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Sensor-Only Noncausal Blind Identification of Pseudo Transfer Functions ⋆

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Abstract: Motivated by passive health monitoring applications, we consider blind identification where only sensor measurements are available. The goal is to identify a pseudo transfer function (PTF) between two sensors in the presence of an unknown initial state and unknown exogenous input. For this problem, we choose one sensor to be the pseudo input to the system and we delay the second sensor, treating it as the pseudo output. We show that the order of the pseudo-transfer function is no larger than one higher than the order of the system. We demonstrate this method on a two-degree-of-freedom mass-spring-damper system and validate the identified PTFs by comparing them with analytical results.

Keywords: Passive Health Monitoring; Sensor-Only System Identification.

1. INTRODUCTION

In many applications of system identification, the system is driven by external signals that are not measured. In this situation, blind identification techniques are used to obtain estimates of the system dynamics [1],[2]. Since the input signal is unknown, its statistical properties are usually assumed to be known in order to compensate for lack of knowledge of its time history.

In the present paper we develop a blind identification technique that uses multiple sensors but does not require assumptions about the statistical properties of the external signal. For the system identification, we designate one sensor signal as the pseudo input and the other as the pseudo output. Since causality between sensor signals may fail (i.e., the transfer function between the pseudo input and the pseudo output may be noncausal), we delay the pseudo output signal prior to parameter estimation. The resulting *pseudo transfer function* (PTF) thus provides a causal map between the sensor signals. Identification of noncausal models is considered in [3, 4, 5].

To illustrate the notion of a PTF, consider a system with one input u and two outputs y_1 and y_2, as shown in Figure 1. Assuming that the system is linear, the time-series model relating u to y_i is given by G_i. To account for the initial state and resulting transient response, we cast the dynamics in terms of the forward shift operator $\mathbf{q}$, which yields

$$y_i(k) = G_i(\mathbf{q})u(k) = \frac{\eta_i(\mathbf{q})}{\delta(\mathbf{q})}u(k), \tag{1}$$

and thus the PTF from y_1 to y_2 is given by

$$y_2(k) = \frac{\eta_1(\mathbf{q})\delta(\mathbf{q})}{\eta_2(\mathbf{q})\delta(\mathbf{q})}y_1(k). \tag{2}$$

A useful aspect of the PTF is that it is independent of the both the input u and the initial condition $x(0)$, and therefore facilitates blind identification given lack of knowledge of u or its statistical properties.

This formulation accounts for both the free and forced responses since the equations are cast in terms of the forward shift operator rather than the z-transform. Therefore, we cannot immediately cancel the common factor $\delta(\mathbf{q})$. However, we show through some surprising identities that this factor can indeed be canceled, which results in a PTF whose order does not exceed the order of the plant by more than 1.

Estimates of a PTF do not provide a full picture of the dynamics of the system. In fact, since the PTF is the ratio of transfer functions from the same input to different outputs, pole information is generally lost, whereas zero information is retained. The motivation for sensor-only noncausal blind identification (SONBI) is to use changes in this zero information for health monitoring and fault detection [11].

Here we use PTFs for SONBI and focus on obtaining a PTF using only output data. To determine a PTF, a possibly unknown input excites the system, and output data is collected from the sensors. Once the output data has been collected, one set of output data (here the data from sensor 2) is delayed. This delay is represented by

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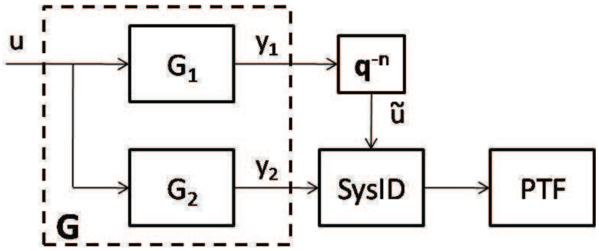

Fig. 1. Sensor-only noncausal blind identification method for identifying pseudo transfer functions (PTFs).

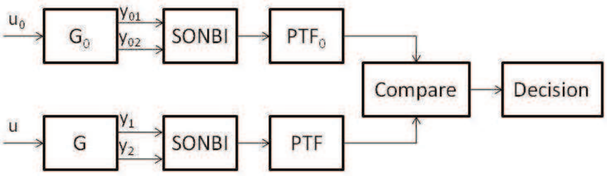

Fig. 2. Fault-detection architecture.

the $\mathbf{q}^{-n}$ block in Figure 1. Then, a system identification algorithm is used to identify the PTF from the non-delayed (pseudo input) data of y_1 to the delayed (pseudo output) data of y_2.

A fault detection architecture based on SONBI is shown in Figure 2, where the upper path uses SONBI for the nominal system, and the bottom half uses SONBI for the possibly faulty system. Comparing estimates of the PTFs, possibly as characterized by their Markov parameters, provides a technique for fault detection. This health monitoring may be either active or passive, where "passive" refers to the fact that the external driving signal need not be known and may arise from ambient disturbances.

2. MARKOV PARAMETERS

The Markov parameters provide essential information about the system dynamics [6]. Numerical results [7] suggest that consistent estimates of the Markov parameters can be obtained from μ-Markov models. Consequently, to facilitate on-line implementation, we use recursive least squares (RLS) with the μ-Markov structure [7]

$$y(k) = -\sum_{j=1}^{n} a_j^{(\mu-1)} y(k-j-\mu+1) + \sum_{j=1}^{\mu} H_{j-1} u(k-j+1) + \sum_{j=1}^{n} b_j^{(\mu-1)} u(k-j-\mu+1), \quad (3)$$

which has the advantage over a standard IIR model of explicitly displaying the Markov parameters $H_0 \ldots H_{\mu-1}$. The μ-Markov structure is an IIR model with a μ-step ahead predictor.

3. DERIVATION OF PSEUDO TRANSFER FUNCTIONS

3.1 Derivation of Transfer Functions Using the $\mathbf{q}$-Operator

We consider the strictly proper linear time-invariant discrete-time system

$$x(k+1) = Ax(k) + Bu(k), \quad (4)$$

$$y(k) = Cx(k) + Du(k), \quad (5)$$

where $x(k) \in \mathbb{R}^n, u(k) \in \mathbb{R}$, and $y(k) \in \mathbb{R}^2$. We reformulate (4) using the forward-shift $\mathbf{q}$ operator so that

$$\mathbf{q}x(k) = Ax(k) + Bu(k). \quad (6)$$

Solving for $x(k)$ in terms of $u(k)$, we obtain

$$x(k) = (\mathbf{q}I - A)^{-1}Bu(k), \quad (7)$$

and thus

$$y(k) = \left[C(\mathbf{q}I - A)^{-1}B + D\right] u(k). \quad (8)$$

We therefore have the time-series model

$$\delta(\mathbf{q})y(k) = \eta(\mathbf{q})u(k), \quad (9)$$

where

$$\delta(\mathbf{q}) \triangleq \det(\mathbf{q}I - A), \quad (10)$$

$$\eta(\mathbf{q}) \triangleq C\mathrm{adj}(\mathbf{q}I - A)B + D, \quad (11)$$

and adj$(\cdot)$ denotes the *adjugate* operator.

3.2 Extension to Pseudo Transfer Functions

Let $C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$ and let the scalar outputs $y_1(k)$ and $y_2(k)$ be given by

$$y_1(k) = \left[C_1(\mathbf{q}I - A)^{-1}B + D_1\right] u(k), \quad (12)$$

$$y_2(k) = \left[C_2(\mathbf{q}I - A)^{-1}B + D_2\right] u(k). \quad (13)$$

We can express (12) and (13) as

$$\delta(\mathbf{q})y_1(k) = \eta_1(\mathbf{q})u(k), \quad (14)$$

$$\delta(\mathbf{q})y_2(k) = \eta_2(\mathbf{q})u(k), \quad (15)$$

where, for $i = 1, 2$,

$$\eta_i(\mathbf{q}) \triangleq C_i\mathrm{adj}(\mathbf{q}I - A)B + D_i. \quad (16)$$

Multiplying (14) by $\eta_2(\mathbf{q})$ and (15) by $\eta_1(\mathbf{q})$ yields

$$\eta_2(\mathbf{q})\delta(\mathbf{q})y_1(k) = \eta_2(\mathbf{q})\eta_1(\mathbf{q})u(k), \quad (17)$$

$$\eta_1(\mathbf{q})\delta(\mathbf{q})y_2(k) = \eta_1(\mathbf{q})\eta_2(\mathbf{q})u(k). \quad (18)$$

Since polynomials in $\mathbf{q}$ commute, (17) can be re-written as

$$\delta(\mathbf{q})\eta_2(\mathbf{q})y_1(k) = \eta_1(\mathbf{q})\eta_2(\mathbf{q})u(k). \quad (19)$$

Furthermore, (18) can be re-written as

$$\delta(\mathbf{q})\eta_1(\mathbf{q})y_2(k) = \eta_1(\mathbf{q})\eta_2(\mathbf{q})u(k). \quad (20)$$

Then, subtracting (20) from (19), we have

$$\delta(\mathbf{q})\left[\eta_2(\mathbf{q})y_1(k) - \eta_1(\mathbf{q})y_2(k)\right] = 0. \quad (21)$$

We thus have the pseudo transfer function

$$y_1(k) = \frac{\delta(\mathbf{q})\eta_1(\mathbf{q})}{\delta(\mathbf{q})\eta_2(\mathbf{q})} y_2(k). \quad (22)$$

In the following subsections, we show that the common factor $\delta(\mathbf{q})$ can be canceled; that is, we show that the outputs $y_1(k)$ and $y_2(k)$ satisfy

$$y_1(k) = \frac{\eta_1(\mathbf{q})}{\eta_2(\mathbf{q})} y_2(k). \quad (23)$$

3.3 Matrix Formulation

Since $A \in \mathbb{R}^{n\times n}$, we can write

$$\eta_1(\mathbf{q}) \triangleq \sum_{i=0}^{n} n_1(i)\mathbf{q}^i, \tag{24}$$

$$\eta_2(\mathbf{q}) \triangleq \sum_{i=0}^{n} n_2(i)\mathbf{q}^i, \tag{25}$$

and

$$\delta(\mathbf{q}) = \sum_{i=0}^{n} d(i)\mathbf{q}^i. \tag{26}$$

We then express (21) as

$$\Delta v = 0, \tag{27}$$

where, for $l > 2n$ data, $\Delta \in \mathbb{R}^{(l-2n)\times(l-n)}$ is defined by

$$\Delta \triangleq \begin{bmatrix} d(0) & \dots & d(n) & 0 & \dots & 0 \\ 0 & d(0) & \dots & d(n) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & d(0) & \dots & d(n) \end{bmatrix}. \tag{28}$$

Furthermore, $v \in \mathbb{R}^{l-n}$ is given by

$$v = NY, \tag{29}$$

where $N \in \mathbb{R}^{(l-n)\times 2l}$ is given by

$$N \triangleq [\, N_2 \ -N_1 \,], \tag{30}$$

and N_1, $N_2 \in \mathbb{R}^{(l-n)\times l}$ are defined by

$$N_i \triangleq \begin{bmatrix} n_i(0) & \dots & n_i(n) & 0 & \dots & 0 \\ 0 & n_i(0) & \dots & n_i(n) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & n_i(0) & \dots & n_i(n) \end{bmatrix}. \tag{31}$$

Finally, $Y \in \mathbb{R}^{2l}$ is given by

$$Y \triangleq \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \triangleq \begin{bmatrix} y_1(0) \\ \vdots \\ y_1(l-1) \\ y_2(0) \\ \vdots \\ y_2(l-1) \end{bmatrix}. \tag{32}$$

Combining (27) and (29) yields

$$\Delta N Y = 0, \tag{33}$$

which is an equivalent matrix formulation of (22). The following result gives a sufficient condition for (33) to imply $NY = 0$, which is an equivalent matrix formulation of (23).

Proposition 3.1. The following statements are equivalent:

i) $NY \in \mathcal{N}(\Delta)^{\perp}$.
ii) $NY \in \mathcal{R}(\Delta^{\mathrm{T}})$.
iii) $\operatorname{rank}\left[\, \Delta^{\mathrm{T}} \ NY \,\right] = \operatorname{rank}(\Delta)$.

In this case, (33) implies that $NY = 0$.

Proof 1. Note that $\mathcal{N}(\Delta)^{\perp} = \mathcal{R}(\Delta^{\mathrm{T}})$; see [9, p. 31]. Now, assume that $\Delta NY = 0$ and let $z \in \mathbb{R}^{l-2n}$ be such that $NY = \Delta^{\mathrm{T}} z$. Then, $\Delta\Delta^{\mathrm{T}} z = 0$, which implies $z^{\mathrm{T}}\Delta\Delta^{\mathrm{T}} z = 0$, and therefore $\Delta^{\mathrm{T}} z = 0$. Then $NY = 0$. □

3.4 Free and Forced Responses

For $i = 1, 2$, it follows from (4) and (5) that (see [8, p. 129]),

$$Y_i = \Gamma_i x_0 + \mathcal{H}_i U, \tag{34}$$

where

$$\Gamma_i \triangleq \begin{bmatrix} C_i \\ C_i A \\ \vdots \\ C_i A^{l-1} \end{bmatrix} \in \mathbb{R}^{l\times n}, \tag{35}$$

$$\mathcal{H}_i \triangleq \begin{bmatrix} D_i & 0 & & \dots & 0 \\ C_i B & D_i & 0 & \dots & 0 \\ C_i AB & C_i B & D_i & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ C_i A^{l-2} B & \dots & & C_i B & D_i \end{bmatrix} \in \mathbb{R}^{l\times l}, \tag{36}$$

and

$$U \triangleq \begin{bmatrix} u(0) \\ \vdots \\ u(l-1) \end{bmatrix} \in \mathbb{R}^{l}. \tag{37}$$

Then we define

$$Y_{i,\mathrm{free}} \triangleq \Gamma_i x(0) \tag{38}$$

and

$$Y_{i,\mathrm{forced}} \triangleq \mathcal{H}_i U, \tag{39}$$

so that

$$Y_i = Y_{i,\mathrm{free}} + Y_{i,\mathrm{forced}}. \tag{40}$$

Furthermore,

$$Y = \Gamma x_0 + \mathcal{H} U, \tag{41}$$

where

$$\Gamma \triangleq \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix}, \quad \mathcal{H} \triangleq \begin{bmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{bmatrix}. \tag{42}$$

Finally,

$$Y = Y_{\mathrm{free}} + Y_{\mathrm{forced}}, \tag{43}$$

where

$$Y_{\mathrm{free}} \triangleq \begin{bmatrix} Y_{1,\mathrm{free}} \\ Y_{2,\mathrm{free}} \end{bmatrix}, \quad Y_{\mathrm{forced}} \triangleq \begin{bmatrix} Y_{1,\mathrm{forced}} \\ Y_{2,\mathrm{forced}} \end{bmatrix}. \tag{44}$$

Hence, (43) can be written as

$$\Delta N (Y_{\mathrm{free}} + Y_{\mathrm{forced}}) = 0. \tag{45}$$

Lemma 3.2.

$$N_2 \mathcal{H}_1 = N_1 \mathcal{H}_2. \tag{46}$$

Proof 2. **FIR case:** Assume the system given by (4)-(5) is FIR with $n < k < l-2$ and $A^k = 0$. Then N_i is composed of Markov Parameters $H_{i,j}$ such that

$$N_i = \begin{bmatrix} H_{i,n} & \dots & H_{i,0} & 0 & \dots & 0 \\ 0 & H_{i,n} & \dots & H_{i,0} & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & H_{i,n} & \dots & H_{i,0} \end{bmatrix}. \tag{47}$$

Noting $H_{i,j} \in \mathbb{R}$,

$$N_1\mathcal{H}_2 = \begin{bmatrix} \sigma_1 & \sigma_2 & \dots & \sigma_3 & 0 & \dots & \dots & \dots & 0 \\ \sigma_4 & \sigma_1 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \sigma_5 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sigma_5 & \dots & \sigma_4 & \sigma_1 & \sigma_2 \dots & \sigma_3 \end{bmatrix} = N_2\mathcal{H}_1,$$

where

$$\sigma_1 = H_{1,n}H_{2,0} + \ldots + H_{1,0}H_{2,n},$$
$$\sigma_2 = H_{1,n-1}H_{2,0} + \ldots + H_{1,0}H_{2,n-1},$$
$$\sigma_3 = H_{1,0}H_{2,0},$$
$$\sigma_4 = H_{1,n+1}H_{2,0} + \ldots + H_{1,1}H_{2,n},$$
$$\sigma_5 = H_{1,n}H_{2,k}. \qquad \square$$

Proposition 3.3.

$$NY_{\text{forced}} = 0. \tag{48}$$

Proof 3. With $x(0) = 0$, (34) implies

$$N_2Y_{1,\text{forced}} = N_2\mathcal{H}_1U, \tag{49}$$

$$N_1Y_{2,\text{forced}} = N_1\mathcal{H}_2U. \tag{50}$$

Subtracting (50) from (49) and invoking (46), we have

$$\begin{aligned} NY_{\text{forced}} &= N_2Y_{1,\text{forced}} - N_1Y_{2,\text{forced}} \\ &= N_2\mathcal{H}_1U - N_1\mathcal{H}_2U \\ &= 0. \end{aligned} \qquad \square$$

Lemma 3.4.

$$N_2\Gamma_1 = N_1\Gamma_2. \tag{51}$$

Example 3.5. **FIR case, n = 2:** Let

$$C_1 = [\, c_{11} \;\; c_{12} \,], \quad C_2 = [\, c_{21} \;\; c_{22} \,],$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Then, noting $A^2 = 0$, we have

$$N_1\Gamma_2 = \begin{bmatrix} C_1AB & C_1B & D_1 & 0 & \dots & 0 \\ 0 & C_1AB & C_1B & D_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & C_1AB & C_1B & D_1 \end{bmatrix} \begin{bmatrix} C_2 \\ C_2A \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} C_1ABC_2 + C_1BC_2A \\ C_1ABC_2A \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} c_{21}c_{11} & c_{22}c_{11} + c_{12}c_{21} \\ 0 & c_{21}c_{11} \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}$$

$$= N_2\Gamma_1.$$

Proposition 3.6.

$$NY_{\text{free}} = 0. \tag{52}$$

Proof 4. With $u(k) \equiv 0$, (34) implies

$$N_2Y_{1,\text{free}} = N_2\Gamma_1x(0), \tag{53}$$

$$N_1Y_{2,\text{free}} = N_1\Gamma_2x(0). \tag{54}$$

Subtracting (54) from (53) and invoking (51), we have

$$\begin{aligned} NY_{\text{free}} &= N_2Y_{1,\text{free}} - N_1Y_{2,\text{free}} \\ &= N_2\Gamma_1x(0) - N_1\Gamma_2x(0) \\ &= 0. \end{aligned} \qquad \square$$

Example 3.7. **IIR Case:** Consider

$$y_1(k) = \frac{1}{\mathbf{q}^2 - b\mathbf{q} - a}u(k), \quad y_2(k) = \frac{\mathbf{q}}{\mathbf{q}^2 - b\mathbf{q} - a}u(k).$$

Therefore,

$$A = \begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = [\, 1 \;\; 0 \,],$$

$$C_2 = [\, 0 \;\; 1 \,], \quad D = 0.$$

Finally, let $l = 5 > 2n = 4$. Then

$$N_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\Gamma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ a & b \\ ab & a + b^2 \\ a^2 + ab^2 & 2ab + b^3 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 1 \\ a & b \\ ab & a + b^2 \\ a^2 + ab^2 & 2ab + b^3 \\ 2a^2b + ab^3 & a^2 + 3ab^2 + b^4 \end{bmatrix},$$

$$\mathcal{H}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ b & 1 & 0 & 0 & 0 \\ a + b^2 & b & 1 & 0 & 0 \end{bmatrix}, \quad \mathcal{H}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ b & 1 & 0 & 0 & 0 \\ a + b^2 & b & 1 & 0 & 0 \\ 2ab + b^3 & a + b^2 & b & 1 & 0 \end{bmatrix}.$$

Hence,

$$N_2\mathcal{H}_1 = N_1\mathcal{H}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ b & 1 & 0 & 0 & 0 \end{bmatrix},$$

which confirms (46). Furthermore,

$$N_2\Gamma_1 = N_1\Gamma_2 = \begin{bmatrix} 0 & 1 \\ a & b \\ ab & a + b^2 \end{bmatrix},$$

which confirms (51).

Propositions 3.3 and 3.6 yield the following result, which confirms the validity of (23).

Theorem 3.8.

$$NY = 0. \tag{55}$$

Therefore, (55) implies that cancelation of the $\delta(\mathbf{q})$ in the numerator and denominator of (22) is valid. In this case, the order of the pseudo transfer function does not exceed the plant order by more than 1. Furthermore, the order of the pseudo transfer function is less than or equal to the plant order if the individual transfer functions are strictly proper.

3.5 Estimation of a Pseudo Transfer Function

To identify a pseudo transfer function from $y_2(k)$ to $y_1(k)$, we divide (23) by $\mathbf{q}$ to obtain

$$y_1(k-1) = \frac{\eta_1(\mathbf{q})}{\mathbf{q}\eta_2(\mathbf{q})}y_2(k). \tag{56}$$

Therefore, causal identification increases the order of the PTF by 1. Because $\eta_i(\mathbf{q})$ has order less than or equal to $\delta(\mathbf{q})$, the order of the causal PTF is at most one greater than the order of the system.

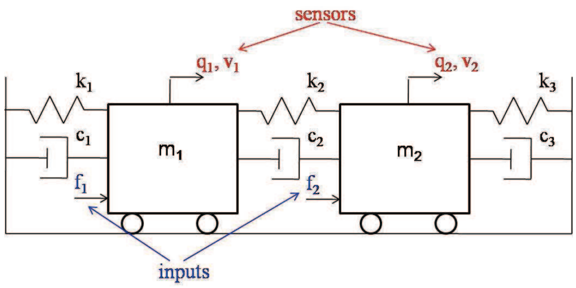

Fig. 3. Mass-spring-damper structure.

4. PTF DERIVATION FOR A MASS-SPRING-DAMPER STRUCTURE

Consider the mass-spring-damper structure shown in Figure 3. For this example, we derive the PTF from the position q_1 of mass m_1 to a delayed measurement of the velocity v_2 of mass m_2.

The equations of motion are given by

$$M\ddot{x} + C\dot{x} + Kx = F, \tag{57}$$

where

$$x = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix},\ M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix},\ C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix},$$
$$K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix},\ F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} u. \tag{58}$$

The compliance transfer function from f to q_1 is given by

$$\frac{q_1(s)}{f(s)} = \frac{m_2 s^2 + (2c_2 + c_3)s + (2k_2 + k_3)}{D(s)}, \tag{59}$$

where

$$D(s) = [m_1 s^2 + (c_1 + c_2)s + (k_1 + k_2)][m_2 s^2 + (c_2 + c_3)s + (k_2 + k_3)] - (c_2 s + k_2)^2. \tag{60}$$

Similarly,

$$\frac{v_2(s)}{f(s)} = \frac{m_1 s^3 + (c_1 + 2c_2)s^2 + (k_1 + 2k_2)s}{D(s)}. \tag{61}$$

Dividing (61) by (59) yields the PTF from q_1 to v_2, given by

$$\frac{v_2(s)}{q_1(s)} = \frac{m_1 s^3 + (c_1 + 2c_2)s^2 + (k_1 + 2k_2)s}{m_2 s^2 + (2c_2 + c_3)s + (2k_2 + k_3)}. \tag{62}$$

We discretize (62) using zero-order hold, and, to reflect a delay of one time step between the pseudo input data q_1 and the pseudo output data v_2, we multiply the discretized version of (62) by $\mathbf{q}^{-1}$.

5. NUMERICAL EXAMPLES

We identify PTFs for the analytical model described by (62) and Figure 3. We assume that the system is excited by an unknown exogenous input, and we use measurement data for x_1 and v_2. We then identify the PTF from q_1 to v_2 using a μ-Markov structure RLS algorithm, taking advantage of zero buffering [10]. To verify the identified PTFs, we identify the first 4 Markov parameters and compare these with their corresponding analytical values.

5.1 White Exogenous Input, Zero Initial Conditions

Setting the initial condition of the system to zero, we excite the system with an unknown exogenous white noise signal.

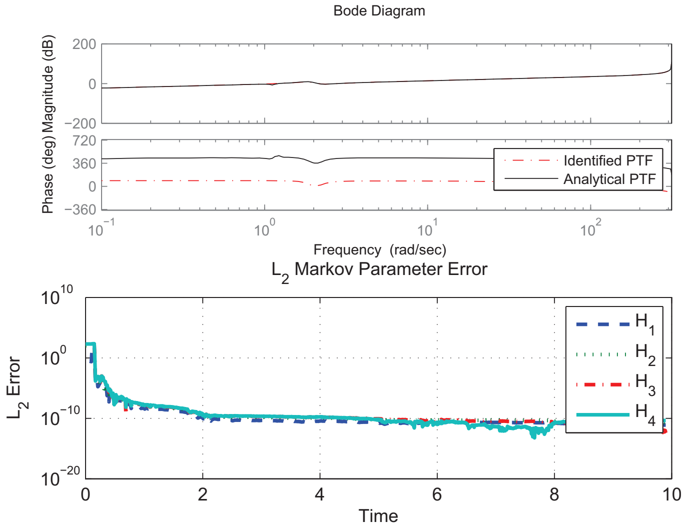

Fig. 4. Numerical simulation with zero initial conditions. The unknown exogenous input is $u_k \sim N(0,1)$, and $x_0 = [0,\ 0,\ 0,\ 0]^{\mathrm{T}}$. The performance of the identification is evaluated by comparing the frequency response and Markov parameters of the true PTF and the identified PTF.

The upper plot of Figure 4 compares the frequency response of the identified PTF with the analytical result. The lower plot compares the L_2 error of the identified Markov parameters with the analytically derived Markov parameters. From Figure 4, we see that the identified PTF closely matches the analytical PTF, the apparent difference between the identified and analytical phase plots is 360°.

5.2 White Exogenous Input, Non-Zero Initial Conditions

Choosing a nonzero initial condition, we again excite the system with an unknown exogenous white noise signal.

The upper plot of Figure 5 compares the frequency response of the identified PTF with the analytical result. The lower plot compares the L_2 error of the identified Markov parameters with the analytically derived Markov parameters. From Figure 5, we see that the identified PTF closely approximates the analytical PTF. The difference between the true PTF phase and the identified PTF phase is 360°.

5.3 System Parameter Change Detection for a White Exogenous Input and Non-Zero Initial Conditions

We investigate whether changes in the PTFs can be used to detect changes in system parameters. Therefore, we simulate 10 seconds of the response of (57) to white noise input. At $t = 5$ seconds, the stiffness coefficients in K are reduced by a factor of 3 and the damping coefficients in C are increased by a factor of 6.

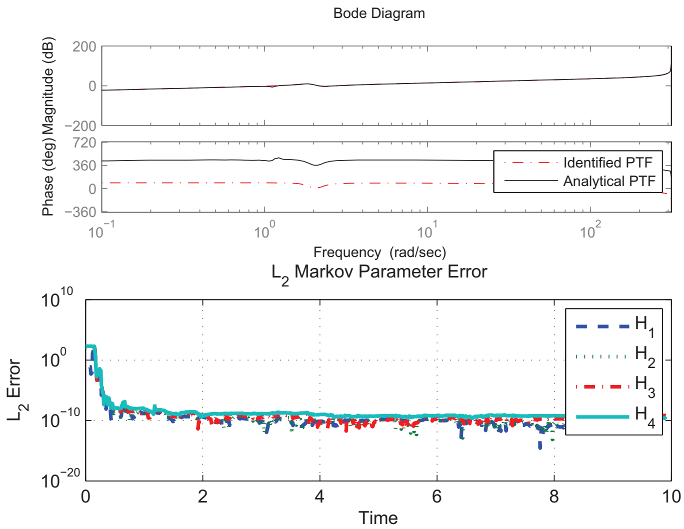

Fig. 5. Numerical simulation with nonzero initial conditions. The unknown exogenous input is $u_k \sim N(0,1)$, and $x_0 = [1,\ 2,\ 3,\ 4]^{\mathrm{T}}$.The performance of the identification is evaluated by comparing the frequency response and Markov parameters of the true PTF and the identified PTF.

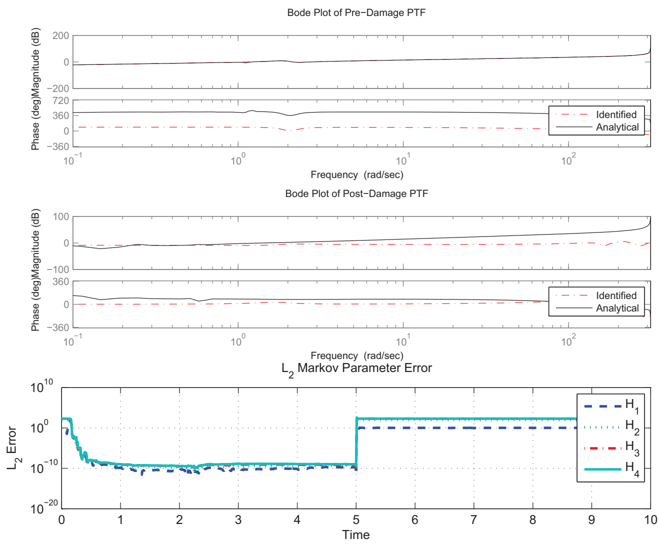

Fig. 6. In this example the stiffness and damping coefficients are changed at t = 5 s to simulate damage. The unknown exogenous input is $u \sim N(0,1)$, and the initial state is $x_0 = [1,\ -2,\ -1,\ 3]^{\mathrm{T}}$. The bottom plot shows the time history of the identified Markov parameter error, the increase in error at the time instance when damage occurs is clearly seen at $t = 5s$.

From the lower plot of Figure 6, we observe an abrupt increase in Markov parameter error at the time of damage, indicating that the system model has changed. Furthermore, the upper plot of Figure 6 shows that we can accurately identify the PTF associated with the pre-damage model. However, the center plot of Figure 6 shows that the identified PTF does not match the analytical post-damage model, this is because the identification contains data from the pre-damaged and post-damaged systems. To obtain an accurate model of the post-damaged system the identification algorithm must be reinitialized.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we investigate an approach to model-based, data-driven fault detection using only output data to construct a PTF associated with a system. We show that the PTF is independent of the input and initial condition, and has order no greater than one higher than the system. We validate the accuracy of the identified PTF for a linear two DOF system and use the identified PTF to detect changes in the system parameters, which could result from structural damage.

Future work will focus on proving that the SONBI technique is valid for IIR models, extending the technique to multiple-input systems, and investigating potential implications for nonlinear systems.

7. ACKNOWLEDGMENTS

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