Ivory Ghost

In the August 2007 issue of *IEEE Control Systems Magazine*, we inaugurated this department by asking Gene Franklin to explain how to choose the sample rate for a digital control system. In the present issue, Dennis Bernstein answers the longstanding question, "What is hysteresis?"

Readers are invited to submit technical questions, which will be directed to experts in the field. Please write to us about any topic, problem, or question relating to control-system technology.

Q. In my controls course, the professor said that a thermostat is a "hysteretic controller." In my structures course, the instructor mentioned "hysteretic damping." How could two things that are so different be related? What is hysteresis, anyway?

Dennis: You're asking an excellent question, especially since hysteresis is relevant to many control applications, and, like any good elephant, is multifaceted in its appearance. Hysteresis arises in an amazingly wide range of applications. The logic hysteresis in a thermostat is designed to prevent the furnace from chattering on and off. On the other hand, hysteresis arises naturally in the deformation of smart materials, in magnetism, and even in biological systems. Despite this diversity, hysteresis arises from the same underlying mechanism, which can best be understood from a systems point of view, that is, by searching for its roots rather than focusing on its leaves.

Many books represent hysteresis as a kind of directed loop, a closed curve with arrows. Other books describe a hysteretic system as a system with memory. Unfortunately, since pictures are only suggestive and every dynamic system has memory, neither definition clarifies what hysteresis really is or what causes it.

The name "hysteresis" comes from the Greek word "υστερεσυσ," which means lag. However, the name is a misnomer, since (as I discuss below) hysteresis is not due to delay in a system, although it is related to the notion of phase shift, which has some delay-like features. Let me explain.

Imagine that you have an asymptotically stable linear system so that, if you take the input to be a sinusoid at a given frequency, the response converges to a sinusoidal signal at the same frequency. The amplitude of the output may be different from the amplitude of the input—in fact, the input and output may have different physical dimensions so that directly comparing amplitudes doesn't make sense—and the output may be phase shifted relative to the input. Now, if you plot the output as a function of the input you will see a loop, which is either a circle or an ellipse. Such a loop, constructed from two sinusoids, is a Lissajous figure. However, in two very special cases, namely, a phase shift of 0° or 180°, the ellipse collapses to a line segment. The point of this discussion is that the appearance of a loop indicates the presence of phase shift due to dynamics.

Now suppose that you plot the loop for a sequence of frequencies that approach zero. What happens? What you observe is that the loop begins to narrow until, in the dc limit, the loop collapses to a line segment. Hence, the loop disappears as the input becomes slower and slower. The loop collapses for the simple reason that the dc phase shift of a linear system is either 0° or 180°.

Next, imagine that you have a very special system—necessarily nonlinear—with the property that, as the input frequency approaches dc, the loop (which is not necessarily an ellipse since the system is nonlinear) does *not* collapse. In other words, the loop persists in the dc limit. That persistent loop is a *hysteresis loop*, while a system with such a loop is *hysteretic* [1]. No linear system can be hysteretic.

Let me stress a key distinction: Any input-output loop due to a time-varying input is a natural consequence of the dynamics of the system and the fact that all dynamical systems effectively possess phase shift—literally for linear systems and by analogy for nonlinear systems. So hysteresis does not represent the kind of phase shift that one observes in a dynamic system during dynamic operation; rather, hysteresis represents the ghostly image of phase shift that persists as the dynamic nature of the inputs vanishes in the static limit.

Here is a physical example. Consider a mass on a spring with a viscous (that is, linear) dashpot. This system is linear and thus cannot be hysteretic. Now consider the transfer function from force input to position displacement. If the force is sinusoidal, then you obtain an input-output loop at each frequency, and the area of the loop, which has the units of energy, is precisely the amount of energy dissipated by the dashpot during each cycle of oscillation. As the forcing frequency approaches zero, the mass move more slowly, and less energy is dissipated during each cycle, which agrees with the fact that the loop collapses.

But now consider a different scenario. Consider a book on your desk. Suppose that you slide the book 1 foot to the right, and then you slide it back to where

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it started. Imagine that you can compute the energy dissipated by the frictional contact between the book and the table during one cycle of motion. Now, repeat the experiment, but do it more slowly. And then even more slowly. Do you expect that the energy dissipated during a cycle approaches zero as the speed with which you move the book decreases to zero? Intuitively, you believe (and experiments confirm) that some energy is dissipated no matter how slowly you move the book, which suggests that the plot of position versus force has positive area that does not vanish as the frequency of forcing goes to zero. The input-output loop is persistent, and thus the system is hysteretic.

In a hysteretic system, the shape of the hysteresis loop is generally different from the shape of the input-output loop during dynamic forcing. In some very special cases, however, such as in ferromagnetic materials, the shape of the input-output loop under dynamic forcing is exactly the same at every frequency; thus, the loop persists in the dc limit, and the shape of the dc-limiting loop is identical to the shape of the loop under dynamic forcing. Such systems have *rate-independent hysteresis*, while systems in which the shape of the dc-limiting hysteresis loop is different from the shape of the loop under dynamic forcing have *rate-dependent hysteresis*. This terminology sometimes causes confusion since, strictly speaking, hysteresis refers only to the persistent loop in the dc limit.

But what *causes* hysteresis? Since hysteresis concerns only what happens near dc, consider a system under very, very slow forcing. As the input signal changes its value, imagine that the state of the system (which is fast compared to the input) has sufficient time to converge to an equilibrium. Then, when the input changes to a new value, the input converges to, another, possibly different, equilibrium. You can imagine that, under very slow forcing, the state of the system is always at or nearly at an equilibrium. If the system has exactly one equilibrium for each constant value of the input, then, for each value of the input, the system's equilibrium state is the same whether the input is increasing or decreasing.

On the other hand, imagine that the system has two or more attracting equilibria for some value of the input. It might be the case that, as the input increases past that input value, the state of the system is attracted to one equilibrium, while, when the input decreases past the same value, the state of the system is attracted to another equilibrium. Consequently, the output signal while the input is increasing may be different from the output signal while the input is decreasing. Plotting the output versus the input in such cases gives rise to a loop whose vertical width is the "gap" between the two output values for the same input value. In the dc limit, this loop is the hysteresis loop. In order for this scenario to occur, the system must have multiple attracting equilibria for a constant input value; a system with this property is called *multistable* [2].

So what are the equilibria in our examples? In the case of the sliding book, every location of the book on your desk obviously corresponds to a position equilibrium when the book's velocity is instantaneously zero. As you push the book slowly, you can imagine that you are moving it infinitesimally from one position equilibrium to another. What is not obvious is that each location of the book corresponds to more than one equilibrium of the frictional system. Imagine that the surface of your desk is a very fine brush; the bristles of the brush are bent to the right as the book moves to the right and bent to the left as the book moves to the left. Consequently, as the book moves slowly through a cycle of motion, the frictional system (book plus bristles) is in two different equilibrium states for each location of the book. In fact, the bristles of the brush buckle in different ways, which provides the multistability mechanism.

For the thermostat example, you can imagine the thermostat as the system, the slowly varying house temperature as the input, and the furnace state (on or off) as the output. If you set the thermostat at 70 °F, the furnace is turned on when the thermostat drops to 68 °F, and it turns off when the temperature rises to 72 °F. When the house is at 70 °F the furnace can be either on or off, depending on whether the temperature is increasing or decreasing. The fact that the output of the thermostat takes on two different output values for the same input value during very slow input changes shows that the system must have multiple attracting equilibria. In fact, if you dissected and modeled an old-fashioned thermostat with a pivoted glass bulb containing a fluid, you would obtain a nonlinear differential equation with multiple equilibria.

So what causes hysteresis? A hysteretic system must be multistable; conversely, a multistable system is hysteretic if increasing and decreasing input signals cause the state to be attracted to different equilibria that give rise to different outputs. In the case of friction, the system has a continuum of equilibria; in contrast, the thermostat has a finite set of equilibria. But the principle is the same.

A familiar example of multistability is an optical illusion. We have all seen drawings of a candlestick, which, when viewed "differently," is seen to be the space between the profiles of two faces. Your brain gets stuck in two different attracting interpretations (equilibria) of the same image. Psychologists have shown that these multiple equilibria give rise to hysteresis [3].

Hysteresis is an elephant with many facets. Energy dissipation during slow operation as well as the operation of a thermostat are—on a system-theoretic level—the same phenomenon. Neither is due to lag but both are manifestations of phase shift, indeed, a stubborn kind of quasi-static phase shift that refuses to vanish in the dc limit.

REFERENCES

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