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# Retrospective Cost Adaptive Control of Spacecraft Attitude Using Magnetic Actuators

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**We apply retrospective cost adaptive control (RCAC) to spacecraft attitude control with magnetic actuators. We compare two approaches to address the rank deficiency and time-varying nature of the input matrix. The first approach utilizes an average of the magnetic field based on a-priori knowledge, whereas the second approach uses three multi-input, single-output controllers. RCAC uses no information about the spacecraft inertia, and model information is limited to the input-output relation given by the first Markov parameter, which is computed from an inertia-free linearization of Euler's and Poisson's equations. We examine two problems for each of the controllers. For both problems, the spacecraft has an arbitrary initial angular rate and initial attitude. The objective for the first problem is to bring the spacecraft to rest at a specified attitude, while the second problem seeks to bring the spacecraft to spin about an inertially pointed body axis.**

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# I. Introduction

The attitude of a spacecraft in low-Earth orbit can be either passively stabilized or actively controlled. Passive methods exploit environmental torques to achieve a stable attitude. However, the range of reachable attitudes is limited by the specific method utilized [1]. Active control actuators generate torques that provide three-axis control and are able to reach any attitude. However, the cost of actuators such as reaction wheels and thrusters for use in small spacecraft is high and their reliability is unproven. Alternatively, magnetic coils, produce torque by creating a magnetic field that interacts with the Earth's local magnetic field.

Magnetic coils are commonly used to reduce momentum in spacecraft that use momentum storage devices such as reaction wheels [2]. Magnetic actuators are also used to de-spin spacecraft after launch vehicle separation [3]. However, the torque produced by the coils is constrained to the plane orthogonal to the Earth's local magnetic field vector. This lack of instantaneous controllability along with low-torque capability, and low pointing accuracy make magnetic coils impractical for three-axis attitude control of large spacecraft. Yet, as the size of the spacecraft decreases and pointing accuracy requirements are relaxed, the benefits of magnetic coils, such as small size, ease of manufacturing, and low power consumption, outweigh the challenges in the design and operation of these control systems [4]. Thus, the application of active magnetic coils for three-axis attitude control of small spacecraft has gained interest in recent years [5].

Attitude regulation methods for magnetic control typically rely on a model of the spacecraft dynamics and kinematics, the spacecraft mass properties, and a model of the magnetic field. Control techniques include proportional-derivative control, optimal control, and nonlinear methods [6]. However, these methods may fail when accurate modeling information is not available. Thus, a control law that reduces the required modeling information is desired.

In this paper we develop a controller that utilizes measurements of the local magnetic field without knowledge of the mass properties. This control law is based on retrospective cost adaptive control (RCAC) which is a multi-input, multi-output direct adaptive controller. RCAC utilizes the input and output history of the system combined with Markov parameters to update teh control law and determine the next control input [7], [8], [9], [10]. Previous applications of RCAC to spacecraft attitude control have been limited to thruster and reaction-wheel actuation [11], [12], [13]. In these applications, RCAC was applied to motion-to-rest (M2R) maneuvers, where the controller brings the body to rest at a specified attitude and motion-to-spin maneuvers (M2S), where the spacecraft is brought to spin about an inertially pointed body axis.

We apply RCAC to the attitude regulation of a rigid body spacecraft using magnetic actuators. First, we develop the nonlinear equations of motion for a rigid body spacecraft with magnetic actuators. Then, we describe the RCAC algorithm based on a linear time-invariant system. Next, we modify the RCAC formulation to accommodate the nonlinear equations, matrix-valued attitude state, and the rank deficiency of the input matrix. Finally, we present numerical results for M2R and M2S maneuvers for spacecraft with various inertia matrices in high-inclination orbits.

# II. Spacecraft Model

We consider a rigid body controlled by magnetic torque actuators. The rotational motion is described by Euler's equation, and the kinematics are given by Poisson's equation. We define a body-fixed frame for the spacecraft, with the origin located at the center of mass, and we use an Earth-centered inertial (ECI) frame to determine the attitude of the spacecraft. Thus, the spacecraft motion is described by

$$J_{\mathrm{SC}}\dot{\omega} = (J_{\mathrm{SC}}\omega) \times \omega + B_{\mathrm{SC}}u + z_{\mathrm{dist}}, \tag{1}$$

$$\dot{R} = R\omega^{\times}, \tag{2}$$



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where $\omega \in \mathbb{R}^3$ is the angular velocity of the body frame with respect to the ECI frame resolved in the spacecraft frame and $J_{\rm SC} \in \mathbb{R}^{3\times 3}$ is the constant inertia dyadic of the spacecraft relative to the spacecraft center of mass resolved in the spacecraft frame. The proper orthogonal matrix (that is, the rotation matrix) $R \in \mathbb{R}^{3\times 3}$ transforms the components of a vector resolved in the spacecraft frame into the components of the same vector resolved in the inertial frame, and $\omega^\times$ is the skew-symmetric cross-product matrix of $\omega$.

The product $B_{\rm SC}u$, where $B_{\rm SC} \in \mathbb{R}^{3\times l_u}$, determines the applied torque about each axis of the spacecraft frame due to the control input vector $u \in \mathbb{R}^{l_u}$. The vector $z_d$ represents disturbance torques, that is, all internal and external torques applied to the spacecraft aside from control torques. These disturbances may be due to onboard components, gravity gradients, solar pressure, atmospheric drag, or the ambient magnetic field. For convenience in (1) and (2) we omit the argument $t$, recognizing that $\omega, R, u$, and $z_d$ are time-varying quantities.

We assume that both rate (inertial) and attitude (noninertial) measurements are available. Gyro measurements $y_{\rm rate} \in \mathbb{R}^3$ provide measurements of the angular velocity resolved in the spacecraft frame, that is,

$$y_{\rm rate} = \omega. \tag{3}$$

Attitude is measured indirectly using sensors such as star trackers. The attitude measurement is determined to be

$$y_{\rm attitude} = R. \tag{4}$$

For simplicity, we assume that both rate and attitude measurements are available without noise and that gyro bias, if present, has been corrected.

The objective of the attitude control problem is to determine control inputs such that the spacecraft attitude $R$ follows a commanded attitude trajectory given by a possibly time-varying $\mathrm{C}^1$ rotation matrix $R_{\rm d}(t)$. For $t \geq 0$, $R_{\rm d}(t)$ is given by

$$\dot{R}_{\rm d}(t) = R_{\rm d}(t)\omega_{\rm d}(t)^\times, \tag{5}$$

$$R_{\rm d}(0) = R_{\rm d0}, \tag{6}$$

where $\omega_{\rm d}$ is the desired, possibly time-varying angular velocity. The attitude error, that is, the rotation between $R(t)$ and $R_{\rm d}(t)$, is given by

$$\tilde{R} \triangleq R_{\rm d}^{\rm T} R, \tag{7}$$

which satisfies Poisson's equation

$$\dot{\tilde{R}} = \tilde{R}\tilde{\omega}^\times, \tag{8}$$

where the angular velocity error $\tilde{\omega}$ is defined by

$$\tilde{\omega} \triangleq \omega - \tilde{R}^{\rm T}\omega_{\rm d}. \tag{9}$$

# III. The RCAC Algorithm

RCAC is a discrete-time output-feedback controller that minimizes the command-following error corresponding to the performance variable $z$. The algorithm does not require detailed plant information, instead, RCAC uses knowledge of the system's input response as described by Markov parameters. Although RCAC is derived for linear systems, we apply it to the nonlinear spacecraft model by using Markov parameters from the linearized dynamics.



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## A. The Extended System and Retrospective Cost

Consider the MIMO discrete-time linear system

$$x(k+1) = Ax(k) + Bu(k), \tag{10}$$
$$z(k) = E_1 x(k) - E_0 r(k), \tag{11}$$

where $x(k) \in \mathbb{R}^{l_x}$, $z(k) \in \mathbb{R}^{l_z}$, $u(k) \in \mathbb{R}^{l_u}$, $r(k) \in \mathbb{R}^{l_r}$, and $k \geq 0$. We can rewrite (11) as

$$z(k) = E_1 A x(k-1) + E_1 B u(k-1) - E_0 r(k). \tag{12}$$

Then, we collect the terms in (12) such that

$$z(k) = S(k) + \mathcal{H} u(k-1), \tag{13}$$

where

$$S(k) = E_1 A x(k-1) - E_0 r(k) \tag{14}$$

and $\mathcal{H} = E_1 B$ is the first Markov parameter of the system.

For a positive integer $s$, define the extended performance as

$$\begin{aligned} \bar{Z}(k) &= \begin{bmatrix} z(k) \\ z(k-1) \\ \vdots \\ z(k-s) \end{bmatrix} \\ &= \bar{S}(k) + \bar{H}\bar{U}(k-1), \end{aligned} \tag{15}$$

where

$$\bar{S}(k) = \begin{bmatrix} S(k) \\ S(k-1) \\ \vdots \\ S(k-s) \end{bmatrix} \in \mathbb{R}^{sl_z}, \tag{16}$$

$$\bar{H} = \begin{bmatrix} \mathcal{H} & & & \\ & \mathcal{H} & & \\ & & \ddots & \\ & & & \mathcal{H} \end{bmatrix} \in \mathbb{R}^{sl_z \times sl_u}, \tag{17}$$

$$\bar{U}(k-1) = \begin{bmatrix} u(k-1) \\ u(k-2) \\ \vdots \\ u(k-s) \end{bmatrix} \in \mathbb{R}^{sl_u}. \tag{18}$$

We replace the control inputs in (15) with the retrospective controls $\hat{\bar{U}}(k-1)$ and define the *extended retrospective performance*

$$\hat{\bar{Z}}(k) = \bar{S}(k) + \bar{H}\hat{\bar{U}}(k-1). \tag{19}$$



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Next, we subtract the extended performance in (15) from the extended retrospective performance in (19) and obtain

$$\hat{Z}(k) = Z(k) - H\bar{U}(k-1) + \hat{\bar{U}}(k-1). \tag{20}$$

We wish to find the retrospective control inputs $\hat{\bar{U}}(k-1)$ that minimize the retrospective performance $\hat{\bar{Z}}(k)$.

Thus, we define the *retrospective cost function*

$$J(\hat{\bar{U}}(k-1), k) \triangleq \hat{\bar{Z}}^{\mathrm{T}}(k)R_Z(k)\hat{\bar{Z}}(k) + \hat{\bar{U}}(k-1)^{\mathrm{T}}R_U(k)\hat{\bar{U}}(k-1)Z(k), \tag{21}$$

where $R_Z(k) \in \mathbb{R}^{sl_z \times sl_z}$ and $R_U(k) \in \mathbb{R}^{sl_u \times sl_u}$ are positive-definite weighting matrices on the performance and the control respectively. Using (20) we rewrite the cost function as

$$J(\hat{\bar{U}}(k-1), k) = \hat{\bar{U}}(k-1)^{\mathrm{T}}\mathcal{A}(k)\hat{\bar{U}}(k-1) + \hat{\bar{U}}^{\mathrm{T}}(k-1)\mathcal{B}^{\mathrm{T}}(k)\hat{\bar{U}} + \mathcal{C}(k), \tag{22}$$

where

$$\mathcal{A}(k) \triangleq \bar{H}^{\mathrm{T}}R_Z(k)\bar{H} + R_U(k), \tag{23}$$

$$\mathcal{B}(k) \triangleq 2\bar{H}^{\mathrm{T}}R_Z(k)[\bar{Z}(k) - \bar{H}\bar{U}(k-1)], \tag{24}$$

$$\mathcal{C}(k) \triangleq \bar{Z}^{\mathrm{T}}(k)R_Z(k)\bar{Z}(k) - 2\bar{Z}^{\mathrm{T}}(k)R_Z(k)\bar{H}\bar{U}(k-1) + \bar{U}^{\mathrm{T}}(k-1)\bar{H}^{\mathrm{T}}R_Z(k)\bar{H}\bar{U}(k-1). \tag{25}$$

If $\mathcal{A}(k)$ is positive definite, the unique minimizer for $J(\bar{U}(k-1), k)$ is

$$\hat{\bar{U}}(k-1) = -\frac{1}{2}\mathcal{A}^{-1}(k)\mathcal{B}(k). \tag{26}$$

## B. Controller Construction

We utilize the retrospective controls $\hat{\bar{U}}(k-1)$ in order to compute the next control input $u(k)$. We design a strictly proper time-series controller of order $n_{\mathrm{c}}$ given by

$$u(k) = \sum_{i=1}^{n_{\mathrm{c}}} M_i(k)u(k-i) + \sum_{i=1}^{n_{\mathrm{c}}} N_i(k)z(k-i), \tag{27}$$

where, for all $i = 1, \ldots, n_{\mathrm{c}}$, $M_i(k) \in \mathbb{R}^{l_u \times l_u}$ and $N_i(k) \in \mathbb{R}^{l_u \times l_z}$. The control (27) can be expressed as

$$u(k) = \theta(k)\phi(k-1), \tag{28}$$

where

$$\theta(k) \triangleq [M_1(k) \ \cdots \ M_{n_{\mathrm{c}}}(k) \ N_1(k) \ \cdots \ N_{n_{\mathrm{c}}}(k)] \in \mathbb{R}^{l_u \times n_{\mathrm{c}}(l_u + l_z)}, \tag{29}$$

and

$$\phi(k-1) \triangleq \begin{bmatrix} u(k-1) \\ \vdots \\ u(k-n_{\mathrm{c}}) \\ z(k-1) \\ \vdots \\ z(k-n_{\mathrm{c}}) \end{bmatrix} \in \mathbb{R}^{n_{\mathrm{c}}(l_u + l_z)}. \tag{30}$$



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### C. Recursive Least Squares Update of $\theta(k)$

We compute the parameter $\theta(k)$ by solving a recursive least squares problem. In order for the applied controls $\bar{U}(k)$ to approach the retrospective controls $\hat{\bar{U}}(k)$ we minimize the cost function

$$\begin{aligned} J(\theta(k)) &\triangleq [\hat{u}(k) - u(k)]^{\mathrm{T}}[\hat{u}(k) - u(k)], \\ &= [\hat{u}(k) - \theta(k)\phi(k-1)]^{\mathrm{T}}[\hat{u}(k) - \theta(k)\phi(k-1)], \\ &= \|\hat{u}(k) - \theta(k)\phi(k-1)\|^2. \end{aligned} \tag{31}$$

The minimizer for (31) is

$$\begin{aligned} \theta^{\mathrm{T}}(k) \triangleq\ & \theta^{\mathrm{T}}(k-1) + P(k-1)\phi(k-q_g-1)[\phi^{\mathrm{T}}(k-q_g-1)P(k-1)\phi(k-q_g-1)]^{-1} \\ & \cdot [\theta(k-1)\phi(k-q_g-1) - \hat{u}(k-q_g)]^{\mathrm{T}}. \end{aligned} \tag{32}$$

The error covariance is updated by

$$\begin{aligned} P(k) \triangleq\ & P(k-1) - P(k-1)\phi(k-q_g-1) \\ & \cdot [\phi^{\mathrm{T}}(k-q_g-1)P(k-1)\phi(k-q_g-1)]^{-1}\phi^{\mathrm{T}}(k-q_g-1)P(k-1). \end{aligned} \tag{33}$$

## IV. Modifications to RCAC for Magnetic Control of Spacecraft Attitude

RCAC is applied to discrete-time linear systems where the state $x(k)$ in (10) is a vector. However, the spacecraft equations (1), (2) are nonlinear, and the attitude state $R$ is a matrix. Thus, the spacecraft equations must be modified in order to apply RCAC as developed in Section III. Furthermore, the magnetic constraints on the control torque introduce additional difficulties in the computation of the retrospective controls $\hat{\bar{U}}(k-1)$ in (26) and the implementation of the control input $u(k)$ in (28).

Thus, we develop the equations for a vector performance variable $z$ and obtain a suitable matrix $\mathcal{H}$ for the nonlinear spacecraft equations based on the linearized system. Then, we present a torque-allocation scheme to transform the torques computed by RCAC into magnetic dipoles. Finally, we develop two methods for managing the rank deficiency in $\mathcal{H}$ caused by the singular input matrix.

### A. Performance Variable for Attitude Control

We express the rotation matrix $\tilde{R}$ as

$$\tilde{R} = \begin{bmatrix} \tilde{r}_1 \\ \tilde{r}_2 \\ \tilde{r}_3 \end{bmatrix}, \tag{34}$$

where, for $i = 1, 2, 3$, $\tilde{r}_i \in \mathbb{R}^{1\times 3}$ is a row of $\tilde{R}$. As in [11], we define the vector state

$$\tilde{r} \triangleq \begin{bmatrix} \tilde{r}_1 & \tilde{r}_2 & \tilde{r}_3 \end{bmatrix}^{\mathrm{T}}. \tag{35}$$

Next, we utilize the attitude error vector in [14] given by

$$S \triangleq \sum_{1}^{3} a_i \left(\tilde{R}^{\mathrm{T}} e_i\right) \times e_i = -M_a R_a \tilde{r}, \tag{36}$$



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where the constants $a_i$ are positive and distinct, the vector $e_i \in \mathbb{R}^3$ for $i = 1, 2, 3$ is the $i$th column of the $3 \times 3$ identity matrix, and

$$M_a = \left[\begin{array}{ccc} e_1^\times & e_2^\times & e_3^\times \end{array}\right], \in \mathbb{R}^{3\times 9} \tag{37}$$

$$R_a = \left[\begin{array}{ccc} a_1 I_3 & & \\ & a_2 I_3 & \\ & & a_3 I_3 \end{array}\right] \in \mathbb{R}^{9\times 9}. \tag{38}$$

Including the angular velocity error in (9) yields the performance variable

$$z = \begin{bmatrix} \tilde{\omega} \\ S \end{bmatrix}. \tag{39}$$

## B. Markov parameter

We rewrite (1) and (2) using the errors $\tilde{r}$ and $\tilde{\omega}$ such that

$$\dot{\tilde{\omega}} = J_{\mathrm{SC}}^{-1}\left[\left[J_{\mathrm{SC}}\left(\tilde{\omega} + \mathcal{D}(\omega_d)r\right)\right] \times \left(\tilde{\omega} + \mathcal{D}(\omega_d)r\right)\right] + \tilde{\omega} \times \left[\mathcal{D}(\omega_d)r\right] - \mathcal{D}(\dot{\omega}_d)r + J_{\mathrm{SC}}^{-1}\left(B_{\mathrm{SC}}u + z_d\right), \tag{40}$$

where, for $x \in \mathbb{R}^3$ with components $x_1, x_2, x_3$,

$$\mathcal{D}(x) \triangleq \left[\begin{array}{ccc} x_1 I_3 & x_2 I_3 & x_3 I_3 \end{array}\right] \tag{41}$$

and

$$\dot{\tilde{r}} = \begin{bmatrix} -\tilde{\omega}^\times & & \\ & -\tilde{\omega}^\times & \\ & & -\tilde{\omega}^\times \end{bmatrix} \tilde{r}. \tag{42}$$

We compute the discrete-time system matrices $A, B, E_1$ by linearizing (40) and (42) about $\tilde{\omega}_e = 0$ and $\tilde{R} = I_3$ followed by discretization with the controller time step $h$. Linearization yields the continuous-time dynamics matrices

$$A_c = \left[\begin{array}{cc} 0_{3\times 3} & 0_{3\times 9} \\ -M_a^{\mathrm{T}} & 0_{9\times 9} \end{array}\right], \tag{43}$$

$$B_c = \left[\begin{array}{c} J_{\mathrm{SC}}^{-1} B_{\mathrm{SC}} \\ 0_{9\times 3} \end{array}\right], \tag{44}$$

$$C_c = \left[\begin{array}{cc} I_3 & 0_{3\times 9} \\ 0_{3\times 3} & -M_a R_a \end{array}\right]. \tag{45}$$

Discretization of $A_c, B_c, C_c$ with the controller time step $h$ yields

$$A = e^{A_c h} = \left[\begin{array}{cc} I_3 & 0_{3\times 9} \\ -hM_a^{\mathrm{T}} & I_9 \end{array}\right], \tag{46}$$

$$B = \left(\int_0^h A d\tau\right) B_c = \left[\begin{array}{c} hJ_{\mathrm{SC}}^{-1} B_{\mathrm{SC}} \\ -\frac{h^2}{2} M_a^{\mathrm{T}} J_{\mathrm{SC}}^{-1} B_{\mathrm{SC}} \end{array}\right], \tag{47}$$

$$E_1 = C_c. \tag{48}$$

The Markov parameter corresponding to the performance in (39) is given by

$$\mathcal{H} = E_1 B = \left[\begin{array}{c} hJ_{\mathrm{SC}}^{-1} B_{\mathrm{SC}} \\ \frac{1}{2} h^2 M_a R_a M_a^{\mathrm{T}} J_{\mathrm{SC}}^{-1} B_{\mathrm{SC}} \end{array}\right]. \tag{49}$$



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As in [11], we remove the inertia information from $\mathcal{H}$ and define

$$H \triangleq \alpha \begin{bmatrix} hB_{\rm SC} \\ \frac{1}{2}h^2 M_a R_a M_a^{\rm T} B_{\rm SC} \end{bmatrix}, \tag{50}$$

where $\alpha$ is a positive scalar.

### C. Magnetic Dipole Allocation

The command $u$ computed by RCAC is the desired torque. However, the magnetic coils must generate these torques using a magnetic dipole command $d$. Given the control torque $u$ commanded by RCAC we must compute the required dipole $d$. The resulting dipole creates a torque vector that is orthogonal to the local, time-varying magnetic field $b(t) \in \mathbb{R}^3$.

The torque obtained from a magnetic dipole $d(t)$ and the Earth's magnetic field $b(t)$ is given by

$$\tau(t) = -b(t)^{\times} d(t). \tag{51}$$

We replace $\tau$ in (51) with the desired control torque $u$, and solve formally for $d$ by using the generalized inverse of the skew-symmetric matrix $b(t)^{\times}$,

$$b(t)^{\times +} = -\frac{b(t)^{\times}}{b(t)^{\rm T} b(t)} \tag{52}$$

and obtain

$$d(t) = -b(t)^{\times +} u = \frac{b(t)^{\times}}{b(t)^{\rm T} b(t)} u. \tag{53}$$

The generalized inverse projects the desired torque onto the plane orthogonal to $b(t)$ and allocates the necessary dipole $d(t)$. Thus, the control torque applied to the spacecraft is

$$\tau(t) = -b(t)^{\times} d(t) = B_{\rm SC}(t) u, \tag{54}$$

where

$$B_{\rm SC}(t) \triangleq \frac{-b(t)^{\times} b(t)^{\times}}{b(t)^{\rm T} b(t)}. \tag{55}$$

Note that, at each time instant, the rank of $B_{\rm SC}(t)$ is 2.

## V. Rank deficiency of $B_{\rm SC}$

In previous approaches to spacecraft attitude control [11], [12], [13], RCAC was set up as a multi-input, multi-output controller. The input matrix $B_{\rm SC}(t)$ in (1) is used to compute the Markov parameter matrix $\bar{H}$ in (17). However, since $b(t)^{\times}$ is skew symmetric, $\bar{H}$ is rank deficient, which prevents the inversion of $\mathcal{A}$ in (26) in the absence of the control weighting matrix $R_U$. Although it is possible to create a full rank $\mathcal{A}$ by using the control weighting matrix $R_U$, numerical studies suggest that this does not result in a successful control law.

Thus, to ensure that the product $\bar{H}^{\rm T} R_Z \bar{H}$ in (23) is invertible, we propose two modifications to the previous attitude control RCAC implementations. The first approach utilizes the average of the input matrix $B_{\rm SC}(t)$. This average matrix is shown in [15] to have full-rank for orbits that are non-equatorial, thus the resulting Markov parameter is left invertible. The second approach uses an alternate control architecture including three separate multi-input, single-output RCAC controllers instead of one multi-input, multi-output controller.



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## A. Averaged Markov parameter

Define the input matrix $B_{\rm SC}(t)$ resolved in the ECI frame as

$$B_{\rm SC}(t)\Big|_{\rm ECI} = B'_{\rm SC}(t) \triangleq \frac{-b'(t)^{\times}b'(t)^{\times}}{b'(t)^{\rm T}b'(t)} \tag{56}$$

where $b'(t)$ is the magnetic field vector resolved in the ECI frame. Next, we compute the average of $B'_{\rm SC}(t)$ over several orbits

$$\tilde{B}'_{\rm SC} = \lim_{T\to\infty}\frac{1}{T}\int_0^T B'_{\rm SC}(t)dt. \tag{57}$$

The averaged input matrix (57) is used to prove controllability and stability in [15]. We use this approach to obtain a full rank Markov parameter for RCAC. First, we transform the averaged input matrix into the spacecraft body frame

$$\tilde{B}_{\rm SC} \triangleq \tilde{B}'_{\rm SC}\Big|_{B} = R^{\rm T}\tilde{B}'_{\rm SC}R. \tag{58}$$

Then, using the average matrix in resolved in the body frame we construct the average Markov parameter

$$\tilde{\mathcal{H}} = \alpha\begin{bmatrix} hJ_{\rm SC}^{-1}\tilde{B}_{\rm SC} \\ \frac{1}{2}h^2M_aR_aM_a^{\rm T}J_{\rm SC}^{-1}\tilde{B}_{\rm SC}\end{bmatrix}. \tag{59}$$

As in (50), we remove the inertia and obtain,

$$\tilde{H} = \alpha\begin{bmatrix} h\tilde{B}_{\rm SC} \\ \frac{1}{2}h^2M_aR_aM_a^{\rm T}\tilde{B}_{\rm SC}\end{bmatrix}, \tag{60}$$

which is left invertible for non-equatorial orbits.

## B. Decentralized RCAC

For the second method, we synthesize the desired torque $u$ using three independent multi-input single-output RCAC control loops. A similar architecture is used in [16] for angular velocity control using a heuristic approach to controller construction. We define the performance for each RCAC block as

$$z_i(k) \triangleq \begin{bmatrix}\tilde{\omega}_i \\ S_i\end{bmatrix} = C'_i z(k), \tag{61}$$

where $S_i$ and $\tilde{\omega}_i$ are the $i$th components of $S$ and $\tilde{\omega}$, respectively, and

$$C'_i \triangleq \begin{bmatrix} e_i^{\rm T} & 0_{1\times3} \\ 0_{1\times3} & e_i^{\rm T}\end{bmatrix}. \tag{62}$$

We rewrite (61) using (10) and obtain

$$z_i(k) = C'_iE_1Ax(k-1) + \sum_{j=1}^{3}C'_iE_1Be_ju_j(k-1), \tag{63}$$

where $u_j(k)$ is the $j$th component of $u(k)$.



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| Parameter | Value |
|---|---|
| Inclination | $87°$ |
| Radius | 450 [km] |
| Right ascension of ascending node | 0 |
| Argument of perigee | 0 |
| Mean anomaly | 0 |
| Period ($T_{\rm orbit}$) | 5615 [sec] |

Table 1: Orbital parameters.

| parameter | Value |
|---|---|
| $n_c$ | 10 |
| $P_0$ | $100I$ |
| $R_U$ | $10I$ |
| $s$ | $\frac{T_{\rm orbit}}{h}$ |
| $\theta_0$ | 0 |
| $a_1$ | 1 |
| $a_2$ | 2 |
| $a_3$ | 3 |
| $h$ | 10 [sec] |
| $\alpha$ | 0.1 |

Table 2: RCAC parameters.

To compute the Markov parameter we assume that the sensor and actuator frames are aligned and that each component of the performance $z(k)$ is only affected by the corresponding component of the control $u(k-1)$ such that

$$z_i(k) \approx C_i' E_1 A x(k-1) + C_i' E_1 B e_i u_i(k-1). \tag{64}$$

Thus, the Markov parameter for the $i$th multi-input, single-output RCAC is

$$\mathcal{H}_i(t) = C_i' E_1 B e_i \tag{65}$$

$$= \begin{bmatrix} h e_i^{\rm T} J_{\rm SC}^{-1} B_{\rm SC}(t) \\ \frac{1}{2} h^2 e_i^{\rm T} M_a R_a M_a^{\rm T} J_{\rm SC}^{-1} B_{\rm SC}(t) \end{bmatrix} e_i. \tag{66}$$

Removing the inertia yields

$$H_i'(t) \triangleq \begin{bmatrix} h e_i^{\rm T} B_{\rm SC}(t) \\ \frac{1}{2} h^2 e_i^{\rm T} M_a R_a M_a^{\rm T} B_{\rm SC}(t) \end{bmatrix} e_i, \tag{67}$$

which is left invertible. This approach ignores the coupling between axes and only requires knowledge of the alignment between actuators and sensors.

# VI. Numerical Examples

Consider a rigid spacecraft around a high-inclination circular orbit given by the orbital parameters in Table 1. The magnetic field is computed using the International Geomagnetic Reference Field (IGRF)



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model [17]. Furthermore, all disturbances $z_{\rm d}$ are assumed to be zero. The RCAC parameters used are shown in Table 2. Furthermore, the performance weighting is given by

$$R_Z = \begin{bmatrix} \epsilon I_3 & 0 \\ 0 & \epsilon^2 I_3 \end{bmatrix}, \tag{68}$$

where $\epsilon$ is a positive number. The scaling requirements between the angular velocity and attitude error terms is explained in [15]. For the M2R examples, we set $\epsilon = 10^{-5}$.

## A. M2R Examples

Let the initial motion of the spacecraft be described by

$$\omega(0) = 0.001 \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^{\rm T} \text{rad/sec.} \tag{69}$$

We describe the initial and desired attitudes using eigenaxis rotations as defined by Rodrigues' equation

$$\mathcal{R}(\theta_e, \hat{n}_e) = \cos(\theta_e) I_3 + (1 - \cos(\theta_e)) n_e n_e^{\rm T} + \sin(\theta_e) n_e^{\times}, \tag{70}$$

where $\theta_e$ is the eigenangle and $\hat{n}_e \in \mathbb{R}^3$ is the eigenaxis. Thus, the initial attitude be given by an eigenaxis rotation of $\theta_0 = 90^\circ$ about the vector

$$n_0 = \begin{bmatrix} -0.03 & -0.9 & 0.03 \end{bmatrix}^{\rm T}. \tag{71}$$

The goal of the controller is to bring the spacecraft to rest, that is, $\omega_d = 0$, at the inertial attitude given by the eigenaxis rotation of $\theta_d = 96^\circ$ about the vector

$$n_d = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}^{\rm T}. \tag{72}$$

We test the M2R maneuver on three different rigid bodies, namely, a sphere, a cylinder, and an arbitrary body.

We assume that the sensor and actuator axes are aligned such that the inertia matrices resolved in the body frame are given by

$$J_{\rm sphere} = {\rm diag}(10, 10, 10) \text{ kg-m}^2, \tag{73}$$

$$J_{\rm cylinder} = {\rm diag}(10, 10, 5) \text{ kg-m}^2, \tag{74}$$

$$J_{\rm arbitrary} = \begin{bmatrix} 5 & -0.1 & -0.5 \\ -0.1 & 2 & 1 \\ -0.5 & 1 & 3.5 \end{bmatrix} \text{kg-m}^2. \tag{75}$$

We compare the performance of both approaches to magnetic control for the M2R maneuver using the inertias in (73), (74), and (75). Figure 1 shows the results for the sphere inertia, Figure 2 shows the results for the cylinder inertia, and Figure 3 shows the results for the arbitrary inertia. The dipoles shown in the examples indicate that both implementations, averaged and decentralized, command dipoles of the same magnitude given the same tuning parameters in Table 2. Thus, we compare both approaches based on settling time of the eigenaxis attitude error $\theta_{\rm eig}$. For all three inertias, the centralized approach based on the average Markov parameter $\tilde{H}$ settles faster than the decentralized version based on the Markov parameter $H_i'$.



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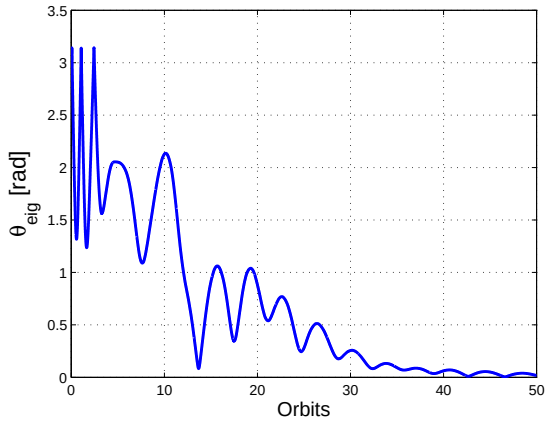
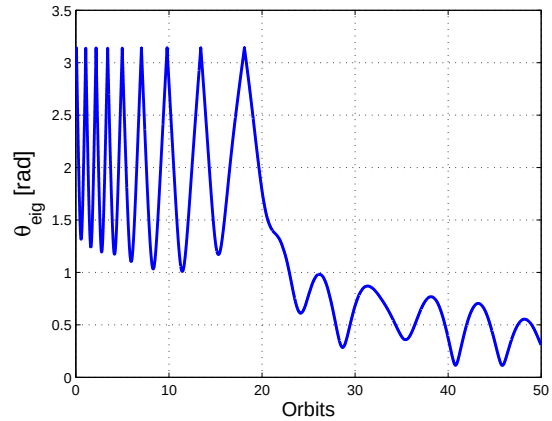
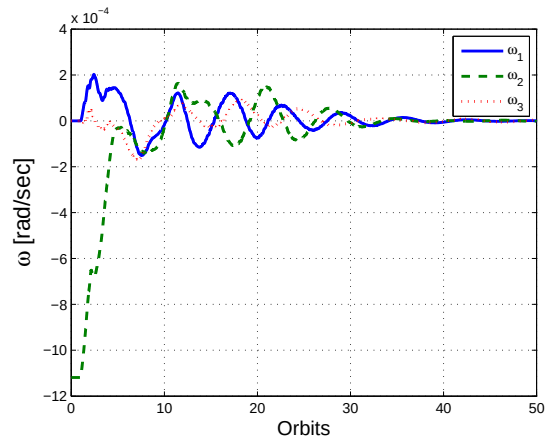


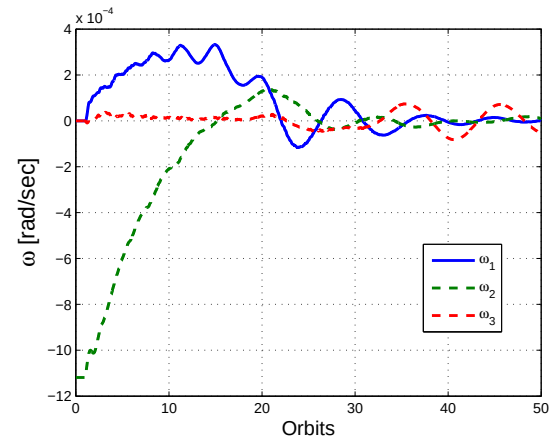

(a) Eigenaxis attitude errorfor MIMO RCAC using the average Markov parameter $\tilde{H}$



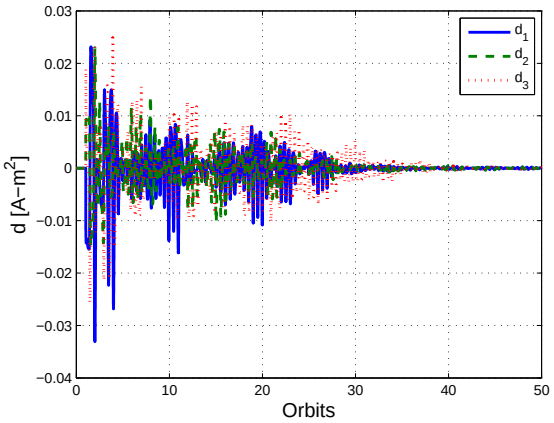

(b) Eigenaxis attitude error for decentralized RCAC



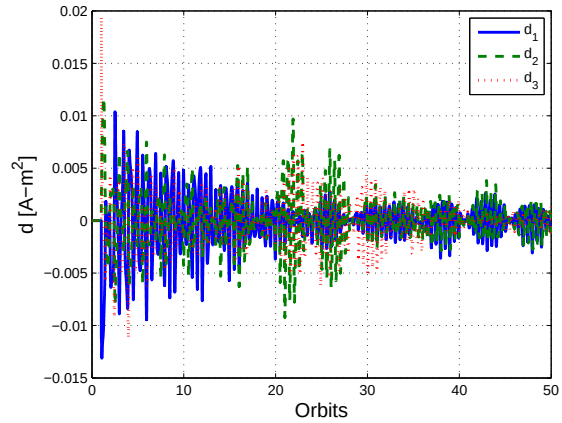

(c) Angular velocity for MIMO RCAC using the average Markov parameter $\tilde{H}$




(d) Angular velocity for decentralized RCAC




(e) Magnetic dipole for MIMO RCAC using the average Markov parameter $\tilde{H}$




(f) Magnetic dipole for decentralized RCAC

Figure 1: Performance comparison of RCAC using the average Markov parameter $\tilde{H}$ versus the decentralized approach for the M2R maneuver for a spherical spacecraft with inertia $J_{\text{sphere}}$ in (73).



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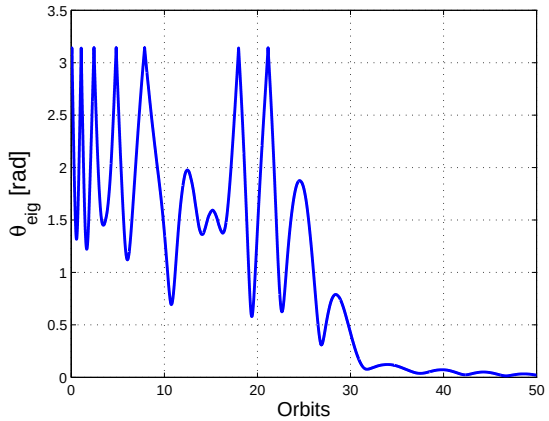
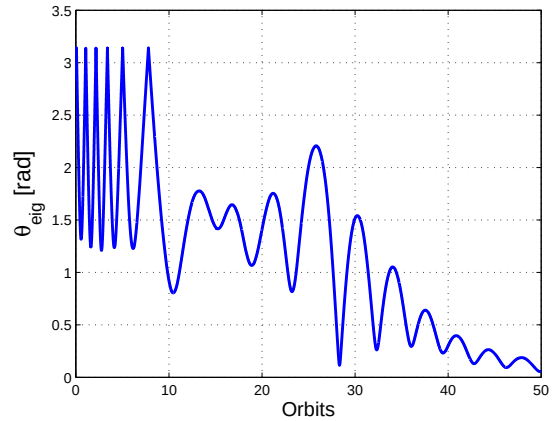
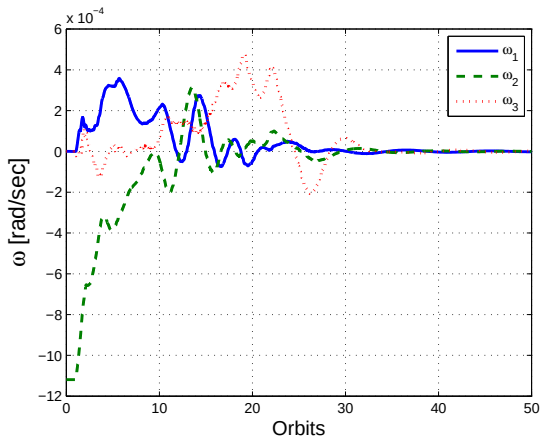


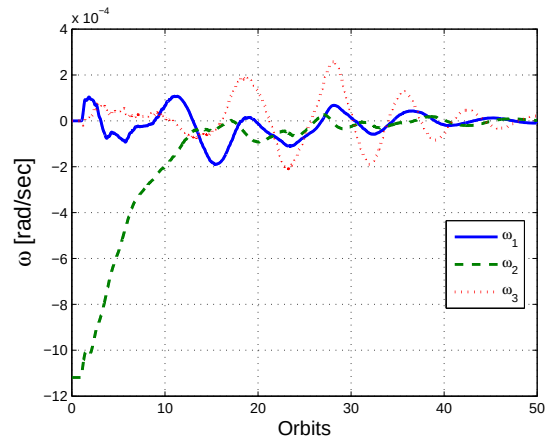

(a) Eigenaxis attitude errorfor MIMO RCAC using the average Markov parameter $\tilde{H}$



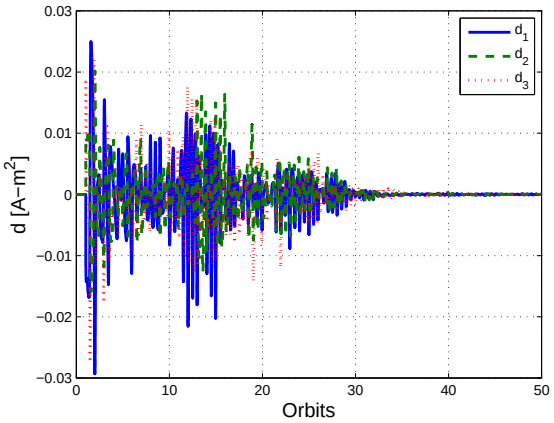

(b) Eigenaxis attitude error for decentralized RCAC



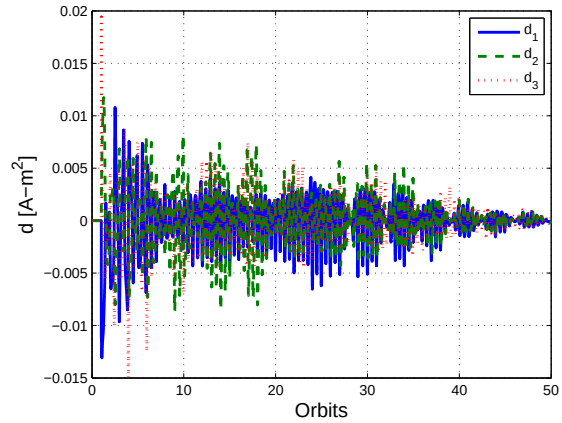

(c) Angular velocity for MIMO RCAC using the average Markov parameter $\tilde{H}$




(d) Angular velocity for decentralized RCAC




(e) Magnetic dipole for MIMO RCAC using the average Markov parameter $\tilde{H}$




(f) Magnetic dipole for decentralized RCAC

Figure 2: Performance comparison of RCAC using the average Markov parameter $\tilde{H}$ versus the decentralized approach for the M2R maneuver for a cylindrical spacecraft with inertia $J_{\text{cylinder}}$ in (74).



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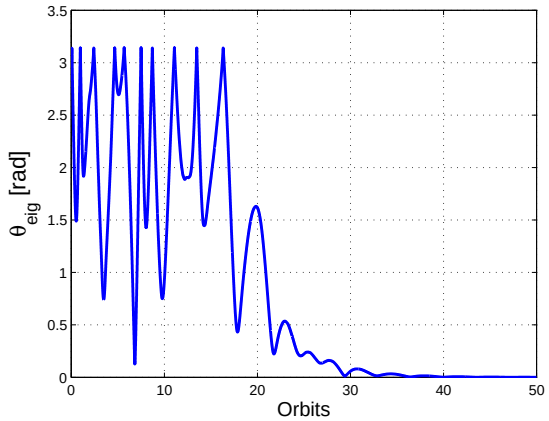
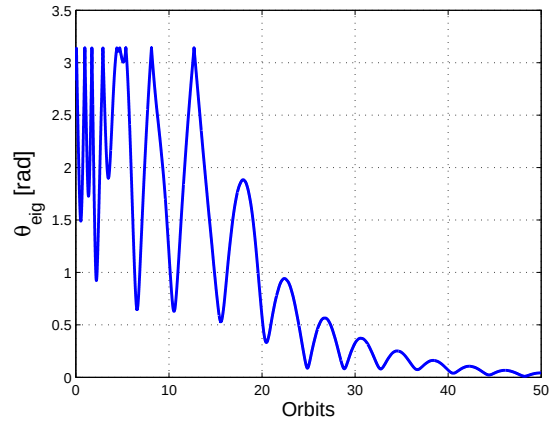
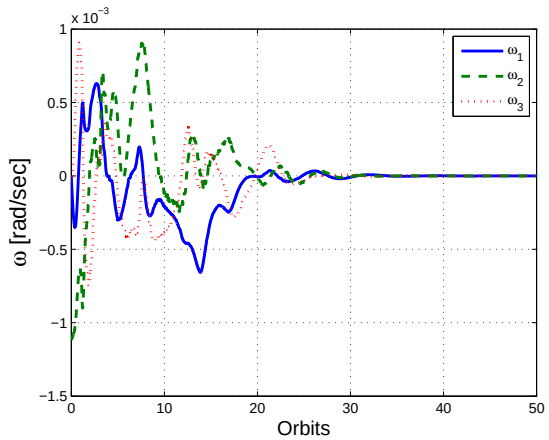


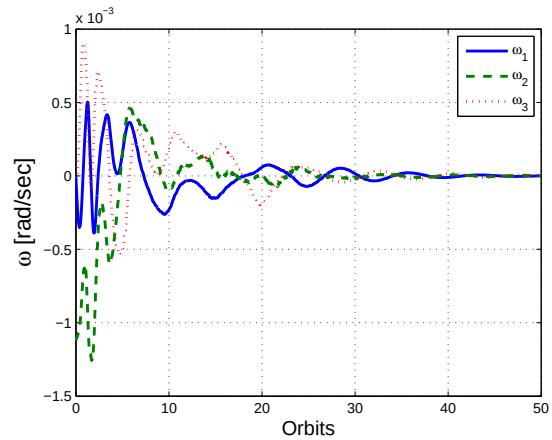

(a) Eigenaxis attitude errorfor MIMO RCAC using the average Markov parameter $\tilde{H}$



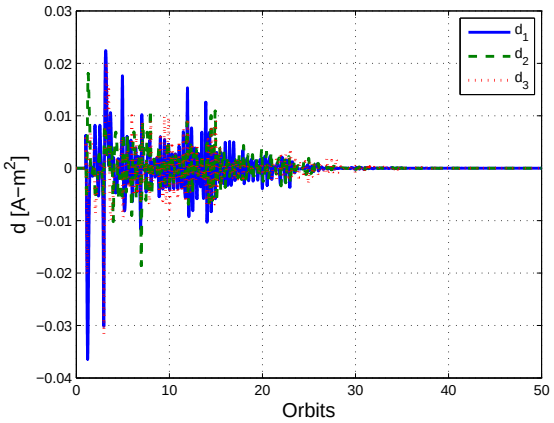

(b) Eigenaxis attitude error for decentralized RCAC



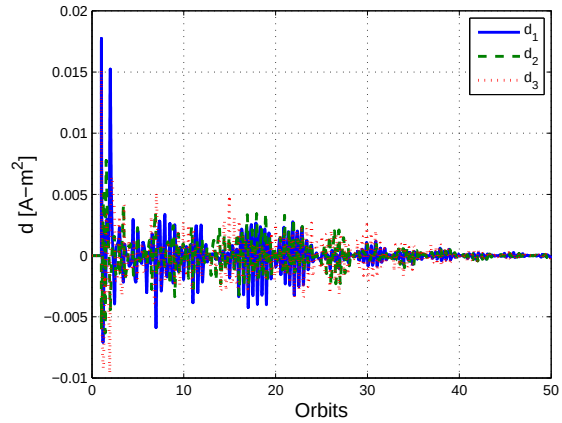

(c) Angular velocity for MIMO RCAC using the average Markov parameter $\tilde{H}$




(d) Angular velocity for decentralized RCAC




(e) Magnetic dipole for MIMO RCAC using the average Markov parameter $\tilde{H}$




(f) Magnetic dipole for decentralized RCAC

Figure 3: Performance comparison of RCAC using the average Markov parameter $\tilde{H}$ versus the decentralized approach for the M2R maneuver for the arbitrary inertia $J_{\text{arbitrary}}$ in (75).



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The advantage of the centralized controller can be attributed to the dimension of the controller parameter $\theta$ and to the information about coupling of the axes in the average input matrix $\tilde{B}_{\mathrm{SC}}$. The loss of information caused by decoupling the input and output relations in (67) increases the settling time of the decentralized architecture in the presence of the real time input matrix $B_{\mathrm{SC}}(t)$.

We can also compare these approaches based on algorithm complexity and execution time. The dimension of the control parameter $\theta$ in (28) affects the memory requirements and time required to compute each control iteration. This is an important factor for the application of these control laws on small spacecraft. For a controller of order $n_c$, the averaged Markov parameter method needs to compute $l_u n_c(l_u + l_z) = 27n_c$ entries of $\theta$. In contrast, the decentralized approach requires $3l_u n_c(l_u + l_z) = 9n_c$ entries. Thus, the decentralized approach yields similar settling times for M2R maneuvers using a third of the computational cost.

## B. M2S Examples

We command the spacecraft to spin about a body axis aligned in a specific inertial direction. Let the initial angular velocity and attitude of the spacecraft be as in Section VI. A. The desired angular rate

$$\omega_d = 0.001 \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^{\mathrm{T}} \text{rad/sec} \tag{76}$$

corresponds to an Earth-pointing attitude. The desired attitude evolves over time according to (6) where the initial desired attitude $R_d(0)$ is described by an eigenaxis rotation of $\theta_d(0) = 96^\circ$ about the vector $n_d$ in (72). We set the controller parameters as in Table 2. Unlike the M2S examples, we set $\alpha = 1$ in (60) and (67) and $\epsilon = 5 \times 10^{-5}$ in (68).

Figures 4 and 5 show that the decentralized approach is able to bring the both the spherical and cylindrical spacecraft into a Nadir pointing attitude, that is, a spin about an inertially pointed body axis.



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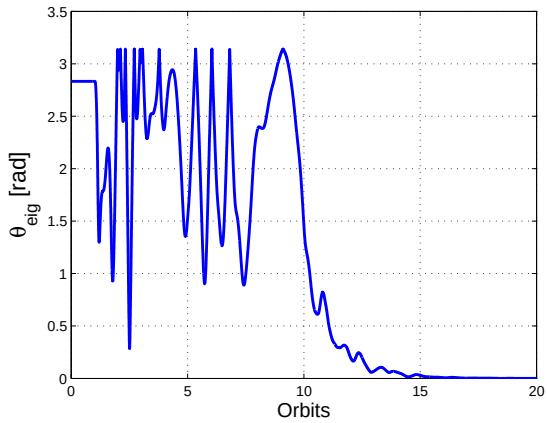
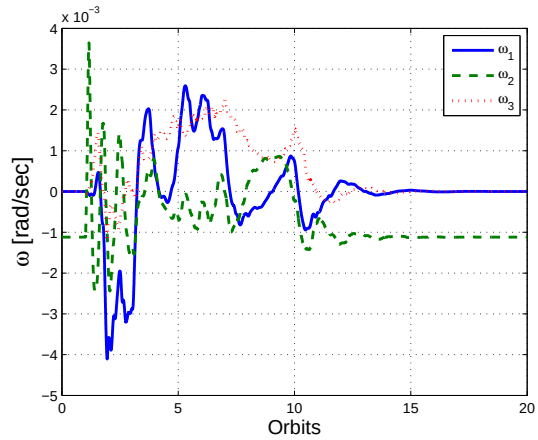
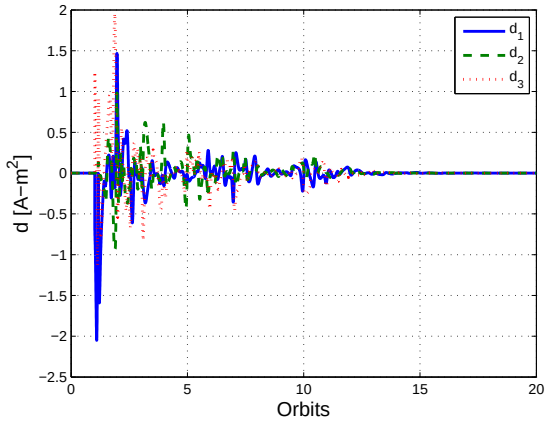



(a) Eigenaxis attitude error for decentralized RCAC




(b) Angular velocity for decentralized RCAC




(c) Magnetic dipole for decentralized RCAC

Figure 4: Performance of RCAC using the decentralized approach for the M2S maneuver for the spherical spacecraft inertia $J_{\mathrm{sphere}}$ in (73).



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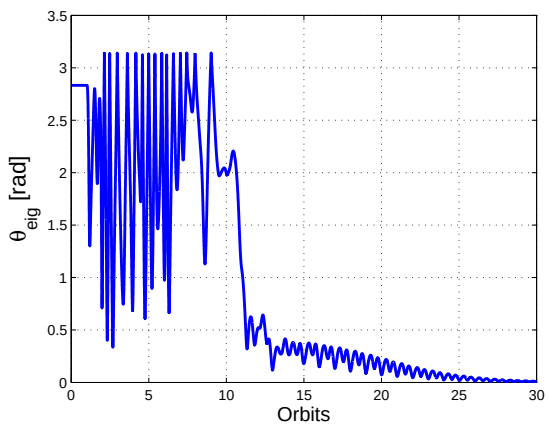
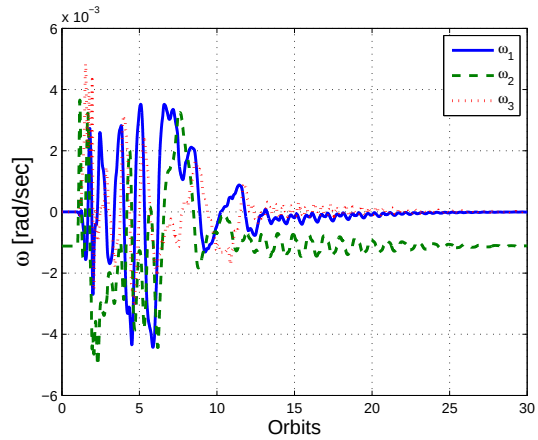
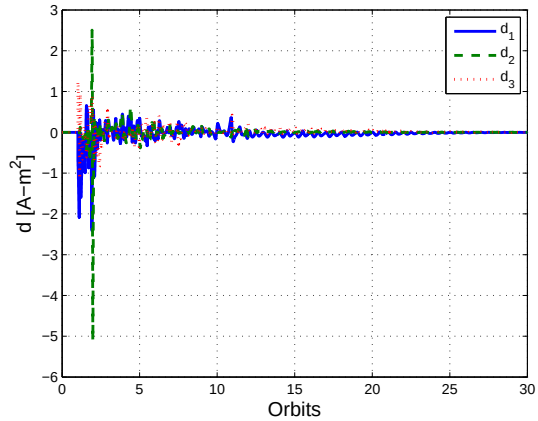



(a) Eigenaxis attitude error for decentralized RCAC




(b) Angular velocity for decentralized RCAC




(c) Magnetic dipole for decentralized RCAC

Figure 5: Performance of RCAC using the decentralized approach for the M2S maneuver for the cylindrical spacecraft inertia $J_{\text{cylinder}}$ in (74).



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# VII. Conclusions and Future Research

The RCAC algorithm was used to control spacecraft angular rate and attitude using magnetic torque actuators. The torque command computed by RCAC was allocated into magnetic dipoles based on the generalized inverse of the skew symetric cross-product matrix of the magnetic field vector. In order to utilize the RCAC framework, the nonlinear, continuous spacecraft dynamics were linearized to obtain Markov parameters. The Markov parameter obtained was then made left invertible through two different approaches, averaging and decentralized control.

The average of the input matrix used to compute the Markov parameter is positive definite for the high inclination orbits which are of interest for most low-earth-orbit missions. Using the averaged Markov parameter an inertia free stabilizing control law was developed. Numerical simulations show that the algorithm can achieve a M2R maneuver and bring different spacecraft to rest at an inertial attitude.

The decentralized control approach assumes that the sensor and actuator axes are aligned. The three control inputs are computed by independent RCAC controllers using different performance variables. This architecture results in a multi-input, single-output system with left invertible Markov parameters. The decentralized RCAC approach was also shown to complete the M2R maneuver for different inertias.

Comparison of the numerical results indicate that the averaged Markov parameter approach has better settling time characteristics than the decentralized approach given similar RCAC tunings. However, the decentralized method uses one third of the computational capacity of the averaged Markov parameter approach. Thus, the decentralized method is better suitable for applications where computational capacity is limited and settling time requirements are flexible. Furthermore, the settling time of the decentralized approach could be improved by modifying the performance variable to account for the coupling present in the magnetic control formulation.

Future work will focus on extending the implementation of RCAC to use additional Markov parameters. Furthermore, the relation between the performance and controller weighting matrices and the magnitude of the control input will be investigated. A stability analysis of the full closed loop system is also of interest. We also wish to investigate the effects of noise on magnetic field measurement and to address the problem of attitude-only output feedback control using RCAC. The problem of momentum dumping using magnetic torque is also of interest. Finally, we will test the RCAC on a higher fidelity simulator which includes disturbances such as aerodynamic drag, gravity gradient, and solar pressure.

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