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Adaptive Non-Bayesian State Estimation

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Abstract— **This paper presents an adaptive retrospective cost state estimation (RCSE) algorithm that uses no knowledge of the statistics of the process and measurement noise. To demonstrate the method, we investigate three cases for comparison with the Kalman filter, namely, known dynamics with known noise statistics, uncertain dynamics with known noise statistics, and uncertain dynamics with uncertain noise statistics. For numerical illustration, we apply RCSE to a damped oscillator, a damped rigid body, and a lateral aircraft model.**

I. Introduction

Among the numerous tools and techniques that have emerged from systems and control theory during the last 55 years, state estimation is likely the most successful. In particular, the Kalman filter and its nonlinear variants use knowledge of system dynamics and noise statistics to provide estimates of plant states that are not directly measured. These estimates are obtained by combining model-based prediction (forecast) with statistically optimal data-based correction (data assimilation).

State estimators thus depend on two key types of information, namely, knowledge of the noise statistics and knowledge of the dynamics. In practice, however, this knowledge may be uncertain. For example, for the case where the statistics of the process noise are unknown, techniques are available for estimating the process noise covariance during operation [1]–[6]. Likewise, if the dynamics are uncertain, then robust estimation methods can be used [7]–[11].

The goal of the present paper is to develop an alternative approach to state estimation in the absence of statistical knowledge of the process and measurement noise as well as uncertainty in the dynamics. To do this, we propose an adaptive state estimation algorithm that uses the innovations as the error signal for updating the estimator. The estimator update is based on a retrospective cost function as used in [12] for adaptive control and in [13] for state and input estimation. As discussed below, this estimator differs from the technique used in [13].

The *retrospective cost state estimation* (RCSE) technique proposed in the present paper is based on an adaptive filter whose input is the innovations. The output of the adaptive filter plays the role of the static (memoryless) innovations term $K(y-C\hat{x})$, which appears in the classical Kalman filter. Aside from the fact that the adaptive filter has memory, the coefficients of the adaptive filter are updated by optimizing the retrospective cost based on the innovations. The output of the adaptive filter then directly updates each state estimate.

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This approach differs substantially from the technique used in [13]. In particular, the technique in [13] estimates the unknown input and, by injecting the input estimate into the estimator, indirectly influences the state estimates. In the present paper, however, all of the state estimates are updated directly. Unlike [13], however, the present paper does not attempt to estimate the unknown input.

II. RCSE Formulation

Consider the linear time-invariant system

$$x(k+1) = Ax(k) + Bu(k) + D_1 w(k), \tag{1}$$

$$y(k) = Cx(k) + D_2 v(k), \tag{2}$$

where $x(k) \in \mathbb{R}^{l_x}$ is the unknown state, $u(k) \in \mathbb{R}^{l_u}$ is the known input, $D_1 w(k) \in \mathbb{R}^{l_x}$ is the process noise with covariance $V_1 \triangleq D_1 D_1^{\mathrm{T}} \in \mathbb{R}^{l_x \times l_x}$, $y(k) \in \mathbb{R}^{l_y}$ is the measured output, and $D_2 v(k) \in \mathbb{R}^{l_v}$ is the measurement noise with covariance $V_2 \triangleq D_2 D_2^{\mathrm{T}} \in \mathbb{R}^{l_y \times l_y}$. The matrices $A \in \mathbb{R}^{l_x \times l_x}$, $B \in \mathbb{R}^{l_x \times l_u}$, and $C \in \mathbb{R}^{l_y \times l_x}$ may be known or uncertain.

In order to estimate the state $x(k)$, we consider the state estimate update equations

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + x_{\mathrm{c}}(k), \tag{3}$$

$$\hat{y}(k) = C\hat{x}(k), \tag{4}$$

$$z(k) = \hat{y}(k) - y(k), \tag{5}$$

where $\hat{x}(k) \in \mathbb{R}^{l_x}$ is the state estimate, $x_{\mathrm{c}}(k) \in \mathbb{R}^{l_x}$ is the state correction, and $z(k) \in \mathbb{R}^{l_y}$ is the innovations error. In (3), (4) it is assumed that A, B, and C are known. Later in the paper, these matrices may be uncertain. The goal is to develop an adaptive estimator that minimizes z.

A. Adaptive Estimator

Define the adaptive-corrector subsystem

$$x_{\mathrm{c}}(k) = \sum_{i=1}^{n_{\mathrm{c}}} P_i(k) x_{\mathrm{c}}(k-i) + \sum_{i=k_0}^{n_{\mathrm{c}}} Q_i(k) \xi(k-i), \tag{6}$$

where $P_i(k) \in \mathbb{R}^{l_x \times l_x}, Q_i(k) \in \mathbb{R}^{l_x \times l_\xi}$ are the corrector coefficient matrices, $k_0 \geq 0$, and $\xi(k) \in \mathbb{R}^{l_\xi}$ consists of components of $\hat{x}$ and z. RCSE minimizes $z(k)$ by updating $P_i(k)$ and $Q_i(k)$. Fig. 1 shows the structure of (1)–(6).

The coefficients $P_i(k)$ and $Q_i(k)$ of the adaptive-corrector subsystem in (6) are updated in order to minimize $z(k)$. We rewrite (6) as

$$x_{\mathrm{c}}(k) = \Phi(k)\theta(k), \tag{7}$$

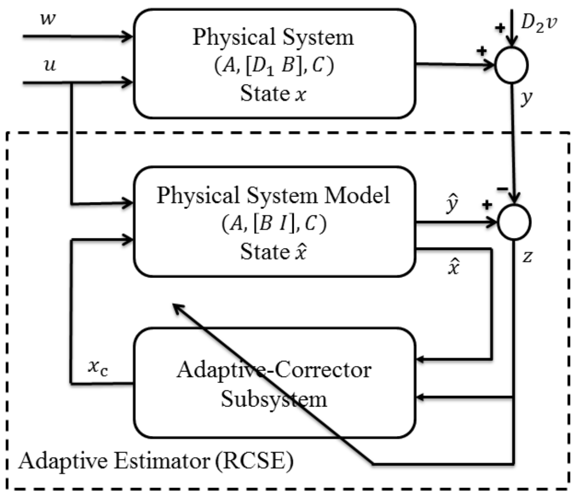

Fig. 1: Adaptive estimator architecture. RCSE uses z to update the adaptive-corrector subsystem with inputs $\hat{x}$ and z in order to generate the output x_c that minimizes z. Consequently, the state $\hat{x}$ of the physical system model is an estimate of the state x of the physical system.

where the regressor matrix $\Phi(k)$ is defined by

$$\Phi(k) \triangleq \begin{bmatrix} x_c(k-1) \\ \vdots \\ x_c(k-n_c) \\ \xi(k-k_0) \\ \vdots \\ \xi(k-n_c) \end{bmatrix}^{\mathrm{T}} \otimes I_{l_x} \in \mathbb{R}^{l_x \times l_\theta},$$

and

$$\theta(k) \triangleq \mathrm{vec}\left[\ P_1(k) \cdots P_{n_c}(k)\ Q_{k_0}(k) \cdots Q_{n_c}(k)\ \right] \in \mathbb{R}^{l_\theta},$$

where $l_\theta \triangleq l_x^2 n_c + l_x l_\xi (n_c + 1 - k_0)$, "$\otimes$" is the Kronecker product, and "vec" is the column-stacking operator. Note that $k_0 = 0$ yields an exactly proper corrector, whereas $k_0 \geq 1$ yields a strictly proper corrector.

B. Retrospective Performance Variable

We define the retrospective correction as

$$\hat{x}_c(k) = \Phi(k)\hat{\theta} \tag{8}$$

and the corresponding retrospective performance variable as

$$\hat{z}(k) \triangleq z(k) + \Phi_f(k)\hat{\theta} - x_{c_f}(k), \tag{9}$$

where $\hat{\theta} \in \mathbb{R}^{l_\theta}$ is determined by optimization below, and $\Phi_f(k) \in \mathbb{R}^{l_y \times l_\theta}, x_{c_f}(k) \in \mathbb{R}^{l_y}$ are filtered versions of $\Phi(k), x_c(k)$, respectively, defined by

$$\Phi_f(k) \triangleq G_f(\mathrm{q})\Phi(k), \tag{10}$$

$$x_{c_f}(k) \triangleq G_f(\mathrm{q})x_c(k). \tag{11}$$

The filter G_f is an $l_z \times l_x$ transfer matrix of the form

$$G_f(\mathrm{q}) \triangleq D_f^{-1}(\mathrm{q})N_f(\mathrm{q}), \tag{12}$$

where q is the forward shift operator, D_f and N_f are polynomial matrices, and D_f is monic. The choice of G_f is discussed in section II-D.

C. State Residual Dynamics

Define the state residual

$$\eta(k) \triangleq \hat{x}(k) - x(k). \tag{13}$$

Using (1)-(5),(13), the state residual dynamics is given as

$$\eta(k+1) = A\eta(k) + x_c(k) - D_1 w(k), \tag{14}$$

$$z(k) = C\eta(k) - D_2 v(k). \tag{15}$$

Using (14)-(15), $z(k)$ can be expanded as

$$\begin{aligned} z(k) &= CA^{n_f}\eta(k-n_f) + C\sum_{i=1}^{n_f} A^{i-1}x_c(k-i) \\ &\quad - C\sum_{i=1}^{n_f} A^{i-1}D_1 w(k-i) - D_2 v(k), \\ &= CA^{n_f}\eta(k-n_f) + \sum_{i=1}^{n_f} H_i x_c(k-i) \\ &\quad - \sum_{i=1}^{n_f} H_i' w(k-i) - D_2 v(k), \\ &= \frac{CA^{n_f}}{\mathrm{q}^{n_f}}\eta(k) + \sum_{i=1}^{n_f}\frac{H_i}{\mathrm{q}^i}x_c(k) - \sum_{i=1}^{n_f}\frac{H_i'}{\mathrm{q}^i}w(k) \\ &\quad - D_2 v(k), \end{aligned} \tag{16}$$

where, for $i = 1, \ldots, n_f, H_i$ and H_i' are the Markov parameters of the transfer matrices G_{zx_c} and G_{zw}, respectively, which are defined as

$$H_i \triangleq CA^{i-1}, \tag{17}$$

$$H_i' \triangleq -H_i D_1, \tag{18}$$

$$G_{zx_c}(\mathrm{q}) \triangleq C(\mathrm{q}I - A)^{-1}, \tag{19}$$

$$G_{zw}(\mathrm{q}) \triangleq -C(\mathrm{q}I - A)^{-1}D_1. \tag{20}$$

D. Markov-Parameter-based FIR Filter Construction

To construct G_f, we write the retrospective performance using (8) and (16) as

$$\hat{z}(k) = \frac{CA^{n_f}}{\mathrm{q}^{n_f}}\eta(k) + \sum_{i=1}^{n_f}\frac{H_i}{\mathrm{q}^i}\hat{x}_c(k) - \sum_{i=1}^{n_f}\frac{H_i'}{\mathrm{q}^i}w(k) - D_2 v(k). \tag{21}$$

We rewrite (21) using (16) as

$$\hat{z}(k) = z(k) + \sum_{i=1}^{n_f}\frac{H_i}{\mathrm{q}^i}\Phi(k)\hat{\theta} - \sum_{i=1}^{n_f}\frac{H_i}{\mathrm{q}^i}x_c(k). \tag{22}$$

Thus, $G_f(\mathrm{q})$ is the FIR filter,

$$G_f(\mathrm{q}) = \sum_{i=1}^{n_f}\frac{H_i}{\mathrm{q}^i}. \tag{23}$$

E. Cumulative Cost and RCSE Update Law

For $k > k_0$, we define the cumulative cost function

$$\begin{aligned} J(k,\hat{\theta}) \triangleq \sum_{i=k_0}^{k} &[\hat{z}(i)^{\mathrm{T}}R_z\hat{z}(i) + [\Phi(i)\hat{\theta}]^{\mathrm{T}}R_x\Phi(i)\hat{\theta}] \\ &+ [\hat{\theta} - \theta(0)]^{\mathrm{T}}R_\theta[\hat{\theta} - \theta(0)], \end{aligned} \tag{24}$$

where R_θ, R_z, and R_x are positive definite. Let $P(0) = R_\theta^{-1}$ and $\theta(0) = \theta_0$. Then, for all $k \geq k_0$, the cumulative cost function (24) has the unique global minimizer $\theta(k)$ given by the RLS update

$$\theta(k) = \theta(k-1) - P(k-1)\tilde{\Phi}(k)^{\mathrm{T}}\Gamma(k)^{-1} \cdot [\tilde{\Phi}(k)\theta(k-1) + \tilde{z}(k)], \quad (25)$$

where $P(k)$ satisfies

$$P(k) = P(k-1) - P(k-1)\tilde{\Phi}(k)^{\mathrm{T}}\Gamma(k)^{-1}\tilde{\Phi}(k)P(k-1), \quad (26)$$

$$\tilde{\Phi}(k) \triangleq \begin{bmatrix} \Phi_{\mathrm{f}}(k) \\ \Phi(k) \end{bmatrix} \in \mathbb{R}^{(l_z+l_x)\times l_\theta},$$

$$\tilde{R}(k) \triangleq \begin{bmatrix} R_z(k) & 0 \\ 0 & R_x(k) \end{bmatrix} \in \mathbb{R}^{(l_z+l_x)\times(l_z+l_x)},$$

$$\tilde{z}(k) \triangleq \begin{bmatrix} z(k) - x_{\mathrm{cf}}(k) \\ 0 \end{bmatrix} \in \mathbb{R}^{l_z+l_x},$$

$$\Gamma(k) \triangleq \tilde{R}(k)^{-1} + \tilde{\Phi}(k)P(k-1)\tilde{\Phi}(k)^{\mathrm{T}}.$$

III. Examples

To assess the accuracy of the state estimate given by (3), we use the error metric

$$e_i(k) \triangleq \frac{1}{N_{\mathrm{trial}}}\sqrt{\sum_{i=1}^{N_{\mathrm{trial}}} [\hat{x}_i(k) - x_i(k)]^2}, \quad (27)$$

where i is the state component and N_{trial} is the number of trials. To compare estimation accuracy, we plot the error for the optimal filter (OPT), the Kalman filter (KF), and RCSE. For OPT, the noise statistics and dynamics are known. For KF, either the noise statistics or the dynamics are uncertain. RCSE uses no knowledge of the noise statistics, and the dynamics may or may not be uncertain.

For KF, we model the uncertainty in the noise covariances as

$$\bar{V}_1 = (M_0 M_0^{\mathrm{T}}) * V_1, \quad (28)$$

$$\bar{V}_2 = (N_0 N_0^{\mathrm{T}}) * V_2, \quad (29)$$

where $*$ denotes entry-wise multiplication, $M_0 \in \mathbb{R}^{l_x}$, $N_0 \in \mathbb{R}^{l_y}$, $\bar{V}_1$ is the modeled process noise covariance, and $\bar{V}_2$ is the modeled measurement noise covariance. The entry-wise multiplication in (28) and (29) ensures that $\bar{V}_1$ and $\bar{V}_2$ are positive semidefinite. Note that, if there is no uncertainty in the noise statistics and dynamics, then KF coincides with OPT.

A. Damped Oscillator

We consider the damped oscillator

$$\dot{x} = A_{\mathrm{c}}x + B_{\mathrm{c}}u = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{C}{M} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} u, \quad (30)$$

where x_1 is the position (m), x_2 is the velocity (m/s), and u is the input force (N). We choose $M = 1\,\mathrm{kg}, K = 1\,\mathrm{N/m}$, and $C = 0.1\,\mathrm{kg/s}$. We discretize (30) as

$$A = e^{A_{\mathrm{c}}T_{\mathrm{s}}}, \quad B = A_{\mathrm{c}}^{-1}(A_{\mathrm{c}} - I)B_{\mathrm{c}},$$

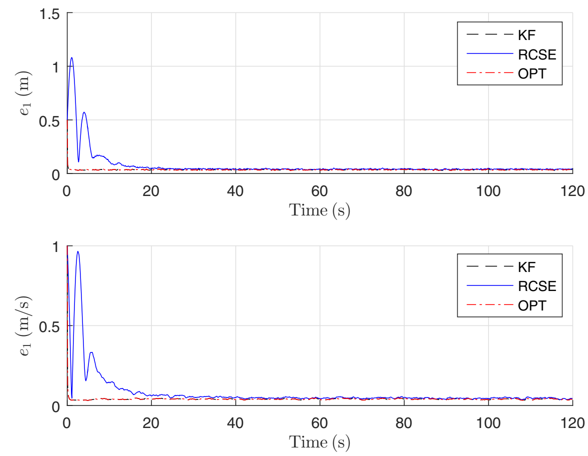

Fig. 2: Error in the state estimates with known noise statistics and known dynamics. Note that the errors for KF and OPT are identical, whereas, after initial transients, the errors for RCSE are close to the errors for OPT.

where $T_{\mathrm{s}} = 0.1\,\mathrm{s}$ is the sampling time. We choose $C = [1\ 0]$, and thus we measure position and estimate velocity. For each numerical example, we set $\hat{x}(0) = [0\ 0]^{\mathrm{T}}$, $D_1 = \mathrm{diag}(0.02, 0.01)$, and $D_2 = 0.1$. We drive the system with white process noise, non-zero initial conditions, and white input u with standard deviation 1 N. We choose $N_{\mathrm{trial}} = 100, k_0 = 0, n_{\mathrm{c}} = 2, n_{\mathrm{f}} = 24, R_\theta = 10^{-6}I_{l_\theta}, R_x = 10^2 I_{lx}$, and $R_z = 1$.

1) Known noise statistics and known dynamics: We consider the case where the noise statistics and dynamics are known. As shown in Fig. 2, the errors for KF and OPT are identical, whereas the errors for RCSE converge close to the errors for OPT.

2) Known noise statistics with uncertain dynamics: We consider the case where the noise covariances V_1 and V_2 are known, but the dynamics are uncertain with estimated $\hat{K} = 1.44\,\mathrm{N/m}$. Note that due to this uncertainty, the values of H_i used in (23) are not accurate. Fig. 3 shows that the error in the velocity estimate for KF converges to 0.3 m/s, whereas the error in the velocity estimate for RCSE is less than the error for KF and converges to 0.09 m/s.

3) Uncertain noise statistics with uncertain dynamics: We reconsider 2) with uncertainty in the noise covariances V_1 and V_2 modeled by (28), (29) with

$$M_0 = \begin{bmatrix} 0.4 & 1.5 \end{bmatrix}^{\mathrm{T}}, \quad N_0 = 2. \quad (31)$$

Fig. 4 shows that the error in the position estimate for KF is degraded and converges to 0.2 m, whereas the error for RCSE remains about the same as in Fig. 3.

B. Damped Rigid Body

We now reconsider the dynamical system (30) with $K = 0\,\mathrm{N/m}$. RCIE weightings, input u, initial conditions, and noise settings are chosen as in section III-A.

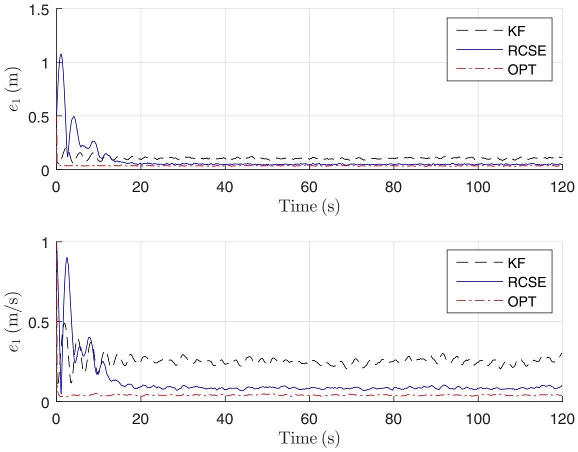

Fig. 3: Error in the state estimates with known noise statistics and uncertain dynamics. The error in the velocity estimate for KF converges to 0.3 m/s, whereas the error in the velocity estimate for RCSE is less than the error for KF and converges to 0.09 m/s.

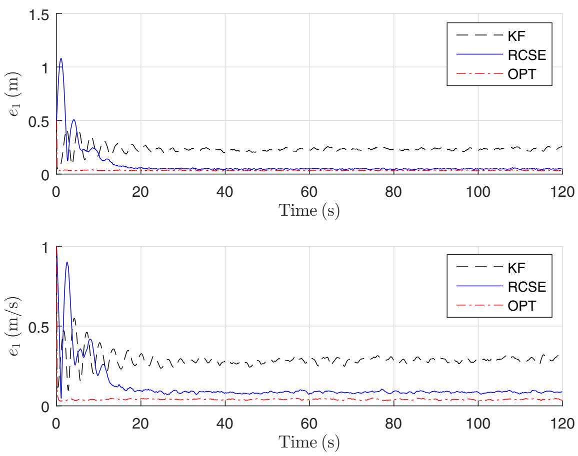

Fig. 4: Error in the state estimates with uncertain noise statistics and uncertain dynamics. The error in the position estimate for KF is degraded and converges to 0.2 m, whereas the error for RCSE remains about the same as in Fig. 3.

1) Known noise statistics and known dynamics: We consider the case where the noise statistics and dynamics are known. As shown in Fig. 5, the errors for KF and OPT are identical, whereas the errors for RCSE converge close to the errors for OPT.

2) Known noise statistics with uncertain dynamics: We consider the case where the noise covariances V_1 and V_2 are known, but the dynamics are uncertain with estimated $\hat{C} = 0.3\,\mathrm{kg/s}$. Note that due to this uncertainty, the values of H_i used in (23) are not accurate. Fig. 6 shows that the error in the velocity estimate for KF converges to 0.2 m/s, whereas the error in the velocity estimate for RCSE is less than the error for KF and converges to 0.09 m/s.

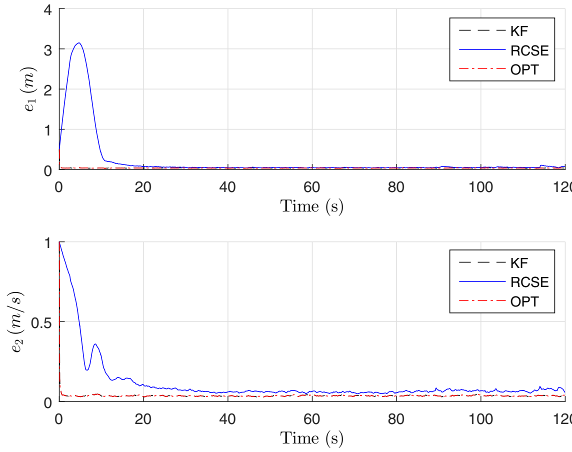

Fig. 5: Error in the state estimates with known noise statistics and known dynamics. Note that the errors for KF and OPT are identical, whereas, after initial transients, the errors for RCSE are close to the errors for OPT.

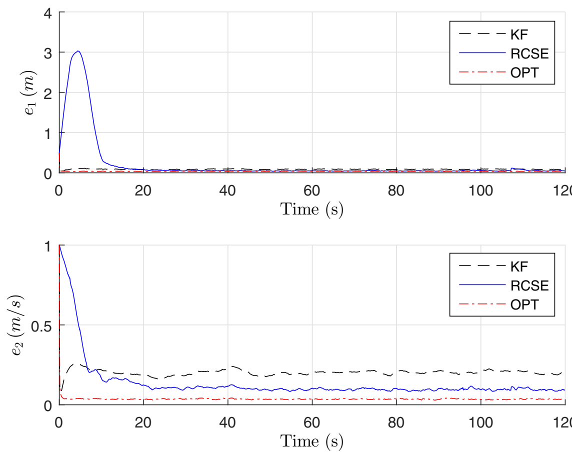

Fig. 6: Error in the state estimates with known noise statistics and uncertain dynamics. The error in the velocity estimate for KF converges to 0.2 m/s, whereas the error in the velocity estimate for RCSE is less than the error for KF and converges to 0.09 m/s.

3) Uncertain noise statistics with uncertain dynamics: We reconsider 2) with uncertainty in the noise covariances V_1 and V_2 modeled by (28), (29) with

$$M_0 = \begin{bmatrix} 0.58 & 0.24 \end{bmatrix}^{\mathrm{T}}, \quad N_0 = 1.5. \tag{32}$$

Fig. 7 shows that the errors in the position and velocity estimate for KF are degraded and converge to 0.2 m and 0.3 m/s, respectively, whereas the errors for RCSE remain about the same as in Fig. 6.

C. Lateral Aircraft Dynamics Model

Output-feedback adaptive roll-angle control of an aircraft that transitions from minimum phase (MP) to nonminimum-phase (NMP) dynamics with roll-angle measurement is considered in [14]. In this section, we estimate the roll angle using rate gyro measurements P and R.

We call the MP asymptotically stable plant the *nominal* plant and the NMP asymptotically stable plant the *off-*

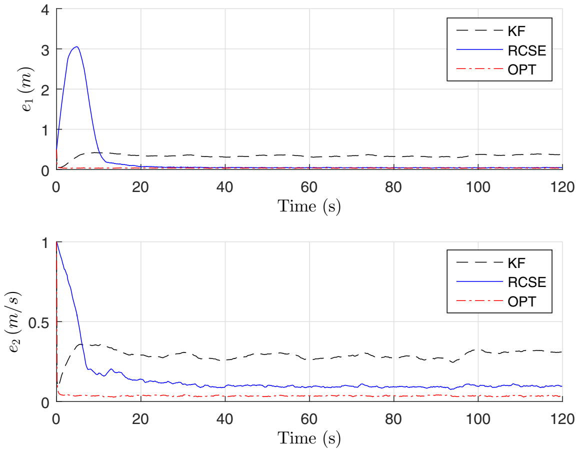

Fig. 7: Error in the state estimates with uncertain noise statistics and uncertain dynamics. The errors in the position and velocity estimate for KF are degraded and converge to 0.2 m and 0.3 m/s, respectively, whereas the error for RCSE remains about the same as in Fig. 6.

nominal plant. The discretized nominal and off-nominal plants with sampling time $T_\mathrm{s} = 0.1$ s are given by

$$A_\mathrm{nom} = \begin{bmatrix} 0.9553 & 0.0265 & -0.0934 & 0.0039 \\ -2.5210 & 0.9680 & 0.1310 & -0.0050 \\ 0.0551 & 0.0045 & 0.9708 & 0.0001 \\ -0.1282 & 0.0989 & -0.0369 & 1.0004 \end{bmatrix},$$

$$A_\mathrm{off} = \begin{bmatrix} 0.8482 & 0.0255 & -0.0900 & 0.0038 \\ -10.3212 & 0.8595 & 0.5152 & -0.0210 \\ 0.0186 & 0.0041 & 0.9723 & 0.0001 \\ -0.5304 & 0.0953 & -0.0239 & 0.9999 \end{bmatrix},$$

$$B_\mathrm{nom} = \begin{bmatrix} 0.0034 \\ 0.2492 \\ -0.0017 \\ 0.0126 \end{bmatrix}, \quad B_\mathrm{off} = \begin{bmatrix} 0.0036 \\ 0.2390 \\ -0.0061 \\ 0.0124 \end{bmatrix},$$

where the control input is the elevon deflection and $x = \begin{bmatrix} \beta & P & R & \phi_\mathrm{roll} \end{bmatrix}^\mathrm{T}$, that is, sideslip angle (rad), roll rate (rad/s), yaw rate (rad/s), and roll angle (rad).

For each numerical example, we set $\hat{x}(0) = [0\ 0\ 0\ 0]^\mathrm{T}$, $D_1 = 0.001\ \mathrm{diag}(1,2,2,1)$, and $D_2 = 0.001\ \mathrm{diag}(1,1)$. We choose $N_\mathrm{trial} = 40, n_\mathrm{c} = 4, n_\mathrm{f} = 48, R_\theta = 10^{-8} I_{l_\theta}, R_x = 10^2 I_{lx}$, and $R_z = I_{lz}$.

1) Known noise statistics and known dynamics: We consider the case where the noise statistics and dynamics are known. Fig. 8 shows that the error e_4 in the roll-angle estimate for KF and OPT are identical, whereas, after an initial transient, the error for RCSE is close to the error for OPT.

2) Known noise statistics with uncertain dynamics: We consider the case where the noise statistics are known, but the dynamics are uncertain. The uncertainty in the dynamics is due to the transition from nominal to off-nominal dynamics. The dynamics are uncertain for both KF and RCSE, whereas the noise statistics are known for KF. Fig. 9 shows the case where the states are driven by the process noise and nonzero initial conditions, whereas Fig. 10 shows the case where the states are driven by white input u with standard deviation 0.05 rad, white process noise, and nonzero initial conditions. Note that in both cases, the error in the roll-angle estimate for RCSE is less than the error for KF and converges to 0.02 rad.

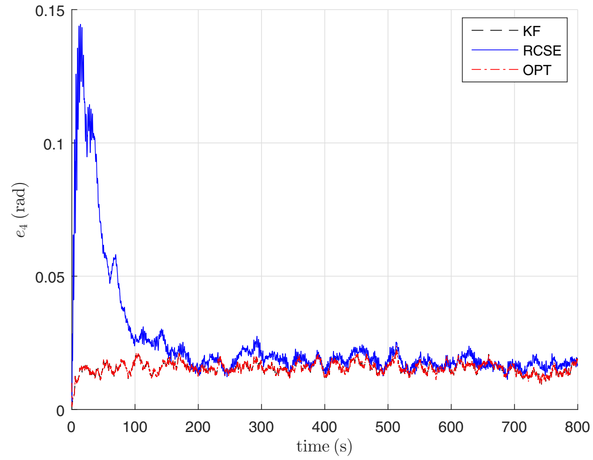

Fig. 8: Error in the roll-angle estimate with known noise statistics and known dynamics. Note that the errors for KF and OPT are identical, whereas, after an initial transient, the error for RCSE is close to the error for OPT.

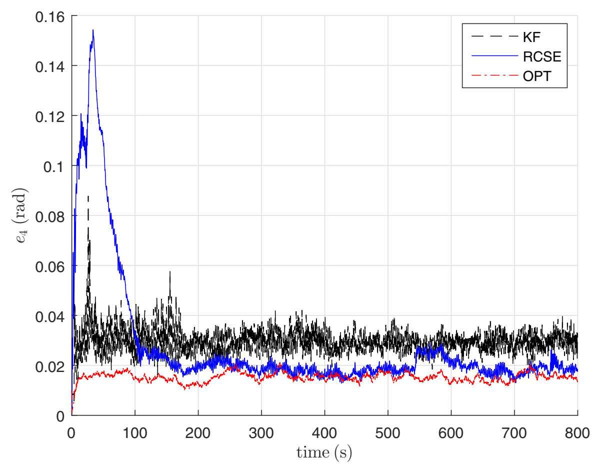

Fig. 9: Error in the roll-angle estimate with known noise statistics and uncertain dynamics. The states are driven by the white process noise, and nonzero initial conditions. Note that the error for RCSE is less than the error for KF.

3) Uncertain noise statistics with uncertain dynamics: We reconsider 2) with uncertainty in the noise covariances modeled by (28), (29) with

$$M_0 = \begin{bmatrix} 0.4 & 0.2 & 0.4 & 0.1 \end{bmatrix}^\mathrm{T}, \quad N_0 = \begin{bmatrix} 0.6 & 0.2 \end{bmatrix}^\mathrm{T}. \quad (33)$$

Fig. 11 shows the case where the states are driven by the process noise and nonzero initial conditions, whereas Fig. 12 shows the case where the states are driven by white input u with standard deviation 0.05 rad, white process noise, and nonzero initial conditions. Note that, in both cases, the error in the roll-angle estimate for KF is degraded by the uncertain statistics, whereas the error for RCSE is close to the error for OPT.

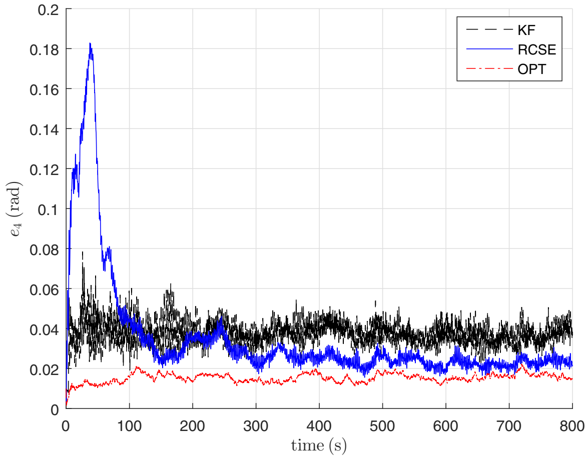

Fig. 10: Error in the roll-angle estimate with known noise statistics and uncertain dynamics. The states are driven by white input, white process noise, and nonzero initial conditions. Note that the error for RCSE is less than the error for KF.

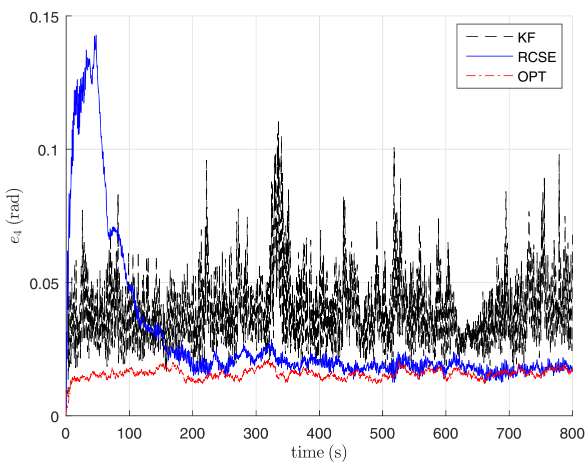

Fig. 11: Error in the roll-angle estimate with uncertain noise statistics and uncertain dynamics. The states are driven by white process noise, and nonzero initial conditions. Note that the error for KF estimate is degraded by the uncertain noise statistics, whereas the error for RCSE is close to the error for OPT.

IV. Conclusion

In this paper, we presented an adaptive non-Bayesian state estimation technique based on retrospective cost optimization. We developed retrospective cost state estimation (RSCE) in the absence of statistical knowledge of the process and measurement noise. To demonstrate the method, we applied RCSE to a damped oscillator, a damped rigid body, and a lateral aircraft model. For these examples, with uncertainty in the noise statistics or dynamics, the errors for RCSE are less than the errors for the Kalman filter. Moreover, for the case of known dynamics, the errors for RCSE are close to the errors for the optimal filter.

V. Future Work

The next step in this research is to extend RCSE to systems with unknown inputs as an extension of [13].

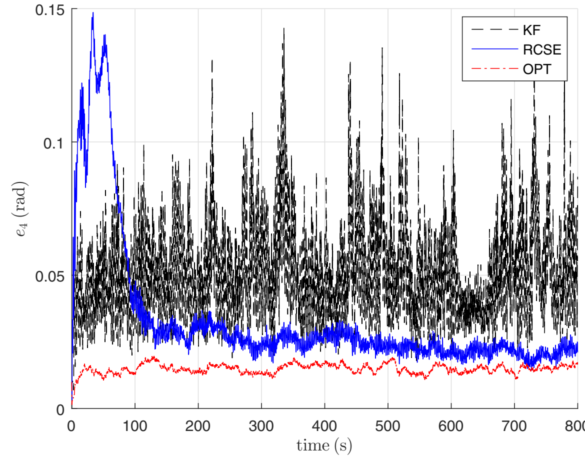

Fig. 12: Error in the roll-angle estimate with uncertain noise statistics and uncertain dynamics. The states are driven by white input u, white process noise, and nonzero initial conditions. Note that the error for KF estimate is degraded by the uncertain noise statistics, whereas the error for RCSE is close to the error for OPT.

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