Calculation of Tripartite Entangled States Generated by Spontaneous Two-Photon Cascade Emission

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Abstract: Tripartite entangled states are generated from spontaneous two-photon cascade emission in three-level systems with spin–$1/2$ ground states. Prototypical W and GHZ states are produced for certain initial conditions and photon emission directions.
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Multipartite entangled states with portable (photonic) and stationary (matter) qubits are crucial for the realization of quantum computation and communication [1, 2]. As an example, three-qubit W states are central to a quantum router protocol where a single qubit is teleported to one of two physically separated recipients [3]. The recipient who ultimately receives the qubit can be arbitrarily chosen at a later time.

Two-photon cascade decays terminating in a spin–$1/2$ ground state generate entangled states comprising three qubits: the polarizations of the two emitted photons and the spin of the ground state. Depending on the initial state preparation and photon emission directions, the final state can display (i) full tripartite entanglement, including ideal W and GHZ states [4, 5], (ii) bipartite entanglement, including Bell states, or (iii) no entanglement, i.e. a product state of the three qubits.

Two-photon cascade decays were explored for the five allowed electric dipole decay channels that terminate in a spin–$1/2$ final state, i.e. $\{F'' = \frac{5}{2}, \frac{3}{2}, \frac{1}{2}\} \rightarrow F' = \frac{3}{2} \rightarrow F = \frac{1}{2}$ and $\{F'' = \frac{3}{2}, \frac{1}{2}\} \rightarrow F' = \frac{1}{2} \rightarrow F = \frac{1}{2}$ decays. The initial excited state wavefunction was taken to be $\cos\left(\frac{\alpha}{2}\right)|F'', +\frac{1}{2}\rangle + \sin\left(\frac{\alpha}{2}\right)e^{+i\beta}|F'', -\frac{1}{2}\rangle$, which can be generated by optically pumping the spin–$1/2$ ground state with circularly polarized light propagating with polar angle α and azimuthal angle β and exciting the ground state with light propagating in the xy-plane that is linearly polarized along the z-axis, see Fig. 1.

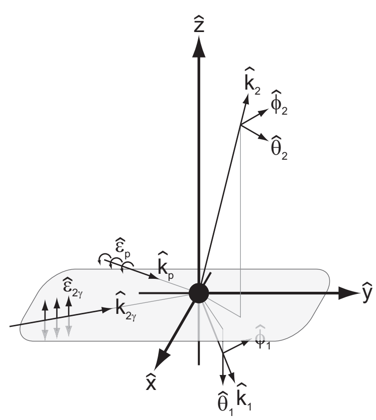

Figure 1: Two-photon cascade geometry. The z-axis is taken as the quantization axis for describing the system. The initial state of the spin–$1/2$ ground state is prepared by optical pumping. The pumping laser has wavevector $\hat{k}_p$ propagating with polar angle $\theta_p = \alpha$ and azimuthal angle $\phi_p = \beta$. The excited state is then populated by driving a two-photon transition with a pair of degenerate photons from a laser with wavevector $\hat{k}_{2\gamma}$ in the xy-plane and polarization $\hat{\varepsilon}_{2\gamma} = \hat{z}$. The pair of spontaneously emitted photons have wavevectors $\hat{k}_{1,2}$ and polarizations that can be written as linear superpositions of $\hat{\theta}_{1,2}$ and $\hat{\phi}_{1,2}$.

Cascades from the excited state will produce a final wavefunction $|\Psi\rangle$ with amplitudes ξ in one of eight possible three-qubit states comprised of the atomic spin qubit and the two polarization qubits.

$$|\Psi\rangle = \sum_{m=-\frac{1}{2},+\frac{1}{2}} \sum_{\hat{\varepsilon}_1=\hat{\theta},\hat{\phi}} \sum_{\hat{\varepsilon}_2=\hat{\theta},\hat{\phi}} \xi_{m\hat{\varepsilon}_1\hat{\varepsilon}_2} |F = \frac{1}{2}, m\rangle|\hat{\varepsilon}_1\rangle|\hat{\varepsilon}_2\rangle \tag{1}$$

The final three-qubit state's amplitudes depend only on the initial excited state amplitudes, where $A_{|F'',+\frac{1}{2}\rangle} = \cos(\frac{\alpha}{2})$ and $A_{|F'',-\frac{1}{2}\rangle} = \sin(\frac{\alpha}{2})e^{+i\beta}$, and the propagation directions $\hat{k}_2(\theta_2,\phi_2),\ \hat{k}_1(\theta_1,\phi_1)$ of photons radiated in the upper ($F'' \to F'$) and lower ($F' \to F$) decay channels:

$$\xi_{m\hat{\varepsilon}_1\hat{\varepsilon}_2} = \sum_{q=-1,0,+1}\ \sum_{m''=-\frac{1}{2},+\frac{1}{2}} A_{|F'',m''\rangle}\langle\frac{1}{2}, m|\hat{\varepsilon}_1\cdot\vec{D}_1|F', m' = m''+q\rangle\langle F', m' = m''+q|\hat{\varepsilon}_2\cdot\vec{D}_2|F'', m''\rangle, \quad (2)$$

where the sum is over all possible initial (F'', m'') and intermediate (F', m') states. The operator $\hat{\varepsilon}\cdot\vec{D}$ is the electric dipole operator for a polarization $\hat{\varepsilon}$. Characterization of various three-qubit final states is summarized in Table 1.

Table 1: Selected three-qubit states produced from the five allowed two-photon electric dipole decay channels that terminate in a spin–$1/2$ ground state, i.e. $\{F'' = \frac{5}{2},\frac{3}{2},\frac{1}{2}\} \to F' = \frac{3}{2} \to F = \frac{1}{2}$ and $\{F'' = \frac{3}{2},\frac{1}{2}\} \to F' = \frac{1}{2} \to F = \frac{1}{2}$ decays. The spin–$1/2$ ground state is optically pumped by a circularly polarized laser with wavevector $\hat{k}_p$ and the pair of spontaneously emitted photons have wavevectors $\hat{k}_{1,2}$. Tripartite entangled states belong to the W or GHZ class. The ideal forms of these states are denoted by W and GHZ, and states which are equivalent to W or GHZ by SLOCC in accordance with Refs. [4, 5] are denoted as W* and GHZ*. The remaining classes are bipartite entangled states which include the ideal two-qubit Bell states, and product states which display no entanglement between any of the three qubits. The eGHZ state from [4] does not satisfy the conditions in [5]. Cylindrical symmetry about the z-axis allows the 14 sets of $\{\hat{k}_p,\hat{k}_1,\hat{k}_2\}$ listed below to summarize the behavior of all 27 possible combinations of Cartesian units vectors. The remaining 13 combinations are equivalent to one of the first 14 entries in the table.

	$F'' \to F' \to F$				
$\hat{k}_p,\hat{k}_1,\hat{k}_2$	$\frac{5}{2}\to\frac{3}{2}\to\frac{1}{2}$	$\frac{3}{2}\to\frac{3}{2}\to\frac{1}{2}$	$\frac{1}{2}\to\frac{3}{2}\to\frac{1}{2}$	$\frac{3}{2}\to\frac{1}{2}\to\frac{1}{2}$	$\frac{1}{2}\to\frac{1}{2}\to\frac{1}{2}$
$\hat{x},\hat{x},\hat{x}$	bipartite	bipartite	bipartite	product	product
$\hat{x},\hat{x},\hat{y}$	GHZ*	GHZ*	GHZ*	GHZ	GHZ
$\hat{x},\hat{x},\hat{z}$	GHZ*	GHZ*	GHZ*	GHZ	GHZ
$\hat{x},\hat{y},\hat{x}$	GHZ*	GHZ*	GHZ*	bipartite	bipartite
$\hat{x},\hat{y},\hat{y}$					
$\hat{x},\hat{y},\hat{z}$	W			GHZ	
$\hat{x},\hat{z},\hat{x}$	GHZ*	GHZ*	GHZ*	bipartite	bipartite
$\hat{x},\hat{z},\hat{y}$	W			GHZ*	GHZ
$\hat{x},\hat{z},\hat{z}$	bipartite			GHZ	GHZ
$\hat{z},\hat{x},\hat{x}$			eGHZ	GHZ*	GHZ
$\hat{z},\hat{x},\hat{y}$	W*			GHZ*	GHZ
$\hat{z},\hat{x},\hat{z}$	GHZ*	GHZ*	GHZ*	bipartite	bipartite
$\hat{z},\hat{z},\hat{x}$	GHZ*	GHZ*	GHZ*	GHZ	GHZ
$\hat{z},\hat{z},\hat{z}$	bipartite	bipartite	bipartite	product	product

In conclusion, maximally-entangled tripartite states can be generated from spontaneous two-photon cascade emission in three-level systems with spin–$1/2$ ground states. Experimental work is underway to realize the various entangled states in Table 1 using the $^3D_2(F'' = \{\frac{5}{2},\frac{3}{2}\}) \to {}^3P_1(F' = \{\frac{3}{2},\frac{1}{2}\}) \to {}^1S_0(F = \frac{1}{2})$ two-photon cascade decay in ^{171}Yb. For this system, the photonic qubits are at telecommunication (1479 nm) and visible (556 nm) wavelengths, and the matter qubit is the well-shielded spin–$1/2$ ^{171}Yb nucleus.

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