Optimization-Based Calibration of a Triaxial Accelerometer-Magnetometer

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I. INTRODUCTION

In this paper, we use a robotic arm to calibrate a triaxial accelerometer and magnetometer. In particular, we estimate the sensitivity and bias of each of the three accelerometers and magnetometers that comprise the triaxial sensor. We also determine the angles between each pair of accelerometers and each pair of magnetometers.

To perform this calibration, the sensor is mounted to the end effector of a robotic arm. The robotic arm has six joints, or links. Accelerometer measurements are recorded as the robot moves three of the six joints, performing rotational movements in roll, pitch, and yaw, which correspond to rotations about the x-axis, y-axis, and z-axis of a cartesian frame, respectively. A related method for calibrating a sensor was used in [1], where a calibration platform for an inertial measurement unit with 4 GPS antennas was designed using a carpal wrist and turntable.

The triaxial accelerometer and magnetometer considered in this paper are encompassed in a 3DM Solid State 3-axis Pitch, Roll, and Yaw Sensor supplied by Microstrain, Inc. Taps were added to the sensor by the vendor, allowing us to obtain analog voltage readings directly rather than post-processed digital readings. The taps are connected to a Q8 High-Performance H.I.L. Control Board manufactured by Quanser, Inc., which implements analog-to-digital conversion for data acquisition. The board has 14-bit resolution and an input range of ±10 V. The robotic arm is a six-degree-of-freedom A465 Robot Arm manufactured by CRS Robotics Corporation.

II. KINEMATIC MODEL

We first consider the problem of calibrating the accelerometers, beginning with the derivation of a kinematic model. Let the subscript rb denote the robot base frame, let the subscript ee denote the end effector frame, and let

\[
\begin{bmatrix}
\hat{i}_{ee} \\
\hat{j}_{ee} \\
\hat{k}_{ee}
\end{bmatrix}, \quad \begin{bmatrix}
\hat{i}_{rb} \\
\hat{j}_{rb} \\
\hat{k}_{rb}
\end{bmatrix}
\]

(1)

denote mutually orthogonal unit coordinate vectors fixed to the end effector frame and fixed to the robot base frame, respectively. For \(i = 1, \ldots, 6\), let

\[
\begin{bmatrix}
\hat{i}_x \\
\hat{j}_y \\
\hat{k}_z
\end{bmatrix}
\]

(2)
denote mutually orthogonal unit coordinate vectors fixed to the \(i^{th}\) link frame of the robot. Furthermore, assume that the base of the robot, which is not necessarily level, is located at the origin of an inertial frame. All motion is sufficiently slow that the only acceleration that each accelerometer senses is due to gravity. Let \(\vec{g}\) denote the gravitational acceleration vector, which, in the robot base frame, is resolved as

\[
\vec{g}_{rb} = \begin{bmatrix}
g_x \\
g_y \\
g_z
\end{bmatrix}
\]

(3)

The gravity vector is not a parameter of interest in our accelerometer calibration. However, its direction does affect the estimates of other parameters that are of concern. To illustrate, suppose that the gravity vector points in the \(-\hat{k}_{rb}\) direction and that an accelerometer’s axis points in the \(+\hat{k}_{rb}\) direction so that the accelerometer measures 1 g. Rotate the orientation vector of the accelerometer 180 deg about the \(\hat{k}_{rb}\)-axis, followed by a 90 degree rotation about the \(\hat{i}_{rb}\)-axis. The accelerometer now measures 0 g. Now, assume that gravity points in the \(-\hat{i}_{rb}\) direction and the accelerometer’s axis points in the \(+\hat{j}_{rb}\) direction so that the accelerometer again measures 1 g. After performing the same sequence of rotations, the accelerometer reads \(-1\) g. Since this reading is inconsistent with the previous reading after the 2 rotations, the direction in which gravity points does affect the data readings and must be taken into account in formulating the system model. Similar comments apply to the magnetometer and the Earth’s magnetic field. Since we did not attempt to precisely level the robot base, the direction of gravity is unknown, and thus we estimate the direction of gravity as part of the calibration.

Each accelerometer measures a scalar component of the acceleration vector due to gravity in the direction of its axis (referred to as an orientation vector) resolved in the end effector frame. We resolve the orientation vector of each accelerometer in the robot base frame by transforming each vector from the end effector frame.

Let the subscript \(k = 1, 2, 3\) denote an accelerometer. Rotating the unit vector \(\hat{i}_{ee}\) through an angle \(\beta\) about the \(\hat{j}_{ee}\)-axis and then through an angle \(\gamma\) about the \(\hat{k}_{ee}\)-axis yields the unit orientation vector \(\hat{P}_k\) of the \(k^{th}\) accelerometer in the end effector frame. For the \(k^{th}\) accelerometer, the
Therefore, the unit orientation vector of the rotation matrix from a varied, and voltage readings are recorded. The general form for a rotation about the \( k \)-th axis for a rotation about the \( i \)-th axis of the \( i \)-th link is given by

\[
R_{\hat{i}}(\alpha_i) = \begin{bmatrix}
\cos(\alpha_i) & -\sin(\alpha_i) & 0 \\
\sin(\alpha_i) & \cos(\alpha_i) & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]

Therefore, the orientation vector \( \hat{P}_k \) resolved in the end effector frame can be transformed into the robot base frame as

\[
\hat{P}_{k\text{base}} = R_{\hat{k}}(\gamma_k)R_{\hat{i}}(\beta_k)\hat{P}_{k\text{effector}}.
\]

For the \( k \)-th accelerometer, let \( S_k \) and \( \delta_k \) denote the sensitivity and offset, respectively, and let \( a_k \) denote the component of acceleration in the direction of the \( k \)-th accelerometer’s axis. Then, the voltage reading \( V_k \) for the \( k \)-th accelerometer is modeled as

\[
V_k = S_k a_k + \delta_k.
\]

Since, by assumption, the only acceleration felt by the accelerometers is due to gravity, the component of gravity measured by the \( k \)-th accelerometer is

\[
a_k = |\vec{g}| \hat{P}_k \cos \phi_k = |\vec{g}|_{\text{rb}} \hat{P}_{k\text{rb}},
\]

where \( \phi_k \) is the angle between \( \vec{g}_{\text{rb}} \) and \( \hat{P}_{k\text{rb}} \). Therefore, the voltage reading of the \( k \)-th accelerometer is given by

\[
V_k = S_k |\vec{g}|_{\text{rb}} \hat{P}_{k\text{rb}} + \delta_k,
\]

where \( \hat{P}_{k\text{rb}} \) and \( \hat{P}_{k\text{effector}} \) are defined in (8) and (6), respectively.

### III. Robot Kinematic Maneuver

To obtain data for calibration, we rotate the accelerometer in roll, pitch, and yaw and collect voltage readings. As a result of (10) and (11), the voltage reading of each accelerometer is proportional to the cosine of the angle between the accelerometer’s orientation vector and the gravitational vector. Since the magnitude of the cosine function’s derivative is greatest when \( \phi_k \) is close to 90 deg, the voltage readings are most responsive to changes in sensor orientation when \( \phi_k \) is close to 90 deg. Hence in this range, the voltage readings are most useful for data analysis. Therefore, it is necessary to rotate the arm so that \( \phi_k \) is close to 90 deg for each accelerometer. However, the orientation vector of each accelerometer is unknown with respect to the end effector frame. We thus devise a maneuver that allows each accelerometer to provide a large number of voltage readings where \( \cos \phi_k \) is close to 0 deg.

In our experiment, Link 1 of the robot arm is rotated \( \theta_1 \) degrees about the \( \hat{k}_{1\text{rb}} \)-axis, performing yaw. As Link 1 is rotated, Link 5 of the robot arm is rotated \( \theta_5 \) degrees in pitch while Link 6 is rotated \( \theta_6 \) degrees, performing roll. The average angular velocities for links 1, 5, and 6 are 3.1 deg/s, 0.87 deg/s, and 13.9 deg/s, respectively. The entire maneuver takes 27.74 minutes to complete. Fig. ?? shows a graphical representation of this movement.

Data is collected at 166,400 link orientations at a sampling rate of 100 Hz. When analyzing the data, every 25th data point is taken so that the sampling rate is effectively 4 Hz. Since each subsequent orientation of the robot arm is close to that of the previous one, we assume that the robot arm is static, which allows us to ignore translational acceleration and angular velocity.

Each accelerometer has a range of \( \pm 2 \) g with an output reading of between 0 V and 5 V. In addition, accelerometers...
1, 2, and 3 have RMS noise levels of approximately 13.4 mV, 13.6 mV, 14.6 mV, respectively. These noise values were found by locking the arm in an orientation for 200 seconds at a sampling rate of 1 kHz. Performing this procedure for three different orientations of the robot arm, the RMS noise was then calculated for each of the three orientations, and the mean of these three values was taken as an approximation of the RMS noise level. From these values, accelerometers 1, 2, and 3 are chosen so that each accelerometer’s orientation vector of the parameters. The initial estimates of the parameters, links, the function to be optimized, and an initial estimate the accelerometers, the angular orientations of the robot program thus utilizes the voltage readings from quasi-Newton minimization with numerical gradients. The problem using (11). Function minimization is performed using the fminunc program in Matlab, which employs (13). The notation

\[
J_k(S_k, \hat{g}_k|\text{rb}, \hat{\Phi}_k|\text{rb}, \delta_k) = \sum_{j=1}^{5} [V_k,j - (9.8 S_k \hat{g}_k|\text{rb}, \hat{P}_k|\text{rb})^2].
\]

Here \( V_{k,j} \) is the \( j \)-th voltage reading from the \( k \)-th accelerometer for a set of angles \((\theta_1, \theta_2, \theta_3)\) that determine the orientation of the end effector with respect to the robot base frame, taken \( n \) times, and \( \hat{P}_k|\text{rb} \) is defined by (6) and (8).

B. Joint Accelerometer Calibration

After calibrating the accelerometers separately and examining the results, we calibrate the three accelerometers jointly. Therefore, the estimate of the direction of gravity is the same for each accelerometer; that is, there is only one unknown unit gravity vector in the optimization instead of three as in the separate calibration case. The accelerometer measurement model is the same as in (14), and the function to be minimized with respect to 14 unknown parameters is

\[
J(S_1, S_2, S_3, \hat{g}_1|\text{rb}, \hat{P}_1|\text{rb}, \hat{P}_2|\text{rb}, \hat{P}_3|\text{rb}, \delta_1, \delta_2, \delta_3) = \sum_{k=1}^{3} J_k.
\]

V. OPTIMIZATION WITH SIMULATED DATA

To assess the effectiveness of the optimization approach for obtaining parameter estimates, we calibrate the sensors using simulated data. We specify the sensitivity, bias, and orientation of each accelerometer, as well as the direction of gravity. We use these specified parameters to generate simulated voltage readings at 6656 link orientations. In addition, we add white noise to the simulated measurements. The resulting parameter estimates are compared to the actual parameter values to assess their accuracy.

A. Separate Accelerometer Calibration

First, we consider the precision of the optimization involving the function (15). For convenience, let gravity point in the \(-\hat{k}_{\text{rb}}\) direction, although this direction is unknown to the optimization procedure. Assume there is no noise in the measurements. The parameters used to generate the simulated data are given in Table II. When there is no noise, the optimization yields highly accurate parameter estimates.

Next, white noise is added to the simulated data. For comparison purposes, the optimization is performed with noise that is distributed normally on an interval centered about zero chosen to have a standard deviation equal in size to the RMS noise level of the respective accelerometer and then with the standard deviation increased by a factor of 10. The notation \( N(a, b) \) is used to denote the normal distribution with mean \( a \) and standard deviation \( b \). For accelerometer 1, the two noise distributions used are \( N(0, 0.0134) \) and \( N(0, 0.134) \).
Optimization with noisy simulated data is performed for an increasing number of data points. If the parameter estimates converge to the actual parameter values as the number of data points used in the optimization increases, then the estimator may be consistent [4].

The estimates of $S_1$ and $\delta_1$ nearly converge to the actual estimates when the noise has a distribution of $N(0, 0.0134)$. For the sensitivity of accelerometer 1, the estimate is close to the actual value at about 1000 data points. In addition, the offset estimate of accelerometer 1 appears to be converging to the actual parameter value. From about 2500 data points onward, the estimate for the offset stays within 0.0001 V of the actual value. Therefore, when we calibrate the accelerometers using this procedure, the estimator might be slightly inconsistent. For the case in which the noise has a distribution of $N(0, 0.134)$, the estimates appear to be converging.

**B. Joint Accelerometer Calibration**

For the function defined in (16), we examine the consistency of the parameter estimates in the same manner. The actual parameters are those given in Table II, and the measurement noise is the same as for the separate accelerometer calibration. When there is no noise, the optimization provides accurate estimates with small errors, as it did for separate calibration. In addition, when white noise is added to the simulated data, the estimates appear to converge to the actual parameter values.

When the noise of accelerometer 1 has a distribution of $N(0, 0.0134)$, the sensitivity estimates reach the actual parameter value at about 1000 data points. The offset estimate is within 0.0003 V of the actual parameter value at about 6500 data points. These results suggest that this estimate may be slightly inconsistent. If the noise distribution has a greater standard deviation, then the accuracy of the estimates degrades.

**VI. OPTIMIZATION WITH EXPERIMENTAL DATA**

We now calibrate each accelerometer with experimental data. Using the data generated by the kinematic maneuver, we optimize the functions given in Section 4 to estimate the sensitivity, bias, and unit orientation vector of each accelerometer, as well as the direction of gravity.

**A. Separate Accelerometer Calibration**

Optimizing (15) for each accelerometer yields the results shown in Table III. The data fit errors in the parameter estimates for accelerometers 1, 2, and 3 are 18 mV, 26 mV, and 26 mV, respectively. Note that the estimates of the angles $\phi_k$ and $\psi_k$ that determine the direction of gravity vary slightly. However, since $\phi_k$ varies by less than 0.25 deg $\hat{g}_k|_{rb}$ is close to the same direction for each accelerometer. Therefore, it appears that our model may not have a fundamental problem discernable from separate estimation of $\hat{g}_k|_{rb}$, and we now calibrate the accelerometers jointly.

**B. Joint Accelerometer Calibration**

Minimizing the function (16) yields the results displayed in Table IV. The data fit error in the parameter estimates for accelerometers 1, 2, and 3 is 41 mV. This error is less than the sum of the errors in the separate calibration case, suggesting that the joint calibration parameter estimates may be more accurate.

To analyze the dependence of the parameter estimates in Table IV on the chosen data subset, we perform the same optimization 25 times. Specifically, we take each 25th data point of the entire set of 166,400 data points, perform the optimization, and then repeat with an offset data set. The
TABLE IV
OPTIMIZATION RESULTS FOR JOINT ACCELEROMETER CALIBRATION.

<table>
<thead>
<tr>
<th>accelerometer</th>
<th>( \beta_k ) (deg)</th>
<th>( \gamma_k ) (deg)</th>
<th>( S_k ) (V/s/m)</th>
<th>( \delta_k ) (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85.2089</td>
<td>89.8537</td>
<td>0.1425</td>
<td>2.3754</td>
</tr>
<tr>
<td>2</td>
<td>-1.2394</td>
<td>0.8943</td>
<td>0.1425</td>
<td>2.4491</td>
</tr>
<tr>
<td>3</td>
<td>1.2829</td>
<td>272.1489</td>
<td>0.14198</td>
<td>2.1096</td>
</tr>
</tbody>
</table>

TABLE V
ANALYSIS OF THE JOINT ACCELEROMETER CALIBRATION USING 25 DATA SETS.

<table>
<thead>
<tr>
<th>accelerometers</th>
<th>parameter</th>
<th>mean</th>
<th>std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( S_1 ) (V/s/m)</td>
<td>0.1426</td>
<td>2.8356 \times 10^{-3}</td>
</tr>
<tr>
<td>1</td>
<td>( \delta_1 ) (V)</td>
<td>2.3755</td>
<td>1.8572 \times 10^{-4}</td>
</tr>
<tr>
<td>1</td>
<td>( \rho_1 ) (deg)</td>
<td>88.3442</td>
<td>0.0170</td>
</tr>
<tr>
<td>2</td>
<td>( S_2 ) (V/s/m)</td>
<td>0.1425</td>
<td>4.1989 \times 10^{-3}</td>
</tr>
<tr>
<td>2</td>
<td>( \delta_2 ) (V)</td>
<td>2.4199</td>
<td>2.6057 \times 10^{-4}</td>
</tr>
<tr>
<td>2</td>
<td>( \rho_2 ) (deg)</td>
<td>-1.2566</td>
<td>0.0230</td>
</tr>
<tr>
<td>2</td>
<td>( \gamma_2 ) (deg)</td>
<td>0.8681</td>
<td>0.0265</td>
</tr>
<tr>
<td>3</td>
<td>( S_3 ) (V/s/m)</td>
<td>0.1421</td>
<td>5.0368 \times 10^{-3}</td>
</tr>
<tr>
<td>3</td>
<td>( \delta_3 ) (V)</td>
<td>2.1093</td>
<td>2.8925 \times 10^{-4}</td>
</tr>
<tr>
<td>3</td>
<td>( \rho_3 ) (deg)</td>
<td>1.3023</td>
<td>0.0199</td>
</tr>
<tr>
<td>gravity</td>
<td>( \phi ) (deg)</td>
<td>272.1241</td>
<td>0.0320</td>
</tr>
<tr>
<td>gravity</td>
<td>( \psi ) (deg)</td>
<td>134.1036</td>
<td>88.7530</td>
</tr>
</tbody>
</table>

The magnitude of the Earth’s magnetic field felt by the magnetometers is assumed for convenience to be 1 G, where G denotes Gauss. The voltage reading \( V_k \) of the \( k^{th} \) magnetometer is

\[
V_k = S_k B_k + \delta_k, \quad (18)
\]

where \( S_k, \delta_k, \) and \( B_k \) are the sensitivity of the \( k^{th} \) magnetometer, the offset of the \( k^{th} \) magnetometer, and the Earth’s magnetic field at the axis of the \( k^{th} \) magnetometer, respectively. Let the unit orientation \( \hat{P}_{kk}^T \) of each magnetometer resolved in the robot base frame be defined by (6) and (8), and let \( \phi_k \) be the angle between \( B_k \) and \( \hat{P}_{kk}^T \). Therefore, the component of the Earth’s magnetic field felt by the \( k^{th} \) magnetometer is given by

\[
\vec{B}_k = \vec{B}_k^T \hat{P}_{kk}^T \hat{P}_{kk} = |\vec{B}_k| |\hat{P}_{kk}| \cos \phi_k, \quad (19)
\]

and the voltage reading \( V_k \) for the \( k^{th} \) magnetometer is

\[
V_k = S_k |\vec{B}_k^T \hat{P}_{kk}^T \hat{P}_{kk}| + \delta_k. \quad (20)
\]

A. Magnetometer Calibration

To estimate the unknown magnetometer parameters \( S_k, \delta_k, \vec{B}_k \), and \( \hat{P}_{kk} \), we perform two different calibrations, as for the accelerometers. Again, this approach provides a check of the fundamental structure of the model and integrity of the data. The kinematic maneuver described in Section 3 is used to generate the experimental data used in the optimizations.

First, we define a function to separately calibrate the magnetometers. Therefore, the Earth’s magnetic field located at the magnetometer’s axis is estimated in each function optimization. The function to be minimized for the \( k^{th} \) magnetometer with 6 unknown parameters is

\[
J_k(S_k, \vec{B}_k, \hat{P}_k, \delta_k) \triangleq \sum_{j=1}^{n} [V_{k,j} - (S_k |\vec{B}_k|^T \hat{P}_k + \delta_k)]^2. \quad (21)
\]

Optimization of (21) results in the parameter estimates displayed in Table VI. The data fit errors in the parameter estimates of magnetometers 1, 2, and 3 are 0.1179 V, 0.150 V, and 0.1257 V, respectively. These error values are all approximately 0.1 V greater than the corresponding errors for the separately calibrated accelerometers, suggesting the
TABLE VII
PARAMETER ESTIMATES USING JOINT MAGNETOMETER CALIBRATION.

| magnetometers | | | | |
|---------------|---------------|---------------|---------------|
| 1             | | | | |
| 2             | | | | |
| 3             | | | | |
| 1             | -0.96237      | 270.2059      | 1.3006        | 2.4216        |
| 2             | -85.6307      | 185.8359      | 1.2235        | 2.3876        |
| 3             | -4.0172       | 1.2153        | 1.3342        | 2.7643        |

parameter estimates for the magnetometers may not be as accurate as the parameter estimates for the accelerometers. One cause of the greater data fit error could be due to stray magnetic fields from which we could not isolate the sensor. The estimates of $\phi_k$ and $\psi_k$ are within 0.26 deg and 0.72 deg, respectively.

Next, we calibrate the magnetometers jointly. One function is formulated and optimized for all 3 magnetometers, and one Earth’s magnetic field vector is estimated. Similar to the case of joint accelerometer calibration, the resulting cost function with 14 unknown parameters is

$$J(S_1,S_2,S_3,B_1|\phi,\psi,P_1|\phi,\psi,P_2|\phi,\psi,P_3|\phi,\psi,\delta_1,\delta_2,\delta_3) \triangleq \sum_{k=1}^3 J_k(S_k,B_k|\phi,\psi,P_k|\phi,\psi,\delta)$$  (22)

Minimizing (22) produces the parameter estimates displayed in Table VII. The error in the parameter estimates is 0.2285 V. This error is less than the sum of the errors in the separate magnetometer calibrations. This finding suggests that calibrating the magnetometers jointly produces more accurate parameter estimates with a smaller error than calibrating the magnetometers separately.

To analyze the dependence of the parameter estimates resulting from the optimization of (22) on the data subset, we perform the optimization 25 times using the same procedure as for the accelerometers. The mean and standard deviation of the parameter estimates are displayed in Table VIII. The parameter estimates in Table VII are within 1 standard deviation of the corresponding mean estimates in Table VIII. This closeness suggests that the estimates are independent of the data subset.

VIII. CONCLUSIONS

In this paper, we derived a kinematic model and kinematic maneuver to calibrate a triaxial accelerometer and magnetometer. The calibration was accomplished by implementing this maneuver on a six-degree-of-freedom robotic arm. Analysis of the accuracy of our experimental results was performed by generating simulated data, formulating two optimization problems, and computing the data fit error of the parameter estimates. By first calibrating the accelerometers with simulated data, we evaluated the possibility of inconsistency in our estimator. By calibrating the accelerometer and magnetometer separately, we were able to check for discrepancies in the data and model. The joint calibrations produced smaller errors for both the magnetometer and accelerometer than did the separate calibrations, implying that the results from the joint calibrations may be more accurate. In our results, we tested the dependence of the parameter estimates on the data subset by repeating the joint calibrations of the accelerometer and magnetometer 25 times with different data sets and, then, computing the mean and standard deviation of the 25 estimates. The results from this analysis suggest that varying the data set may not change our estimate by a relatively large amount, suggesting that the parameter estimates for the joint calibration are independent of the chosen data subset the collected data.

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REFERENCES