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PARADOXES OF INTENSIONALITY

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Abstract. We identify a class of paradoxes that is neither set-theoretical nor semantical, but that seems to depend on intensionality. In particular, these paradoxes arise out of plausible properties of propositional attitudes and their objects. We try to explain why logicians have neglected these paradoxes, and to show that, like the Russell Paradox and the direct discourse Liar Paradox, these intensional paradoxes are recalcitrant and challenge logical analysis. Indeed, when we take these paradoxes seriously, we may need to rethink the commonly accepted methods for dealing with the logical paradoxes.

§1. Introduction With few exceptions,[1] contemporary work on the paradoxes of logic and set theory is framed by ideas that go back to the 1920s and 1930s. In 1925, Frank Ramsey divided the paradoxes into those (like the Russell Paradox) that arise within mathematics, and those (like the Liar Paradox) that do not.

Having made his division of the paradoxes, Ramsey took the Liar and related paradoxes to be extralogical, involving an "empirical" linguistic element. But later, in work published in 1936, Alfred Tarski showed that the Liar Paradox arises even in rigorously presented semantic theories of formalized languages, making a compelling case that the enterprise of producing these theories belongs to logic. At the same time, he demonstrated how the paradox could be avoided by relativizing truth to a language and invoking a linguistic hierarchy in which no language could serve as its own semantic metalanguage.

Together, Ramsey and Tarski suggest an attractive picture of the general problem posed by the family of paradoxes resembling the Russell Paradox and the Liar. In the terminology that became current after 1936, the *set-theoretical* paradoxes belong to the foundations of mathematics, and are the proper concern of set theory and related areas of mathematics. On the other hand, the *semantic paradoxes* belong to the foundations of semantics. Along with the distinction goes a division of labor: most mathematicians working with semantic theories (and, in particular, most model theorists) can afford to ignore the semantic paradoxes, but they remain a problem for a group of philosophers and mathematicians concerned with the foundations of semantics.

This picture, and the division of labor that goes along with it, had become well accepted by the 1960s,[2] and is still presupposed in most contemporary work on the paradoxes. Since the publication of Kripke (1975) the Liar Paradox has received a great deal of attention; we know of ten books published after 1980 that deal with this topic. And almost all of

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[1] The most notable of these is the paraconsistent approach advocated by Graham Priest. See Priest (2005).

[2] For documentation of this point, see Fraenkel & Bar-Hillel (1958, pp. 5–14), Beth (1959, §171), Quine (1963, pp. 254–255), and Kneale & Kneale (1962, pp. 664–665).

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this work on the Liar Paradox takes the metalinguistic formalization of the paradox for granted.[3]

The purpose of this paper is to question this cluster of assumptions. We believe that Ramsey's distinction is not exhaustive. There are, for instance, versions of the Liar Paradox that are not metalinguistic in any obvious way.[4] And indeed, reflection on these examples suggests that Ramsey's categories, and the division of labor that goes along with them, may be misguided. Although, of course, in the last sixty years we have learned much about specific formalizations of certain paradoxes, the general foundational problem presented by the paradoxes is rather neglected, and calls for radical reassessment. In fact, we may have to reset the clock back to 1900, and to rethink the entire problem in the light of what has been learned since.

§2. Two kinds of truth Jespersen (1965) devotes a chapter to direct and indirect speech, introducing the topic as follows:

> When one wishes to report what someone else says or has said (thinks or has thought)—or what one has said or thought oneself on a previous occasion—two ways are open to one.
>
> Either one gives, or purports to give, the exact words of the speaker (or writer): *direct speech* (oratio recta).
>
> Or else one adapts the words according to the circumstances in which they are now quoted: *indirect speech* (oratio obliqua).
> (Jespersen, 1965, p. 290)

Thus, wishing to report what Bert said to her yesterday, Alice can say either (2.1) or (2.2).

(2.1) Bert said 'I understand you'.

(2.2) Bert said that he understood me.

Jespersen presents the two forms as stylistic alternatives,[5] and gives evidence from several languages, showing that even in written language the two forms can blend and mingle. (Quotation marks can be omitted in constructions that are partly or entirely direct, and there are exceptions to the rule that tense and person are shifted in indirect but not in direct discourse.)

Philosophers and semanticists sharpen the distinction, and take it much more seriously than speakers of language seem to. During much of the twentieth century, analytic philosophers were frequently admonished not to confuse use and mention.

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3 Barwise & Etchemendy (1987) is a notable exception to this generalization.

4 We are not the first to question the exhaustiveness of Ramsey's division. See, for instance, Sullivan (2003). The failure of Ramsey's categories to deal with the particular paradoxes we discuss, however, has been largely overlooked.

5 For linguistic data on the stylistic differences, see Clark & Gerrig (1990). The evidence certainly seems to support a difference in logical form between direct and indirect discourse, although not necessarily a simple one according to which in direct discourse a linguistic expression is mentioned.

Is the adjective ‘true’ like the verb ‘say’, in supporting both direct and indirect usages? It is hard to say. Naturally occurring examples of ‘true’ with indirect discourse are easy to find, such as the following one from the Brown Corpus:[6]

(2.3) It may be true that pool lighting dramatizes an evening scene, but . . . .

Natural examples of ‘true’ with direct discourse are, apparently, much rarer, if they exist at all. The Brown Corpus exhibits 110 occurrences of ‘true’ for which the distinction between direct and indirect discourse might arise. (The other 95 occurrences are adjectival or adverbial.) Of these 110, only 26 are explicitly indirect. The most common usage (64 examples) makes reference to a previous or subsequent claim made in the text without anything explicit to indicate whether a sentence or a proposition is intended. A typical example is:

(2.4) High-level abstractions are always difficult to pin down with precision. That is particularly true of sovereignty when it is applied to democratic . . . .

In anaphoric cases like this, both the explicit words that have been used and the claim or proposition that has been made are salient, and there is no simple way to tell which of these is demonstrated by ‘that’. The fact that an elaboration like ‘that claim’ sounds more natural than ‘that sentence’ here may provide weak evidence for a propositional interpretation.

Cases where ‘true’ is predicated of explicitly quoted material are rare: there are only 2 of them in the Brown Corpus. Here is an example.

(2.5) And a witty American journalist remarked over a century ago what is even more true today, “Many a writer seems to think he is never profound except when he can’t understand his own meaning”.

But in ordinary usage, quotes are not an unambiguous sign of reference to a linguistic expression, and even cases like (2.5) don’t provide altogether convincing instances of ‘true’ predicated of a sentence: observe that (2.6a) is a much more natural elaboration of (2.5) than (2.6b).

(2.6a) And a witty American journalist remarked over a century ago what is even more true today, and what many contemporary journalists believe as well, . . .

(2.6b) And a witty American journalist remarked over a century ago what is even more true today, and what consists of eighteen words, . . .

As we said, however, the language that is used to deal with truth in philosophical and logical work since the 1970s is more regimented than this, and here at least you can clearly distinguish between indirect discourse forms like (2.7) and (2.8).[7]

[6] This is a corpus collected in 1961, containing over a million words of representative English prose from various genres.

[7] (2.7a) and (2.7b) are synonymous. (2.7a) is the less natural form with a ‘that’ clause in subject position. (2.7b) is the more natural extraposed form with expletive ‘it’.

(2.7a) That $5 + 7 = 12$ is true.

(2.7b) It is true that $5 + 7 = 12$.

(2.8) '$5 + 7 = 12$' is true.

When (2.7) and (2.8) are formalized, the differences between the two constructions are sharpened. The formalization of (2.8) is unproblematic and metalinguistic: when formalized, (2.8) has the form

(2.9) $T(\ulcorner\phi\urcorner)$,

where T is a first-order predicate, ϕ is a sentence, and $\ulcorner\phi\urcorner$ is an individual term serving as a canonical name of a linguistic expression—in this case, of '$5+7 = 12$'.[8] This means not only that $\ulcorner 5{+}7{=}12 \urcorner$ names '$5 + 7 = 12$', but that $\ulcorner 5{+}7{=}12 \urcorner$ should integrate with syntactic predicates if they are present, or if they are added. So if a syntactic theory is added to the formalization language, we should expect the resulting theory to account for how the structure of the formula '$5 + 7 = 12$' can be recovered from the name $\ulcorner 5 + 7 = 12 \urcorner$. For instance, there will be sentences of the theory involving $\ulcorner 5 + 7 = 12 \urcorner$ saying that '$5 + 7 = 12$' consists of a certain number of symbols in a certain order; and if the axioms of the syntactic theory are adequate, such a sentence will be be provable if it is true, and disprovable if it is false.

There is no universally agreed-on policy for the formalization of indirect discourse forms like (2.7a) and (2.7b). If we follow the policy used in most versions of modal logic, where

(2.10) It is necessary that $5 + 7 = 12$

would be formalized as

(2.11) $\Box(5 + 7 = 12)$,

then we would formalize (2.7a) and (2.7b) as

(2.12) $T(5 + 7 = 12)$,

where now T is a modal operator rather than a one-place first-order predicate.

Formalizations like (2.12) are automatically consistent as additions to modal logics of the familiar sort. In a Kripke frame that uses the identity relation over worlds to interpret T, the analog (2.13) of Convention T is valid.

(2.13) $T(\phi) \leftrightarrow \phi$

The valid formulas of a theory that has models will be consistent. Kripke models of modal logics therefore provide a guarantee that indirect discourse versions of the Liar Paradox cannot arise when truth is formalized as in (2.13). This protection against paradox applies to any extension of propositional modal logic that has Kripke models; in particular, it applies to type-theoretic extensions like Montague's Intensional Logic (Montague, 1970; Gallin, 1975) with quantification over propositional types.

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[8] We here follow the practice of allowing formulas to name themselves. If we marked the distinction between use and mention explicitly, we would have to write, "... where 'T' is a first-order predicate, ϕ is a sentence, and '$\ulcorner\phi\urcorner$' is an individual term ...."

In Montague's Intensional Logic, (2.7) would be formalized as

(2.14) $T({}^\wedge[5+7=12])$.

Given types s of possible worlds and t of truth-values, T has the type $\langle\langle s,t\rangle,t\rangle$, and so denotes a function from sets of worlds[9] to truth-values. ${}^\wedge\phi$ denotes the function that takes a possible world into the denotation of ϕ in that world, so that when ϕ has type t, as in this case, ${}^\wedge\phi$ has type $\langle s,t\rangle$, and denotes a set of worlds. Formulas like (2.15) are admitted

(2.15) $\forall x\,[T(x) \rightarrow {}^\vee x]$,

where the variable x has type $\langle s,t\rangle$ and so ranges over sets of worlds. More generally, where ϕ has type $\langle s,\tau\rangle$, ${}^\vee\phi$ has type τ and denotes, relative to a world w, the denotation of ϕ in w.

As with ordinary modal logic, consistency is not a problem. Under the same interpretation of T using the identity relation,

(2.16) $\forall x\,[T(x) \leftrightarrow {}^\vee x]$

will be valid in this enriched logic. (Here, as before, x is a variable of type $\langle s,t\rangle$.)

Modal logic with propositions interpreted as sets of possible worlds therefore provides a paradox-free setting for intensional logic.[10] However, the modal approach is committed to the closure of propositional attitudes under logical equivalence: where μ is any propositional operator of a modal logic, $\mu(\phi) \leftrightarrow \mu(\psi)$ is valid if $\phi \leftrightarrow \psi$ is valid. This makes modality unsatisfactory as a general treatment of propositional attitudes.

The discussion of this issue in the philosophical literature goes back to Kathleen Johnson Wu's critique of Jaakko Hintikka's defense of "logical omniscience" in epistemic logic; see Wu (1970) and Hintikka (1970). Since then, an extensive literature on the topic has developed in philosophy and in computer science, where the problem is particularly acute because computer scientists are interested in applications of epistemic logic to agents with limited reasoning power.[11]

Some philosophers, especially Robert Stalnaker, have defended modal logic by claiming that propositional attitudes, if properly understood, do exhibit logical omniscience; see Stalnaker (1984). But it is hard to see how to extend defenses of this kind, which depend on contextual parameters, to cases like (2.17). And approaches to intensionality based on modal logic over Kripke frames are clearly unsuited to many computational applications.

(2.17a) Liz is aware that $5+7=12$.

(2.17b) Liz is aware that for all positive integers n, if there are positive integers i, j, and k such that $i^n + j^n = k^n$ then $n \leq 2$.

9 Here, sets of worlds are themselves functions from worlds to truth-values.

10 Paradox-free in a sense, at least. There are still intuitively consistent assumptions with inconsistent consequences. But unlike most resolutions of the paradoxes, which seek to block the derivation of a contradiction, a modal logic with possible-worlds propositions simply insists that the assumptions are actually inconsistent. This might leave one wondering where our intuitions have gone wrong, but it makes resolving the paradoxes, in the sense of avoiding the contradictions, entirely trivial.

11 See, for instance, Fagin *et al.* (1995, chap. 9).

But if we are willing to work with formalizations of intensionality that, unlike modal logic with Kripke frames, fail to validate logical omniscience, the situation with regard to the paradoxes is no longer so straightforward, especially when we work without well-behaved models and introduce axioms that are inconsistent with logical omniscience. Here, we will have to reassess the question of freedom from paradoxicality.

To recapitulate: the direct and indirect formalizations of truth predications in (2.9) and (2.12) have much in common. They share the form

$$(2.18)\quad T(\langle\!\langle 5+7=12 \rangle\!\rangle),$$

where $\langle\!\langle\ \rangle\!\rangle$ creates a syntactic environment that is either like regimented quotation, or like the 'that' of indirect discourse. Also, both formalizations allow quantification into the argument position of the truth predicate. In principle, they both allow free iteration of the T, producing *semantic closure* for quotational truth, and *modal* closure for modal truth.

But these two formalizations differ dramatically with regard to the Liar Paradox. In the presence of an adequate syntactic theory, semantic closure is inconsistent with Convention T when $\langle\!\langle\ \rangle\!\rangle$ is interpreted as quotation. But even in the presence of an adequate theory of sets of worlds, modal closure is perfectly consistent with Convention T.

The concept of a proposition that comes to us from Frege and Russell is not syntactic, but it is clearly intended to admit the possibility of different propositions that are true in the same possible worlds. As we said, there are good reasons, motivated by the semantic behavior of propositional attitudes, for taking these more finely individuated propositions seriously.

Therefore, the question arises whether, when we interpret $\langle\!\langle\ \rangle\!\rangle$ as creating references to propositions of this sort, quantification over propositions and Convention T or similar schemes will produce paradoxes.

In at least some cases, we know that this can happen. In Myhill (1958), John Myhill showed that Alonzo Church's first formalization of the Logic of Sense and Denotation[12] was inconsistent, using an argument based on cardinalities. But the general case has not been much explored, and it is worthwhile to ask whether a formalization of propositions that does justice to the requirements of propositional attitudes and that allows unrestricted quantification over propositions can hope to avoid paradoxes like the Liar.

Unfortunately, it will turn out that paradoxes do arise very generally in formal settings of this type, and that in fact some of these paradoxes are very like traditional formulations of the Liar Paradox.

Furthermore, these "paradoxes of intensionality" lie outside of Ramsey's classification of the paradoxes and the toolkit of solutions that go along with this classification. Paradoxes of intensionality are not resolved by appeals to a metalinguistic hierarchy, unless we take an otherwise not well-motivated syntactic approach to propositional attitudes. Nor are they resolved by any of the set-theoretic solutions to the Russell Paradox.[13]

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[12] In Church (1951).

[13] These paradoxes have not been entirely ignored, but almost all the attention they have received has either been too narrow or not systematic enough to satisfactorily account for the phenomena. See, e.g., Church (1993, p. 152), Anderson (1987), Kaplan (1995), Bealer (1982, pp. 98–100), Lindström (2003a, 2003b), Klement (2002), and Burge (1979, 1984). For discussions of related forms of the Liar paradox, see Barwise & Etchemendy (1987) and Groeneveld (1994) on the one hand and Kneale (1972), Parsons (1974), and Glanzberg (2004) on the other.

§3. The empirical paradoxes It seems likely that the earliest formulations of the Liar Paradox have an empirical component: for instance, a Cretan says that everything a Cretan says is false. Whether this is paradoxical depends on the facts about what Cretans say. The intensional paradoxes generalize the Epimenides, by allowing propositional attitudes other than "saying that," and by elaborating the empirical presuppositions. Unlike formulations of the direct discourse Liar Paradox that rely only on the presence of a syntactic theory, these intensional paradoxes presuppose the possibility of agents having certain attitudes: for instance, the possibility of a Cretan believing certain things.

Prior (1961) provided an extended discussion of these paradoxes. This paper is unusual (almost unique) in concentrating on the intensional paradoxes.

It is (let us take this for granted) a matter of fact that Epimenides was a Cretan, and it *seems* to be a matter of fact that he said that everything a Cretan (ever) says is false. We may not believe *what* Epimenides said (we had better not, if we can reason with propositional quantifiers). But at least we believe that he said it.

Let us reiterate what we said about direct and indirect discourse in Section . Although the Epimenides Paradox, as we have stated it, is often mentioned in the literature on the Liar Paradox, it is usually not distinguished from the version of the Liar that runs "This sentence is false." But the two forms are not at all equivalent: (3.1) is (certainly) false, because Epimenides didn't speak English, whereas (3.2) is (probably) true, if we can trust the historical sources.

(3.1) Epimenides said 'Everything a Cretan said is false'.

(3.2) Epimenides said that everything a Cretan said is false.

We will use the type framework of Thomason (1980) to formalize this paradox. The type system resembles that of Montague's Intensional Logic, but intensionality is introduced with a primitive type p of propositions. Since this framework is neutral as to what propositions are (they could be truth-values, sets of possible worlds, sentences from a "language of thought," Fregean senses, or platonic abstractions of some other sort), it provides a conveniently general medium for this purpose that is ontologically neutral in at least some important respects. Where α is an expression of type p, ${}^{\vee}\alpha$ now denotes the truth-value of the proposition denoted by α.

As a first step in representing the Epimenides Paradox, we must have a way to represent the *proposition* saying that all propositions a Cretan says are false. For this purpose, we need intensional analogs of extensional logical operators. For example, to formulate *the proposition* that if $1 > 0$ then $2 > 0$ we will need a conditional operator $\leadsto$ of type $\langle p, \langle p, p\rangle\rangle$. If we use '$\phi \leadsto \psi$' to abbreviate '$[\leadsto(\phi)](\psi)$', then $[1 > 0 \leadsto 2 > 0]$ denotes the proposition that if $1 > 0$ then $2 > 0$, and ${}^{\vee}[1 > 0 \leadsto 2 > 0]$ denotes the truth value of this proposition. We will introduce the suite of intensional operators shown in Figure 1.

In the absence of a specific reification of propositions, there are few if any plausible intensional constraints to be placed on the intensional operators. However, it is reasonable to require that these operators reduce to the corresponding extensional operators. The following *extensional homomorphism* principles do this.

(3.3) Extensional homomorphism principles for $\approx$, $\sim$, $\leadsto$, $\cap$, $\cup$, and $\uplus$:

(3.3a) ${}^{\vee}[\alpha \approx \beta] \leftrightarrow [\alpha = \beta]$

(3.3b) ${}^{\vee}[\sim\phi] \leftrightarrow [\neg{}^{\vee}\phi]$

Intensional $=$ (over objects of type τ):	$\approx$, type $\langle \tau, \langle \tau, p \rangle\rangle$
Intensional $\neg$:	$\sim$, type $\langle p, p\rangle$
Intensional $\wedge$:	$\cap$, type $\langle p, \langle p, p\rangle\rangle$
Intensional $\vee$:	$\cup$, type $\langle p, \langle p, p\rangle\rangle$
Intensional $\rightarrow$:	$\rightsquigarrow$, type $\langle p, \langle p, p\rangle\rangle$
Intensional $\leftrightarrow$:	$\leftrightsquigarrow$, type $\langle p, \langle p, p\rangle\rangle$
Intensional $\forall$: (over objects of type τ):	$\biguplus$, type $\langle\langle \tau, p\rangle, p\rangle$

Fig. 1. Intensional operators

(3.3c) $^{\vee}[\phi \cap \psi] \leftrightarrow [^{\vee}\phi \wedge {}^{\vee}\psi]$
(3.3d) $^{\vee}[\phi \cup \psi] \leftrightarrow [^{\vee}\phi \vee {}^{\vee}\psi]$
(3.3e) $^{\vee}[\phi \rightsquigarrow \psi] \leftrightarrow [^{\vee}\phi \rightarrow {}^{\vee}\psi]$
(3.3f) $^{\vee}[\phi \leftrightsquigarrow \psi] \leftrightarrow [^{\vee}\phi \leftrightarrow {}^{\vee}\psi]$
(3.3g) $^{\vee}[\biguplus x^{\tau}\phi] \leftrightarrow \forall x^{\tau}\, {}^{\vee}\phi$

In (3.3a–g), ϕ and ψ are arbitrary expressions

In reproducing the Epimenides Paradox, we will think of saying as a propositional function that inputs a proposition and returns the proposition that the input was said: it is a function of type $\langle p, p\rangle$. If this were a function of type $\langle p, t\rangle$, there would be no way to speak sensibly of *the* proposition that everything that has been said is false. Prior uses propositional quantification to formalize (3.2). With our new connectives, the paradox takes the following form:[14]

(3.4) $^{\vee}[\mathbf{Say}(\biguplus x\,[\mathbf{Say}(x) \rightsquigarrow {\sim}x\,])]$

Here, **Say** has type $\langle p, p\rangle$ and x has type p.

In this sort of type theory, universal quantification appears as a typed operator, and formulas like (3.4) are more properly formulated using lambda abstraction:

(3.5) $^{\vee}[\mathbf{Say}(\biguplus(\lambda x\,[\mathbf{Say}(x) \rightsquigarrow {\sim}x\,]))]$

Here, $\biguplus$ has type $\langle\langle p, p\rangle, p\rangle$.

At this point, we begin to use a convention of using superscripts to type the first occurrence of a constant or variable in a formula. The general form of Prior's Epimenides Paradox is then:

(3.6) $^{\vee}[\,F^{\langle p,p\rangle}(\biguplus x^{p}\,[\,F(x) \rightsquigarrow {\sim}x\,])]$

Prior himself makes no distinction between expressions of type p (expressions that denote propositions) and expressions of type t (expressions that denote truth-values), or

[14] Prior assumes propositional quantification throughout Prior (1961).

between boolean operators and the corresponding intensional operators. We believe that it is helpful to make these distinctions explicit, and will work with formalizations like (3.6).

Formula (3.6) denotes a truth-value. If, for example, F denotes the propositional function of being said by a Cretan, (3.6) will denote the truth-value of the proposition that a Cretan says that everything a Cretan says is false.

We continue to follow Prior's train of thought, using this notation to formalize it. First, (3.6) has the following two consequences:[15]

(3.7) $\neg^{\vee}[\,\forall x^{p}\,[\,F(x) \rightsquigarrow {\sim}x\,]\,]$

(3.8) $\exists x^{p}\,[\,{}^{\vee}F(x) \wedge {}^{\vee}x\,]$

These two formulas are logically equivalent; we mention them both only because the first makes it evident that the argument of F in (3.6) is false, while the second shows that some Cretan saying must be true.

Deriving (3.7) from (3.6) is a simple exercise, but it does involve instantiating the variable x in ${}^{\vee}[\,\forall x^{p}\,[\,F(x) \rightsquigarrow {\sim}x\,]\,]$ with the formula $\forall x^{p}\,[\,F(x) \rightsquigarrow {\sim}x\,]$.

The empirical paradoxes depend on a contingent premise, which may in fact be false, but intuitively could be true. To simplify things, suppose that as a matter of fact, Cretans are very laconic—the only other thing a Cretan ever says is that $7+5=11$. If (3.6) were true, then because of (3.8) some proposition a Cretan says must be true. But we have assumed that the only propositions a Cretan says are that $7 + 5 = 11$ and the proposition denoted by $\forall x^{p}\,[\,F(x) \rightsquigarrow {\sim}x\,]$, i.e., the proposition that everything a Cretan says is false. And we know independently that it is false that $7 + 5 = 11$, while in view of (3.7) it must be false that everything a Cretan says is false. That is, (3.6) is false.

In view of this argument, Prior concludes that, in a world in which a Cretan has said that $7+5 = 11$ and no Cretan has yet said anything else, it is impossible for a Cretan to say that everything a Cretan says is false. That is, there can be empirical situations which prevent a Cretan from saying something (from being in the appropriate relation to a proposition) or more generally, which can prevent an agent from having a propositional attitude to a proposition, even though the usual prerequisites for that circumstance are present. (We can assume, in this hypothetical situation, that Epimenides uttered the appropriate words.) Epimenides must not have said anything on the problematic occasion. Prior accepts this conclusion somewhat reluctantly, having this to say about it.

> ... I must confess that all I can say to allay the misgivings expressed in the past four sections is that so far as I have been able to find out, my terms are the best at present offering. I have been driven to my conclusion very unwillingly, and have as it were wrested from Logic the very most that I can for myself and others who feel as I do. So far as I can see, we must just accept the fact that thinking, fearing, etc., because they are attitudes in which we put ourselves in relation to the real world, must from time to time be oddly blocked by factors in the world, and we must just let Logic teach us where these blockages will be encountered. (Prior, 1961, p. 32).

15 That is, (3.7) and (3.8) must be true in any model of the type theory that satisfies (3.6). We are appealing here to the model theory of Thomason (1980).

It may be easier for us to accept this conclusion now than when Prior wrote his paper. Hilary Putnam, David Kaplan, and many other philosophers of language have urged that what you say or think depends on general on the circumstances, and that the "internal relations" of the speaker will not always suffice to fix a reference. If this is accepted, it may not be surprising that *whether* anything is said or thought could also be risky.

You can even use Prior's techniques to construct Putnam-like examples, without having to resort to science-fiction-like hypotheticals. Imagine that for some reason Ralph, who is in Room 17 but doesn't realize that this is where he is, thinks to himself to the effect that[16] whatever anyone in Room 17 thinks to himself then is false. Unknown to Ralph, someone else—Annie—is hiding in the room. There are two cases: (1) Annie thinks to herself that $7 + 5 = 12$, and (2) Annie thinks to herself that $7 + 5 = 11$. According to Prior, Ralph is thinking something in case (1), but in case (2) he isn't. But nothing about Ralph's internal state will reveal this.

With this example, we begin to see the generality of (3.6) and its logical consequences as a source of problematic examples. F in ${}^{\vee}F^{\langle p,p\rangle}(\forall x^{p}\,[\,F(x) \rightsquigarrow {\sim}x\,])$ can be instantiated with any propositional attitude. We can start with a general attitude type, like thinking or expecting, and qualify it in any way we like—restricting the agent, the time, the place, and any other circumstances we care to choose. If we can do this in such a way that all the other instances in which the qualified attitude is instantiated are false, we have an empirical paradox.

Prior (1961, p. 29) recounts an elaboration which he attributes to Michael Dummett. According to one popular view of what happened when Epimenides spoke, he uttered certain words (of Greek) that in virtue of the conventional rules of the language are associated in each context of utterance with a proposition. To simplify things, we can suppose that there are no indexicals in Epimenides' hypothetical sentence; then we can forget the context of utterance. But, although Epimenides' *words* are conventionally associated with the proposition that everything a Cretan says is false, we know that speech acts can misfire in various ways. Prior postulates a *logical* misfire in the case of Epimenides' utterance, which prevents him from saying anything when he makes the utterance.

As Prior presents it, Dummett's idea is to let F in (3.6) stand for 'Epimenides speaks words of Greek that conventionally signify (in Greek) ...'. (The dots here stand for an argument position of type p.)

It seems to follow that Epimenides *can't even utter the words.* This, of course, is unacceptable. Prior's (1961, p. 29) response is to suggest that signifying "can't be infallibly effected by our conventions." As far as we can see, this would rule out a theoretical approach to semantics. You can't put semantics on a proper footing without some way of drawing the encyclopedia/dictionary distinction—some way of making it possible to allow semantics to assign interpretations to phrases—and propositions to sentences—by local rules that are not forced to appeal to arbitrary and apparently irrelevant contingencies.

Dummett's example doesn't strike us as calling for such drastic measures at all, though in the present context it may give this appearance. We need to remember that whoever adopts Simple Type Theory is likely to have the Tarski hierarchy in his repertoire of puzzle

[16] We use the awkward phrase "thinks to himself to the effect that" to indicate the motions that someone would go through normally in thinking something, and that would create the presumption that in going through these motions they had indeed thought something. The phrase sounds so awkward because there is no reason in the ordinary course of affairs to distinguish between going through the motions of thinking something and actually thinking it.

solving devices. And this case is well suited to a Tarskian cure. Of course, '___ utters words of Greek that conventionally signify (in Greek) ...' is a relation between an individual and a proposition: its type is $\langle p, \langle e, p\rangle\rangle$, the same as that of '___ believes ...'. But it is a *semantical* relation.[17] If we can convince ourselves that the **L**-expression relation, which relates an individual (a sentence) to a proposition, and so has type $\langle e, p\rangle$, is not definable in **L**, similar considerations should persuade us that Dummett's relation isn't definable in **L**.

But Prior considers another elaboration that has nothing metalinguistic about it and that is potentially much more damaging.[18] Consider an example in which Tarski thinks to himself: "Snow is white." Ordinarily, you'd suppose that Tarski has thought that snow is white. But unfortunately, someone else (whom we will call "Gödel") gets there first. Just before Tarski's act of thought, Gödel thinks to himself: "Whatever I am now thinking is false just in case whatever Tarski thinks immediately afterwards is true."

We can formalize the proposition that Gödel thinks as follows.

$$(3.9)\ \ \forall\!\!\!\!\forall x^p\,[\,G(x) \rightsquigarrow {\sim}x\,] \leftrightsquigarrow \forall\!\!\!\!\forall x^p\,[\,T(x) \rightsquigarrow x\,]$$

Suppose as a matter of fact that Gödel thinks this, and nothing else; that immediately afterwards Tarski thinks that snow is white, and nothing else; and that snow is white. We can formalize these facts as follows, where s^p denotes the proposition that snow is white.

$$(3.10a)\ \ \forall x^p\,[\,{}^{\vee}G(x) \leftrightarrow x = [\,\forall\!\!\!\!\forall x^p\,[\,G(x) \rightsquigarrow {\sim}x\,] \leftrightsquigarrow \forall\!\!\!\!\forall x^p\,[\,T(x) \rightsquigarrow x\,]\,]\,]$$

$$(3.10b)\ \ \forall x^p\,[\,{}^{\vee}T(x) \leftrightarrow x = s\,]$$

$$(3.10c)\ \ {}^{\vee}s$$

From (3.10a–c), we can prove that the proposition expressed by (3.9) is both true and false—using (3.3b,e–g), that (3.11) denotes both $\top$ and $\bot$.

$$(3.11)\ \ \forall x^p\,[\,{}^{\vee}G(x) \rightarrow \neg{}^{\vee}x\,] \leftrightarrow \forall x^p\,[\,{}^{\vee}T(x) \rightarrow {}^{\vee}x\,]$$

We begin by observing that, by (3.10b) and (3.10c), the right-hand side of (3.11) must denote $\top$. Now suppose that (3.11) denotes $\top$; we then know that the left-hand side also denotes $\top$, i.e., that everything Gödel says is false. By (a), we then know that the proposition expressed by (3.9) must be false—that (3.11) must denote $\bot$—contra our supposition.

Thus, (3.11) cannot denote $\top$; it must denote $\bot$. But then, since the right-hand side still denotes $\top$, the left-hand side must denote $\bot$—Gödel must say something true. By (a), the only possible witness is the proposition denoted by (3.9), so it must be true—(3.11) must denote $\top$.

This is a contradiction; our empirical assumptions (3.10a–c) are inconsistent. But it is difficult to say which of them is wrong, and it is cases like this that lead Prior to the uncomfortable solution he offers in the passage we quoted above.

In this example, Prior's explanation is that Tarski can't have managed to think anything after all, despite the apparently innocuous *content* of what he tried to think. Prior doesn't back his diagnosis up with a detailed account of the conditions under which agents can successfully have attitudes, but the general idea seems to be that Gödel's act of thought

[17] Whatever this means. We are very much in need of tests (even relatively unreliable ones) that can help us to tell which predicates are semantical.

[18] Prior attributes examples of this kind to Jean Buridan.

trumps Tarski's because Gödel gets his thought in first. Perhaps the idea is that propositions are served out on a first-come first-served basis, and so a seemingly innocuous attempt to think something can be blocked by logic from being thought in paradoxical circumstances like this.

But if this sort of theory were right, elaborations of the Gödel-Tarski Paradox give malicious preemptors far too much scope. These elaborations don't even have to be hypothetical. For instance, you, the reader, may have felt as you read this paper that you were having thoughts, and that some of these were true. We can now reveal that you were mistaken. We are, of course, now writing this paper before you have had a chance to read it. And one of us is now thinking that what he is thinking is false just in case somebody else thinks something true while reading this paper. On Prior's account, you can't succeed in having any true thoughts while reading this paper. And it is too late for you to do anything about this.[19]

Indeed, for reasons like this, Prior's account seems to imply that we could never be sure, when we seek to engage a proposition with a propositional attitude, that we have actually managed to relate ourselves to the proposition we had in mind. Unlucky enough to have a malicious precursor, a person could go through an entire life without ever thinking, suspecting, or doubting anything.

Furthermore, our attempt to state what happens when someone attempts to engage a proposition but fails is subject to the same sort of paradoxical argument that any other attitude is. Prior wants to say that Epimenides didn't in fact say[20] anything. But (perhaps as part of an explanation of why he failed to say anything) we need to say what did he do.

We are tempted to say that Epimenides tried (unsuccessfully) to say something, or that he made as if to say something, or that he simulated saying something. In each case, we can reintroduce the paradox by substituting for F in (3.6) the predicate that we obtain by deleting 'something' from these formulations. This would lead to consider the case of a Cretan who, for instance, tries to say that everything a Cretan tries to say is false. Epimenides can't try to say that everything a Cretan tries to say is false. It can't seem to Prior that Epimenides can try to say that everything it seems to Prior that Epimenides can try to say is false. But then we are left with no very good way to describe what the person who is logically blocked from relating successfully to a proposition does do—or else we are left with a problematic regress of "trying to say" or "making as if to say."

We conclude that Prior's way out of the paradox is hopeless. Tentatively, or perhaps as a challenge to any philosopher who wants to work out such a theory, we suggest that attempts to develop, within the framework of a Simple Type Theory, a plausible theory of "propositional acts" or relations of epistemic agents to propositions that will resolve these paradoxes are likewise hopeless.

§4. Some possible solutions You can't help feeling that there is an asymmetry in content between sentences like 'Everything a schizophrenic fears is false' and '$7 + 5 = 12$', and that Prior's solution is flawed in allowing Tarski's relatively simple thought in the Gödel-Tarski Paradox to be blocked by Gödel's complex thought, which involves

[19] The situation is even worse: One of us has thought that what he is thinking is false just in case somebody else bears a propositional attitude toward something. On Prior's account, nobody else can ever think, say, fear, etc. anything at all, unless this thought was itself preempted by someone else.

[20] This is the 'say' of indirect discourse.

propositional quantification. The restricted comprehension axiom of Zermelo set theory suggests an approach that would do more justice to this difference. Just as the set theorist errs in assuming unrestricted set comprehension, in the form

(4.1) $\exists x \forall y\, [\, y \in x \leftrightarrow \phi\,]$,

we could try to trace these intensional paradoxes to the following principle, which says that every propositional function of type $\langle p, p \rangle$ possesses a logical product.

(4.2) $\forall f^{\langle p,p \rangle} \exists x^{p}\, [\, x = \biguplus y^{p}\, [\, f(y) \rightsquigarrow y\,]\,]$

Of course, there are differences as well as similarities in the set-theoretical analogy: (4.1) is not a principle of logic. Since (4.2) is validated by the semantics of quantification in Simple Type Theory, we can't do away with it without adjusting the logic of Simple Type Theory.

The following argument, whose last step is (4.2), indicates what will have to be discarded.

(4.3) (1) $\biguplus y^{p}\, [\, f(y) \rightsquigarrow y\,] = \biguplus y^{p}\, [\, f(y) \rightsquigarrow y\,]$

(2) $\exists x^{p}\, [\, x = \biguplus y^{p}\, [\, f(y) \rightsquigarrow y\,]\,]$

(3) $\forall f^{\langle p,p \rangle} \exists x^{p}\, [\, x = \biguplus y^{p}\, [\, f(y) \rightsquigarrow y\,]\,]$

If we think of the intensional paradoxes as arising from a discrepancy between the domain of propositions (the values of propositional variables) and the language's ability to form expressions of propositional type, the most natural object of suspicion is the inference from Step (1) to Step (2). Invalidating this inference[21] would result in a logic of *partial* propositional functions.

This seems in some ways like an attractive idea. If the proposition that everything a Cretan says is false doesn't exist in the paradoxical situation, then this can explain why Epimenides has said nothing. But if the facts are different, and on another occasion a Cretan has said that $7 + 5 = 12$, then a Cretan has said something true and we are inclined to conclude that here, Epimenides has said something false. Surely, however, he couldn't have said something false without saying something.

We have been maneuvered at this point into saying that the existence of propositions is a contingent affair. This is likely to complicate our theory of set existence.

As usual, we can make matters worse by elaborating the empirical paradox. Let's go back to the original Epimenides, and imagine that a non-Cretan kibitzer says that everything a Cretan says is false. Epimenides, we agreed, said nothing. And the only thing that Cretans ever say is that $5 + 7 = 11$. (We have *supposed* this, in setting up the problem.) But it is false that $5 + 7 = 11$, and so it follows that everything a Cretan says is false. So the kibitzer has said something true, and unproblematic. But this is just the proposition that we had to rule out of existence, in order to prevent Epimenides from saying anything. Now we seem forced to say either that the kibitzer has in fact said nothing or that the existence of propositions is not only contingent, but speaker-relative as well.

[21] The mechanics of this are fairly straightforward, and we will not go into details here. It is only necessary to allow models in which the domain of propositions is not closed under all the operations definable in the logical language, and to adopt one of the standard policies for dealing with the resulting truth-value gaps.

Also (and this is typical too of similar approaches to the direct discourse semantic paradoxes), if we develop a logic of partial propositional functions with a proof theory, it will be very difficult to avoid the existence of formulas ϕ of type p such that both ${}^{\vee}\phi$ and $\neg\exists x^p\,[x = \phi]$ are provable. Such cases tend to undermine the motivation of the theory.

This approach looks more promising than Prior's, but on the whole it is still pretty dismal.

An alternative approach, and one well worth considering, is to explore the idea that the problem with the empirical intensional paradoxes is unrestricted quantification over propositions. To do this is to reopen the intensional ramified theory of types as a serious logical alternative. This amounts to restricting our comprehension principle (4.2) by insisting that the x therein ranges over a larger domain than the y.[22]

This idea leads to a project that is beyond the scope of this paper. But we do wish to point out that rehabilitating Ramified Type Theory is not as hopeless a suggestion now as it would have been, say, in 1950. Attitudes towards intensionality are not as hostile now as they were then. And there is no compelling reason to complicate matters by associating a revived Ramified Type theory with a foundational program for mathematics. Although some people may perhaps take the logicist program as seriously as it was in Russell's day, it is less easy nowadays to take it entirely seriously. Even if we do choose to be logicists, we can still be *extensional logicists*. We can take set-theoretical formalisms based on extensional logics to be adequate for the formalization of mathematics. This leaves us perfectly free to explore Ramified Type Theory as a basis for formalizing intensional phenomena without having to invoke Reducibility or calling into question any of the work that has gone into formalizing analysis and other areas of mathematics.

Ramified Type Theory was also shunned because of its complexity, but since 1925 we have learned a great deal about how to develop complex logics in a way that makes them intelligible and even useful. Perhaps we can do the same for Ramified Type Theory.[23]

Finally, one could pursue a sort of truth-value gap resolution, according to which the proposition that Epimenides said is neither true nor false. Again, the details are beyond the scope of this paper, but it is worth observing that such a project differs from familiar truth-value gap resolutions of the direct discourse Liar, such as Kripke (1975), in at least two ways. First, resolutions like Kripke's rely on syntactic features of formulas to determine which ones are neither satisfied nor not satisfied, and there is no guarantee that propositions have analogous structure. Second, we might think that sentences lack truth values because they fail to express propositions, but a truth-value gap approach to the intensional paradoxes must embrace truth-valueless *propositions*.

§5. Concerns about ramification The rehabilitation of ramification solves many problems, but doesn't automatically provide a general solution to the logical paradoxes. Tucker (2011) mentions two intensional paradoxes that Ramified Type Theory does not appear to solve: an intensional version of Yablo's Paradox, formulated in Yablo (1993), and the version of the Liar in Barwise & Etchemendy (1987) in which a proposition is identical to its own negation. Bearing in mind cases like these, a divide-and-conquer approach to the

[22] See Church (1976) for a precise statement of the restricted comprehension principle.

[23] Church (1976) provides a good starting point for this project. Church does not there provide a semantics for his reformulation of Ramified Type Theory, though Anderson (1980) later provides models for a ramified form of Church's Logic of Sense and Denotation, carrying out a suggestion of Church's.

intensional paradoxes might be appropriate. But in view of the failure of Ramsey's division of the paradoxes, we would need to find a way to convince ourselves that the classification is exhaustive.

An attempt to analyze the Epimenides Paradox from a ramified standpoint precipitates other, potentially more serious worries. According to ramification, no propositions are properly expressed with an unrestricted propositional quantifier. Thus, for instance, Epimenides can only have said that everything up to some order n that a Cretan says is false. One might ask whether this does violence to Epimenides' intentions—surely Epimenides was *trying* to quantify over every proposition. But ramification must extend to propositional attitudes. Epimenides cannot have intended to express an order-unrestricted proposition any more than he can have expressed such a proposition.

Reflecting on what we have just said raises a familiar worry. In attempting to explain the commitments of ramification, we said "no propositions are properly expressed with an unrestricted quantifier." But if every propositional quantifier is restricted, we could only have been saying, for some n, "no propositions of order n are properly expressed with an unrestricted quantifier." The problem is not quite that we cannot state the ramified theory of types from within the ramified theory of types, because strictly speaking, the ramified theory of types is not a theory of languages. It is, rather, a theory of *propositions*, and the real problem is that the ramified theory of types cannot give an account of the propositions we express when we describe the ramified theory of types. It is not merely that there is no sentence that properly expresses the claims of ramification—it is that there is nothing to express in the first place, because there can be no proposition that is true when and only when ramified type theory is true.

This is a serious issue, but it would be unfair to pin the blame for it on ramification; the problem is more general. Unhappiness with the fact that solutions to the paradoxes provide no standpoint for the theorist who wishes to formulate them have motivated some logicians—especially Frederick B. Fitch—to search for universal metalanguages. (See, e.g., Fitch, 1964.) Fitch, however, was concerned not with ramification, but with the Tarski hierarchy; and the case he makes is general enough to apply to any hierarchical attempt to solve the paradoxes. Thus, although we do not pretend to have an answer to Fitch's worries, they don't appear to raise problems that are specific to solutions that use ramification.

§6. Conclusion In Ramsey (1925), Ramsey begins a trend of extensionalization that typifies later formalizations of mathematics. In particular, Quine is much more explicit about the connection between extensionality and the dispensability of Reducibility. In an extended discussion of these matters Quine (1963, §35), he says that the relevance of ramification to paradoxes concerning falsehood or denotation is dissolved in the presence of a careful distinction between use and mention, and especially between propositional functions and open sentences. And the foundations of mathematics do not need orders and ramification because they do not need intensionality.

Although in this work, which is primarily concerned with the theory of sets, Quine does not have much to say about intensionality, he evidently feels that intensional notions such as propositional functions are more problematic than sets and open sentences, that they are not required for the logical treatment of mathematics, and that the regrettable need for Reducibility was due to foolishly incorporating intensionality into the foundations of logic, and was compounded by a pervasive confusion of use and mention. In his later anti-intensional writings, Quine appeals to altogether different considerations, but his discussion of reducibility in Quine (1963) makes it clear that systematic concerns having to do with the paradoxes and the formalization of mathematics also motivate the banishment of intensionality from logic.

Quine's rejection of intensionality may therefore originate as much in the logical paradoxes, as in a merely philosophical abhorrence for things mental. It served a logical purpose, removing the threat of the paradoxes like those on which we have concentrated in this paper. In fact, the classical areas of mathematics do seem to ignore intensionality. If, taking a cue from this, intensionality is ruled out of bounds as a subject for logic, the Ramsey-Tarski approach to the paradoxes seems to work.

But contemporary logicians, unlike Quine, can't really afford to deny legitimacy to intensional logics. The status of logic has changed dramatically since the 1960s. Logic is no longer merely a "foundation" for mathematics (if it ever was such a thing), but is a source of formalisms that are widely used in philosophy, linguistics, economics, and computer science. Even if intensionality is marginal for "pure" mathematics, it is not marginal in these other areas.

In the absence, however, of a ramification revival, genuine (as opposed to merely linguistic) truth-value gaps, or some alternative that has not occurred to us, we are not left with a comfortable strategy for dealing with the logical and set-theoretical paradoxes, particularly if we want a strategy that is supported by a rationale that makes it seem general as well as plausible. Despite the problems with the Vicious Circle Principle, there is a great deal to be said for Russell's attempt to diagnose the general cause of the paradoxes and to use this diagnosis to produce an equally general cure.

If we accept Ramsey's twofold classification of the paradoxes, along with generally accepted formalization methods for dealing with both of Ramsey's categories, then it may well seem unnecessary to seek a general, principled solution to the logical paradoxes. However, as we have seen, Ramsey's distinction not only fails to be exhaustive, but leaves out some particularly challenging paradoxes that have no very appealing solution method.

Even if a special-purpose method can be found for these paradoxes, the idea of replacing Ramsey's two-part distinction with a many-fold distinction does not seem very appealing, in the absence of reasons to suppose that the distinction is exhaustive. But we see no way to produce such reasons without a general diagnosis of the logical paradoxes.

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