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Retrospective Cost Adaptive NARMAX Control of Hammerstein Systems with Ersatz Nonlinearities

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In this paper, we generalize retrospective cost adaptive NARMAX control (RCANC) to a command-following problem for uncertain Hammerstein systems. In particular, RCANC with ersatz nonlinearities is applied to linear systems cascaded with input nonlinearities. We assume that one Markov parameter of the linear plant is known. RCANC also uses knowledge of the monotonicity properties of the input nonlinearity to select the ersatz nonlinearity. The goal is to determine whether RCANC can improve the command-following performance compared to the linear RCAC controller.

I. Introduction

While nonlinear control techniques have been extensively developed, the vast majority of modern methods assume the availability of full-state measurements. This is largely due to the fact that optimal control methods produce control laws that depend on full-state feedback as well as the fact that output-feedback control laws consisting of nonlinear observers combined with full-state feedback control laws may not be stabilizing. The lack of a widely applicable separation principle within a nonlinear setting thus remains an impediment to nonlinear output-feedback control [1].

In the present paper we focus on Hammerstein systems, which comprise a class of nonlinear systems consisting of an input nonlinearity cascaded with linear dynamics. These systems encompass plants that involve linear dynamics with, for example, saturation [2], deadzone, or on-off input nonlinearities. Identification of Hammerstein systems is widely studied [3–5], while control of Hammerstein systems includes the entire literature on control of linear systems with saturation [6] and actuator nonlinearities [7, 8].

For command-following problems, performance is degraded by the input nonlinearity in various ways. If the range of the input nonlinearity is insufficient for the plant output to follow the command, then the performance error is unavoidable; this is the case with saturation, which can also cause instability due to windup. On the other hand, if the range of the input nonlinearity is sufficiently large for the output to follow the command, performance degradation may result from the distortion introduced by the shape of the input nonlinearity. If the input nonlinearity is known, then this effect can be mitigated or removed by inversion; if the input nonlinearity is uncertain, or has a critical point, then adaptive inversion may be feasible [9].

In the present paper we take an unconventional approach to nonlinear output feedback control of Hammerstein systems by using adaptive control to directly update the gains of a NARMAX controller. A NARMAX model is a discrete-time ARMAX system in which the past output and inputs appear as arguments of basis functions. These functions are chosen by the user, and the controller coefficients appear linearly.

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The constraint that the controller coefficients appear linearly implies that the basis function functions are fixed a priori and thus cannot be modified as part of the adaptation process. NARMAX models have been applied to nonlinear system identification [10, 11].

For adaptive NARMAX control, we apply retrospective cost adaptive control (RCAC). RCAC has been developed in [12–16] and applied to Hammerstein systems in [17, 18] and NARMAX control in [19]. The present paper extends and improves the results of [17–19] by modifying the adaptation mechanism to include a nonlinear adaptation mechanism. This modification ensures that the retrospective optimization accounts for the presence of the input nonlinearity. To account for the case in which the input nonlinearity is uncertain, we investigate the performance of RCNAC control in the case of uncertainty. In particular, we determine the minimal modeling information about the input nonlinearity that RCANC requires; once this information is known, an approximate input nonlinearity, called the ersatz nonlinearity, can be used by RCANC for adaptation.

II. Hammerstein Command-Following Problem

Consider the MIMO discrete-time Hammerstein system

$$x(k+1) = Ax(k) + B\mathcal{N}(u(k)) + D_1 w(k), \tag{1}$$

$$y(k) = Cx(k) + D_2 w(k), \tag{2}$$

$$z(k) = E_1 x(k) + E_0 w(k), \tag{3}$$

where $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^{l_y}$, $z(k) \in \mathbb{R}^{l_z}$, $w(k) \in \mathbb{R}^{l_w}$, $u(k) \in \mathbb{R}^{l_u}$, $\mathcal{N} : \mathbb{R}^{l_u} \to \mathbb{R}^{l_u}$, and $k \geq 0$. The goal is to develop an adaptive output feedback controller that minimizes the command-following error z with minimal modeling information about the dynamics, and input nonlinearity $\mathcal{N}$. We assume that measurements of $z(k)$ are available for feedback; however, measurements of $v(k) = \mathcal{N}(u(k))$ are not available. A block diagram for (1)-(3) is shown in Figure 1.

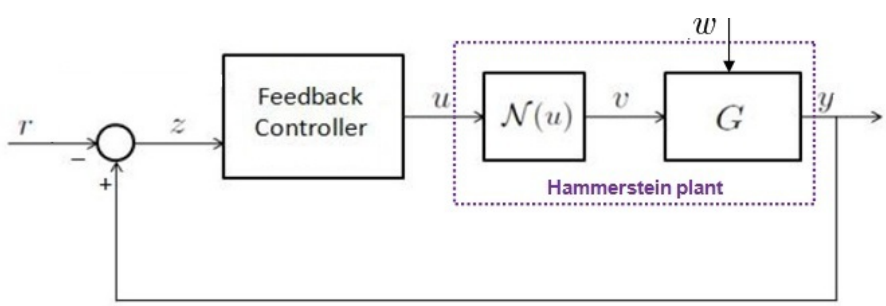

Figure 1. Adaptive command-following problem for a Hammerstein plant with input nonlinearity $\mathcal{N}$. We assume that measurements of $z(k)$ are available for feedback; however, measurements of $v(k) = \mathcal{N}(u(k))$ and $w(k)$ are not available.

III. Retrospective-Cost Adaptive NARMAX Control

III.A. ARMAX Modeling

Consider the ARMAX representation of (1)–(3) given by

$$z(k) = \sum_{i=1}^{n} -\alpha_i z(k-i) + \sum_{i=d}^{n} \beta_i \mathrm{Sat}(u(k-i)) + \sum_{i=0}^{n} \gamma_i w(k-i), \tag{4}$$

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where $\alpha_1, \ldots, \alpha_n \in \mathbb{R}$, $\beta_1, \ldots, \beta_n \in \mathbb{R}^{l_z \times l_u}$, $\gamma_0, \ldots, \gamma_n \in \mathbb{R}^{l_z \times l_w}$, and d is the relative degree. Next, let $v(k) \stackrel{\triangle}{=} \mathrm{Sat}(u(k))$, and define the transfer function

$$G_{zv}(\mathbf{q}) \stackrel{\triangle}{=} E_1(\mathbf{q}I - A)^{-1}B = \sum_{i=d}^{\infty} \mathbf{q}^{-i} H_i = H_d \frac{\alpha(\mathbf{q})}{\beta(\mathbf{q})}, \tag{5}$$

where $\mathbf{q}$ is the forward shift operator and, for each positive integer i, the Markov parameter H_i of G_{zv} is defined by

$$H_i \stackrel{\triangle}{=} E_1 A^{i-1} B \in \mathbb{R}^{l_z \times l_u}. \tag{6}$$

Note that, if $d = 1$, then $H_1 = \beta_1$, whereas, if $d \geq 2$, then

$$\beta_1 = \cdots = \beta_{d-1} = H_1 = \cdots = H_{d-1} = 0 \tag{7}$$

and $H_d = \beta_d$. The polynomials $\alpha(\mathbf{q})$ and $\beta(\mathbf{q})$ have the form

$$\alpha(\mathbf{q}) = \mathbf{q}^{n-1} + \alpha_1 \mathbf{q}^{n-1} + \cdots + \alpha_{n-1}\mathbf{q} + \alpha_n, \tag{8}$$

$$\beta(\mathbf{q}) = \mathbf{q}^{n-d} + \beta_{d+1}\mathbf{q}^{n-d-1} + \cdots + \beta_{n-1}\mathbf{q} + \beta_n. \tag{9}$$

Next, define the *extended performance* $Z(k) \in \mathbb{R}^{pl_z}$ and *extended plant input* $V(k) \in \mathbb{R}^{q_c l_u}$ by

$$Z(k) \stackrel{\triangle}{=} \begin{bmatrix} z(k) \\ \vdots \\ z(k-p+1) \end{bmatrix}, \quad V(k) \stackrel{\triangle}{=} \begin{bmatrix} v(k-1) \\ \vdots \\ v(k-q_c) \end{bmatrix} = \begin{bmatrix} \mathrm{Sat}(u(k-1)) \\ \vdots \\ \mathrm{Sat}(u(k-q_c)) \end{bmatrix}, \tag{10}$$

where the data window size p is a positive integer, and $q_c \stackrel{\triangle}{=} n + p - 1$. Therefore (10) can be expressed as

$$Z(k) = W_{zw}\phi_{zw}(k) + B_f V(k), \tag{11}$$

where

$$W_{zw} \stackrel{\triangle}{=} \begin{bmatrix} -\alpha_1 I_{l_z} & \cdots & -\alpha_n I_{l_z} & 0_{l_z\times l_z} & \cdots & 0_{l_z\times l_z} & \gamma_0 & \cdots & \gamma_n & 0_{l_z\times l_w} & \cdots & 0_{l_z\times l_w} \\ 0_{l_z\times l_z} & \ddots & & \ddots & \ddots & \vdots & 0_{l_z\times l_w} & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & 0_{l_z\times l_z} & \vdots & \ddots & \ddots & & \ddots & 0_{l_z\times l_w} \\ 0_{l_z\times l_z} & \cdots & 0_{l_z\times l_z} & -\alpha_1 I_{l_z} & \cdots & -\alpha_n I_{l_z} & 0_{l_z\times l_w} & \cdots & 0_{l_z\times l_w} & \gamma_0 & \cdots & \gamma_n \end{bmatrix}$$
$$\in \mathbb{R}^{pl_z \times [q_c l_z + (q_c+1) l_w]}, \tag{12}$$

$$B_f \stackrel{\triangle}{=} \begin{bmatrix} \beta_1 & \cdots & \beta_n & 0_{l_z\times l_u} & \cdots & 0_{l_z\times l_u} \\ 0_{l_z\times l_u} & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & 0_{l_z\times l_u} \\ 0_{l_z\times l_u} & \cdots & 0_{l_z\times l_u} & \beta_1 & \cdots & \beta_n \end{bmatrix} \in \mathbb{R}^{pl_z \times q_c l_u}, \tag{13}$$

and

$$\phi_{zw}(k) \stackrel{\triangle}{=} \begin{bmatrix} z(k-1) \\ \vdots \\ z(k-p-n+1) \\ w(k) \\ \vdots \\ w(k-p-n+1) \end{bmatrix} \in \mathbb{R}^{q_c l_z + (q_c+1) l_w}. \tag{14}$$

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Note that W_{zw} includes modeling information about the poles of G_{zv} and the exogenous signals, while B_f includes modeling information about the zeros of G_{zv}.

For the open-loop system (4), we make the following assumptions:

1. The relative degree d is known.
2. The first nonzero Markov parameter H_d is known.
3. There exists an integer $\bar{n}$ such that $n < \bar{n}$ and $\bar{n}$ is known.
4. If $\zeta \in \mathbb{C}$, $|\zeta| > 1$, and $\beta(\zeta) = 0$, then the spectral radius of A is less than 1.
5. The performance variable $z(k)$ is measured and available for feedback.
6. The exogenous signal $w(k)$ is generated by

$$x_w(k+1) = A_w x_w(k), \tag{15}$$
$$w(k) = C_w x_w(k), \tag{16}$$

   where $x_w \in \mathbb{R}^{l_w}$ and all of the eigenvalues of A_w are on the unit circle and do not coincide with the transmission zeros of G_{zv}.
7. There exists an integer $\bar{n}_w$ such that $n_w < \bar{n}_w$ and $\bar{n}_w$ is known.
8. The exogenous signal $w(k)$ is not measured.
9. $\alpha(\mathbf{z}), \beta(\mathbf{z}), n,$ and $x(0)$ are unknown.

III.B. NARMAX Controller Construction

In this section, we assume a NARMAX structure for the adaptive controller, which uses a nonlinear difference equation to model the relation between the input z and output u of the controller. The nonlinear controller may include nonlinearities on the input to the controller (NARMAX/I), the output of the controller (NARMAX/O), or both (NARMAX/IO). The NARMAX controller structure is linear in the controller parameters, and linear regression is used to update the controller coefficients.

The control $u(k)$ is given by the strictly proper time-series controller of order n_c written as

$$u(k) = \sum_{j=1}^{s}\sum_{i=1}^{n_c} M_{ji}(k) f_j(u(k-i)) + \sum_{j=1}^{t}\sum_{i=1}^{n_c} N_{ji}(k) g_j(y(k-i)), \tag{17}$$

where, for all $j = 1, \ldots, s$, $i = 1, \ldots, n_c$, $s \in \mathbb{Z}^+$, $M_{ji}(k) \in \mathbb{R}^{l_u \times l_u}$, and for all $j = 1, \ldots, t$, $i = 1, \ldots, n_c$, $t \in \mathbb{Z}^+$, $N_{ji}(k) \in \mathbb{R}^{l_u \times l_y}$. The control (17) can be expressed as

$$u(k) = \theta(k)\phi(k-1),$$

where

$$\theta(k) \triangleq \begin{bmatrix} M_{11}(k) & \cdots & M_{sn_c}(k) & N_{11}(k) & \cdots & N_{tn_c}(k) \end{bmatrix} \in \mathbb{R}^{l_u \times n_c(sl_u + tl_y)}$$

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and

$$\phi(k-1) \triangleq \begin{bmatrix} f_1(u(k-1)) \\ \vdots \\ f_s(u(k-n_{\rm c})) \\ g_1(y(k-1)) \\ \vdots \\ g_t(y(k-n_{\rm c})) \end{bmatrix} \in \mathbb{R}^{n_{\rm c}(sl_u+tl_y)}. \tag{18}$$

To illustrate the NARMAX/O controller structure, let $f_1(u) = u$, $f_2(u) = u^2$, and $f_3(u) = u^3$. Then $\theta(k)$ and $\phi(k-1)$ can be expressed as

$$\theta(k) \triangleq \begin{bmatrix} M_1(k) & \cdots & M_{n_{\rm c}}(k) & M_{n_{\rm c}+1}(k) & \cdots & M_{2n_{\rm c}}(k) & M_{2n_{\rm c}+1}(k) & \cdots & M_{3n_{\rm c}}(k) & N_1(k) & \cdots & N_{n_{\rm c}}(k) \end{bmatrix} \in \mathbb{R}^{l_u \times n_{\rm c}(3l_u+l_y)}$$

and

$$\phi(k-1) \triangleq \begin{bmatrix} u(k-1) & \cdots & u(k-n_{\rm c}) & u^2(k-1) & \cdots & u^2(k-n_{\rm c}) & u^3(k-1) & \cdots & u^3(k-n_{\rm c}) & y(k-1) & \cdots & y(k-n_{\rm c}) \end{bmatrix}^{\rm T} \in \mathbb{R}^{n_{\rm c}(3l_u+l_y)}.$$

To illustrate the NARMAX/I controller structure, let $g_1(y) = y$ and $g_2(y) = y^2$. Then $\theta(k)$ and $\phi(k-1)$ can be expressed as

$$\theta(k) \triangleq \begin{bmatrix} M_1(k) & \cdots & M_{n_{\rm c}}(k) & N_1(k) & \cdots & N_{n_{\rm c}}(k) & N_{n_{\rm c}+1}(k) & \cdots & N_{2n_{\rm c}}(k) \end{bmatrix} \in \mathbb{R}^{l_u \times n_{\rm c}(l_u+2l_y)}$$

and

$$\phi(k-1) \triangleq [u(k-1) \quad \cdots \quad u(k-n_{\rm c})) \quad y(k-1) \quad \cdots \quad y(k-n_{\rm c}) \quad y^2(k-1) \quad \cdots \quad y^2(k-n_{\rm c})]^{\rm T} \in \mathbb{R}^{n_{\rm c}(l_u+2l_y)}.$$

III.C. Retrospective Performance

Define the *retrospective performance* $\hat{Z}(k) \in \mathbb{R}^{pl_z}$ by

$$\hat{Z}(k) \triangleq W_{zw}\phi_{zw}(k) + B_f \mathcal{N}(U(k)) + \bar{B}_f[\tilde{\mathcal{N}}(\hat{U}(k)) - \tilde{\mathcal{N}}(U(k))], \tag{19}$$

where $\bar{B}_f \in \mathbb{R}^{pl_z \times p_c l_u}$ is the retrospective input matrix, $\tilde{\mathcal{N}} : \mathbb{R} \to \mathbb{R}$ is the ersatz nonlinearity, $\tilde{\mathcal{N}}(\hat{U}(k))$ means componentwise evaluation, and $\hat{U}(k) \in \mathbb{R}^{q_c l_u}$ is the recomputed extended control vector, the components of $\hat{U}(k)$ are the recomputed control $\hat{u}(k-1) \ldots \hat{u}(k-q_c)$ ordered in the same way as the components in (10). Subtracting (11) from (19) yields

$$\hat{Z}(k) = Z(k) + \bar{B}_f[\tilde{\mathcal{N}}(\hat{U}(k)) - \tilde{\mathcal{N}}(U(k))]. \tag{20}$$

Note that the retrospective performance $\hat{Z}(k)$ does not depend on W_{zw} or the exogenous signal w. For the disturbance rejection problem, we do not need to assume that the disturbance is known; for the command-following problem, the command w can be unknown. Therefore, only limited model information is needed. The model information matrix $\bar{B}_f$ is discussed in Section IV, and the construction of ersatz nonlinearity $\tilde{\mathcal{N}}$ is discussed in Section V.

III.D. Retrospective Cost and Recursive Least Square (RLS) Update Law

III.D.1. Retrospective Cost

We define the *retrospective cost function*

$$J(\tilde{\mathcal{N}}(\hat{U}(k)), k) \triangleq \hat{Z}^{\rm T}(k)R(k)\hat{Z}(k), \tag{21}$$

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where $R(k) \in \mathbb{R}^{pl_z \times pl_z}$ is a positive-definite performance weighting. The goal is to determine retrospectively optimized controls $\hat{U}(k)$ that would have provided better performance than the controls $U(k)$ that were applied to the system. The retrospectively optimized control values $\hat{U}(k)$ are subsequently used to update the controller.

Next, to ensure that (21) has a global minimizer, we consider the regularized cost

$$\bar{J}(\tilde{\mathcal{N}}(\hat{U}(k)), k) \triangleq \hat{Z}^{\mathrm{T}}(k)R(k)\hat{Z}(k) + \eta(k)\tilde{\mathcal{N}}^{\mathrm{T}}(\hat{U}(k))\tilde{\mathcal{N}}(\hat{U}(k)), \tag{22}$$

where $\eta(k) \geq 0$. Substituting (11) into (22) yields

$$\bar{J}(\tilde{\mathcal{N}}(\hat{U}(k)), k) = \tilde{\mathcal{N}}^{\mathrm{T}}(\hat{U}(k))\mathcal{A}\tilde{\mathcal{N}}(\hat{U}(k)) + \mathcal{B}(k)\tilde{\mathcal{N}}(\hat{U}(k)) + \mathcal{C}(k),$$

where

$$\begin{aligned}
\mathcal{A}(k) &\triangleq \bar{B}_f^{\mathrm{T}}R(k)\bar{B}_f + \eta(k)I_{q_c l_u \times q_c l_u},\\
\mathcal{B}(k) &\triangleq 2\bar{B}_f^{\mathrm{T}}R(k)[Z(k) - \bar{B}_f\tilde{\mathcal{N}}(U(k))],\\
\mathcal{C}(k) &\triangleq Z^{\mathrm{T}}(k)R(k)Z(k) - 2Z^{\mathrm{T}}(k)R(k)\bar{B}_f\tilde{\mathcal{N}}(U(k)) + \tilde{\mathcal{N}}^{\mathrm{T}}(U(k))\bar{B}_f^{\mathrm{T}}R(k)\bar{B}_f\tilde{\mathcal{N}}(U(k)).
\end{aligned}$$

If either $\bar{B}_f$ has full column rank or $\eta(k) > 0$, then $\mathcal{A}(k)$ is positive definite. In this case, $\bar{J}(\tilde{\mathcal{N}}(\hat{U}(k)), k)$ has the unique global minimizer

$$\tilde{\mathcal{N}}(\hat{U}(k)) = -\frac{1}{2}\mathcal{A}^{-1}(k)\mathcal{B}(k). \tag{23}$$

If $\tilde{\mathcal{N}}$ is not onto, then $\hat{U}(k)$ in (23) may not have a solution. Hence, we take

$$\hat{U}(k) = \operatorname{argmin} \left\| \tilde{\mathcal{N}}(\hat{U}(k)) + \frac{1}{2}\mathcal{A}^{-1}(k)\mathcal{B}(k) \right\|_2. \tag{24}$$

An arbitrary choice is made if the argmin in (24) is not unique.

III.D.2. Cumulative Cost and RLS Update

Define the *cumulative cost function*

$$\begin{aligned}
J_{\mathrm{cum}}(\theta, k) \triangleq \sum_{i=d+1}^{k} &\lambda^{k-i}\|\phi^{\mathrm{T}}(i-d-1)\theta(i-1) - \hat{u}_k(i-d)\|^2\\
&+ \lambda^k[\theta(k) - \theta(0)]^{\mathrm{T}}P_0^{-1}[\theta(k) - \theta(0)],
\end{aligned} \tag{25}$$

where $\|\cdot\|$ is the Euclidean norm, $P_0 \in \mathbb{R}^{l_u[n_c l_u + (n_c+1)l_z] \times [n_c l_u + (n_c+1)l_z]}$ is positive definite, and $\lambda \in (0, 1]$ is the forgetting factor. The next result follows from standard recursive least-squares (RLS) theory [20, 21].

Lemma III.1. *For each $k \geq d$, the unique global minimizer of the cumulative retrospective cost function (25) is given by*

$$\theta(k) = \theta(k-1) + \frac{P(k-1)\phi(k-d)\varepsilon(k-1)}{\lambda + \phi^{\mathrm{T}}(k-d)P(k-1)\phi(k-d)}, \tag{26}$$

where

$$P(k) = \frac{1}{\lambda}\left[P(k-1) - \frac{P(k-1)\phi(k-d)\phi^{\mathrm{T}}(k-d)P(k-1)}{\lambda + \phi^{\mathrm{T}}(k-d)P(k-1)\phi(k-d)}\right], \tag{27}$$

$P(0) = P_0$, *and* $\varepsilon(k-1) \triangleq \phi^{\mathrm{T}}(k-d-1)\theta(k-1) - \hat{u}(k-d)$.

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IV. Model Information $\bar{B}_f$

For SISO asymptotically stable linear plants, if the open-loop linear plant is minimum-phase, then using the first nonzero Markov parameter in RCAC yields asymptotic convergence of z to zero. In this case, let $\bar{B}_f = \begin{bmatrix} 0_{1\times d-1} & H_d \end{bmatrix}$ [14, 16]. Furthermore, if the open-loop linear plant is nonminimum-phase and the absolute values of all nonminimum-phase zeros are less than the plant's spectral radius, a sufficient number of Markov parameters can be used to approximate the nonminimum-phase zeros [14]. Alternatively, a phase-mismatching condition $\Delta(\theta) \leq 90$ is given in [22, 23] to construct $\bar{B}_f$.

For MIMO Lyapunov stable linear plants, an extension of the phase-matching-based method is discussed in [24].

For unstable and nonminimum-phase plants, knowledge of the locations of the nonminimum-phase zeros is needed to construct $\bar{B}_f$. For details, see [14, 25].

In this paper, we assume that the Hammerstein system is Lyapunov stable, and we choose $\bar{B}_f = \begin{bmatrix} 0_{1\times d-1} & H_d \end{bmatrix}$, that is, the first nonzero Markov parameter of G.

V. Construction of Ersatz Nonlinearity $\tilde{\mathcal{N}}$

In this section, we investigate the performance of various constructions for the ersatz nonlinearity $\tilde{\mathcal{N}}$. The objective is to determine the effect of model error in identifying $\mathcal{N}$. We consider the asymptotically stable, minimum-phase plant

$$G(z) = \frac{(z-0.5)(z-0.9)}{(z-0.7)(z-0.5-\jmath 0.5)(z-0.5+\jmath 0.5)}, \tag{28}$$

with the input nonlinearity

$$\mathcal{N}(u) = (u-2)^2 - 3. \tag{29}$$

We consider the sinusoidal command $r(k) = \sin(\Omega_1 k)$, where $\Omega_1 = \pi/5$ rad/sample. Let the controller structure be NARMAX/IO with $s = t = 6$ in (17). In particular, we choose $f_1(u) = u$, $f_2(u) = \exp(-(u+0.2)^2)$, $f_3(u) = \exp(-(u-0.2)^2)$, $f_4(u) = \exp(-(u+0.4)^2)$, $f_5(u) = \exp(-(u-0.4)^2)$, $f_6(u) = \exp(-u^2)$, and $g_1(y) = y$, $g_2(y) = \exp(-(y+0.2)^2)$, $g_3(y) = \exp(-(y-0.2)^2)$, $g_4(y) = \exp(-(y+0.4)^2)$, $g_5(y) = \exp(-(y-0.4)^2)$, $g_6(y) = \exp(-y^2)$ for the NARMAX/IO model. Furthermore, we let $n_{\rm c} = 10$, $P_0 = 10 I_{12n_{\rm c}}$, $\eta_0 = 0$, and $\bar{B}_f = H_1 = 1$ as the required linear plant information.

We consider various choices of the ersatz nonlinearity $\tilde{\mathcal{N}}$ in order to elicit the required minimum model information of the input nonlinearity $\mathcal{N}$. First, we consider the ersatz nonlinearity $\tilde{\mathcal{N}}(u) = (u-2)^2$. The closed-loop response is shown in Figure 2. Note that the steady-state average performance $z_{\rm ss,avg} = 5.7154 \times 10^{-4}$. Note that RCANC compensates for the unknown bias in $\mathcal{N}$.

Next, we consider the ersatz nonlinearity $\tilde{\mathcal{N}}(u) = u^2$, and note that the intervals of monotonicity of $\tilde{\mathcal{N}}$ and $\mathcal{N}$ are different. As shown in Figure 3, RCANC is not able to follow the command.

Furthermore, consider the ersatz nonlinearity $\tilde{\mathcal{N}}(u) = 5(u-2)^2$. As shown in Figure 4, the steady-state average performance $z_{\rm ss,avg} = 9.0578 \times 10^{-4}$, and the performance degradation is 58.48%.

Last, consider the ersatz nonlinearity $\tilde{\mathcal{N}}(u) = |u-2|$, which matches the monotonicity but not the shape of $\mathcal{N}$. The closed-loop response is shown in Figure 5. Note that the steady-state average performance $z_{\rm ss,avg} = 0.0241$, which represents two orders of magnitude degradation.

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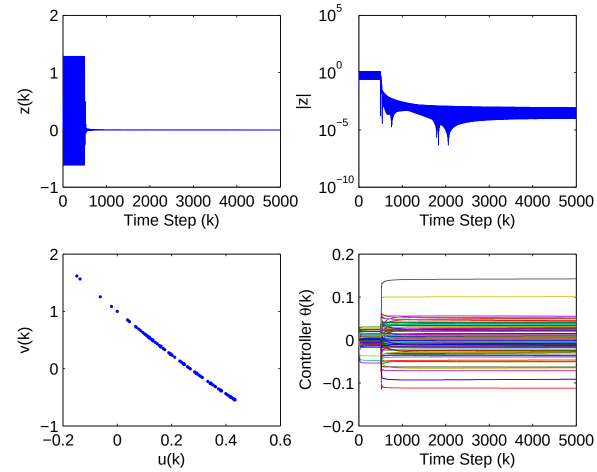

Figure 2. Response of the reference signal $r(k) = \sin(\pi/5k)$ with input nonlinearity $\mathcal{N}(u) = (u-2)^2 - 3$. We consider $\tilde{\mathcal{N}}(u) = (u-2)^2$ and the steady-state average performance $z_{\rm ss,avg} = 5.7154 \times 10^{-4}$. Note that RCANC compensates for the unknown bias in $\mathcal{N}$.

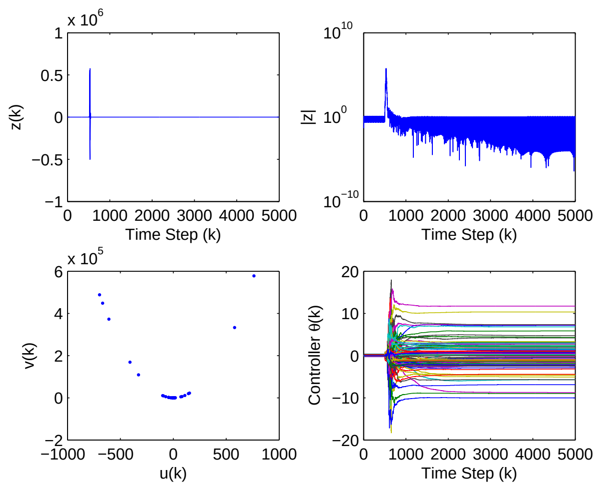

Figure 3. Response of the reference signal $r(k) = \sin(\pi/5k)$ with input nonlinearity $\mathcal{N}(u) = (u-2)^2 - 3$. We consider $\tilde{\mathcal{N}}(u) = u^2$ and note that the intervals of monotonicity of $\tilde{\mathcal{N}}$ and $\mathcal{N}$ are different. As shown in Fig 3, RCANC is not able to follow the command.

These examples suggest that the monotonicity intervals of $\mathcal{N}$ are needed to construct $\tilde{\mathcal{N}}$, and the more accurately $\tilde{\mathcal{N}}$ approximates $\mathcal{N}$, the better the performance is.

VI. Effect of Basis Functions

We now present numerical examples to illustrate the response of the RCANC with different basis functions. We assume that first nonzero Markov parameter of G and the monotonicity of $\mathcal{N}$ are known. For convenience, each example is constructed such that the first nonzero Markov parameter $H_d = 1$, where d is the relative degree of G. All examples assume $y = z$, with $\phi(k)$ given by (18), where f and g are chosen based on the choice of NARMAX structure. In all cases, we initialize the adaptive controller to be zero, that is, $\theta(0) = 0$. We let $\lambda = 1$ for all examples.

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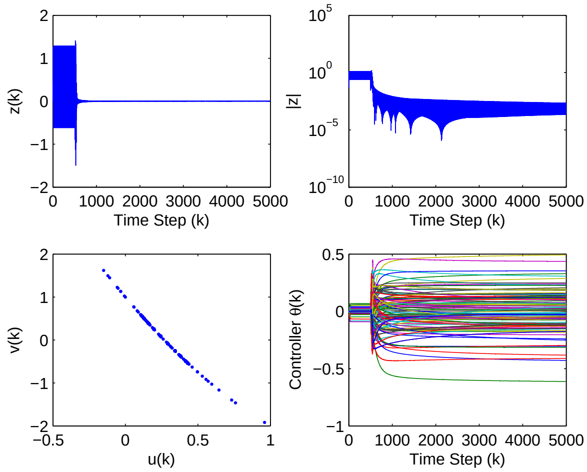

Figure 4. Response of the reference signal $r(k) = \sin(\pi/5k)$ with input nonlinearity $\mathcal{N}(u) = (u-2)^2 - 3$. We consider $\tilde{\mathcal{N}}(u) = 5(u-2)^2$ and the steady-state average performance $z_{\rm ss,avg} = 9.0578 \times 10^{-4}$. Note that the performance degradation is 58.48%.

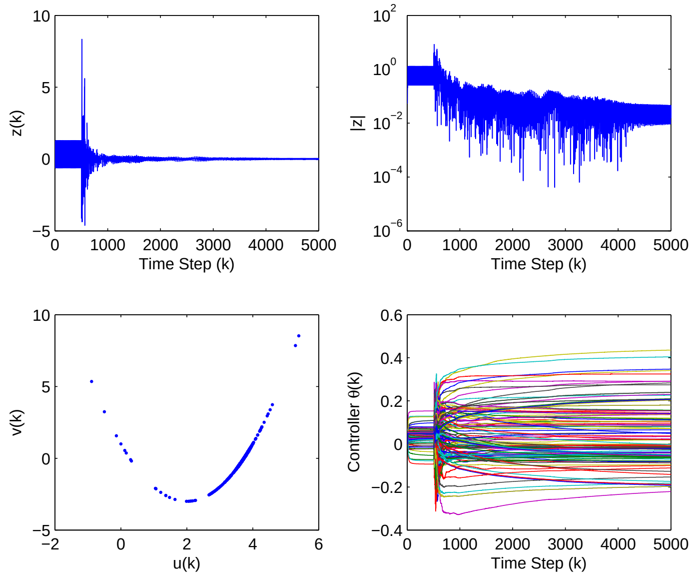

Figure 5. Response of the reference signal $r(k) = \sin(\pi/5k)$ with input nonlinearity $\mathcal{N}(u) = (u-2)^2-3$. Using $\tilde{\mathcal{N}}(u) = |u-2|$, the steady-state average performance $z_{\rm ss,avg} = 0.0241$ and the performance degradation is of two orders of magnitude. In this case, the ersatz nonlinearity $\tilde{\mathcal{N}}$ matches the monotonicity but not the shape of $\mathcal{N}$.

We consider the asymptotically stable, minimum-phase plant

$$G(z) = \frac{(z-0.5)}{z^2}, \tag{30}$$

with the input nonlinearity

$$\mathcal{N}(u) = (u-2)^2 - 3. \tag{31}$$

We consider the sinusoidal command $r(k) = \sin(\Omega_1 k)$, where $\Omega_1 = \pi/5$ rad/sample. We choose the ersatz nonlinearity $\tilde{\mathcal{N}}(u) = (u-2)^2$. Furthermore, we let $n_{\rm c} = 10$, $P_0 = I_{(s+t)n_{\rm c}}$, $\eta_0 = 0.011$, and select $\bar{B}_f = H_1 = 1$ as the required linear plant information.

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First, we consider a linear controller structure, that is, $f(u) = u$ and $f(y) = y$. The closed-loop response is shown in Figure 6. In this case, the steady-state average performance $z_{\mathrm{ss,avg}} = 0.0110$.

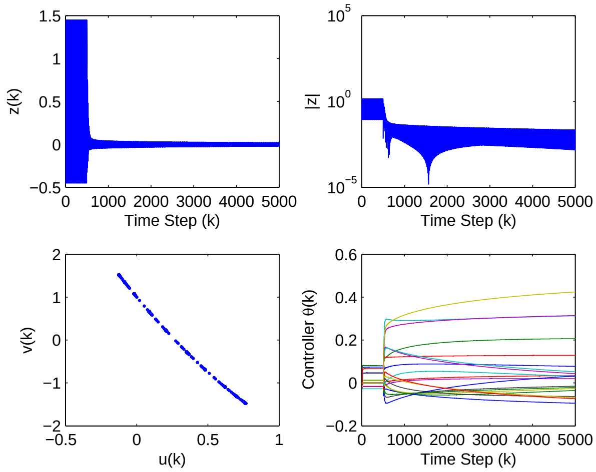

Figure 6. Response of reference signal $r(k) = \sin(\pi/5k)$ with input nonlinearity $\mathcal{N}(u) = (u-2)^2 - 3$. We consider $\tilde{\mathcal{N}}(u) = (u-2)^2$ with a linear controller structure and the steady-state average performance $|z_{\mathrm{ss,avg}}| = 0.0110$.

Next, we consider NARMAX controllers with four types of nonlinear functions, namely Fourier basis function, radial basis function [26], logistic basis function [26], and triangular basis function. In all simulations, we compute the closed-loop steady-state average performance $|z_{\mathrm{ss,avg}}|$ as we increase the number of basis functions using NARMAX/O, NARMAX/I, and NARMAX/IO structures.

VI.A. Fourier basis function

Consider sine and cosine functions of increasing frequency

$$f_i(u) = u, \sin(\frac{1}{4}u), \cos(\frac{1}{4}u), \sin(\frac{1}{2}u), \cos(\frac{1}{2}u), \sin u, \cos u, \ldots,$$
$$g_j(y) = y, \sin(\frac{1}{4}y), \cos(\frac{1}{4}y), \sin(\frac{1}{2}y), \cos(\frac{1}{2}y), \sin y, \cos y, \ldots,$$

For NARMAX/O controller structure, we let $g(y) = y$, that is, $t = 1$ in (17), and increase the number of basis functions in $f(u)$. Figure 7 shows the closed-loop steady-state average performance $|z_{\mathrm{ss,avg}}|$ decreases as we increase the number of basis functions in $f(u)$ using the NARMAX/O structure. Following the same procedure, the closed-loop steady-state average performance $|z_{\mathrm{ss,avg}}|$ for NARMAX/I and NARMAX/IO structures are shown in Figure 7. Note that overall NARMAX/O structure provides the best steady-state average performance $|z_{\mathrm{ss,avg}}|$.

VI.B. Radial Basis Function

Consider the radial basis functions

$$f_i(u) = u, e^{-u^2}, e^{-(u-0.2)^2}, e^{-(u+0.2)^2}, e^{-(u-0.4)^2}, e^{-(u+0.4)^2}, \ldots,$$
$$g_j(y) = y, e^{-y^2}, e^{-(y-0.2)^2}, e^{-(y+0.2)^2}, e^{-(y-0.4)^2}, e^{-(y+0.4)^2}, \ldots,$$

Following the same procedures, the closed-loop steady-state average performance $|z_{\mathrm{ss,avg}}|$ for NARMAX/O, NARMAX/I, and NARMAX/IO structures are shown in Figure 8. Note that overall NARMAXI/O structure provides the best steady-state average performance $|z_{\mathrm{ss,avg}}|$.

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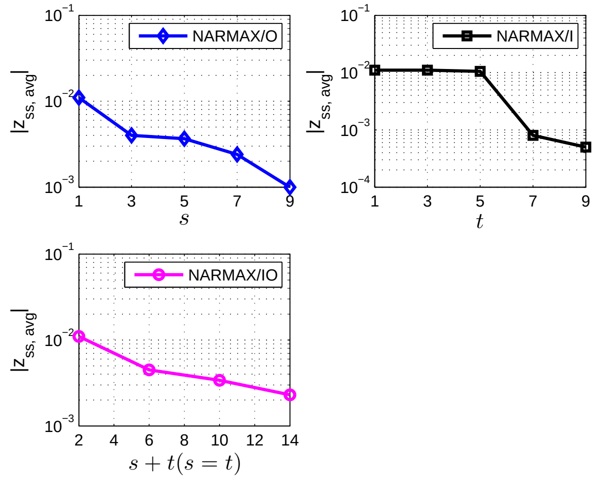

Figure 7. closed-loop steady-state average performance $|z_{\rm ss,avg}|$ with Fourier basis function for NARMAX/O, NARMAX/I and NARMAX/IO structure. $|z_{\rm ss,avg}|$ decreases as we increase the number of basis functions for all three cases. Note that NARMAX/O structure provides the best steady-state average performance $|z_{\rm ss,avg}|$.

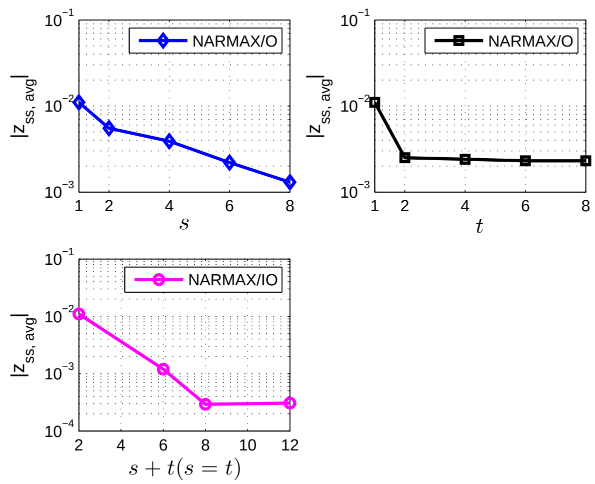

Figure 8. closed-loop steady-state average performance $|z_{\rm ss,avg}|$ with RBF for NARMAX/O, NARMAX/I and NARMAX/IO structure. $|z_{\rm ss,avg}|$ decreases as we increase the number of basis functions for all the cases. Note that overall NARMAX/IO structure provides the best steady-state average performance $|z_{\rm ss,avg}|$.

VI.C. Logistic Basis Function

Consider the logistic basis functions

$$f_i(u) = u, \frac{1}{1+e^{-u}}, \frac{1}{1+e^{-(u-0.2)}}, \frac{1}{1+e^{-(u+0.2)}}, \frac{1}{1+e^{-(u-0.4)}}, \frac{1}{1+e^{-(u+0.4)}}, \ldots,$$
$$g_j(y) = y, \frac{1}{1+e^{-y}}, \frac{1}{1+e^{-(y-0.2)}}, \frac{1}{1+e^{-(y+0.2)}}, \frac{1}{1+e^{-(y-0.4)}}, \frac{1}{1+e^{-(y+0.4)}}, \ldots,$$

Following the same procedures, the closed-loop steady-state average performance $|z_{\rm ss,avg}|$ for NARMAX/O, NARMAX/I, and NARMAX/IO structures are shown in Figure 9. Note that overall NARMAX/O structure provides the best steady-state average performance $|z_{\rm ss,avg}|$.

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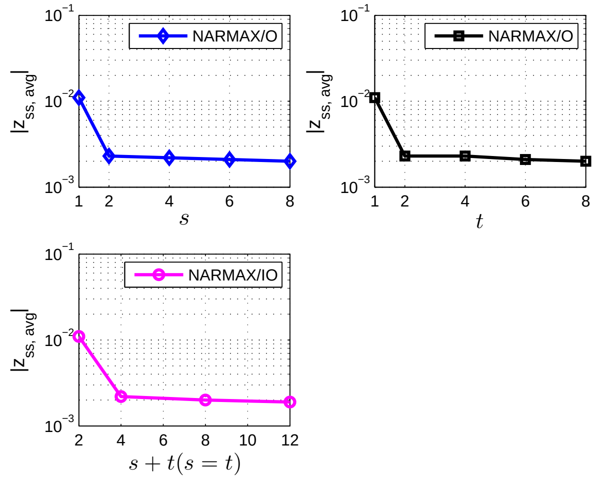

Figure 9. closed-loop steady-state average performance $|z_{\rm ss,avg}|$ with logistic basis function for NARMAX/O, NARMAX/I and NARMAX/IO structure. $|z_{\rm ss,avg}|$ decreases as we increase the number of basis functions for all three cases. Note that NARMAX/O structure provides the best steady-state average performance $|z_{\rm ss,avg}|$.

VI.D. Triangular Basis Function

Consider the triangular basis functions

$$f_i(u) = u, 1 - \max(1 - |u|, 0), 1 - \max(1 - |u - 0.2|, 0), 1 - \max(1 - |u + 0.2|, 0), \ldots,$$
$$g_j(y) = y, 1 - \max(1 - |y|, 0), 1 - \max(1 - |y - 0.2|, 0), 1 - \max(1 - |y + 0.2|, 0), \ldots,$$

Following the same procedures, the closed-loop steady-state average performance $|z_{\rm ss,avg}|$ for NARMAX/O, NARMAX/I, and NARMAX/IO structures are shown in Figure 10. Note that overall NARMAX/O structure provides the best steady-state average performance $|z_{\rm ss,avg}|$.

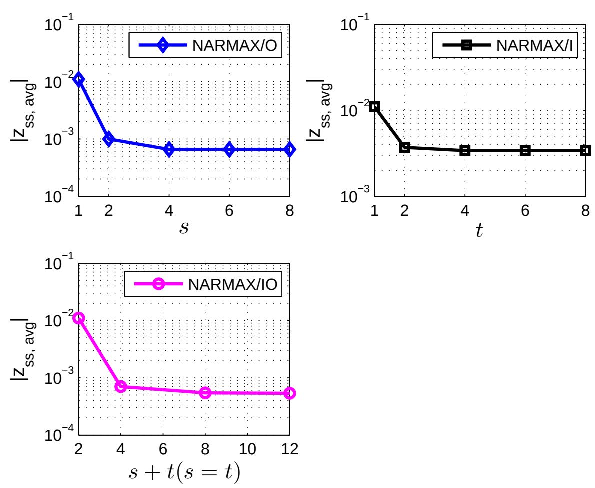

Figure 10. closed-loop steady-state average performance $|z_{\rm ss,avg}|$ with triangular basis function for NARMAX/O, NARMAX/I and NARMAX/IO structure. $|z_{\rm ss,avg}|$ decreases as we increase the number of basis functions for all the cases. Note that overall NARMAX/O structure provides the best steady-state average performance $|z_{\rm ss,avg}|$.

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VI.E. Numerical Example Summary

RCANC can improve the command-following performance for the Hammerstein systems over the linear controller structure and the closed-loop steady-state average performance decreases as we increase the number of basis functions for all three controller structures. Simulation also demonstrates that NARMAX/O and MARMAX/IO provides better command-following performance compared with NARMAX/I. However, for NARAMX/IO, the number of parameters in θ is much larger than the number of parameters for NARMAX/O, which is more computational expansive. Therefore, NARMAX/O controller structure is recommended for Hammerstein systems.

VII. Conclusions

Retrospective cost adaptive NARMAX control (RCANC) was applied to command following for Hammerstein systems. RCANC was used with limited modeling information. In particular, RCANC uses knowledge of the first nonzero Markov parameter of the linear system and the monotonicity intervals of the input nonlinearity to construct the ersatz nonlinearity. To handle the effect of the input nonlinearity, we numerically demonstrated that RCANC can improve the command-following performance for the Hammerstein systems over the linear controller structure for compensating performance distortion caused by the input nonlinearity. Future research will focus on choosing the ersatz nonlinearity and basis functions for RCANC based on limited knowledge of Hammerstein nonlinearities.

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