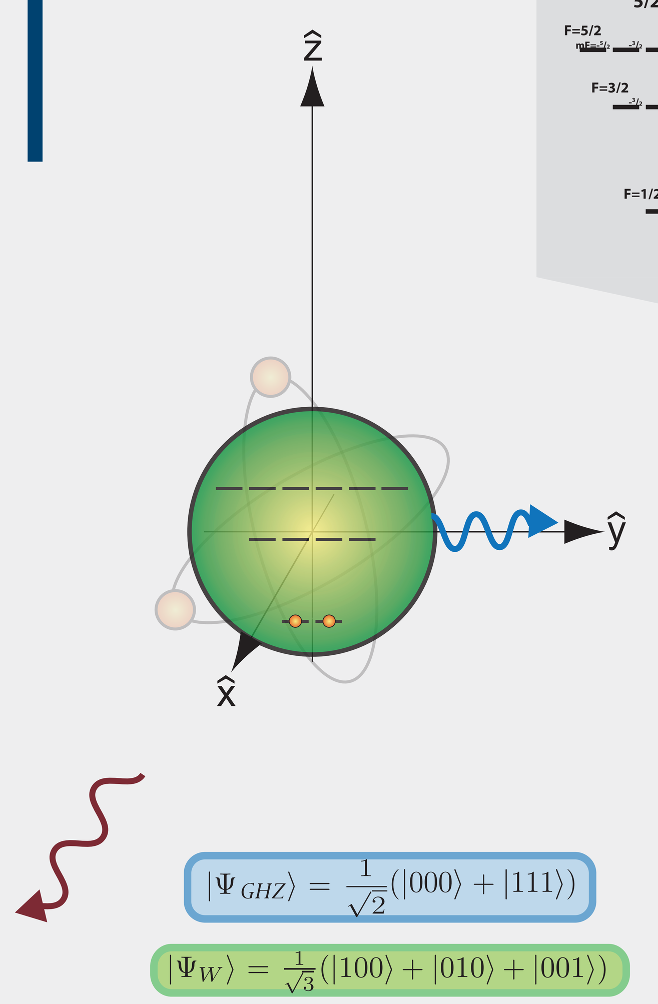
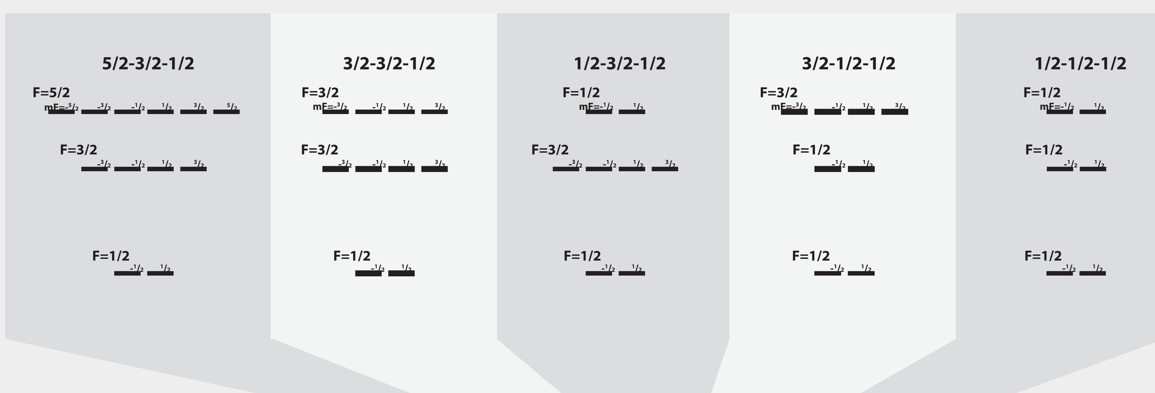
Tripartite Entangled States in Two-Photon Cascades

There are two classes of entangled tripartite states, W and GHZ type. Both of these classes can be immediately generated from two-photon cascades in atomic systems.



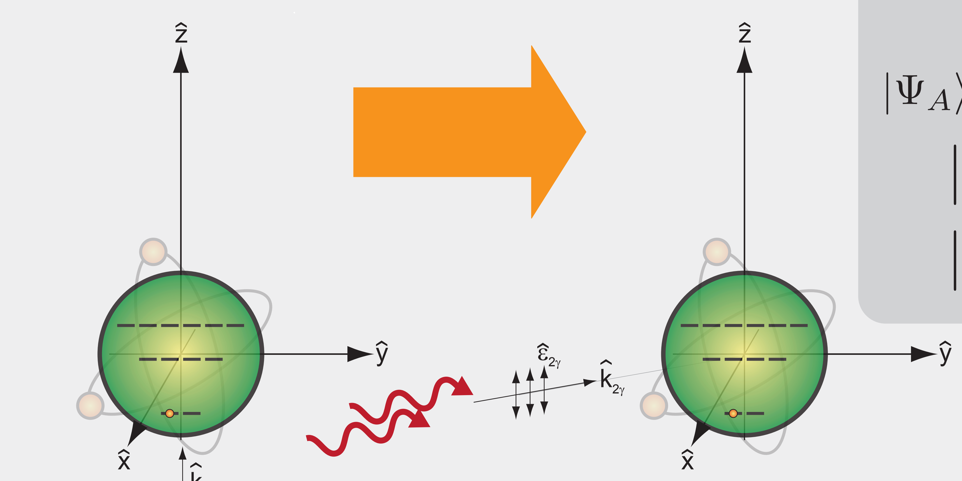


$\hat{kp}, \hat{k1}, \hat{k2}$	$\frac{5}{2}-\frac{3}{2}-\frac{1}{2}$	$\frac{3}{2}-\frac{3}{2}-\frac{1}{2}$	$\frac{1}{2}-\frac{3}{2}-\frac{1}{2}$	$\frac{3}{2}-\frac{1}{2}-\frac{1}{2}$	$\frac{1}{2}-\frac{1}{2}-\frac{1}{2}$
$\hat{x},\hat{x},\hat{x}$	A-BC	A-BC	A-BC	A-B-C	A-B-C
$\hat{x},\hat{x},\hat{y}$	GHZ	GHZ	GHZ	GHZ	GHZ
$\hat{x},\hat{x},\hat{z}$	GHZ	GHZ	GHZ	GHZ	GHZ
$\hat{x},\hat{y},\hat{x}$	GHZ	GHZ	GHZ	C-AB	C-AB
$\hat{x},\hat{y},\hat{y}$	GHZ	GHZ	GHZ	GHZ	GHZ
$\hat{x},\hat{y},\hat{z}$	W	GHZ	GHZ	GHZ	GHZ
$\hat{x},\hat{z},\hat{x}$	GHZ	GHZ	GHZ	C-AB	C-AB
$\hat{x},\hat{z},\hat{y}$	W	GHZ	GHZ	GHZ	GHZ
$\hat{x},\hat{z},\hat{z}$	A-BC	GHZ	GHZ	GHZ	GHZ
$\hat{z},\hat{x},\hat{x}$	GHZ	GHZ	GHZ	GHZ	GHZ
$\hat{z},\hat{x},\hat{y}$	W	GHZ	GHZ	GHZ	GHZ
$\hat{z},\hat{x},\hat{z}$	GHZ	GHZ	GHZ	C-AB	C-AB
$\hat{z},\hat{z},\hat{x}$	GHZ	GHZ	GHZ	GHZ	GHZ
$\hat{z},\hat{z},\hat{z}$	A-BC	A-BC	A-BC	A-B-C	A-B-C

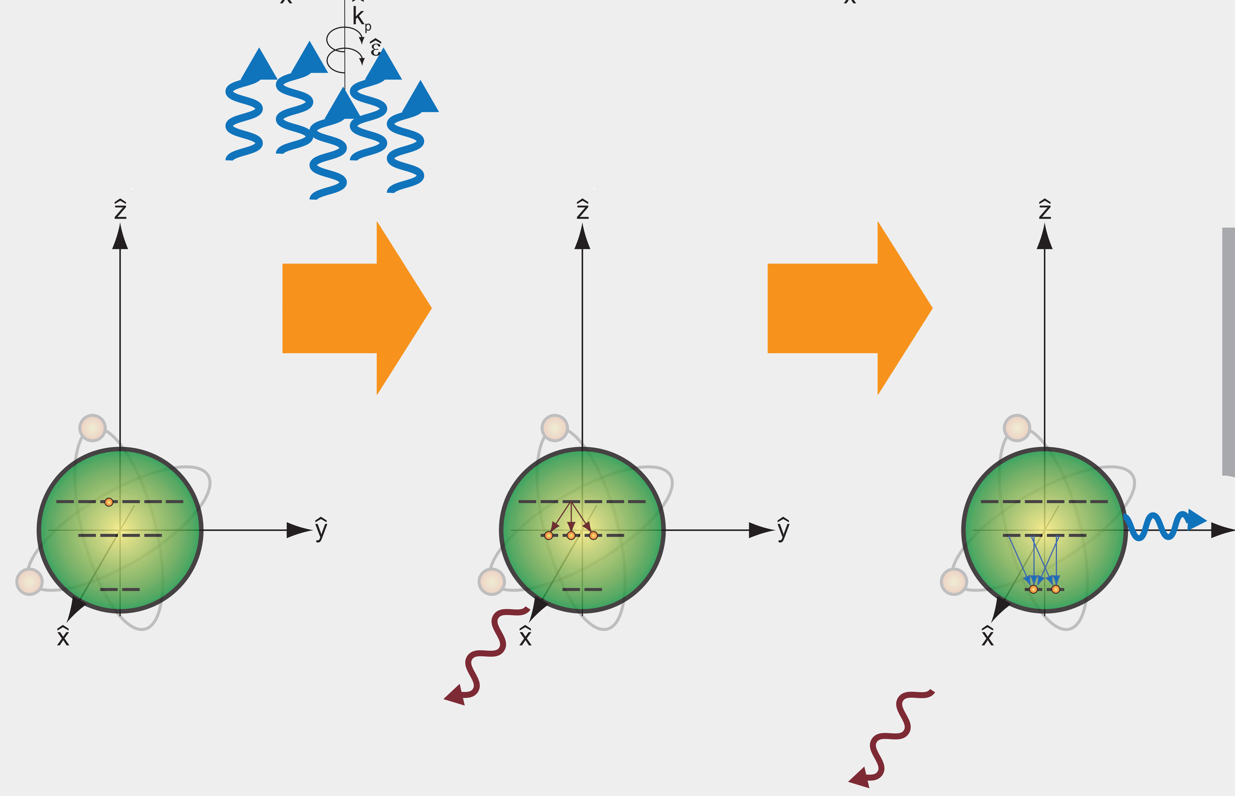
Dipole selection rules only allow ΔF=±1,0. Levels must connect to mF=1/2 ground state.

These are the classes of tripartite state generated by a two photon cascade with radiation on the k1,k2 directions.

Atom	Photon	Photon
$\lvert\Psi_A\rangle = c_{i\downarrow}\lvert\frac{1}{2},-\frac{1}{2}\rangle + c_{i\uparrow}\lvert\frac{1}{2},\frac{1}{2}\rangle$	$\lvert\Psi_B\rangle = c_{j\uparrow}\lvert\widehat{\theta_1}\rangle + c_{j\downarrow}\lvert\widehat{\phi_1}\rangle$	$\lvert\Psi_C\rangle = c_{k\uparrow}\lvert\widehat{\theta_2}\rangle + c_{k\downarrow}\lvert\widehat{\phi_2}\rangle$
$\lvert 0\rangle_A \equiv \lvert\frac{1}{2},\frac{1}{2}\rangle$	$\lvert 0\rangle_B \equiv \lvert\widehat{\theta_1}\rangle$	$\lvert 0\rangle_C \equiv \lvert\widehat{\theta_2}\rangle$
$\lvert 1\rangle_A \equiv \lvert\frac{1}{2},-\frac{1}{2}\rangle$	$\lvert 1\rangle_B \equiv \lvert\widehat{\phi_1}\rangle$	$\lvert 1\rangle_C \equiv \lvert\widehat{\phi_2}\rangle$



$$|\Psi\rangle = \sqrt{\frac{2}{3}}|000\rangle - \frac{1}{\sqrt{6}}|101\rangle + \frac{i}{\sqrt{6}}|110\rangle$$

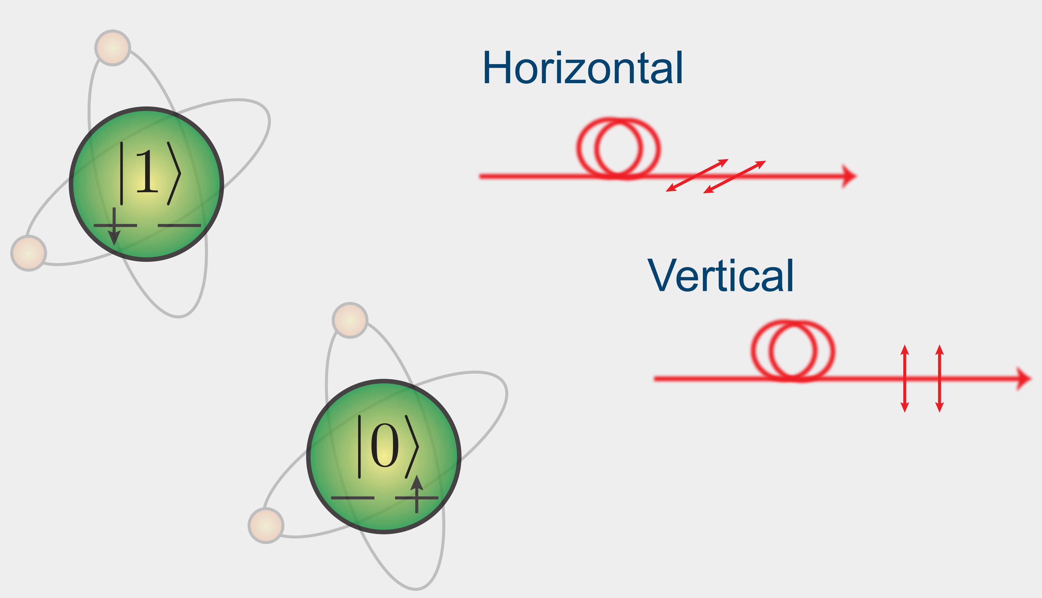


$$U_A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad U_B = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \quad U_C = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$|\Psi\rangle = \sqrt{\frac{2}{3}}|100\rangle + \frac{1}{\sqrt{6}}|010\rangle + \frac{1}{\sqrt{6}}|001\rangle$$

Meet the Qubits

The constituent qubits of this scheme are the ground state mF=1/2 atomic spin, and the polarization of each of the two cascade photons.



Classifying Tripartite States

Tripartite states can be quickly classified by computing the reduced density matrices of each qubit and the tangle of the tripartite state.

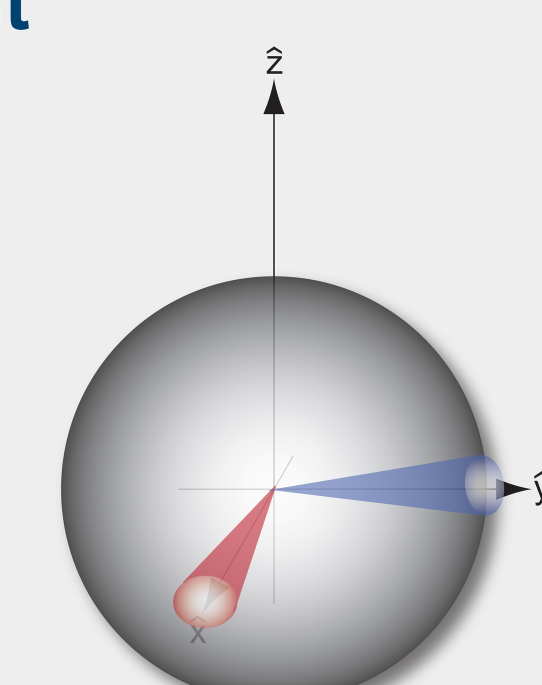
	S_A	S_B	S_C	τ
Product (A-B-C)	0	0	0	0
Bipartite (A-BC)	0	>0	>0	0
Bipartite (B-AC)	>0	0	>0	0
Bipartite (C-AB)	>0	>0	0	0
W	>0	>0	>0	0
GHZ	>0	>0	>0	>0

Feasibility of Experiment

Frequency of Events
-GHZ PD = 1
-W PD = 0.675

Quality of Entanglement
W and GHZ type cascades with acceptance angles up to 12°
-maintain 0.9 overlap
-scattering probabilities of 1E-4

For an atom with decay rate 1E6 Hz, we expect 100Hz entangling cascade rates



Unique Representation of Tripartite States

A three-qubit state can clearly be represented by these eight complex coefficients, but one state can represented many ways.

$$t_{000}|000\rangle + t_{001}|001\rangle + t_{010}|010\rangle + t_{011}|011\rangle + t_{100}|100\rangle + t_{101}|101\rangle + t_{110}|110\rangle + t_{111}|111\rangle$$

Acín et al. showed that (2) can uniquely represent any state. Carteret et al. proved that (1) could represent any state, but did not prove uniqueness. (1) is interesting because of its aesthetic appeal.

In order to compare and analyze states, we want to write them in a form independent of the measurement bases. A set of five basis states can do this compactly with four real parameters and a phase, but only for specific sets.

$$1: \{|000\rangle, |001\rangle, |010\rangle, |100\rangle, |111\rangle\}$$
$$2: \{|000\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$$

However, we have shown that both classes of tripartite entangled states (W and GHZ) are expressed non-uniquely in (1):

W

$$\frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$

Flip two measurement bases

$$\frac{1}{\sqrt{3}}(|100\rangle + |111\rangle + |001\rangle) \qquad \frac{1}{\sqrt{3}}(|111\rangle + |100\rangle + |010\rangle) \qquad \frac{1}{\sqrt{3}}(|010\rangle + |001\rangle + |111\rangle)$$

GHZ

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

Rotate bases by π/2

$$\frac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle)$$

References

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