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# Experimental Identification of the Spatial Spillover Operator for Systems with Insuppressible Disturbances

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*Abstract*— **The spatial spillover operator quantifies the relative effectiveness of a controller at two different spatial locations. This operator is relevant to noise and vibration control, where the response of the system at a location without a sensor may be unsatisfactory. Identification of the spatial spillover operator based on a representation involving four transfer functions requires measurements of the disturbances, which may not be available in practice. Likewise, identification based on a representation involving two transmissibilities is not feasible in systems where the disturbance cannot be suppressed. To overcome this difficulty in systems with disturbances that are both unmeasured and insuppressible, this paper considers identification based on a hybrid representation involving one transmissibility and two transfer functions. Acoustic data are used to identify the spillover operator and validate the proposed method.**

## I. INTRODUCTION

Active noise and vibration control has been widely studied for several decades, and applications of this technology are becoming increasingly common [1], [2], [3]. Both feedforward and feedback architectures are used in practice depending on the availability of direct or indirect measurements of the disturbance [4], [1].

Despite this progress, a key unsolved problem concerns the fact that a controller designed to suppress noise at one location (the location of the performance sensor) may amplify the noise at another location (which can be viewed as the location of an evaluation sensor). This is called the spatial spillover ("waterbed") effect [5] and is distinct from spectral spillover, which, within the context of feedback control, is unavoidable due to the Bode sensitivity integral [6], [7]. The disturbance amplification at the location of the evaluation sensor may be unacceptable in practice. An example is the active noise control in the interior of a vehicle, where the key locations at which noise should be suppressed are the ears of the occupants but where it is impractical to place microphones. It is therefore of interest to formalize the notion of spatial spillover and determine the extent to which it depends on the dynamics and geometry of the system. In many cases, it is possible to quantify the spatial spillover in the development process by means of test evaluation sensors that cannot be implemented in the operational system.

We note, in passing, that disturbance suppression can be achieved uniformly in space in the special case where the actuation is colocated with the disturbance [6]. However, this geometry is rarely feasible in practice.

The notion of spatial spillover is formalized in [5] in terms of the *spatial spillover operator* $G_{\rm ss}$. In particular, $G_{\rm ss}$ relates the performance of the controlled system relative to the uncontrolled system at the location of the evaluation sensor to the performance of the controlled system relative to the uncontrolled system at the location of the performance sensor. This definition is applicable to both feedback and feedforward control architectures for disturbance rejection. It turns out that, in the case where the control, disturbance, performance, and evaluation are scalar signals, $G_{\rm ss}$ is independent of the controller; otherwise, $G_{\rm ss}$ may depend on the control law.

As shown in [5], in the case where all the signals are scalar, $G_{\rm ss}$ can be written as a ratio involving four transfer functions. This ratio can also be expressed as the ratio of two transmissibilities [8], [9], [10]. This connection provides a natural interpretation for $G_{\rm ss}$.

The objective of the present paper is to experimentally estimate $G_{\rm ss}$. In fact, $G_{\rm ss}$ was estimated in [5] using measurements of all four signals using both feedback and feedforward controllers. These estimates used knowledge of all four signals, including the disturbance. In addition, the estimation in [5] was based on the assumption that the control and disturbance can be applied separately.

The motivation for the present paper is the fact that, in practice, it is often required to perform identification in the presence of disturbances that cannot be measured or suppressed. For the objective of identifying $G_{\rm ss}$, this presents an obvious difficulty due to the fact that two of the four transfer functions appearing in the definition of $G_{\rm ss}$ cannot be estimated if the disturbance cannot be measured. Furthermore, in the expression for $G_{\rm ss}$ involving two transmissibilities, only one of the tranmsmissibilities can be estimated if the disturbance cannot be suppressed. As explained later in the paper, the effect of the insuppressible disturbance can be treated as an errors-in-variables (EIV) regression problem [11], [12], [13], but the EIV noise is colored and correlated and thus the effect of the EIV noise cannot be distinguished from the system dynamics. Consequently, the resulting EIV problem is unsolvable.

To overcome these difficulties, the present paper considers an alternative approach where $G_{\rm ss}$ is expressed in terms

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of one transmissibility (the transmissibility arising from the disturbance) and two transfer functions. Estimation using this expression does not need the measurement or suppression of disturbance. Since the transmissibility arises from the disturbance only, it can be estimated by turning off the control. Also, the effect of the insuppressible disturbance now appears as only output noise in the estimation of the two required transfer functions. Consequently, since no input noise is present, the identification problems are not EIV problems. In order to demonstrate this approach, experiments are conducted on an acoustic setup consisting of speakers and microphones. Using the experimental data, identification of spatial spillover operator is done and it is also shown that the estimation is consistent.

This paper is organized as follows. Section II explains the concept of spatial spillover operator and derives expressions for it. The different approaches for identifying the spatial spillover operator are presented in Section III. Section IV and Section V describe the experimental setup and experimental results, respectively. Section VI contains the conclusions and future work.

## II. SPATIAL SPILLOVER OPERATOR

Consider the feedback control problem shown in Figure 1, where $z \in \mathbb{R}$ is the performance variable, $e \in \mathbb{R}$ is the evaluation variable, $w \in \mathbb{R}$ is the disturbance, and $u \in \mathbb{R}^{l_u}$ is the control input. The dynamics and signals may be either continuous time or discrete time. It follows from Figure 1 that

$$z = G_{zu}u + G_{zw}w, \tag{1}$$
$$e = G_{eu}u + G_{ew}w, \tag{2}$$

where the feedback control $u$ is given by

$$u = G_{\rm c}z. \tag{3}$$

Using (1) and (3) we obtain

$$z = \widetilde{G}_{zw}w, \tag{4}$$

where

$$\widetilde{G}_{zw} \triangleq \frac{G_{zw}}{1 - G_{zu}G_{\rm c}}. \tag{5}$$

In addition, it follows from (2), (3), and (4) that

$$e = \widetilde{G}_{ew}w, \tag{6}$$

where

$$\widetilde{G}_{ew} \triangleq \frac{G_{eu}G_{\rm c}G_{zw}}{1 - G_{zu}G_{\rm c}} + G_{ew}. \tag{7}$$

The spatial spillover operator $G_{\rm ss}$ is defined as

$$G_{\rm ss} \triangleq \frac{\dfrac{\widetilde{G}_{ew}}{G_{ew}} - 1}{\dfrac{\widetilde{G}_{zw}}{G_{zw}} - 1}. \tag{8}$$

It can be seen that $G_{\rm ss}$ relates the performance of the controlled system relative to the uncontrolled system at $e$ to the performance of the controlled system relative to the uncontrolled system at $z$. Assuming that $G_{zu}G_{\rm c} \neq 0$, it follows from (5) and (7) that

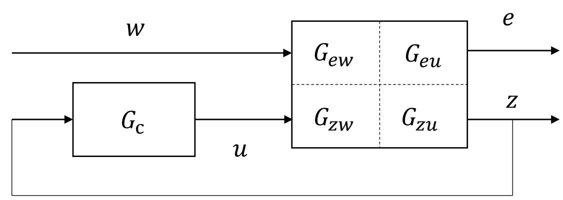


Fig. 1. Feedback control.

$$\frac{\widetilde{G}_{zw}}{G_{zw}} - 1 = \frac{G_{zu}G_{\rm c}}{1 - G_{zu}G_{\rm c}}, \tag{9}$$
$$\frac{\widetilde{G}_{ew}}{G_{ew}} - 1 = \frac{G_{eu}G_{\rm c}G_{zw}}{G_{ew}(1 - G_{zu}G_{\rm c})}. \tag{10}$$

Therefore, (8), (9), and (10) imply that

$$G_{\rm ss} = \frac{G_{eu}G_{\rm c}G_{zw}}{G_{zu}G_{\rm c}G_{ew}}. \tag{11}$$

In this paper, only the case where $u$ is scalar is considered. When $u$ is scalar, (11) becomes

$$G_{\rm ss} = \frac{G_{eu}G_{zw}}{G_{zu}G_{ew}}, \tag{12}$$

which is independent of $G_{\rm c}$.

$G_{\rm ss}$ can also be expressed as the ratio of two transmissibility functions. To do this, we define the notation

$$G_{zw} = \frac{N_{zw}}{D_{zw}}, G_{ew} = \frac{N_{ew}}{D_{ew}}, G_{zu} = \frac{N_{zu}}{D_{zu}}, G_{eu} = \frac{N_{eu}}{D_{eu}}. \tag{13}$$

It is realistic to assume that $D_{zw} = D_{ew}$ and $D_{zu} = D_{eu}$. The transmissibility [8] from $z$ to $e$ driven by $w$ is given by

$$T_{ez,w} \triangleq \frac{G_{ew}}{G_{zw}} = \frac{\dfrac{N_{ew}}{D_{ew}}}{\dfrac{N_{zw}}{D_{zw}}} = \frac{N_{ew}}{N_{zw}}. \tag{14}$$

Similarly, the transmissibility from $z$ to $e$ driven by $u$ is given by

$$T_{ez,u} \triangleq \frac{G_{eu}}{G_{zu}} = \frac{N_{eu}}{N_{zu}}. \tag{15}$$

Therefore, it follows from (12) that

$$G_{\rm ss} = \frac{G_{eu}G_{zw}}{G_{zu}G_{ew}} = \frac{N_{eu}N_{zw}}{N_{zu}N_{ew}} = \frac{\dfrac{N_{eu}}{N_{zu}}}{\dfrac{N_{ew}}{N_{zw}}} = \frac{T_{ez,u}}{T_{ez,w}}. \tag{16}$$

## III. IDENTIFICATION OF THE SPATIAL SPILLOVER OPERATOR

Equations (12) and (16) provide two approaches for identifying the spatial spillover operator. Since these approaches have practical limitations, a third approach, which is a hybrid of the first two, is proposed. All three approaches are explained below.

### A. Transfer Function Approach ($4G$)

In the transfer function approach, $G_{\rm ss}$ is identified using (12). For this, the four transfer functions $G_{eu}, G_{zu}, G_{zw}$, and $G_{ew}$ have to be estimated, which requires knowledge of input signal $u$, disturbance $w$, and output signals $z$ and $e$. In practice, it is difficult to measure $w$. Thus, this method is not useful if measurements of $w$ are not available.

### B. Transmissibility Approach ($2T$)

In the transmissibility approach, $G_{\rm ss}$ is identified using (16). This involves estimation of transmissibilities $T_{ez,u}$ and $T_{ez,w}$. Here, knowledge of signals $u$ and $w$ is not required. Let

$$z_u = G_{zu}u, \tag{17}$$
$$z_w = G_{zw}w, \tag{18}$$
$$e_u = G_{eu}u, \quad \text{and} \tag{19}$$
$$e_w = G_{ew}w. \tag{20}$$

Estimating the rational function from $z_u$ (pseudo input of the transmissibility) to $e_u$ (pseudo output of the transmissibility) gives $T_{ez,u}$, while estimating the rational function from $z_w$ (pseudo input of the transmissibility) to $e_w$ (pseudo output of the transmissibility) gives $T_{ez,w}$. We set $u = 0$ in order to measure $z_w$ and $e_w$. It is desirable to set $w = 0$ for measuring $z_u$ and $e_u$, but in most practical cases, this is not possible. Let $v_z = G_{zw}w$ and $v_e = G_{ew}w$. Then, (1) and (2) can be written as

$$z = z_u + v_z, \tag{21}$$
$$e = e_u + v_e. \tag{22}$$

In the case where $w$ cannot be set to zero, $T_{ez,u}$ must be estimated using $z$ and $e$. Here, $z$ is the pseudo input of the transmissibility and $e$ is the pseudo output of the transmissibility. The signals $v_z$ and $v_e$ can be considered as input noise and output noise, respectively. Hence, this estimation is an errors-in-variables (EIV) problem [11]. Since the input noise and output noise are colored and correlated, it is not possible to distinguish the noise from the actual signal, and thus this identification problem cannot be solved.

### C. Hybrid Approach ($2G + 1T$)

The hybrid approach is useful for identifying $G_{\rm ss}$ in the case where $w$ cannot be measured and $w$ cannot be shut off. The hybrid expression for the spatial spillover operator is given by

$$G_{\rm ss} = \frac{G_{eu}}{G_{zu}T_{ez,w}}. \tag{23}$$

As in the transmissibility approach, $T_{ez,w}$ can be estimated from measurements of $z_w$ and $e_w$. Since $u$ is the control input, we assume that $u$ is known. Note that (1) and (2) can be written as

$$z = G_{zu}u + v_z, \tag{24}$$
$$e = G_{eu}u + v_e, \tag{25}$$

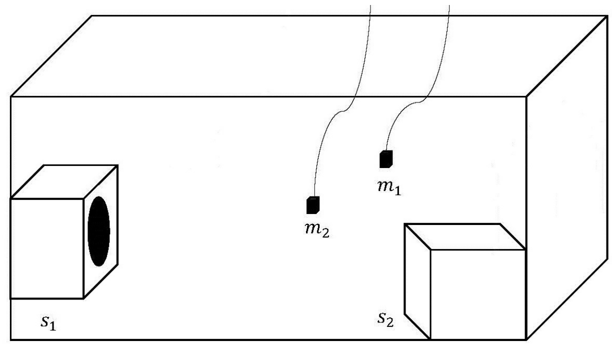

Fig. 2. Experimental setup showing the microphone and speaker placement.

where the signals $z$ and $e$ are viewed as outputs due to the input $u$ and corrupted by the noise signals $v_z$ and $v_e$ respectively. Thus, (24) and (25) define an identification problem where the output signals are corrupted by noise but the input signal $u$ is known. This identification problem is amenable to identification techniques for systems with output noise [14], [15]. Denote the estimates of $G_{zu}$ and $G_{eu}$ obtained from (24) and (25) as $\hat{G}_{zu}$ and $\hat{G}_{eu}$, respectively. Therefore, it is possible to calculate $G_{\rm ss}$ from (23) using $\hat{G}_{zu}$, $\hat{G}_{eu}$, and $T_{ez,w}$.

## IV. EXPERIMENTAL SETUP

An acoustic experiment is used to provide data for identifying the spatial spillover operator. Omni-directional microphones are used as sensors, and mid-bass woofers are used for actuation. Real Time Workshop (RTW) and MATLAB/Simulink are used with a dSPACE DS1104 board to collect data from the sensors and actuators. Additional hardware used in the implementation includes speaker amplifiers, microphone amplifiers, and anti-aliasing filters.

Figure 2 shows the microphone and speaker placement. The approximate dimensions of the acoustic space are 6 ft $\times$ 3 ft $\times$ 3 ft with microphone locations $m_1$ and $m_2$ and speaker locations $s_1$ and $s_2$. One microphone is chosen as the performance microphone $z$, and the other microphone is chosen as the evaluation microphone $e$. In addition, one speaker is chosen to produce the disturbance $w$, and the other speaker is chosen to produce the control $u$. The frequency range used is from 50 Hz to 500 Hz, with data sampled at 1 kHz. The data set consists of 20,000 samples. A photo of the experimental setup is shown in Figure 3.

## V. EXPERIMENTAL RESULTS

Since the disturbance $w$ is produced by a speaker in the experimental setup, the speaker input $w$ can be measured and also the speaker can be shut off. This makes it possible to perform the identification of the spatial spillover operator using all three approaches described in Section III and compare the resulting models. In a real application case, such as the interior of a car where the disturbance due to wind, road, or engine noise cannot be measured or suppressed, only the hybrid ($2G + 1T$) approach can be used to identify the spatial spillover operator.

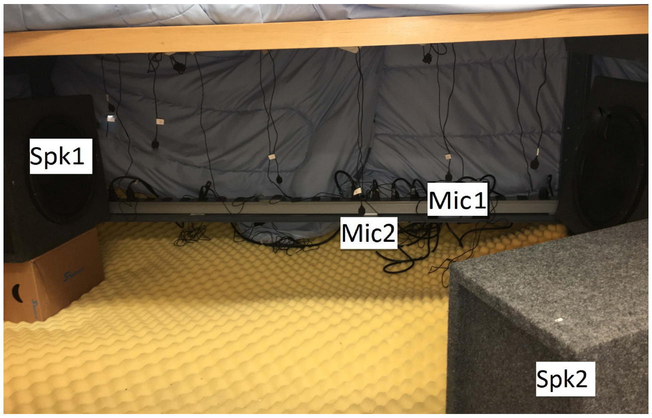


Fig. 3. Experimental setup.

Identification is performed in both the frequency domain and the time domain. For frequency-domain identification, spectral analysis [16] is used, whereas, for time-domain identification, least squares [17] is used. For frequency-domain identification, Figure 4 compares the estimated frequency response of $G_{ss}$ obtained for the $4G$, $2T$, and $2G+1T$ approaches. For time-domain identification, Figure 5 compares the estimated frequency response of $G_{ss}$ obtained for the $4G$, $2T$, and $2G+1T$ approaches. These plots show that the frequency response of the spatial spillover operator estimated using the three approaches are in reasonable agreement. Figure 6 compares frequency-domain identification with time-domain identification for the $2G+1T$ approach.

In order to check whether the estimation using $2G+1T$ method is consistent, we define the error metric

$$E = \sqrt{\sum_{\theta\in[0,\pi]} \left|G_{\rm ss}(e^{\jmath\theta}) - G_{\rm ss,true}(e^{\jmath\theta})\right|^2}, \tag{26}$$

where $G_{\rm ss}$ is estimated using the $2G+1T$ approach as described in Section III.C. In contrast, $G_{\rm ss,true}$ is estimated using the $2G+1T$ approach, but $G_{eu}$ and $G_{zu}$ are estimated with $w$ suppressed (that is, using $z_u$, $e_u$, and $u$). The error $E$ is calculated for an increasing number of data samples from 1000 to 20000 samples. The log of $E$ is plotted versus the log of the number of data samples in Figure 7, which shows that the error decreases with a log-log slope of about $-1/2$. This trend indicates that the estimates are consistent.

## VI. CONCLUSIONS

The notion of spatial spillover was explained, and expressions for the spatial spillover operator in terms of transfer functions and transmissibilities were presented. Three approaches for identifying the spillover operator were described, namely, $4G$, $2T$, and $2G+1T$. It was shown that only the $2G+1T$ approach can be used in the case of unmeasured and insuppressible noise in the system. An experiment was conducted on an acoustic setup, and the three approaches were compared. By varying the size of the data set, it was shown that the $2G+1T$ approach captures the dynamics of the spatial spillover operator with a statistical trend demonstrating consistency.

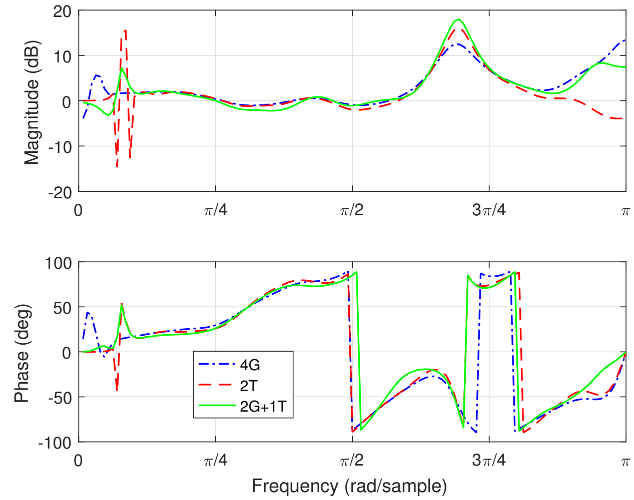


Fig. 4. Frequency response of $G_{\rm ss}$. This plot compares the $4G$, $2T$, and $2G+1T$ approaches. The identification is performed in the frequency domain using spectral analysis.

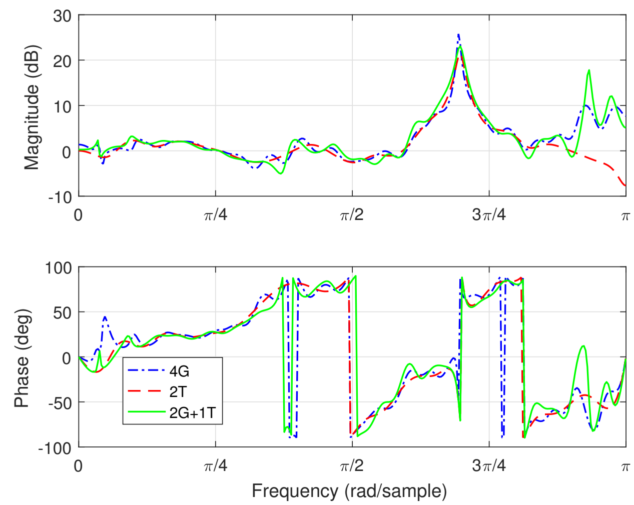


Fig. 5. Frequency response of $G_{\rm ss}$. This plot compares the $4G$, $2T$, and $2G+1T$ approaches. The identification is performed in the time domain using least squares.

Future work involves extension of identification of $G_{\rm ss}$ to MIMO systems. Since $G_{\rm ss}$ depends on the controller for MIMO systems, it may be possible to synthesize controllers so as to attain a desired spatial spillover frequency response. Finally, analysis of how spatial spillover changes with speaker/microphone placement and disturbance location may prove to be helpful in determining acoustic geometries that facilitate spatially uniform disturbance suppression.

## ACKNOWLEDGMENT

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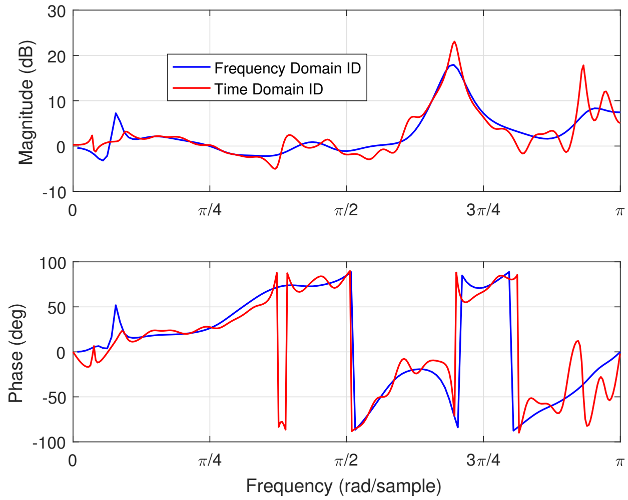


Fig. 6. Frequency response of $G_{\mathrm{ss}}$. This plot compares frequency-domain identification with time-domain identification for the $2G+1T$ approach.

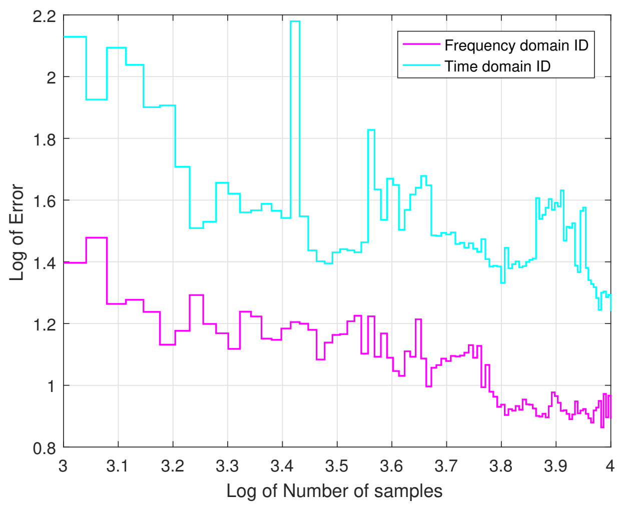


Fig. 7. Log of error versus the log of number of samples. The error $E$ is based on a truth model that is constructed by using the $2G+1T$ method, where $G_{zu}$ and $G_{eu}$ are estimated with $w$ suppressed. The log of error is plotted versus the log of number of samples for both time-domain and frequency-domain identification. The error decreases with a log-log slope of about $-1/2$, which indicates that the estimates are consistent.

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