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Gravity-Gradient-Stabilized Spacecraft Attitude Estimation Using Rate-Gyroscope Measurements

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***Abstract*— This paper considers attitude estimation of a gravity-gradient-stabilized spacecraft. In particular, an observability analysis based on the linearized equations of motion, as well as numerical simulations, show that it is possible to estimate attitude using only a rate gyroscope. The ability to estimate the attitude using only a rate gyroscope reduces the need for additional sensing hardware such as sun sensors and horizon sensors. These savings allow for more mass, volume, and power resources to be devoted to scientific payloads, communications, and other operations, which in the context of Earth orbiting microsatellites and nanosatellites, is highly desirable.**

I. INTRODUCTION

Earth-pointing spacecraft can be designed so that Earth's gravitational field acts to stabilize the spacecraft's attitude through a gravity-gradient torque. One of the attractive features of gravity-gradient stabilization is that it is passive. By exploiting the intrinsic dynamics of the satellite orbiting the Earth, attitude stabilization is achieved without consuming power and without dedicated hardware for sensing or actuation [1, pp. 282]. Numerous spacecraft have been designed, launched, and placed in service that rely on gravity-gradient stabilization including, for example, GEOS-II launched in 1968 [2], the Radio Astronomy Explorer (RAE-1) Satellite launched in 1968 [3], the Long Duration Exposure Facility (LDEF) launched in 1984 [4], and the Orsted satellite launched in 1999 [5].

The recent interest in small spacecraft, such as microsatellites and nanosatellites, have introduced the need for small, light-weight, attitude sensing and control systems that consume little power and require only limited computing requirements. As a result, with respect to attitude sensing and attitude estimation, the use of star-trackers, sun sensors, and horizon sensors, must be carefully budgeted. With respect to attitude control, some research has been done on incorporation of gravity-gradient stabilization into the design of small satellites [6], [7], [8], [9]. By reducing the hardware and software resources required for attitude sensing and control, more resources can be devoted to mission operations.

The main contribution of this paper is demonstrating that the restorative nature of the gravity-gradient torque enables the attitude of the spacecraft to be estimated using a rate gyroscope (rate gyro) and an extended Kalman filter (EKF). In Section II, the nonlinear attitude dynamics of a spacecraft under the influence of a gravity-gradient torque is reviewed and the linearized equations of motion assuming small angles about an Earth-pointing attitude are presented. Section II also presents an observability analysis based on the linearized equations of motion showing that with angular rate measurements, the spacecraft's attitude is observable. Section III outlines the linear Kalman filter and the EKF. Finally, Section IV presents simulation results. Specifically, the attitude of the linearized system and the attitude of the nonlinear system are successfully estimated using a Kalman filter and an EKF, respectively.

II. LINEARIZED ATTITUDE DYNAMICS

Although the kinematics and dynamics for a gravity-gradient-stabilized spacecraft are well known [1], [10], [11], for completeness, derivation of the relevant equations of motion will be represented here. The following derivation follows that of [10, pp. 268-272]. Consider a spacecraft in a circular low-Earth orbit, with orbit radius r, with orbital rate $\omega_o = \sqrt{\frac{\mu}{r^3}}$, and with an inertia matrix resolved in the spacecraft's body-fixed frame, denoted $\mathcal{F}_b$, given by $\mathbf{I}_b = \operatorname{diag}\{I_1, I_2, I_3\}$. Let $\underrightarrow{b}_1$, $\underrightarrow{b}_2$, and $\underrightarrow{b}_3$ represent the physical basis vectors of $\mathcal{F}_b$. In addition to $\mathcal{F}_b$, consider also the frames $\mathcal{F}_i$ and $\mathcal{F}_o$ where $\mathcal{F}_i$ is the Earth-centered inertial (ECI) frame and $\mathcal{F}_o$ is the orbit frame. The physical basis vectors composing $\mathcal{F}_o$ are $\underrightarrow{o}_1$, $\underrightarrow{o}_2$, and $\underrightarrow{o}_3$, as illustrated in Fig. 1. In Fig. 1, $\underrightarrow{r}$ is the spacecraft position relative to the center of the Earth. The nonlinear attitude dynamics including the gravity gradient torque are

$$\mathbf{I}_b\dot{\boldsymbol{\omega}}_b^{bi} + \boldsymbol{\omega}_b^{bi\times}\mathbf{I}_b\boldsymbol{\omega}_b^{bi} = \boldsymbol{\tau}_b^{gg} + \boldsymbol{\tau}_b^{c} + \mathbf{w}, \qquad (1)$$

where $\boldsymbol{\omega}_b^{bi}$ is the angular velocity of the spacecraft body frame relative to the ECI frame resolved in the body frame, $(\cdot)^\times$ is the cross matrix [1, pp. 546], $\boldsymbol{\tau}_b^c$ are control torques provided by an actuation system such as

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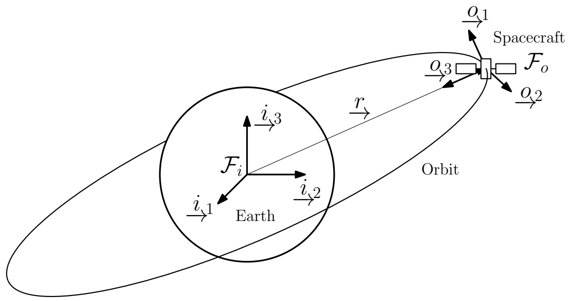

Fig. 1. Spacecraft in orbit around the Earth.

magnetic torque rods, $\mathbf{w}$ is a disturbance torque, and $\boldsymbol{\tau}_b^{gg}$ is the gravity-gradient torque resolved in the spacecraft body frame. The gravity-gradient torque is given by [10, p. 269]

$$\boldsymbol{\tau}_b^{gg} = \frac{3\mu}{r^5}\mathbf{r}_b^{\times}\mathbf{I}_b\mathbf{r}_b = \frac{3\mu}{r^5}\mathbf{C}_{bo}\mathbf{r}_o^{\times}\mathbf{C}_{bo}^{\mathsf{T}}\mathbf{I}_b\mathbf{C}_{bo}\mathbf{r}_o, \tag{2}$$

where μ is Earth's gravitational parameter, $\mathbf{r}_b$ is the spacecraft orbital position resolved in the spacecraft body frame, $\mathbf{C}_{bo}$ is the direction cosine matrix of the spacecraft body frame relative to the orbit frame, and $\mathbf{r}_o = [0 \;\; 0 \;\; -r]^{\mathsf{T}}$ is the position of the spacecraft relative to the center of the Earth resolved in the orbit frame. The control torque $\boldsymbol{\tau}_b^c$ is included for generality. For instance, often natural oscillations associated with a gravity-gradient-stabilized spacecraft are dampened using magnetic torque rods [11, pp. 126-131].

It is desired to determine the attitude of the spacecraft relative to the orbital frame $\mathcal{F}_o$. This attitude described by $\mathbf{C}_{bo}$ can also be parameterized by a 3-2-1 Euler sequence, where ψ is a rotation about $\underrightarrow{o}_3$ of the orbit frame, θ is a rotation about the 2-axis of the intermediate frame, and ϕ is a rotation about $\underrightarrow{b}_1$ of the body-fixed frame. The relationship between $\mathbf{C}_{bo}$ and the Euler angles is given by

$$\begin{aligned}\mathbf{C}_{bo} &= \mathbf{C}_1(\phi)\mathbf{C}_2(\theta)\mathbf{C}_3(\psi)\\ &= \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{bmatrix},\end{aligned}$$

where $s_b = \sin b$ and $c_b = \cos b$. The angular velocity of $\mathcal{F}_b$ relative to $\mathcal{F}_o$ resolved in $\mathcal{F}_b$ is related to the Euler angle rates via

$$\boldsymbol{\omega}_b^{bo} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix}\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \mathbf{S}\dot{\boldsymbol{\theta}}, \tag{3}$$

where $\boldsymbol{\theta} = [\phi \;\; \theta \;\; \psi]^{\mathsf{T}}$ is a column matrix of Euler angles. Using these relations, the angular velocity of the spacecraft body frame relative to the ECI frame resolved in the body frame is

$$\boldsymbol{\omega}_b^{bi} = \boldsymbol{\omega}_b^{bo} + \mathbf{C}_{bo}\boldsymbol{\omega}_o^{oi} \tag{4}$$
$$= \mathbf{S}\dot{\boldsymbol{\theta}} + \mathbf{C}_{bo}\boldsymbol{\omega}_o^{oi}, \tag{5}$$

where the angular velocity of the orbit frame relative to the ECI frame resolved in the orbit frame is $\boldsymbol{\omega}_o^{oi} = [0 \;\; -\omega_o \;\; 0]^{\mathsf{T}}$. For a circular orbit, $\boldsymbol{\omega}_o^{oi}$ is constant. Taking the time derivative of $\boldsymbol{\omega}_b^{bi}$ yields

$$\dot{\boldsymbol{\omega}}_b^{bi} = \dot{\boldsymbol{\omega}}_b^{bo} - \boldsymbol{\omega}_o^{bo\times}\mathbf{C}_{bo}\boldsymbol{\omega}_o^{oi} \tag{6}$$
$$= \mathbf{S}\ddot{\boldsymbol{\theta}} + \dot{\mathbf{S}}\dot{\boldsymbol{\theta}} - \boldsymbol{\omega}_o^{bo\times}\mathbf{C}_{bo}\boldsymbol{\omega}_o^{oi}, \tag{7}$$

where $\dot{\mathbf{C}}_{bo} = -\boldsymbol{\omega}_o^{bo\times}\mathbf{C}_{bo}$ has been used in going from from (4) to (6) [1, p. 23]. Substituting (4) and (6) into (1) and solving for $\dot{\boldsymbol{\omega}}_b^{bo}$ results in

$$\begin{aligned}\dot{\boldsymbol{\omega}}_b^{bo} &= \mathbf{I}_b^{-1}(\boldsymbol{\omega}_b^{bo} + \mathbf{C}_{bo}\boldsymbol{\omega}_o^{oi})^{\times}\mathbf{I}_b(\boldsymbol{\omega}_b^{bo} + \mathbf{C}_{bo}\boldsymbol{\omega}_o^{oi})\\ &\quad + \boldsymbol{\omega}_b^{bo\times}\mathbf{C}_{bo}\boldsymbol{\omega}_o^{oi} + \frac{3\mu}{r^5}\mathbf{I}_b^{-1}(\mathbf{C}_{bo}\mathbf{r}_o)^{\times}\mathbf{I}_b\mathbf{C}_{bo}\mathbf{r}_o\\ &\quad + \mathbf{I}_b^{-1}\boldsymbol{\tau}_b^c + \mathbf{I}_b^{-1}\mathbf{w},\end{aligned} \tag{8}$$

Likewise, substituting (5) and (7) into (1) allows $\ddot{\boldsymbol{\theta}}$ to be solved for. By using a small angle assumption,

$$\begin{aligned}\sin\phi &\approx \phi, & \cos\phi &\approx 1, & \sin\theta &\approx \theta,\\ \cos\theta &\approx 1, & \sin\psi &\approx \psi, & \cos\psi &\approx 1,\end{aligned}$$

the linearized equations of motion for the spacecraft's attitude are then given by [10, pp. 268-272]

$$\begin{bmatrix}\dot{\boldsymbol{\theta}}\\ \ddot{\boldsymbol{\theta}}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{3\times3} & \mathbf{1}_{3\times3}\\ \mathbf{A}_1 & \mathbf{A}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{\theta}\\ \dot{\boldsymbol{\theta}}\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{3\times3}\\ \mathbf{I}_b^{-1}\end{bmatrix}\mathbf{u} + \begin{bmatrix}\mathbf{0}_{3\times3}\\ \mathbf{I}_b^{-1}\end{bmatrix}\mathbf{w}, \tag{9}$$

where

$$\mathbf{A}_1 = \omega_o^2 \operatorname{diag}\left\{4\frac{(I_3 - I_2)}{I_1}, 3\frac{(I_3 - I_1)}{I_2}, \frac{(I_1 - I_2)}{I_3}\right\},$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 0 & \omega_o\frac{I_1 - I_2 + I_3}{I_1} \\ 0 & 0 & 0 \\ -\omega_o\frac{I_1 - I_2 + I_3}{I_3} & 0 & 0 \end{bmatrix},$$

$\mathbf{1}_{n\times n}$ denotes the $n \times n$ identity matrix, $\mathbf{0}_{n\times m}$ denotes a $n \times m$ matrix full of zeros and $\mathbf{u}$ is the linearized system's control input.

Measurements from a rate gyro are of the form

$$\boldsymbol{y} = \boldsymbol{\omega}_b^{bi} + \boldsymbol{\beta} + \boldsymbol{\nu} \tag{10}$$

where $\boldsymbol{\beta}$ is a measurement bias and $\boldsymbol{\nu}$ is measurement noise. In the present analysis, the gyro bias is not considered, but will be considered in future work. Using (4) and neglecting $\boldsymbol{\beta}$, this measurement can be written in terms of $\dot{\boldsymbol{\theta}}$ as

$$\boldsymbol{y} = \boldsymbol{\omega}_o^{bo} + \mathbf{C}_{bo}\boldsymbol{\omega}_o^{oi} + \boldsymbol{\nu} \tag{11}$$
$$= \mathbf{S}\dot{\boldsymbol{\theta}} + \mathbf{C}_{bo}\boldsymbol{\omega}_o^{oi} + \boldsymbol{\nu}.$$

Using a small angle assumption $\mathbf{S} \approx \mathbf{1}_{3\times3}$ and $\mathbf{C} \approx \mathbf{1}_{3\times3}$, and subtracting $\boldsymbol{\omega}_o^{oi}$ from $\boldsymbol{y}$ results in the linearized measurement equation

$$\begin{aligned}\mathbf{y} &= \dot{\boldsymbol{\theta}} + \boldsymbol{\nu} \\ &= \begin{bmatrix}\mathbf{0}_{3\times3} & \mathbf{1}_{3\times3}\end{bmatrix}\begin{bmatrix}\boldsymbol{\theta} \\ \dot{\boldsymbol{\theta}}\end{bmatrix} + \boldsymbol{\nu}. \end{aligned} \tag{12}$$

Equation (9) and (12) can be written succinctly as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \boldsymbol{\Gamma}_w\mathbf{w}, \quad \mathbf{y} = \mathbf{C}\mathbf{x} + \boldsymbol{\nu},$$

where $\mathbf{x} = [\boldsymbol{\theta}^\mathsf{T} \quad \dot{\boldsymbol{\theta}}^\mathsf{T}]^\mathsf{T}$. To verify that this linearized system is observable, the rank of the observability matrix must be n where n is the number of states. The observability matrix is given by

$$\mathcal{O}_{3\times3} = \begin{bmatrix}\mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{n-1}\end{bmatrix}$$

The first two blocks of this matrix are

$$\begin{bmatrix}\mathbf{C} \\ \mathbf{CA}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{3\times3} & \mathbf{1}_{3\times3} \\ \mathbf{A}_1 & \mathbf{A}_2\end{bmatrix}$$

which is already rank 6 provided that $I_1 \neq I_2$, $I_1 \neq I_3$, and $I_2 \neq I_3$. Given that $n = 6$, the system is observable provided the principal inertias are unique.

III. Attitude Estimation

A. Kalman Filter

A Kalman filter is used to estimate the attitude of the linearized spacecraft system using angular rate measurements from a rate gyro. Defining the state of the Kalman filter to be the estimate of the attitude and angular rates, that is $\hat{\mathbf{x}} = [\hat{\boldsymbol{\theta}}^\mathsf{T} \quad \dot{\hat{\boldsymbol{\theta}}}^\mathsf{T}]^\mathsf{T}$, then the estimate of the attitude and angular rates can be computed via [12, pp. 235-236]

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{K}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}), \quad \mathbf{K} = \mathbf{P}\mathbf{C}^\mathsf{T}\mathbf{R}^{-1}. \tag{13}$$

The covariance of the state estimation error, $\mathbf{P}$, evolves according to $\dot{\mathbf{P}} = \mathbf{AP} + \mathbf{PA}^\mathsf{T} - \mathbf{PC}^\mathsf{T}\mathbf{R}^{-1}\mathbf{CP} + \boldsymbol{\Gamma}_w\mathbf{Q}\boldsymbol{\Gamma}_w^\mathsf{T}$, where $\mathbf{Q} = \mathbf{Q}^\mathsf{T} \geq 0$ and $\mathbf{R} = \mathbf{R}^\mathsf{T} > 0$ are the covariance matrices of the process noise and measurement noise, respectively.

B. Extended Kalman Filter

To accurately estimate the state of the nonlinear spacecraft system, the Kalman filter is modified so that the linearized system equations are linearized about the current state estimate. This modification results in an implementation of the extended Kalman filter (EKF). The state estimate is given by [12, pp. 401]

$$\dot{\hat{\boldsymbol{x}}} = \mathbf{f}(\hat{\boldsymbol{x}}, \boldsymbol{u}, \mathbf{0}) + \mathbf{K}(\boldsymbol{y} - \mathbf{h}(\hat{\boldsymbol{x}}, \mathbf{0}))$$

where $\boldsymbol{x} = [\boldsymbol{\theta}^\mathsf{T} \quad \boldsymbol{\omega}_b^{bo\mathsf{T}}]^\mathsf{T}$, and $\boldsymbol{u} = \boldsymbol{\tau}_b^c$ is the control input for the nonlinear system. The expression for $\dot{\boldsymbol{\theta}}$ and the expression for $\boldsymbol{\omega}_b^{bo}$ that compose $\mathbf{f}(\boldsymbol{x}, \boldsymbol{u}, \mathbf{w})$ are taken from (3) and (8), respectively. For the nonlinear system, the measurement $\boldsymbol{y} = \mathbf{h}(\boldsymbol{x}, \boldsymbol{\nu})$ is the angular velocity $\boldsymbol{\omega}_b^{bi}$ as given in (11) rather than the Euler angle rates. The gain matrix $\mathbf{K}$ is given by $\mathbf{K} = \mathbf{P}\mathbf{H}^\mathsf{T}\mathbf{R}^{-1}$ where

$$\mathbf{H} = \left.\frac{\partial \mathbf{h}(\boldsymbol{x}, \boldsymbol{\nu})}{\partial \boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}, \mathbf{0}}$$

is the Jacobian of the measurement model evaluated at the current state estimate. The time-rate-of-change of $\mathbf{P}$ is given by $\dot{\mathbf{P}} = \mathbf{FP} + \mathbf{PF}^\mathsf{T} - \mathbf{PH}^\mathsf{T}\mathbf{R}^{-1}\mathbf{HP} + \mathbf{GQG}^\mathsf{T}$, where

$$\mathbf{F} = \left.\frac{\partial \mathbf{f}(\boldsymbol{x}, \boldsymbol{u}, \mathbf{w})}{\partial \boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}, \boldsymbol{u}, \mathbf{0}} \qquad \mathbf{G} = \left.\frac{\partial \mathbf{f}(\boldsymbol{x}, \boldsymbol{u}, \mathbf{w})}{\partial \mathbf{w}}\right|_{\hat{\boldsymbol{x}}, \boldsymbol{u}, \mathbf{0}}$$

are the Jacobians of the process model evaluated at the current state estimate and control input, and $\mathbf{Q} = \mathbf{Q}^\mathsf{T} \geq 0$ and $\mathbf{R} = \mathbf{R}^\mathsf{T} > 0$ are the covariance matrices of the process noise and measurement noise, respectively.

IV. Simulation Results

To verify that it is indeed possible to estimate the attitude of a gravity-gradient-stabilized spacecraft using rate gyro measurements only, a set of simulated test cases will now be presented. A satellite in a circular Keplerian orbit with orbital radius $r = 6821.2$ [km], inclination $i = 87°$, right ascension of the ascending node $\Omega = \frac{\pi}{2}$ [rad], and initial mean anomaly $M_0 = 0$ [rad] is simulated. The orbital rate is $\omega_o = 1.1207 \times 10^{-3}$ [rad/s], and the spacecraft inertia is $\mathbf{I}_b = \operatorname{diag}\{0.0612, 0.0664, 0.0066\}$ [kg·m^2]. The initial attitude of the spacecraft's body-fixed frame relative to the ECI frame is $\mathbf{q}_{bi}(0) = \begin{bmatrix}0.3207 & -0.4755 & 0.5892 & 0.5690\end{bmatrix}^\mathsf{T}$, where $\mathbf{q}_{bi}$ is the quaternion representation of $\mathbf{C}_{bi}$, and the initial angular velocity of the spacecraft $\boldsymbol{\omega}_b^{bi}(0) = [-4.857 \ -0.181 \ 1.174]^\mathsf{T} \times 10^{-4}$ [rad/s]. Relative to the orbit frame, the initial attitude and angular velocity are $\boldsymbol{\theta}(0) = [-0.1075 \ 0.3742 \ 0.1059]^\mathsf{T}$ [rad] and $\boldsymbol{\omega}_b^{bo}(0) = [-0.375 \ 1.085 \ 0.280]^\mathsf{T} \times 10^{-3}$ [rad/s]. No additional torque inputs are considered, and as such, $\boldsymbol{u} = \mathbf{0}$ and $\mathbf{w} = \mathbf{0}$.

A. Kalman Filter Estimating the State of the Linearized System

The first case considered is the application of a Kalman filter, from (13), to estimate the Euler angles and Euler angle rates associated with the linearized system. The Kalman filter initial estimate is $\hat{\mathbf{x}}_0 = \mathbf{0}$, the initial covariance matrix is $\mathbf{P}_0 = \operatorname{diag}\{\mathbf{1}_{3\times3}, 10^{-4} \times \mathbf{1}_{3\times3}$ [(rad/s)^2]$\}$, $\mathbf{Q} = (1 \times 10^{-7})^2 \times \mathbf{1}_{3\times3}$ [(N·m)^2], and $\mathbf{R} = (5 \times 10^{-6})^2 \times \mathbf{1}_{3\times3}$ [(rad/s)^2]. Measurements are corrupted by a zero-mean Gaussian noise process $\boldsymbol{\nu}$, as indicated in (12). This noise process had variances $\sigma_\nu^2 = (1 \times 10^{-6})^2$ [(rad/s)^2] in each component with a characteristic length-scale of 10 [s], and was modelled using the method outlined in [13, Chap. 2, Sec. 2]. The simulation duration is 5 h. The true Euler angles and rate, provided by simulation, and the estimated Euler angles and rates provided by the Kalman filter are shown in Fig. 2 and Fig. 3. The estimation errors are shown in Fig. 4 and Fig. 5. From these results it is apparent that state estimates converge to the true states. These results indicate it is possible to estimate the attitude for the linearized dynamics of a gravity-gradient-stabilized spacecraft using only angular rate measurements using a Kalman filter.

B. EKF Estimating the State of the Nonlinear System

The second case considered is the application of an EKF to estimate the Euler angles and angular velocity of the nonlinear gravity-gradient-spacecraft system as described by (3) and (8). The EKF initial estimate is $\hat{\boldsymbol{x}}_0 = \mathbf{0}$, the initial covariance matrix is $\mathbf{P}_0 = \text{diag}\{\mathbf{1}_{3\times3}, 10^{-4} \times \mathbf{1}_{3\times3}$ [(rad/s)^2]$\}$, $\mathbf{Q} = (1 \times 10^{-7})^2 \times \mathbf{1}_{3\times3}$ [(N·m)^2], and $\mathbf{R} = (5 \times 10^{-6})^2 \times \mathbf{1}_{3\times3}$ [(rad/s)^2]. As with the linear case, the angular velocity measurement provided by a rate gyro is corrupted by a zero-mean Gaussian process with variances $\sigma_\nu^2 = (1 \times 10^{-6})^2$ [(rad/s)^2] in each component and a characteristic length-scale of 10 [s]. Again, the simulation duration is 5 h. The true Euler angles and angular velocities, provided by simulation, and the estimated Euler angles and angular velocities provided by the EKF are shown in Fig. 6 and Fig. 7. Estimation errors are shown in Fig. 8 and Fig 9. These results show that the state estimates converge to the true states indicating it is also possible to estimate the attitude of a gravity-gradient-stabilized spacecraft using only angular velocity measurements with an EKF.

V. Conclusions

In this study the ability to estimate the attitude of a gravity-gradient-stabilized spacecraft using only a rate gyro is demonstrated. The nonlinear attitude dynamics for a spacecraft under the influence of a gravity-gradient torque are first reviewed. These equations of motion are then linearized about an Earth pointing attitude, and a linear observability analysis demonstrates that the attitude of the linear system is observable using angular rate measurements provided by a rate gyro only. Simulation results show that the attitude estimate of a spacecraft under the influence of a gravity-gradient torque converges to the true attitude for both a Kalman filter implemented on the linearized system and an EKF implemented on the nonlinear system. The ability to estimate the attitude of a satellite using only angular rate measurements from a rate gyro is particularly attractive in the context of microsatellites and nanosatellites. In the

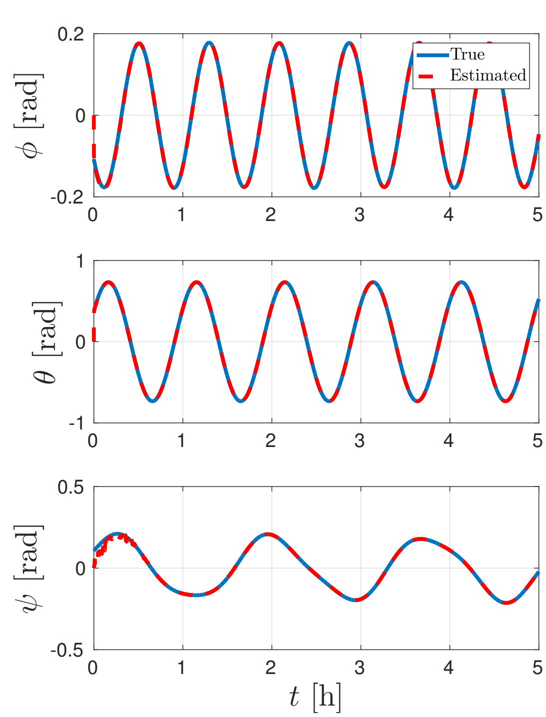

Fig. 2. True and estimated Euler angles for the linearized dynamics using a Kalman filter.

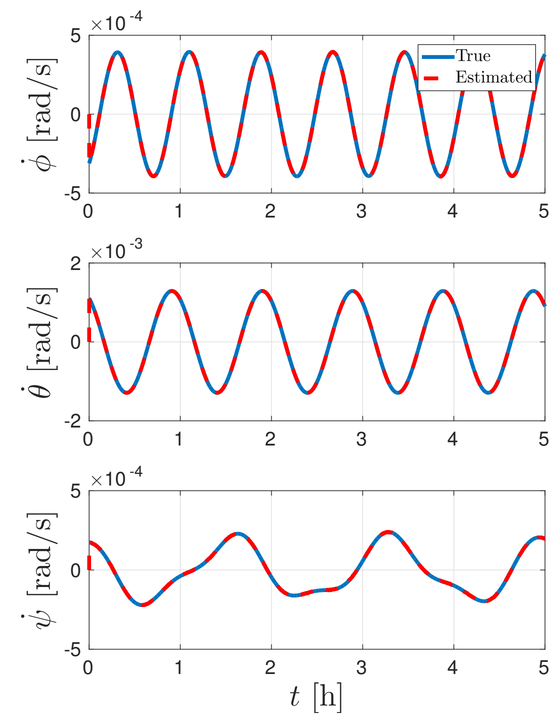

Fig. 3. True and estimated Euler angle rates for the linearized dynamics using a Kalman filter.

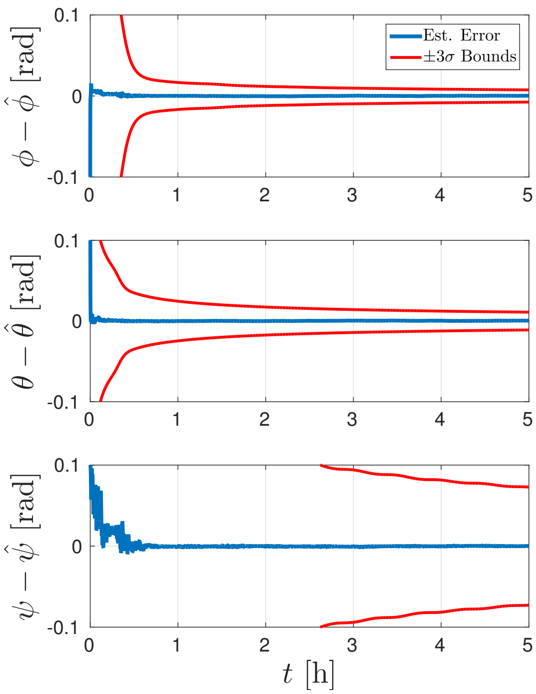

Fig. 4. Estimation error associated with Euler angles for the linearized dynamics using a Kalman filter.

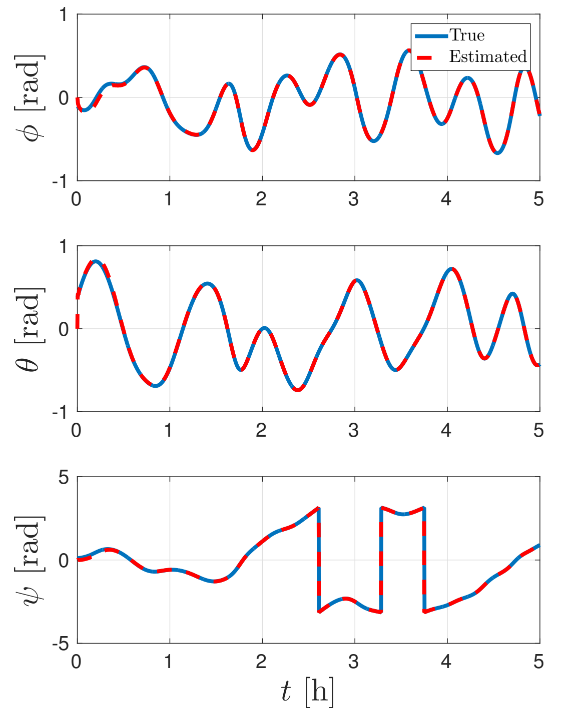

Fig. 6. True and estimated Euler angles for nonlinear dynamics with EKF.

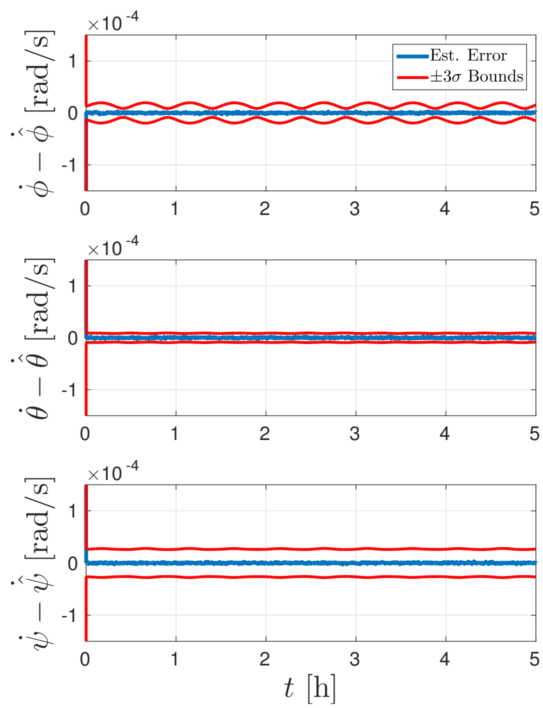

Fig. 5. Estimation error associated with Euler angle rates for the linearized dynamics using a Kalman filter.

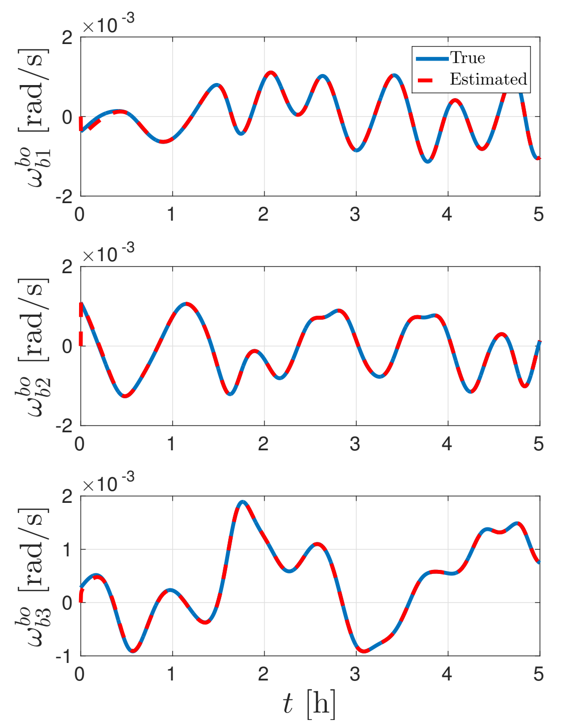

Fig. 7. True and estimated angular velocity for nonlinear dynamics with EKF.

case of small spacecraft, with their very strict mass and power requirements, rate-gyro-based attitude estimation can allow more resources to be budgeted to mission supporting operations instead of attitude estimation. Future work will consider using the attitude estimates acquired via a rate gyro plus EKF for attitude control within a feedback control architecture.

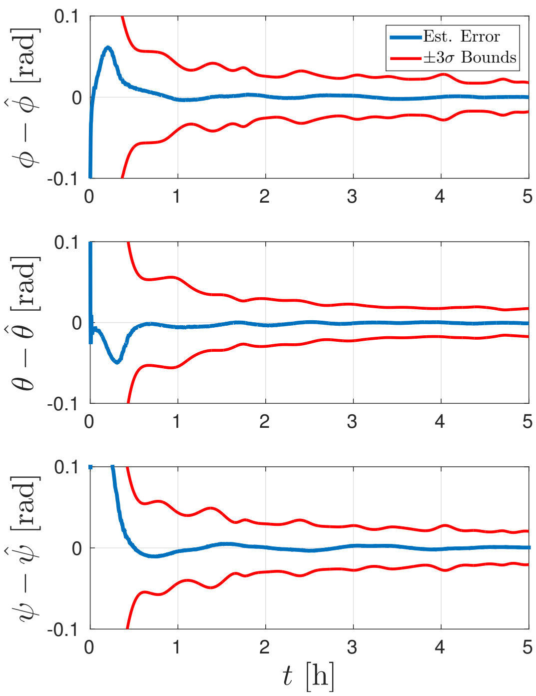

Fig. 8. Estimation error for Euler angles for nonlinear dynamics with EKF.

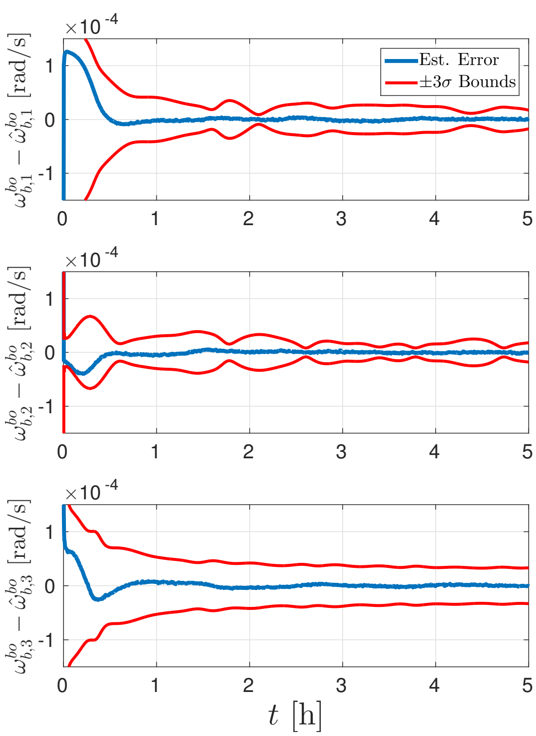

Fig. 9. Estimation error for angular velocities for nonlinear dynamics with EKF.

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