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INTENSIONALITY AND PARADOXES IN RAMSEY'S 'THE FOUNDATIONS OF MATHEMATICS'

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Abstract. In 'The Foundations of Mathematics', Frank Ramsey separates paradoxes into two groups, now taken to be the logical and the semantical. But he also revises the logical system developed in Whitehead and Russell's *Principia Mathematica*, and in particular attempts to provide an alternate resolution of the semantical paradoxes. I reconstruct the logic that he develops for this purpose, and argue that it falls well short of his goals. I then argue that the two groups of paradoxes that Ramsey identifies are not properly thought of as the logical and semantical, and that in particular, the group normally taken to be the semantical paradoxes includes other paradoxes—the intensional paradoxes—which are not resolved by the standard metalinguistic approaches to the semantical paradoxes. It thus seems that if we are to take Ramsey's interest in these problems seriously, then the intensional paradoxes deserve more widespread attention than they have historically received.

§1. Introduction. Frank Ramsey's (1925) 'The Foundations of Mathematics' is remembered almost exclusively for distinguishing two types of paradox:[1] the logical and the semantical. But as influential as this distinction was, its statement takes up less than a page, while the reprinting of the paper in Braithwaite (1931) (and the reprinting in Mellor, 1990, for that matter) is 61 pages long. One might wonder what Ramsey was doing in the other 60 pages.

The title of Ramsey's work makes his purpose clear: he was attempting to construct *the foundations of mathematics*. Specifically, he was attempting to revise the logical system of *Principia Mathematica* (Whitehead & Russell, 1910)[2] in order to avoid what he saw as its three major defects. The first and third are concerned with classes and identity respectively, and I will set them aside. The second defect is, ultimately, the axiom of reducibility. Whitehead and Russell employ a *ramified* theory of types which, among other things, requires that the ranges of bound variables be restricted not only by type, but also by order. Because of this, one cannot talk about, for instance, all functions from individuals to propositions, but only all functions of order n from individuals to propositions. The axiom of reducibility was introduced as an attempt to correct the diminished power of the logic.

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[1] A distinction is sometimes drawn between paradoxes and antinomies, according to which most (but not all) of the problems Ramsey was concerned with are antinomies, not paradoxes. While this distinction is an important one, it is not significant for my purposes in this paper. Thus, for simplicity, I use 'paradox' throughout.

[2] Although I cite the first edition here and throughout the paper (the few page references to *Principia Mathematica* can be used with either edition), it is worth noting that Ramsey was familiar with at least the Introduction to the second edition [p. 25]. All page references, such as the one in the previous sentence, are to the reprinting of Ramsey (1925) in Braithwaite (1931) unless otherwise noted.

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This axiom says that for every higher order function, there is an extensionally equivalent first-order function.

The axiom of reducibility was never popular; even the Introduction to the second edition of *Principia Mathematica* attempts to do without it. Since it is only necessary in a ramified theory of types, Ramsey attempts to develop a nonramified theory of types that can serve as the foundation of mathematics. The cited motivation for the ramified theory of types is a collection of seven paradoxes (Whitehead & Russell, 1910, pp. 60–61). Ramsey observes that these paradoxes can be divided into two classes, which have since come to be called the semantical and the logical,[3] and that ramification is only required for the former. Thus, with the aim of eliminating the axiom of reducibility, he attempts to develop a nonramified theory of types—one that only restricts the ranges of bound variables by type—in which the semantical paradoxes do not arise.

My original aim in studying this section of Ramsey's paper was twofold. (i) I found it surprising that a proposed resolution of the semantical paradoxes by a figure such as Ramsey could have been completely ignored for over 80 years. Even contemporary reviews, such as Church (1932); Russell (1932), say nothing about this resolution, and I have not seen any discussion of it in later work. I had hoped that Ramsey's resolution would provide fresh ideas about the semantical paradoxes—ideas not colored by Tarski's subsequent analysis. At least, I hoped, it might illuminate a more familiar idea in a new way. (ii) I wanted to determine whether his resolution could also handle another class of paradoxes, which are specific to intensional logics (and which I thus call the intensional paradoxes). Again, the hope was that even if it could not resolve them, it would illuminate something or suggest something different from what has been done in the intervening decades.

Unfortunately, my conclusions seem to divest Ramsey's resolution of almost all its interest qua logical system. With respect to (i), I think not only that he fails to resolve the semantical paradoxes, but also that the failure involves such confusion that there is little insight into those paradoxes to be gained from examining his failure. I am loath to attribute such confusion to anybody, of course, and especially to someone like Ramsey, but I think that it is the most charitable reading of his attempted resolution. (ii) fares no better, as I think that there is nothing to be gleaned from his system's inability to avoid these other paradoxes.

In spite of this divestment, I think that it is at least interesting when someone like Ramsey falls into such confusion. And, of course, perhaps my interpretation is incorrect, and Ramsey actually *has* developed a novel and satisfactory resolution of the semantical paradoxes. I might then hope that this exposition could lead to a better understanding of such a resolution. Thus, one main aim of this paper is to reconstruct the logic Ramsey employs in 'The Foundations of Mathematics' and to understand how it is intended to resolve the semantical paradoxes. Unfortunately, his resolution of the paradoxes is far from transparent. It is somewhat reminiscent of Tarski's hierarchy of languages in that Ramsey thinks that the resolution of the paradoxes relies on distinguishing different senses of 'mean', but the details are significantly murkier than those of Tarski's now-classical resolution. Also, Ramsey does not develop a *hierarchy* of meaning relations, and he does not relativize meaning to a language the way Tarski does. Part I is thus devoted to a reconstruction and analysis of Ramsey's logic, the technical consequences of his account

[3] Actually, I do not think that his groups are the same as the semantical and the logical paradoxes; this is the subject of Part II.

of meaning, and the application of those consequences to that logic in an attempt to resolve the semantical paradoxes.

Part II is concerned with a slightly different topic. I have said that 'The Foundations of Mathematics' is largely remembered for its division of paradoxes into two groups, Groups A and B, which are now known as the logical and the semantical respectively. Certainly this is how people have historically taken the distinction Ramsey draws.[4] However, I think that this misrepresents his division. In particular, I think that there are paradoxes in his Group B that are not resolved by the common resolutions forwarded for the semantical paradoxes—that are arguably not semantical paradoxes.[5] Even more particularly, I think that the extra paradoxes in Group B are the intensional paradoxes that I mentioned in (ii) above. My general interest in the intensional paradoxes is that they pose problems for logics attempting to capture propositional attitudes, but I will not argue for that point here. My sole concern in Part II is to argue that Ramsey includes paradoxes in his Group B that are not resolved by metalinguistic resolutions of the semantical paradoxes.

PART I

RAMSEY'S RESOLUTION OF THE SEMANTICAL PARADOXES

§2. Ramsey's intensional logic. Ramsey employs a fairly straightforward form of simple type theory with lambda abstraction. Since the now-standard notation for the lambda operator had not yet been introduced, he writes, for example, '$\phi\hat{x}$' instead of '$\lambda x[\phi(x)]$', but the meaning is the same. I will use the more modern notation.[6] I also replace all scope-disambiguating dots with brackets, use more modern connectives, and write all instances of functional application (other than that of connectives) as the function followed by all of the arguments.[7] Finally, Ramsey's system differs from standard type theories in that well-formed formulas belong to a type p of propositions, rather than to the type t of truth-values.[8] What follows is an explicit characterization of the language that I take Ramsey to be working with.

There are primitive types i and p. If τ and σ are types, then $\langle \tau, \sigma \rangle$ is a type. I sometimes refer to types constructed in this way as *functional types*. For every type τ, there is

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4 See, for example, Fraenkel & Bar-Hillel (1958, pp. 5–14), Beth (1959, §171), Kneale & Kneale (1962, pp. 664–665), Quine (1963, pp. 254–255), Feferman (1984, p. 75), and Priest (1994, pp. 25–26).

5 Sort of not semantical paradoxes, anyway. I argue that, while all of Ramsey's formalizations of the Group B paradoxes will be prohibited by any resolution of the semantical paradoxes, those formalizations are not the only ones we should be concerned with. I then attempt to show that other, better formalizations of those paradoxes do not admit of similar resolutions, and argue that one therefore ought to not be satisfied as soon as one has a resolution of the semantical paradoxes, at least if one takes Ramsey's concern with these paradoxes seriously. These details are the subject of Section 8.

6 Most of the time. But, as in Section 3, it is sometimes easier to use Ramsey's notation when quoting him.

7 I also Curry the logic—I treat, for instance, binary relations as functions to other functions—but this has no substantive impact on the resulting system, and could be eliminated at the cost of only a little simplicity.

8 In my formulation of the logic, there are also well-formed formulas of type i and various functional types (see below). But these well-formed formulas play no role beyond simplifying the formation rules.

an infinite alphabet of variables with superscript τ. These superscripts are often omitted when no undesired ambiguity thereby arises. There are two primitive constants $\sim^{\langle p,p\rangle}$ and $\rightsquigarrow^{\langle p,\langle p,p\rangle\rangle}$ and infinitely many constants $\forall^{\langle\langle\tau,p\rangle,p\rangle}$, one for each type τ. The other primitives are λ, [,], and any primitive constants of determinate type that one wishes to include.

If $\mathbf{f}$ is a variable or constant with superscript τ, it is a well-formed formula of type τ. If $\mathbf{P}$ is a well-formed formula of type τ and $\mathbf{x}$ is a variable with superscript σ, $\lambda\mathbf{x}[\mathbf{P}]$ is a well-formed formula of type $\langle\sigma,\tau\rangle$. If $\mathbf{P}$ is a well-formed formula of type $\langle\tau,\sigma\rangle$ and $\mathbf{Q}$ is a well-formed formula of type τ, then $\mathbf{PQ}$ (often written $\mathbf{P}(\mathbf{Q})$) is a well-formed formula of type σ. I abbreviate $\mathbf{P}(\mathbf{Q})(\mathbf{R})$ (i.e., $\mathbf{PQR}$) with $\mathbf{P}(\mathbf{Q},\mathbf{R})$, $\rightsquigarrow$ $(\mathbf{P},\mathbf{Q})$ with $\mathbf{P} \rightsquigarrow \mathbf{Q}$, and $\forall(\lambda\mathbf{x}[\mathbf{P}])$ with $\forall\mathbf{x}[\mathbf{P}]$. I use square brackets to disambiguate scope when these abbreviations lead to ambiguity.

Ramsey assigns numbers to some functional types: "A function of individuals we will call a function of type 1; a function of functions of individuals, a function of type 2; and so on" [p. 46]. Using the above notation, we can define these recursively. Anything of type $\langle i,p\rangle$, $\langle i,\langle i,p\rangle\rangle$, $\langle i,\langle i,\langle i,p\rangle\rangle\rangle$, and so forth is of Type 1; given types τ_i of type n, anything of type $\langle\tau_1,p\rangle$, $\langle\tau_1,\langle\tau_2,p\rangle\rangle$, $\langle\tau_1,\langle\tau_2,\langle\tau_3,p\rangle\rangle\rangle$, and so forth *or* of type $\langle\tau_1,\tau_2\rangle$, $\langle\tau_1,\langle\tau_2,\tau_3\rangle\rangle$, $\langle\tau_1,\langle\tau_2,\langle\tau_3,\tau_4\rangle\rangle\rangle$, and so forth is of type $n+1$. To simplify later definitions slightly, I will also take anything of type i to be of Type 0.

Ramsey actually takes these numbered collections of types to be types themselves, and does not distinguish between the different types within each collection. Thus, when I indicate types with superscripts below, I often follow Ramsey and ambiguously use numbers as though they denote types rather than collections of types. This undoubtedly raises difficult technical problems, but I think that I can safely ignore them for the purposes of this paper.

The numbered types are the only functional types that Ramsey considers. These are all types of *propositional functions*:[9] the last type symbol appearing in their symbols is always p. This is what I mean when I say that Ramsey's logic is intensional—repeated functional application always results in well-formed formulas of type p rather than of the type t of truth-values. In this respect, Ramsey's logic is similar to the Russellian simple type theory presented in Church (1974). I follow Thomason (1980) in using different symbols for the primitive constants of this logic, replacing the standard truth-functional constants $\neg$, $\rightarrow$, and $\forall$ with my intensional $\sim$, $\rightsquigarrow$, and $\forall$ respectively. The remaining symbols $\vee$, $\wedge$, $\leftrightarrow$, and $\exists$ are replaced with $\cup$, $\cap$, $\leftrightsquigarrow$, and $\exists$ respectively, which are defined in the usual way.[10]

§3. Terminology. I have not tried to provide a model for Ramsey's logic because model theory had not been invented when Ramsey was writing. One probably could construct models for the logic I have just described, but they are not necessary for explaining Ramsey's resolution of the paradoxes, so I will not be concerned to do so. Of course, if his resolution seemed promising, we would need to confirm that he really has provided a

[9] Except for my added Type 0, of course.

[10] One might not want to define $\cup$, $\cap$, $\leftrightsquigarrow$, and $\exists$ in terms of $\sim$, $\rightsquigarrow$, and $\forall$—one might think that, for instance, the proposition that dogs and cats bark is different from the proposition that neither cats nor dogs don't bark. The approach I am taking is a littler simpler, but it is hardly more complex to take all of the connectives as primitive.

solution by constructing a model, but I argue that his resolution is not successful, so this should not be an issue.

This raises some terminological worries, because Ramsey uses 'proposition' and 'function', words that are now associated with models. Ramsey frequently uses these terms to refer to strings of symbols. Thus, 'ϕa' often *is* a proposition—it isn't just a formula of type p—and '$\lambda x[\phi(x)]$' often *is* a function.

Even for Ramsey, this is not the correct use of 'proposition'—'ϕa' is a *propositional symbol*, and propositional symbols are *instances of* propositions. Then "two propositional symbols are to be regarded as instances of the same proposition ... when they express agreement and disagreement with the same sets of truth-possibilities of atomic propositions" [p. 9]. That is, they are instances of the same proposition when their truth tables agree. He reiterates this criterion of sameness at pp. 33, 34 and employs it again at p. 35.[11]

In contrast to propositions, Ramsey, at least most of the time, insists that functions are just symbols. He initially defines 'propositional function' as "an expression of the form '$f\hat{x}$', which is such that it expresses a proposition when any symbol (of a certain appropriate logical type depending on f) is substituted for '$\hat{x}$' " [p. 8]. Similar statements litter 'The Foundations of Mathematics': "a propositional function of individuals [is] a symbol of the form ..." [p. 35], "functions are symbols" [p. 36], and "our definition of a propositional function as itself a symbol" [p. 43] are some examples. Only twice does he suggest that there is a distinction to be drawn between functions and functional symbols, as there was for propositions. As with propositions, it arises when he is explaining identity for functions; thus, for example, he writes,

> Two such symbols [i.e., propositional functional symbols] are regarded as the same function when the substitution of the same set of names in the one and in the other always gives the same proposition. Thus if '$f(a, b, c)$', '$g(a, b, c)$' are the same proposition for any set of a, b, c, '$f(\hat{x}, \hat{y}, \hat{z})$' and '$g(\hat{x}, \hat{y}, \hat{z})$' are the same function, even if they are quite different to look at. [p. 35]

Here it seems as though he has a notion of function that is distinct from the symbols he has been dealing with.

This, however, is an isolated case, and rarely arises. In fact, Ramsey seldom even invokes the distinction between propositions and propositional symbols. He is almost exclusively concerned with propositional and functional symbols[12]—in particular, his discussion and proposed resolution of the semantical paradoxes only make use of these symbolic senses of 'proposition' and 'function'. Still, in what follows, I will use 'propositional symbol' and 'functional symbol' whenever I am talking about the actual formulas, and I will edit quotes from Ramsey accordingly when doing so does not obscure anything relevant. In light of this exclusive concern with propositional and functional symbols, I almost always follow the practice of allowing symbols to name themselves and omit mention (and corner) quotes around them. Section 2 is a good illustration of this practice; the beginning of this section, an illustration of an exception.

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[11] This should sound similar to things like Carnap's state descriptions and more modern possible worlds. I revisit this in Section 7.3.

[12] Ramsey never uses the latter term, but given that propositional symbols are well-formed formulas of type p, I will take functional symbols to be well-formed formulas of any functional type.

§4. Meaning. As mentioned above, Ramsey approaches the semantical paradoxes by way of meaning. He makes a number of claims about meaning at pp. 43–44. At best, what he says is intricate—he uses 'mean' in at least three distinct ways in a passage of less than two pages' length—and at worst, it is poorly thought out—immediately after insisting that "to speak of 'F' as meaning $\lambda x[F(x)]$ at all must appear very odd in view of our definition of a propositional function [such as $\lambda x[F(x)]$] as itself a symbol [as opposed to an object]," he is happy to talk about an "object ... S," where 'S' is "the name of a relation" [p. 43]. I think that one can construct a theory of meaning that supports (most of) what Ramsey says in these pages, but the details are complex.

Luckily, most of what he says about meaning is not strictly relevant to his resolution of the semantical paradoxes. This is because his resolution relies on his notion of orders, which arise only for well-formed formulas and symbols introduced by definitions. His motive for introducing orders relies on his account of meaning, but the formal machinery employed in his resolution of the paradoxes can be presented and understood without appealing to any meaning relation beyond the relation between definiendum and definiens. Put another way, for the purposes of Ramsey's resolution of the paradoxes, his formalization of "the relation of meaning between 'ϕ' and $\lambda x[\phi(x)]$" [p. 42] can be thought of as only taking symbols introduced via definition for its first argument. He formalizes this relation as R. It is not clear whether Ramsey thought seriously about the meanings of any other symbols, but I will not pursue this issue further.[13]

Before moving to the formal machinery just mentioned, it is worth considering what the arguments to R actually are. The first is obviously a symbol. The second seems to obviously be a function. But recall that for Ramsey, there are no functions—there are only functional symbols. The second argument, then, must be a functional symbol, and this is what Ramsey takes it to be. Thus, he writes,

> to speak of 'F' as meaning $\lambda x[F(x)]$ at all must appear very odd in view of our definition of a propositional function [such as $\lambda x[F(x)]$] as itself a symbol. But the expression is merely elliptical. ... [I]t is clearly an impossible simplification to suppose that there is a single object F, which ['F'] means. [p. 43]

The expression that Ramsey is talking about can be written as '$R\big(\text{'}F\text{'}, \lambda x[F(x)]\big)$'. He is saying quite explicitly that *both* arguments are symbols, and that the expression is elliptical for something more complicated. He employs the complicated, possibly confused theory of meaning that I discussed above in explaining what the expression is elliptical for—what 'F' actually does mean, if not a single object F. But since R is the relation that appears in his formalizations of the paradoxes, it is enough for the present purposes to understand the properties of the expression '$R\big(\text{'}F\text{'}, \lambda x[F(x)]\big)$'. Thus we can again set to the side the question of what this expression is elliptical for, the answer to which involves Ramsey's theory of meaning.

§5. Formal machinery.

5.1. Orders. Consider the following definitions. The first two are definitions that Ramsey gives, and the third is at least one he would not object to. (Recall that superscripts indicate a collection of types.)

[13] I do not mean to say that he has no other views about meaning; he does. It is just not clear that those views were well considered.

$$\phi(x) =_{\text{df}} S^1(a, x),$$
$$\psi(x) =_{\text{df}} \forall y^0[S'^1(y, x)],$$
$$\chi(x) =_{\text{df}} \exists \phi^1\big[f^2\big(\lambda z[\phi(z)], x\big)\big].$$

Ramsey thinks that 'ϕ', 'ψ', and 'χ' not only mean different things, but also do so in quite different ways. As I said above, the details of his theory of meaning are not relevant here. What matters is that Ramsey thinks that there is an important difference between 'ϕ', 'ψ', and 'χ' based on the structures of their definientia: the definiens of 'ϕ' contains no bound variables, the definiens of 'ψ' contains a bound variable of Type 0, and the definiens of 'χ' contains a bound variable of Type 1. Ramsey introduces orders to capture this difference.

Ramsey's use of 'order' here is unfortunate, because it recalls Russell's use of the term in connection with ramified type hierarchies. Unlike Russell's orders, Ramsey's are properties of propositional and functional symbols; derivatively, they are also properties of symbols whose definientia are propositional or functional symbols, such as 'ϕ', 'ψ', and 'χ'.[14]

His orders are assigned as follows [pp. 46–47]. The order of a propositional symbol is $n + 1$, where n is the highest type of bound variable appearing in the propositional symbol. If a propositional symbol contains no bound variables, its order is 0.[15] Ramsey does not consider bound variables of any other types. This can be glossed as only allowing bound variables of type i or of a type of propositional functional symbol.[16] Functional symbols are assigned orders in the obvious way: $\lambda x\mathbf{P}$ has the same order as $\mathbf{P}$. For any symbol $\mathbf{f}$ such that either $\mathbf{f} =_{\text{df}} \mathbf{P}$ or $\mathbf{f}(x) =_{\text{df}} \mathbf{P}$, we will say that the order of $\mathbf{f}$ is that of $\mathbf{P}$.[17] From this definition, we can see that 'ϕ' is of Order 0, 'ψ' is of Order 1, and 'χ' is of Order 2.

Ramsey often takes the difference between his orders and those of ramified type theory to be a matter of their definition—his orders, unlike those used for ramification, are determined by the appearance of bound variables in strings of symbols. Now distinguishing propositions from propositional symbols, he writes,

> For me, propositions in themselves have no orders; they are just different truth-functions of atomic propositions—a definite totality, depending only on what atomic propositions there are. Orders and illegitimate totalities only come in with the symbols we use to symbolize the facts in variously complicated ways.[18] [pp. 48–49]

But this is not the most important difference between his orders and Russell's. What keeps Ramsey's type theory from being ramified is his insistence that the ranges of bound variables not be restricted by order. This means that in a propositional symbol such as

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[14] He is not this explicit about the way the orders of definienda are determined, but this is clearly what he has in mind.

[15] As an aside, it is interesting to note that Ramsey has not actually introduced a *hierarchy* of orders: orders are not defined in terms of earlier orders, but of types.

[16] These are the only functions that he considers [p. 35, n. 1].

[17] Technically, this would require that there be mention quotes around the definiendum, as $\mathbf{f}$ is a variable over symbols. In practice, and in keeping with my general laxity about mention quotes, I take $=_{\text{df}}$ to imply mention rather than use and omit these quotes without loss of precision.

[18] One might be concerned here that the distinction between propositions and propositional symbols is playing more of a role in Ramsey's resolution of the paradoxes than I claim, since it is being appealed to here. But, as will be apparent below when the resolution is presented, the distinction is only important in that propositions proper play no role whatsoever in the resolution.

'$\forall\phi[f(\phi)]$', the range of ϕ is "the set of [all] functions [of a specific type], not ... the set of [functions of order 1]" [p. 44, n. 1]. Similarly, Ramsey never allows functional symbols that can only take arguments of certain orders. This is understandable—if he allowed such functional symbols, he would have a hard time keeping orders out of bound variables.

Note that Ramsey characterizes the range of the variable ϕ as a set of functions. But for Ramsey, there are no functions proper, just functional symbols, and in fact, we can interpret this remark as taking the ranges of bound variables of type > 0 to actually be sets of functional symbols (or, equivalently for my purposes, symbols that are short for functional symbols—symbols such as the 'ϕ', 'ψ', and 'χ' introduced above). However implausible this sounds to modern ears, it follows from Ramsey's insistence that there are no functions but functional symbols. And even if it didn't, I think that it plays a crucial role in his analysis of the semantical paradoxes; see Section 6.2.

We can now see why I take the characteristic property of Ramsey's orders to be that they do not restrict the ranges of bound variables beyond their type. One might have thought that there was no such restriction simply in virtue of his definition of orders. He, at least, seems to have thought so. After all, how can properties of symbols restrict the ranges of bound variables, which range over objects? But if I am right that the correct interpretation of his resolution requires that at least some variables range over symbols, then his evasion of such a restriction—and thus his evasion of ramification—is not so immediate. In fact, I argue below that he *does* restrict the ranges of variables in ways that go beyond mere type restrictions (although not, it turns out, by order, which is a separate problem).

5.2. The relations R_i. We still do not have enough to resolve the semantical paradoxes, because we have yet to make any changes to the formalism of simple type theory. Orders have been defined, but they do not place any restrictions on anything. We have not yet captured the motivation for his orders, namely, that 'ϕ', 'ψ', and 'χ' are related to their definientia in importantly different ways.

Ramsey's solution is that there is not just one meaning relation R, but instead one such relation for each order. Then a formula of the form $R(\mathbf{f}, \mathbf{P})$ is true when both $\mathbf{f}$ actually does mean $\mathbf{P}$ (in the elliptical sense mentioned above) and R is the relation corresponding to the order of $\mathbf{f}$. Given the above motivation, this idea is not entirely unreasonable. Although he does not use this notation, I will use R_n to indicate the relation corresponding to order n, so that we can say that $R_n(\mathbf{f}, \mathbf{P})$ is only true when $\mathbf{f}$ is of order n and $\mathbf{f}$ elliptically means $\mathbf{P}$.

As an example of how this works, we can represent the above definitions with the following formulas.

$$\begin{aligned}
&R_0\big(\text{`}\phi\text{'}, \lambda x[S^1(a, x)]\big),\\
&R_1\big(\text{`}\psi\text{'}, \lambda x\big[\forall y^0[S'^1(y, x)]\big]\big),\\
&R_2\Big(\text{`}\chi\text{'}, \lambda x\Big[\exists\phi^1\big[f^2\big(\lambda z[\phi(z)], x\big)\big]\Big]\Big).
\end{aligned}$$

These are true (or, more accurately, denote true propositions), but would be false if the R_is were changed.

Ramsey also considers meaning relations that are appropriate to multiple orders. For instance, we could have a relation R that is only capable of being true when its first argument is either of order n or of order m, which we might represent with '$R_{n,m}$'. For simplicity's sake I will restrict myself to relations only appropriate to a single order. This restriction has no substantive consequences.

It is very important to note that $R_n(\mathbf{f}, \mathbf{P})$ is not ill formed when $\mathbf{f}$ is not of order n, but simply false. Orders cannot render formulas ill formed—to allow that would be to embrace ramification.

§6. The semantical paradoxes.

6.1. Grelling's paradox. With all this machinery in place, we can consider Ramsey's treatment of the semantical paradoxes. He examines Grelling's paradox about 'heterological' (which he incorrectly attributes to Weyl) in the most detail.[19] I will begin, as he does, by formulating the paradox without the various R_i. He begins by defining 'F':

$$F(x) =_{\mathrm{df}} \exists\phi^1\big[R\big(x, \lambda z[\phi(z)]\big) \cap {\sim}\phi(x)\big]. \tag{1}$$

Since '$R(x, y)$' means 'x means y', '$F(x)$' is intended to be 'x is heterological'. Ramsey then thinks that we have

$$R\big(\text{‘}F\text{’}, \lambda x[F(x)]\big). \tag{2}$$

It is not entirely clear to me how he gets to this, but I am going to assume it. He seems to generally have a disquotational theory of meaning, so that, for instance, 'a' means a, where 'a' has been introduced as the name of some object, but I will not pursue the issue further.[20]

From (2) we have

$$\exists\phi^1\big[R\big(\text{‘}F\text{’}, \lambda x[\phi(x)]\big)\big], \tag{3}$$

whence Ramsey concludes

$$F(\text{‘}F\text{’}) \leftrightsquigarrow {\sim}F(\text{‘}F\text{’}). \tag{4}$$

It is worth looking more carefully at (4), because it is not clear that Ramsey can actually derive it from (1)–(3) alone. The right-to-left direction is straightforward. We assume

$${\sim}F(\text{‘}F\text{’}), \tag{5}$$

whence by (1)

$${\sim}\exists\phi^1\big[R\big(\text{‘}F\text{’}, \lambda z[\phi(z)]\big) \cap {\sim}\phi(\text{‘}F\text{’})\big], \tag{6}$$

whence by (2)

$$F(\text{‘}F\text{’}). \tag{7}$$

The left-to-right direction requires an additional assumption, though. There are many premisses that will do, but (8) is a particularly plausible principle.

$$\forall\phi\forall\psi\Big[\big[R\big(\mathbf{f}, \lambda x[\phi(x)]\big) \cap R\big(\mathbf{f}, \lambda x[\psi(x)]\big)\big] \rightsquigarrow \forall x[\phi(x) \leftrightsquigarrow \psi(x)]\Big]. \tag{8}$$

(8) is an axiom schema. It can be glossed as the principle that all meanings of a given symbol $\mathbf{f}$ are coextensive.

Now we can prove the left-to-right direction of (4). Assume

$$F(\text{‘}F\text{’}), \tag{9}$$

whence by (1)

$$\exists\phi^1\big[R\big(\text{‘}F\text{’}, \lambda z[\phi(z)]\big) \cap {\sim}\phi(\text{‘}F\text{’})\big]. \tag{10}$$

[19] Ramsey's formulation of this paradox is at the bottom of p. 42.

[20] This is an example of what I was referring to in note 13.

Now let ψ be a functional symbol such that

$$R\big(\text{‘}F\text{’}, \lambda z[\psi(z)]\big) \cap {\sim}\psi(\text{‘}F\text{’}), \tag{11}$$

whence (via the first conjunct) by (2) and (8), letting **f** be ‘F’,

$$\forall x[F(x) \leftrightsquigarrow \psi(x)], \tag{12}$$

whence by the second conjunct of (11)

$${\sim}F(\text{‘}F\text{’}). \tag{13}$$

Ramsey himself does not give any proof, and proceeds directly to (4) from (3). But, despite not making any use of (3), I think that (1)–(13) are a reasonable reconstruction of the reasoning he intends to employ and make his resolution of Grelling's paradox more transparent.

6.2. Ramsey's resolution of Grelling's paradox. As one might expect, Ramsey's resolution relies on orders [pp. 45–46]. As R is actually ambiguous between the different relations that are appropriate to symbols of different orders, all of the Rs in (1)–(13) have to be replaced by appropriate R_i. To resolve the paradox, Ramsey argues that no matter what R_n is used in (1), (2) will only be true if it employs some R_m, $m > n$. That is, Ramsey claims that no matter what R_n is used in (1), ‘F’ will be of order greater than n.

In eliminating R, we must replace (8) with something like (8′), an axiom schema yielding an axiom for every pair of **f** and n.[21]

$$\forall\phi\forall\psi\Big[\big[R_n\big(\mathbf{f}, \lambda x[\phi(x)]\big) \cap R_n\big(\mathbf{f}, \lambda x[\psi(x)]\big)\big] \rightsquigarrow \forall x[\phi(x) \leftrightsquigarrow \psi(x)]\Big]. \tag{8′}$$

It is easy to see how Ramsey's strategy works when (1) is replaced with

$$F(x) =_{\text{df}} \exists\phi^1\big[R_0\big(x, \lambda z[\phi(z)]\big) \cap {\sim}\phi(x)\big]; \tag{1′}$$

this results in (6) and (11) becoming

$$\forall\phi^1\big[R_0\big(\text{‘}F\text{’}, \lambda z[\phi(z)]\big) \rightsquigarrow \phi(\text{‘}F\text{’})\big] \tag{6′}$$

and

$$R_0\big(\text{‘}F\text{’}, \lambda z[\psi(z)]\big) \cap {\sim}\psi(\text{‘}F\text{’}) \tag{11′}$$

respectively. But since the definiens in (1′) contains a bound variable of Type 1, ‘F’ has Order 2. Thus, as explained above, (2) will only be true if it is replaced with

$$R_2\big(\text{‘}F\text{’}, \lambda x[F(x)]\big). \tag{2′}$$

Clearly, we can derive nothing from (6′), (11′), and (2′), so the proof of (4) no longer goes through.[22]

The obvious response is to change (1′) to

$$F(x) =_{\text{df}} \exists\phi^1\big[R_2\big(x, \lambda z[\phi(z)]\big) \cap {\sim}\phi(x)\big], \tag{1″}$$

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[21] This is the obvious revision of (8), but it is not the only possible one. Perhaps, for instance, we would want to allow the subscripts on the two Rs to be different. This would not defeat Ramsey's resolution, though—see note 22—and I can think of no plausible replacement for (8) that would do so.

[22] If, as I considered in note 21, (8′) allowed different R_is, then the left-to-right direction would still be provable. As it is, we cannot go in either direction.

which results in (6′) and (11′) becoming

$$\forall\phi^1\big[R_2\big(\text{`}F\text{'}, \lambda z[\phi(z)]\big) \rightsquigarrow \phi(\text{`}F\text{'})\big] \tag{6''}$$

and

$$R_2\big(\text{`}F\text{'}, \lambda z[\psi(z)]\big) \cap {\sim}\psi(\text{`}F\text{'}) \tag{11''}$$

respectively. From these and (2′), we can derive (7) and (12), whence we can derive (4). Ramsey considers this case,[23] and argues that now ‘F’ must be of Order 3 because the definiens in (1″) contains a “hidden variable” of Type 2 [pp. 45–46]. If this is right, then the paradox is once again avoided, because (2′) becomes

$$R_3\big(\text{`}F\text{'}, \lambda x[F(x)]\big), \tag{2''}$$

whence we can derive neither (7) nor (12). But his argument that ‘F’ is now of Order 3 is both intricate and dubious.

Ramsey begins by observing that for R_2, $R_2\big(\text{`}\phi_2\text{'}, \lambda x[\phi_2(x)]\big)$. I have not omitted any words there; the quote I am relying on is “this new R, for which ‘ϕ_2’ $R(\phi_2\hat{x})$” [p. 45]. Here, the “new R” is $R_{0,1,2}$, which I am replacing with R_2 for simplicity’s sake. Recalling his use of $\hat{x}$, this remark is equivalent to the statement that began this paragraph.

There is one point to quickly address and set aside: in this part of Ramsey’s paper, subscripts indicate order, so ‘ ‘ϕ_2’ $R(\phi_2\hat{x})$’, that is, ‘$R_2\big(\text{`}\phi_2\text{'}, \lambda x[\phi_2(x)]\big)$’, seems to say nothing more than that there is a symbol of Order 2 that has been introduced via a definition (and thus means something). Ramsey does not actually explain what this formula is supposed to mean, but I do not think that these details are actually of much consequence, so I will not worry about them further.

The main issue raised by this passage in Ramsey is that it is not clear what he is actually saying about R_2. There are at least two plausible interpretations: (i) that $R_2\big(\text{`}\phi_2\text{'}, \lambda x[\phi_2(x)]\big)$ is not ill formed, and (ii) that $R_2\big(\text{`}\phi_2\text{'}, \lambda x[\phi_2(x)]\big)$ is true. As I said earlier, Ramsey has to say that when an R_i is given arguments of the wrong order, it is false, not ill formed, so (ii) had better be what he intends on pain of triviality. Actually, I think that he has (i) in mind, as will become clear later. This should be cause for concern, though, because if he intends (i), then either he is not saying anything of substance—orders are supposed to be irrelevant to well-formedness—or he is reintroducing something dangerously like ramification.

In an attempt to settle this issue, let us turn to what he hopes to conclude from this observation. Immediately after making it, he writes, “since ‘$\phi_2(x)$’ is of some such form as $\exists\phi^1\big[f^2\big(\lambda z[\phi(z)], x\big)\big]$, ... [(1″) involves] at least a variable function $f^2\big(\lambda z[\phi(z)], x\big)$ of functions of individuals” [p. 45].[24] This variable f is the hidden variable because it “is involved in the notion of a variable ‘ϕ_2’, which is involved in the variable ϕ taken in conjunction with R_2 [in (1″)]” [pp. 45–46].[25]

Before we go on, a quick comment should be made about Ramsey’s notation here. Obviously, $f^2\big(\lambda z[\phi(z)], x\big)$ is not actually a functional symbol, and Ramsey should have written $f^2\big(\lambda z[\phi(z)], \hat{x}\big)$, that is, $\lambda x\big[f^2\big(\lambda z[\phi(z)], x\big)\big]$, when talking about the “variable function.” In what follows, I will often prepend the λx where necessary.

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23 Actually, he considers changing the R to $R_{0,1,2}$, but as I said above, restricting ourselves to R_2 here has no substantive consequences.

24 None of the superscripts here or in the later quotes appears in the original, of course.

25 The subscript on the R is not in the original.

With that notational point made, we can return to examining Ramsey's reasoning. The idea here seems to be that since ϕ can be instantiated by something "of some such form as $\exists\phi^1\big[f^2\big(\lambda z[\phi(z)], x\big)\big]$" (or, rather, $\lambda x\Big[\exists\phi^1\big[f^2\big(\lambda z[\phi(z)], x\big)\big]\Big]$), (1″) involves a hidden variable that must be able to range over functions of the same type as f—functions of Type 2.[26]

It is not clear how this is supposed to play out in the formalism, because Ramsey never explains exactly what a hidden variable is or what it means to involve one. But to criticize Ramsey's resolution of the paradoxes, as I wish to do, we need only understand the conditions under which a formula involves a hidden variable, not why it does so. To get a handle on this, it is helpful to return to the observation made above, that variables range over symbols. The principle he is using, as best I can tell, is this: a hidden variable of type n is involved in a formula whenever an explicit variable in that formula is capable of ranging over symbols that contain symbols of type n.[27] In (1″), ϕ could be the symbol $\lambda x\Big[\exists\phi^1\big[f^2\big(\lambda z[\phi(z)], x\big)\big]\Big]$, so according to this principle, (1″) contains (or, again, involves) a hidden variable of Type 2, making 'F' of Order 3.

One ought to be dubious of this principle, and thus of any interpretation of Ramsey that claims that he relies on it. But consider his discussion of the other semantical paradoxes. He does not spend much time on these paradoxes, but his discussion of what he calls the Liar, which I will call the Liar′ (to distinguish it from the modern metalinguistic Liar) seems to rely on the same principle. He formulates the liar sentence as

$$\exists\text{`}p\text{'}\exists p[\mathrm{Say}(\text{`}p\text{'}) \cap R_n(\text{`}p\text{'}, p) \cap {\sim}p]. \tag{14}$$

According to Ramsey, since 'p' is of order n, "'p' may be $\exists\phi^{n-1}[\psi^n(\phi)]$. Hence $\exists$'p' involves $\exists\psi^n$, and 'I am lying' in the sense of 'I am asserting a false proposition of order n' is at least of order $n+1$ and does not contradict itself" [p. 48].[28]

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26 It is not clear whether Ramsey is here saying that *all* functions of Order 2 have to be of this form. They do not, of course; '$\forall\phi^1[\phi(a)]$' is of Order 2 and does not contain a function of Type 2. Ramsey was aware of formulas of this form, too; one appears at the top of p. 42. But whether he is making a mistake here is irrelevant, as he does not need to say that the second argument to R in (1″) *must* contain a function of Type 2. If he did, he would be in trouble, because as mentioned above, the R he actually uses is not just R_2, but $R_{0,1,2}$, "the sum of [R_0, R_1, and R_2]" [p. 45]. See also note 28.

27 Or, if we want variables to range over symbols that are short for functional symbols, whenever an explicit variable in that formula is capable of ranging over symbols that are short for expressions containing symbols of type n. For simplicity's sake, I will assume that variables range over functional symbols directly, but this is clearly not importantly different from assuming that they range over symbols that are defined with functional symbols.

28 Again, Ramsey is working with a 'p' of order n or less, but this is irrelevant. He uses ϕ for both types, but I have changed one to ψ for clarity's sake. Ramsey says that ϕ and ψ are of types n and $n+1$ respectively—and thus that $\exists$'p' involves $\exists\psi^{n+1}$—but this, I think, can only be a simple mistake. Functions of order n contain bound variables of type $n-1$, not type n. If we used Ramsey's n for ϕ, then the R_n would have to be R_{n+1}.
Against the concerns raised in note 26, Ramsey's use of 'may' here is further evidence not only that he need not say that a variable must be instantiated by the right sort of expression, but also that he himself relies on nothing more than that it may be so.
One might be concerned about Ramsey's gloss of 'I am lying' as 'I am asserting a false proposition of order n', since propositions do not have orders. I will set this issue aside for now; I revisit it in note 41.

The first thing that one notices about this account of the Liar′ paradox is $\exists$‘p’ in (14). (Though I have often omitted mention quotes throughout the paper, I especially do so here to avoid confusion about whether they originate in Ramsey's text.) This is the first time that a quoted symbol has appeared immediately following a quantifier in Ramsey's paper, and he does not explain what it means. The natural reading is that the variable in $\exists$‘p’ is a variable that ranges over propositional symbols (or symbols introduced via definitions whose definientia are propositional symbols; see note 27). This might strike one as somewhat odd, since I argued above that bound variables of any type (other than i) range over symbols. If that is right, then there should be no need to have an explicit symbol following a quantifier. But it is actually not so clear that Ramsey is ignoring the difference between propositions and propositional symbols here: he writes, "‘p’ may be $\exists\phi^{n-1}[\psi^n(\phi)]$," whence, it seems, we are supposed to think that p—which ‘p’ means—is the proposition denoted by $\exists\phi^{n-1}[\psi^n(\phi)]$. If this is right, then the variable in $\exists p$ could plausibly range over propositions themselves, rather than propositional symbols, and the R_n in (14) would be different from the R_i above: its second argument would be an object—a proposition—rather than another symbol.[29]

For now, this discussion can be set aside, as Ramsey is clear that the order of (14), which is all that is relevant to his resolution of the Liar′ paradox, is determined by the variable in $\exists$‘p’, not the variable in $\exists p$. (The discussion will return with force in Section 8.) The only point I wish to make here is that Ramsey's reasoning clearly relies on the principle that I stated above: (14) involves a hidden variable function of type n because ‘p’[30] may be short for an expression containing a symbol of type n—because "$\exists$‘p’ involves $\exists\psi^n$"—and it is this hidden variable that forces (14) to be of order $n + 1$.

To recap: I am suggesting that a formula **P** contains a hidden variable of type n just in case there is an explicit variable in **P** that can be instantiated by a formula containing a constant of type n. The order of **P** is then $m + 1$, m the higehst type of variable—explicit or hidden—occurring in **P**.

6.3. Problems for Ramsey's resolution. If this principle is really what Ramsey is relying on, then his resolution of the paradoxes faces serious problems. I hinted at one of these when I pointed out that (i) he seems to be talking as though giving an R_i arguments of the wrong order yields an ill-formed formula. This actually points to a much more serious issue, which I have also hinted at before: (ii) Ramsey seems to actually be reintroducing a restriction on the ranges of bound variables beyond types. This is already problematic, because this is precisely what ramified type theories do, but it will turn out that (iii) Ramsey's restriction is not even as good a restriction as Russellian orders are. Ramsey's restriction, unlike that of ramification, has no basis in the formalism—it is entirely ad hoc. Finally, I argue that (iv) if we are allowed to restrict the ranges of variables in this way, we can reintroduce the paradox.

Of course, Ramsey does not see it this way, so let us start from the beginning and return to (i). I have said repeatedly that he seems to be assuming that formulas like $R_0\big(‘\phi’, \lambda x\big[\biguplus x^0[f(x)]\big]\big)$ are not just false but ill formed. He does not actually have to assume this, but if he doesn't, then the reasoning behind his resolution becomes extremely

29 Indeed, this might not be too surprising. Recall that though Ramsey is almost always insistent that there are no functions but functional symbols, he *does* think that there are propositions independent of propositional symbols. Perhaps that distinction is (probably unconsciously) in play here. This becomes more significant in Section 8.2.

30 As I said above, I am adding no mention quotes here; this is the first variable occurring in (14).

weak. As explained above, he wants to infer the existence of a hidden variable in (1″) from his observation that for R_2, $R_2\big(‘\phi_2’, \lambda x[\phi_2(x)]\big)$. I offered two possible interpretations of “for R_2, $R_2\big(‘\phi_2’, \lambda x[\phi_2(x)]\big)$”: that it is about the truth of $R_2\big(‘\phi_2’, \lambda x[\phi_2(x)]\big)$, and that it is about its well-formedness. Now we can see why I thought that the latter was the better interpretation. If Ramsey means the former, then we have to be able to conclude that ϕ is capable of ranging over symbols containing symbols of Type 2 just because $R_2\big(x, \lambda x[\phi(x)]\big)$ is only possibly true if it is such a symbol. Much more reasonable, it seems, is to conclude this from the observation that $R_2\big(x, \lambda x[\phi(x)]\big)$ would actually be ill formed if ϕ were not of such a form. Of course, to say this is to embrace ramification, so Ramsey seems forced to make the weaker claim about truth.

This poses a problem for motivating his resolution, but no matter which interpretation is correct, point (ii) from above is looming. A ramified theory of types restricts the ranges of bound variables by their order as well as their type. This, I argued, is crucially absent from Ramsey's logic—his variables range over *all* functions (or, rather, functional symbols) of a given type, irrespective of order. But this cannot be quite right. Consider once again the two possible interpretations discussed above. Clearly if we go with the second, Ramsey is forced to restrict the ranges of variables by order as well as by type; this is why he seems to be forced to go with the weaker first interpretation. But it turns out that he actually has to restrict the ranges further, regardless of which interpretation is correct. This is because functional symbols of Order 2 can contain functional symbols of arbitrarily high type. For instance, $\lambda x\Big[\exists\phi^1\big[f^4(g^3) \cap h^2\big(\lambda z[\phi(z)], x\big)\big]\Big]$ is of Order 2, but it contains a functional symbol of Type 4. If the ϕ in (1″) were allowed to range over this expression, then there would be a hidden variable of Type 4, and ‘F’ would be of Order 5. And since one can make the type of f in this expression arbitrarily high, ‘F’ seems to have no determinate order at all—it contains hidden variables of *every* type.

In order to avoid this consequence—in order to retain the ability to assign orders to symbols after hidden variables have been introduced—Ramsey needs to restrict the ranges of bound variables by more than just their type. But the foundation of his restriction has to be more fine grained than even his orders, because he needs to be able to allow formulas like $\lambda x\Big[\exists\phi^1\big[f^2\big(\lambda z[\phi(z)], x\big)\big]\Big]$ while disallowing formulas like $\lambda x\Big[\exists\phi^1\big[f^4(g^3) \cap h^2\big(\lambda z[\phi(z)], x\big)\big]\Big]$.

Such a restriction is problematic for at least two reasons. First, it is precisely the sort of restriction that Ramsey was attempting to avoid. But second and more pressingly, (iii): it has absolutely no basis in the formalism. That is, there is no way to determine the range of a bound variable simply by looking at a formula; one must first decide what order one wants the formula to be, and then decide how to restrict the range of the variable. We can no longer determine what the order of ‘F’ is just by looking at (1″). First, we have to decide that we want it to be of Order 2. Then we know that ϕ must not range over formulas like $\lambda x\Big[\exists\phi^1\big[f^4(g^3) \cap h^2\big(\lambda z[\phi(z)], x\big)\big]\Big]$. In this sense, his resolution relies on an entirely ad hoc restriction on the ranges of bound variables.[31]

[31] Of course, one could add more structure to the logic in order to avoid this worry. One could probably even argue for such additions through an appeal to Ramsey's understanding of meaning. But the other three worries would still stand, and spelling out his account of meaning would be a lengthy digression, so I will not pursue this response.

This leads to one final point, (iv), which is that we can also choose the range of ϕ so that 'F' is only of Order 2, whence we can once again derive (4). To do this, we simply restrict ϕ so that the only second-order functional symbols it can range over are symbols like $\lambda x\big[\forall\phi^1[\phi(x)]\big]$. Prohibiting this restriction would, of course, require still more arbitrary restrictions, this time on what restrictions are permissible. Thus, after all this work, it seems as though his resolution of the paradoxes not only requires an unmotivated and unwanted restriction on the ranges of bound variables, but does not even successfully resolve the paradoxes without even more ad hoc restrictions on how these very restrictions can look.

PART II

RAMSEY'S DIVISION OF THE PARADOXES

At this point, Ramsey's solution seems to be in trouble. He has appealed to a restriction on the ranges of bound variables akin to ramification, he has no basis for that restriction, and anyway we seem to be able to construct paradoxes by employing the restriction. But I now want to set all of that aside and consider his division of the paradoxes, which is the contribution of 'The Foundations of Mathematics' that has had the most lasting impact. The division is now put as a division between the logical and the semantical paradoxes,[32] and indeed Ramsey seems to encourage this reading with his own analysis of the paradoxes: he claims that the contradictions of his Group B "all contain some reference to thought, language, or symbolism" [p. 20] and thinks that the correct formalizations of the Group B paradoxes all involve semantical notions. I think, however, that there are other possible formalizations, which more closely capture the informal statements of the paradoxes and involve no semantical predicates. If this is right, then it is unfortunate that nobody writing about Ramsey's division of the paradoxes looked more closely at the paradoxes he was actually concerned with. In what follows, after introducing some additional connectives, I give an example of the intensional paradoxes I have in mind (Section 7.1); argue that they do not involve any concepts that cannot reasonably be attributed to Ramsey (Section 7.3); and attempt to show that, despite Ramsey's own formalization of the paradoxes he lists, there are more natural formalizations of the informal statements of his Group B paradoxes that behave like the intensional, and not the semantical, paradoxes (Section 8).

§7. The intensional paradoxes. So far, the intensionality of the logic I have been using has played no role: neither fact that well-formed formulas are of type p nor the use of nonstandard logical symbols has been relevant. But paralleling the intensional part of the logic, I now introduce a type t, intuitively of truth-values; the primitive constants $\neg^{\langle t,t\rangle}$ and $\rightarrow^{\langle t,\langle t,t\rangle\rangle}$; and infinitely many constants $\forall^{\langle\langle\tau,t\rangle,t\rangle}$, one for each type τ. The constants $\vee$, $\wedge$, $\leftrightarrow$, and $\exists$ are defined in the usual way. I also introduce infinitely many constants $=^{\langle\tau,\langle\tau,t\rangle\rangle}$ and $\approx^{\langle\tau,\langle\tau,p\rangle\rangle}$, which are the obvious identity relations for each type τ,[33] and the constant $^{\vee\langle p,t\rangle}$, which takes propositions to their truth-values. This suggests

[32] Again, see Fraenkel & Bar-Hillel (1958, pp. 5–14), Beth (1959, §171), Kneale & Kneale (1962, pp. 664–665), Quine (1963, pp. 254–255), Feferman (1984, p. 75), and Priest (1994, pp. 25–26).

[33] If there is a reason to think that Ramsey's logic absolutely prohibits the inclusion of identity, then we can add Church's strict equivalence from Church (1974). The following paradox requires a little more work in that case, but it can still be constructed. However, it seems to me that the reasons Church presents in Church (1974) in favor of strict equivalence work in favor of identity

the following translation principles. (Superscripted τs on the metavariables indicate that there is one such principle for each type τ; the types of the other symbols are fixed by context.)

$$\begin{aligned} {}^{\vee}[\mathbf{P}^{\tau} \approx \mathbf{Q}^{\tau}] &\leftrightarrow [\mathbf{P}^{\tau} = \mathbf{Q}^{\tau}], \\ {}^{\vee}{\sim}\mathbf{P} &\leftrightarrow \neg{}^{\vee}\mathbf{P}, \\ {}^{\vee}[\mathbf{P} \rightsquigarrow \mathbf{Q}] &\leftrightarrow [{}^{\vee}\mathbf{P} \rightarrow {}^{\vee}\mathbf{Q}], \\ {}^{\vee}\forall\mathbf{x}^{\tau}\mathbf{P} &\leftrightarrow \forall\mathbf{x}^{\tau}\,{}^{\vee}\mathbf{P}. \end{aligned}$$

The natural principles for the defined constants fall out of these. With this additional machinery in place, we can construct an intensional paradox. Unlike, say, Grelling's paradox, what follows is not an antinomy: I only show that a contradiction follows from premises that are not obviously contradictory (and, indeed, seem to describe a possible state of the world), and not that the logic just described is inconsistent.

7.1. An intensional paradox. Let a^{i} be Aristotle and $A^{\langle i,\langle p,p\rangle\rangle}(x, y)$ mean that x asserts y.[34] (15) then denotes the proposition that everything Aristotle asserts is false.

$$\forall x^{p}[A(a, x) \rightsquigarrow {\sim}x]. \tag{15}$$

Thus (16) denotes the proposition that Aristotle asserts that everything Aristotle asserts is false.

$$A\big(a, \forall x^{p}[A(a, x) \rightsquigarrow {\sim}x]\big). \tag{16}$$

Once we have this, though, paradox threatens; if the propositions denoted by both (16) and (17) are true, then we can derive a contradiction.

$$\forall x^{p}\Big[A(a, x) \rightsquigarrow \Big[\big[x \approx \forall y^{p}[A(a, y) \rightsquigarrow {\sim}y]\big] \cup {\sim}x\Big]\Big]. \tag{17}$$

(17) denotes the proposition that the only things that Aristotle has asserted (if he has asserted anything at all) are either the proposition denoted by (15) or false propositions. This seems to at least be possibly true, and it does not seem to contradict the denotation of (16), so hypothesizing that the two of them are true should not be problematic. But, of course, it is. Formally, we are supposing

$${}^{\vee}A\big(a, \forall x^{p}[A(a, x) \rightsquigarrow {\sim}x]\big) \tag{18}$$

and

$$\forall x^{p}\Big[{}^{\vee}A(a, x) \rightarrow \Big[\big[x = \forall y^{p}[A(a, y) \rightsquigarrow {\sim}y]\big] \vee \neg{}^{\vee}x\Big]\Big]; \tag{19}$$

these are ${}^{\vee}$(16) and ${}^{\vee}$(17) respectively.

From (18) we have

$$\neg\forall x^{p}[{}^{\vee}A(a, x) \rightarrow \neg{}^{\vee}x], \tag{20}$$

which is simply $\neg{}^{\vee}$(15), almost immediately. The derivation is elementary; one assumes ${}^{\vee}$(15) and, after applying the translation principles, instantiates the variable with (15) itself.

here, and the concerns raised there about using identity instead of strict equivalence do not apply to the present logical system.

34 The example of Aristotle and assertion comes from Church (1974), although Church does not develop any paradoxes.

From (20) we know that there is a constant b^p such that both

$$^{\vee}A(a, b) \tag{21}$$

and

$$^{\vee}b. \tag{22}$$

From (19), (21), and (22) we have

$$b = \forall x^p[A(a, x) \rightsquigarrow \sim x], \tag{23}$$

whence by (22)

$$^{\vee}\forall x^p[A(a, x) \rightsquigarrow \sim x], \tag{24}$$

which is just $^{\vee}(15)$.

(24) and (20) are contradictories, so the supposition of (18) and (19) has gone wrong somewhere. It is hard to see where, though. It is certainly possible for Aristotle to say (the Greek equivalent of) "Everything Aristotle asserts is false" and nothing else, which would, at least prima facie, satisfy both assumptions; such a situation is, at least prima facie, one in which Aristotle has said (and said only) that everything Aristotle says is false.[35]

7.2. The difference between the intensional and the semantical paradoxes. It is tempting to think that the above paradox—call it the Aristotle paradox—will be resolved by whatever one adopts to resolve the Liar (and the Grelling, and maybe the Strengthened Liar, etc.). It certainly feels similar to the semantical paradoxes. But there is a crucial difference. Consider, for example, Tarski's hierarchy of languages. This resolves the semantical paradoxes by prohibiting any language from talking about the semantics of that very language. In particular, no sentence of a given language L_n can contain a satisfaction predicate for L_n, so one cannot construct a sentence that says of itself that it is false.

In the Aristotle paradox, though, no sentence says of itself that it is false. Indeed, there is no appearance of a metalinguistic notion of truth at all. The only notion of truth that appears in the Aristotle paradox is one that applies to propositions. The problematic assumption is not that Aristotle asserts, "Every sentence Aristotle asserts is false"; the problem is that Aristotle asserts the proposition that every proposition Aristotle asserts is false. Thus, any restriction on metalinguistic satisfaction predicates that we wish to include in our logic (such as a Tarskian prohibition on languages containing their own satisfaction predicates) can be taken on board without qualification—it will do nothing to the above derivation of a contradiction, because satisfaction of formulas is completely irrelevant there.

The preceding paragraph is a little imprecise. We can informally state the situation that leads to the Aristotle paradox as one in which (i) Aristotle asserts the proposition that every proposition Aristotle asserts is false and (ii) every proposition Aristotle asserts is either that proposition or false. The 'false's in this informal characterization apply to propositions, and in that sense, as I wrote above, "[t]he only notion of truth that appears in the Aristotle

[35] I do not mean to say that, at the end of the day, this actually is a situation in which (18) and (19) are both true; one way to resolve the paradoxes is to insist that it is not actually such a situation. But if they were not even prima facie true, there would nothing for a logician to do here; there is work to be done precisely because (i) this (clearly possible) situation *seems* to be one in which both (18) and (19) are true and (ii) (18) and (19) *seem* to imply a contradiction. If things did not even seem this way, then there would be no paradox to resolve in the first place.

paradox is one that applies to propositions." But I did not use any truth or falsity predicates when formalizing (i) and (ii), and generally do not do so to translate English sentences containing 'true' or 'false' (in their propositional senses, anyway). I would, for example, represent the proposition that Aristotle asserts a true proposition with '$\exists p[A(a, p) \cap p]$', in which nothing like a truth predicate appears; if we were saying that the proposition is false, I would simply insert a $\sim$ in front of the second conjunct.[36]

While good to clarify—I really was speaking imprecisely—these details are irrelevant to the main point, which is that no metalinguistic predicates of any sort appear in either the informal statements (i) and (ii) or the formalizations thereof. As long as that is true, Tarski's hierarchy of semantical predicates—of satisfaction predicates—will be of no immediate assistance in resolving the Aristotle paradox.[37]

Similarly, a truth-value gap approach to the semantical paradoxes will not help here without adaptation. That approach allows sentences to lack truth-values, but again, truth-values of sentences are irrelevant to the Aristotle paradox; truth-values of propositions are what matter.

I do not mean to say that these are the only two approaches to the semantical paradoxes, or that they cannot be adapted to resolve the intensional paradoxes as well.[38] But such issues are far beyond the scope of this paper, as I am not concerned here with resolutions of the intensional paradoxes. My only concern is to show that as far as Ramsey is concerned, the intensional paradoxes are distinct from the semantical paradoxes, and it is enough for this purpose that resolutions of the latter do not always resolve the former.

7.3. Propositions. "As far as Ramsey is concerned" in the preceding sentence is not innocuous. This distinction between the intensional and the semantical paradoxes might not be all that interesting if one takes propositions to be, or at least be very much like, sentences. Thus, one might worry at this point that I am imposing too specific an account of propositions on Ramsey. Russell, for instance, is notorious for not being clear about the distinction between propositions and the sentences that express them. While I think that the issues raised by the intensional paradoxes are interesting in their own right, they would

[36] This way of formalizing the relevant propositions (without the $^\vee$ constant) has been taken by others working on these paradoxes; Prior (1961) is a notable example. The general approach of translating English sentences containing 'true' and 'false' without truth or falsity predicates is discussed at length in Grover *et al.* (1974).

[37] One might think that $^\vee$ is the relevant truth predicate, contra my claim that no metalinguistic truth predicate appears in my formalizations of (i) and (ii). If it were, then the Aristotle paradox wouldn't be very surprising—it would be arising in a logic that Tarski already proved couldn't have models. But the $^\vee$s in (18) and (19) only serve to say that the propositions expressed by (16) and (17)—which are formalizations of (i) and (ii) respectively and which contain no truth predicates of any sort—are true. That is, if one wants to think of $^\vee$ as a truth predicate, one must think of it as a *propositional*, not *metalinguistic*, truth predicate, and this is precisely the distinction that I am trying to draw between the intensional and semantical paradoxes: the former are concerned with propositional truth; the latter, metalinguistic.

[38] Indeed, the ramified theory of types is not entirely unlike Tarski's hierarchy—especially in light of Church (1976)—and it has been employed to resolve these paradoxes in, for example, Church (1993, p. 152). And there is no reason to think that an approach that posits some sort of proposition gaps would be hopeless, although none has been developed in perfect detail. Parsons argues for such an approach in Parsons (1974), and Bealer (1994, p. 162) at least thinks that positing gaps is the most promising extant route. But what—and, more importantly, where—such gaps are exactly is never very clear, and such an approach has never been worked out in print in any detail (as far as I know).

have little place in this paper if it turned out that Ramsey was equally unclear, or if he clearly took propositions and sentences to be importantly alike.

Luckily, Ramsey is explicit both about the distinction between propositions and formulas and about the nature of propositions themselves. As I said in Section 3, his terminology is somewhat confusing at times, but he seems to be clear on the distinction in practice, even if his notation does not observe it very scrupulously. In fact, I think that his understanding of propositions is very close to the popular modern account of propositions as sets of possible worlds.[39] One of the quotes in Section 3 already suggests this. As I wrote there,

> “two propositional symbols are to be regarded as instances of the same proposition . . . when they express agreement and disagreement with the same sets of truth-possibilities of atomic propositions” [p. 9]. That is, they are instances of the same proposition when their truth tables agree.

Here, I have glossed expressing agreement and disagreement with sets of truth-possibilities of atomic propositions as having truth tables that agree. But it is easy to think of each row of a truth table picking out a set of possible worlds, namely, the worlds at which the atomic propositions in the truth table have the truth-values they are assigned on that row. One can then think of two propositional symbols as instantiating the same proposition, to use Ramsey’s terminology, when they are true at the same possible worlds. This, at least, should behave the same as his notion of agreeing and disagreeing with the same truth-possibilities.

§8. Intensional paradoxes in Group B. I have argued that, given Ramsey’s account of propositions, the intensional paradoxes are distinct from the semantical paradoxes. But this is not enough; I also have to argue that Ramsey was worried (at least de re, if not de dicto) about the intensional paradoxes. The argument here is a bit more intricate than one might expect, because all of Ramsey’s formalizations of the Group B paradoxes actually involve semantical notions: Tarski’s hierarchy, for example, will resolve them all. But this does not mean that the formalizations he provides are the only ones available, or the only ones that he would have endorsed. Indeed, given the informal statements of the paradoxes that both he and Russell—from whom Ramsey inherited all but one of the paradoxes he lists—provide, I think that it is plain that one can construct more faithful formalizations in some cases. In particular, I think that he has shoehorned meaning into his formalization of the Liar′, and that a more natural formalization of even the sentence that he starts with, and definitely the sentence that Russell starts with, involves no semantical predicates. If (i) this is right, and if (ii) we are interested in resolving the (informal statements of the) paradoxes that Ramsey began with, then we need to look beyond the standard resolutions of the semantical paradoxes even to resolve all of the paradoxes that Ramsey himself was worrying about (though we do not need to do so to deal with all of his formalizations of those paradoxes). Given the attention people have paid to Ramsey’s account of the paradoxes, (ii) seems to be obviously true. I hope to show that (i) is as well.

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[39] Or the precursor to that account, Carnap’s notion of state descriptions. In fact, Carnap (1956, p. 9) writes, “Some ideas of Wittgenstein were the starting-point for the development of [state-descriptions],” citing the *Tractatus* in a footnote. Meanwhile, §I of ‘The Foundations of Mathematics’, which contains most of Ramsey’s discussion of propositions, makes frequent reference to Wittgenstein and his *Tractatus*. It is thus not surprising that Ramsey’s account of propositions is similar to Carnap’s state descriptions.

8.1. The division. I quote Ramsey.

> The best known [contradictions] are divided as follows:—
>
> A. (1) The class of all classes which are not members of themselves.
> (2) The relation between two relations when one does not have itself to the other.
> (3) Burali Forti's [sic] contradiction of the greatest ordinal.
>
> B. (4) 'I am lying.'
> (5) The least integer not nameable in fewer than nineteen syllables.
> (6) The least indefinable ordinal.
> (7) Richard's Contradiction.
> (8) Weyl's contradiction about 'heterologisch'.[40]
>
> The principle according to which I have divided them is of fundamental importance. ... [T]he contradictions of Group B ... all contain some reference to thought, language, or symbolism.... So they may be due not to faulty logic or mathematics, but to faulty ideas concerning thought and language. [p. 20]

Beginning with Tarski, we have seen that language and symbolism are very much the concern of logic and mathematics, but that part of Ramsey's principle is not relevant. I contend that the first claim, that "the contradictions of Group B ... all contain some reference to thought, language, or symbolism," is misleading. Well, I don't actually know what it means to contain a reference to thought. But as long as they do not essentially contain a reference to language or symbolism, they will not be resolved by just any resolution of the semantical paradoxes (certainly not by any of the standard ones), and that is enough for my purposes.

Before arguing for this, I should make a note about my terminology. Though Ramsey follows Russell in calling these eight paradoxes 'contradictions', I refer to them as Paradoxes 1–8 in what follows. (I refer to the seven paradoxes presented in Whitehead & Russell (1910) as Contradictions 1–7 in Section 8.3.2 below.) My concern will be with Paradox 4.

8.2. Ramsey's formalization of Paradox 4. When addressing Paradox 4, Ramsey claims that we should analyze 'I am lying' as

$$(\exists`p\text{'}, p) \mathbin{\textbf{:}} \text{I am saying }`p\text{'} \mathbin{\centerdot} `p\text{' means } p \mathbin{\centerdot} {\sim}p. \text{ [p. 48]} \tag{25}$$

I have already presented a translation of (25) into the logic I have constructed, which I reproduce here.

$$\exists`p\text{'}\exists p[\mathrm{Say}(`p\text{'}) \cap R_n(`p\text{'}, p) \cap {\sim}p]. \tag{14}$$

I have not introduced *Say*, but it is intended to be that function that takes a sentence and returns the proposition that I am saying that sentence.

As I said when discussing (14) in Section 6.2, and especially in note 29, I think that the R_n here is one that relates symbols to propositions, rather than symbols to symbols. To distinguish this meaning relation from the one employed in Ramsey's analysis of Grelling's

[40] This is, of course, Grelling's paradox, incorrectly attributed to Weyl.

paradox (Paradox 8), I will represent it with R'_n, and retranslate (25) as

$$\exists`p'\exists p[\mathrm{Say}(`p') \cap R'_n(`p', p) \cap \sim p]. \tag{14'}$$

The idea seems to be that one cannot say the sentence (14′) and nothing else, though there is nothing intuitively contradictory about such a supposition. If we were to take this formalization seriously, we would probably want to rephrase it using more modern techniques for representing self-reference, which had not been developed when Ramsey was writing. As with the Aristotle paradox, one could then spell the supposition out carefully with $^\vee$.

I am not going to take this formalization seriously, though, because it involves a metalinguistic relation, R'_n, and a metalinguistic speech predicate, *Say*. Of course, with such relations present, resolutions of the semantical paradoxes can be easily adapted to prohibit any contradiction from forming. Consider, for example, Tarski's argument that no language can contain its own satisfaction predicate. One can easily adapt that argument to show that no language with a constant like $^\vee$ can contain an expression relation that holds between formulas and the propositions they express. That is, one can show that, on pain of contradiction, no language can include both $^\vee$ and R'_n. If we were to follow Tarski all the way, this would lead us to develop a hierarchy of R'_n relations, in much the same way that Tarski proposes a hierarchy of truth predicates.

This, however, is a very unsatisfactory resolution of Paradox 4, especially in light of the way Ramsey himself glosses 'I am lying' immediately after presenting this resolution. As I quoted in Section 6.2, he thinks that he has shown that " 'I am lying' in the sense of 'I am asserting a false proposition of order *n*' ... does not contradict itself" [p. 48].[41] But surely 'I am asserting a false proposition of order *n*' does not involve any semantical notions—it just involves the notion of assertion as applied to a proposition (a proposition which happens to be about itself). Thus, although Ramsey's own formalization of Paradox 4 will be resolved by, say, an adaptation of Tarski's argument, it does not seem like the best formalization of Paradox 4 in the first place. If we are going to take Paradox 4 seriously, then, we ought to try to construct a more faithful formalization of it, and see if it, too, will turn out to be no different than the semantical paradoxes.

8.3. Better formalizations of Paradox 4 and a related paradox. There are two places we could look for the informal statements that we are attempting to formalize. Ramsey's statement of the paradoxes is, of course, one place. But since he took himself to be simply repeating the list in Whitehead & Russell (1910) (with the exception of Grelling's paradox), one might also look there. If it turns out that there is an important difference between a paradox listed in 'The Foundations of Mathematics' and its precursor in *Principia Mathematica*, then it seems reasonable to think that even Ramsey would be interested in resolving both the paradox he lists and the distinct version from Russell. I think that such a difference does turn up with respect to Paradox 4 and what I will call Contradiction 1, the first of seven contradictions listed at Whitehead & Russell (1910, pp. 60–61); I discuss possible formalizations of both paradoxes (or, more precisely, of the informal statements of both paradoxes).

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41 As I said in note 28, this is not actually a good gloss of his formalization, because propositions do not have orders: it is not the order of the proposition, but the order of the propositional symbol in (25)—and (14′)—that leads to the hidden variable. A better gloss would be 'I am saying a sentence of order *n* that denotes a false proposition'; in addition to attributing the order to the right sort of thing, this captures the essentially semantical nature of (25).

8.3.1. Ramsey's Paradox 4. As I quoted above, Ramsey reads 'I am lying' as 'I am asserting a false proposition (of a certain order)'. I observed in notes 28 and 41 that this is not a satisfactory gloss of Ramsey's formalization of 'I am lying', since it both attributes an order to a proposition and does not involve any semantical relations like R'_n. But it is at least not unreasonable to think that Ramsey would be interested in the actual problem posed by 'I am asserting a false proposition'. After all, this is very similar to the informal formulation of the analogous paradox in Whitehead & Russell (1910, p. 62): 'There is a proposition which I am affirming and which is false'. Thus, in the interest of doing what Ramsey failed to do, one might attempt to formalize 'I am asserting a false proposition'.

One way to read this is as involving a self-referential proposition of some sort. Self-reference is easy and well documented in the case of sentences; though Ramsey did not know how to construct self-referential sentences, we now do. But almost nothing analogous has been done for propositions.[42] Thus, it would be best to find a non–self-referential reading that still goes beyond the power of, say, Tarski's hierarchy.

This is easy to come by: one might also read 'I am asserting a false proposition' as not involving self-reference at all, and take it completely at face value. 'A dog is walking' just means that there is a thing that is a dog and is walking. Similarly, one might think that 'I am asserting a false proposition' just means that there is a proposition which I am asserting and which is false.[43] Formally, where I am c^i,

$$\exists x^p[{}^\vee A(c, x) \wedge \neg{}^\vee x]. \tag{26}$$

Notice that the connectives are all extensional. (26) is not intended to represent the proposition that I am asserting; it simply states a fact about me and what I am asserting. But (26) is not contradictory at all. The contradiction arises when I am asserting (and asserting only) that I am asserting something false. That is, the contradiction arises on an assumption of

$${}^\vee A\big(c, \exists x^p[A(c, x) \cap \sim x]\big) \wedge \forall x^p\big[{}^\vee A(c, x) \leftrightarrow x = \exists x^p[A(c, x) \cap \sim x]\big]. \tag{27}$$

This, of course, is analogous to the assumption of (18) and (19) in the Aristotle paradox, and the derivation of a contradiction would proceed in a similar way.

I think that this is a more faithful formalization of 'I am asserting a false proposition' than anything Ramsey provides. It clearly involves no metalinguistic predicates.[44] Thus, if one thinks that the paradoxes that Ramsey was concerned with are real problems to be solved, then it is misleading to think that all of the paradoxes in Group B can be resolved by the standard resolutions of the semantical paradoxes.

However, one might be a little worried about my formalization for two reasons: (i) there still seems to be something self-referential in 'I am asserting a false proposition' that I have not captured; and (ii) there is an indexical, 'I', in 'I am asserting a false proposition', and there is certainly nothing in (26) or (27) that behaves at all indexically. (27) is clearly a problem for anybody interested in intensional logic, but again, that is not enough for my

[42] Barwise & Etchemendy (1987) addresses self-referential (they prefer 'circular') propositions, but it is highly unusual in this. While I think that the suggestions made in that book deserve closer attention, this is not the place for it.

[43] This is just the way it is put in Whitehead & Russell (1910), as quoted above.

[44] It does involve an assertion relation that relates an individual to a proposition directly, and there is no evidence that Ramsey considered such relations, but this does not strike me as a huge problem. Certainly we now are happy to countenance propositional attitudes, and I see no reason to think that Ramsey would be averse to them.

purposes. Luckily, (27) is very close to a formalization of one of the paradoxes listed in Whitehead & Russell (1910), about which the point can be made without such concerns. As my aim is only to show that there is at least one paradox that Ramsey would have been interested in that is not resolved along with the semantical paradoxes, this should suffice.

8.3.2. Russell's Contradiction 1. There are seven paradoxes, or contradictions, discussed in Whitehead & Russell (1910); they are basically Paradoxes 1–7 in a different order. The first of the seven, which I will call Contradiction 1, is the analogue of Paradox 4. It is introduced:

> Epimenides the Cretan said that all Cretans were liars, and all other statements made by Cretans were certainly lies. Was this a lie? The simplest form of this contradiction is afforded by the man who says "I am lying"; if he is lying, he is speaking the truth, and vice versa. (Whitehead & Russell, 1910, p. 60)

Later, when discussing the resolution of the paradoxes by the ramified theory of types [p. 62], the sentence I quoted above, 'There is a proposition which I am affirming and which is false', is presented. But as all of these ostensible simplifications involve indexicality and possibly self-reference, neither of which is present in the original statement, I want to focus on the Epimenides paradox itself.

The problematic state of affairs here is one in which three things obtain: (i) Epimenides says the proposition that every proposition a Cretan says is false; (ii) Epimenides is a Cretan; and (iii) every other proposition any Cretan has said is false. As I said above, Russell is not known for his care in distinguishing propositions from sentences, so one might worry about my use of 'proposition' in (i) and (iii). But he uses 'proposition' in the simplified version from p. 62, and Ramsey clearly takes those propositions to be distinct from the sentences that denote them in a very modern way, so I do not think that my use of the term is illicit.

It should now be clear how these formalizations go, but for completeness, I include them here. Letting e^i be Epimenides, $S^{\langle i,\langle p,p\rangle\rangle}(x,y)$ mean that x says (the proposition) y, and $C^{\langle i,p\rangle}(x)$ mean that x is a Cretan,

$$^{\vee}S\big(e, \forall\!\!\!\!\forall x^p \forall\!\!\!\!\forall y^i\big[C(y) \rightsquigarrow [S(y,x) \rightsquigarrow \sim x]\big]\big), \tag{28}$$

$$^{\vee}C(e), \tag{29}$$

and

$$\forall x^p \forall y^i \Big[{}^{\vee}C(y) \rightarrow \Big[{}^{\vee}S(y,x) \rightarrow \Big[x = \forall\!\!\!\!\forall x^p \forall\!\!\!\!\forall y^i \big[C(y) \rightsquigarrow [S(y,x) \rightsquigarrow \sim x]\big] \vee \neg{}^{\vee}x\Big]\Big]\Big] \tag{30}$$

are (i), (ii), and (iii) respectively.

The derivation of a contradiction again closely follows that in the Aristotle paradox. The supposition of (28), (29), and (30) seems to me to be precisely the assumption at issue in the first informal statement of Contradiction 1 in *Principia Mathematica*. There is certainly no indexicality or self-reference to create worries. But there are also no metalinguistic predicates. Thus, if we take the problem posed by Contradiction 1 seriously, as it seems Ramsey did, then we seem to have to conclude, contra Ramsey, and contra everybody following Ramsey for the last 80 years, that the paradoxes in Group B do not all essentially "contain some reference to ... language" [p. 20].

§9. Conclusions. In light of much of 'The Foundations of Mathematics', Ramsey might be taken to be uninterested in intensionality. Indeed, he claims that if he is right that the paradoxes of Group B involve extralogical notions of thought or language, then "they [are] not ... relevant to mathematics or logic, if by 'logic' we mean a symbolic system" [p. 21]. But he immediately follows this with "though of course they would be relevant to logic in the sense of the analysis of thought." This sense of 'logic', it seems, is his concern when he analyzes meaning and attempts to formalize and resolve his Group B paradoxes.

Unfortunately, I think that his analysis of meaning would sound naive to modern ears, although I have not said much about it. And the logic itself seems to be somewhat sketchily thought out, as evinced by the number of details I have had to fill in with my reconstruction. Aside from these difficulties, his resolution of the semantical paradoxes seems to rely on a restriction of the ranges of bound variables that is dangerously close to ramification and has no basis in the logic. Worse still, I have argued that it does not successfully avoid all of the paradoxes it is designed to avoid.

But this failure is perhaps not as interesting as the general sketchiness and confusion that seem to pervade §III of 'The Foundations of Mathematics', which contains his treatment of meaning and the Group B paradoxes. Time and again in that section one encounters small mistakes, as with his treatment of Paradox 4, or confusing uses of terms, as with his multiple distinct uses of 'mean' (and 'involve', though I have not said much about it). As I said in Section 1, I am reluctant to impute such confusion to Ramsey, but I hope that the interpretation I have provided in Part I is at least not entirely unsatisfactory, in spite of its generally pessimistic tone. Indeed, I think that the confusion is easily understood: these problems are notoriously difficult, and Ramsey was attempting to formalize concepts that are still to some extent problematic today. It is not surprising that one would not have a clearer idea of what is going on when one only has *Principia Mathematica* (and the *Tractatus*) to work from.

If I am right about this confusion, though, then it is unfortunate that so much weight has been given to Ramsey's characterization of the paradoxes. I tried to show in Part II that we can construct formalizations of some of the Group B paradoxes that are, even by his own lights, more faithful to the informal statements of the paradoxes. If I am right, then it is unfortunate that so many people have taken Ramsey at his word that the paradoxes in Group B are all due to semantical notions, thereby obscuring an important, and possibly importantly different, collection of paradoxes.[45] Again, Ramsey's confusion is understandable, but one might wish that it had not been so influential.

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[45] The intensional paradoxes have not been completely ignored. See, for example, Prior (1961), Church (1993, p. 152), Bealer (1982, pp. 98–100), and Klement (2002). But attention to them beyond these and with the realization that they are importantly different from the semantical paradoxes has been minimal.

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