53rd IEEE Conference on Decision and Control
December 15-17, 2014. Los Angeles, California, USA

Adaptive Control of Uncertain Hammerstein Systems with Hysteretic Nonlinearities

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Abstract— **We numerically investigate the sense in which an adaptive control law achieves internal model control of Hammerstein plants with Prandtl-Ishlinskii hysteresis. We apply retrospective cost adaptive control (RCAC) to a command-following problem for uncertain Hammerstein systems with hysteretic input nonlinearities. The only required modeling information of the linear plant is a single Markov parameter. Describing functions are used to determine whether the adaptive controller inverts the plant at the exogenous frequencies.**

I. Introduction

Considerable effort has been devoted to developing methods that enhance the tracking performance of hysteretic systems [1]-[8]. The control algorithms applied to compensate for the hysteresis nonlinearities could be classified into two broad categories, namely, inverse-based control methods and model-based control methods. Inverse based methods apply the inverse of the hysteresis nonlinearity as a feedforward compensator to compensate for the hysteresis nonlinearity [2]-[4]. Various robust and adaptive control methods have been proposed, see for example [4]. Alternatively, model-based hysteresis compensation methods employ the hysteresis models to construct controllers that compensate for the actuator hysteresis without the explicit goal of hysteresis inversion. These methods include robust adaptive [5], energy-based [6], phase control [7], and hybrid control systems [8], which employ a hysteresis model of the actuator for constructing the controller.

In the present paper we numerically investigate the ability of an adaptive control law to achieve internal model control of Hammerstein plants with unknown input hysteresis. The internal model principle states that a stabilizing control law that achieves asymptotically perfect command following or disturbance rejection must "possess" a model of the exogenous signal [10]-[12]. This principle is the basis of PID control, where the integrator can be viewed as a model of a step command or step disturbance [13]. It is worth noting that, in a classical servo loop, where the objective is command following, the requirement for an internal model in the loop transfer function can be satisfied by the plant itself, but this is not the case for disturbance rejection. For example, asymptotic command following for a step command with a plant that has a pole at zero is achieved by any stabilizing controller, although rejection of a step command requires that the controller provide integral action.

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In the present paper we revisit internal model control within the context of adaptive control of Hammerstein systems. Although we focus on retrospective cost adaptive control (RCAC) [14]-[22], which requires minimal plant modeling information as well as no knowledge of the command or disturbance amplitude, frequency, or phase shift, the methodology that we use to assess the controller action can be applied to any control law that achieves internal stability along with either command following or disturbance rejection. Furthermore, although we focus on discrete-time control of discrete-time (possibly sampled-data) plants, the ideas are applicable to continuous-time systems.

Of special interest is the operation of the control law in terms of phase compensation. Since asymptotically perfect command following requires that the plant output match the phase and amplitude of the command, the plant input must also be a sinusoid whose amplitude and phase are consistent with the magnitude and phase shift of the plant at the command frequency. However, the phase of the control input cannot be determined in terms of the phase shift of the controller due to the fact that an internal model controller has a phase discontinuity at the command frequency. Instead, the frequency response of the transfer function from the command to the plant input is used to determine whether the control law inverts the plant at the command frequency.

The numerical investigation in this paper is intended to motivate future theoretical studies of adaptive control of hysteretic Hammerstein systems with harmonic commands and disturbances. In particular, we use the classical technique of describing functions to determine whether RCAC provides correct phase compensation in the presence of an unknown hysteretic input nonlinearity. The Prandtl-Ishlinskii hysteresis model is used to represent the input nonlinearity.

This paper shows that the classical technique of describing functions can shed light on the performance of adaptive control laws. We stress that the diagnostics that we use are not confined to RCAC, but can be used to investigate the asymptotic properties of any control law that is applicable to either harmonic command following (possibly MRAC) or harmonic disturbance rejection.

II. Background

We begin with nonadaptive control for a servo loop with harmonic commands. For a SISO LTI plant, we choose an internal model control law under the assumption that the command frequency is known. Consider the linear system

$$x(k+1) = Ax(k) + Bu(k), \quad (1)$$

$$y(k) = Cx(k), \quad (2)$$

$$e(k) = y(k) - r(k), \quad (3)$$

where $x(k) \in \mathbb{R}^n$ is the state, $y(k) \in \mathbb{R}$ is the measured output available to the controller, $e(k) \in \mathbb{R}$ is the command-following error, $u(k) \in \mathbb{R}^{l_u}$ is the control, and $r(k) \in \mathbb{R}$ is the command. The goal is to determine u that makes e small. The closed-loop system presented in Figure 1 can be represented by the cascaded system in Figure 2, where

$$G_{ur}(\mathbf{q}) = \frac{G_{\rm c}(\mathbf{q})}{1 + G_{\rm c}(\mathbf{q})G(\mathbf{q})}, \quad (4)$$

where $\mathbf{q}$ is the forward shift operator.

Suppose that the command is the harmonic signal $r(k) = \mathrm{Re}\{A_r e^{(\jmath\Omega k)}\}$, where A_r is a complex number and Ω is the command frequency with units rad/sample. If G_{ur} is asymptotically stable and u is also harmonic, then

$$u(k) = \mathrm{Re}\{A_r |G_{ur}(e^{\jmath\Omega})| e^{\jmath(\Omega k + \angle G_{ur}(e^{\jmath\Omega}))}\}, \quad (5)$$

where $|G_{ur}(e^{\jmath\Omega})|$ and $\angle G_{ur}(e^{\jmath\Omega})$ are, respectively, the magnitude and phase of G_{ur} at the frequency Ω. Then the harmonic steady-state response is given by

$$y(k) = \mathrm{Re}\{A_r |G_{ur}(e^{\jmath\Omega})||G(e^{\jmath\Omega})| e^{\jmath(\Omega k + \angle G_{ur}(e^{\jmath\Omega}) + \angle G(e^{\jmath\Omega}))}\}. \quad (6)$$

The command-following error e is given by

$$e(k) = \mathrm{Re}\{A_r e^{(\jmath\Omega k)}\} - \mathrm{Re}\{A_r |G_{ur}(e^{\jmath\Omega})||G(e^{\jmath\Omega})| e^{\jmath(\Omega k + \angle G_{ur}(e^{\jmath\Omega}) + \angle G(e^{\jmath\Omega}))}\}. \quad (7)$$

Therefore, $e(k) = 0$ if and only if the magnitude and phase of $G_{ur}(e^{\jmath\Omega})$ satisfy

$$|G_{ur}(e^{\jmath\Omega})| = \frac{1}{|G(e^{\jmath\Omega})|}, \quad (8)$$

$$\angle G_{ur}(e^{\jmath\Omega}) = -\angle G(e^{\jmath\Omega}). \quad (9)$$

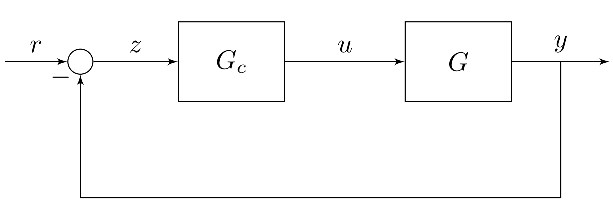

Fig. 1. Command-following problem for the linear plant G with the controller $G_{\rm c}$.

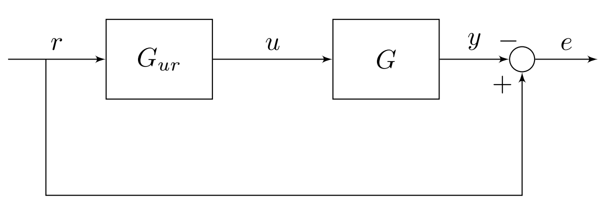

Fig. 2. Representation of the command-following problem as a cascaded system.

III. HAMMERSTEIN SYSTEM WITH INPUT HYSTERESIS

We consider the Hammerstein system shown in Figure 3, where $\mathcal{P}$ is a Prandtl-Ishlinskii hysteresis model.

A. Prandtl-Ishlinskii Hysteresis

The Prandtl-Ishlinskii hysteresis model is used to represent hysteresis in piezoceramic and magnetostrictive actuators [2],[3]. This model is based on a linear combination of play operators. For an input $u(k)$, the output $v(k)$ of the Prandtl-Ishlinskii model is represented by

$$v(k) = \mathcal{P}[u](k) \triangleq \sum_{i=1}^{n} \kappa_i \Phi_{d_i}[u](k), \quad (10)$$

where $\kappa_1, \ldots, \kappa_n$ are positive weights and the backlash operator with threshold d_i is defined by

$$\Phi_{d_i}[u](k) \triangleq \begin{cases} u(k) - d_i, & \text{if} \quad u(k) > d_i \quad \text{and} \quad u(k) > u(k-1), \\ u(k) + d_i, & \text{if} \quad u(k) < d_i \quad \text{and} \quad u(k) < u(k-1), \\ \Phi_{d_i}[u](k-1), & \text{otherwise}, \end{cases} \quad (11)$$

with the initial condition

$$\Phi_{d_i}[u](0) = \begin{cases} u(0) - d_i, & \text{if} \quad u(0) > d_i, \\ u(0) + d_i, & \text{if} \quad u(0) < d_i, \\ 0, & \text{otherwise}. \end{cases} \quad (12)$$

The backlash operator is shown in Figure 4. Since the backlash operator (11) is rate-independent, it follows that the Prandtl-Ishlinskii model is also rate-independent.

B. Problem Reformulation

In place of (1), consider the Hammerstein system consisting of (2), (3) and

$$x(k+1) = Ax(k) + Bv(k), \quad (13)$$

$$v(k) = \mathcal{P}[u](k), \quad (14)$$

where $\mathcal{P}$ is the Prandtl-Ishlinskii hysteresis model. The goal is to determine u that makes e small.

C. A Describing-Function for the Prandtl-Ishlinskii Hysteresis Model

Let $u(k) = \mathrm{Re}\{A_u e^{\jmath\Omega k}\}$, where A_u is a complex number. For $i = 1, \ldots, n$, let

$$v_i(k) = \Phi_{d_i}[u](k). \quad (15)$$

For $|A_u| > d_i$,

$$v_i(k) \cong \mathrm{Re}\{|A_u||F_i(|A_u|)| e^{\jmath(\Omega k + \angle F_i(|A_u|))}\}, \quad (16)$$

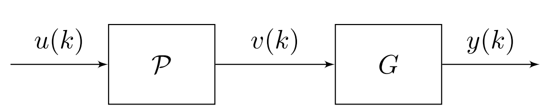

Fig. 3. Hammerstein system with Prandtl-Ishlinskii hysteresis $\mathcal{P}$.

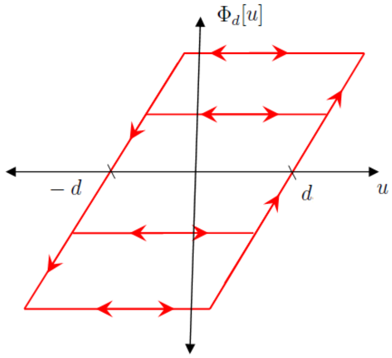

Fig. 4. The play operator with threshold d.

where the amplitude $|F_i(|A_u|)|$ and phase $\angle F_i(|A_u|)$ of the describing function of the backlash operator are given by [23]

$$|F_i(|A_u|)| = \frac{1}{|A_u|}\sqrt{{a_i}^2 + {b_i}^2}, \tag{17}$$

$$\angle F_i(|A_u|) = \tan^{-1}\frac{a_i}{b_i}, \tag{18}$$

where

$$a_i \triangleq \frac{2d_i}{\pi}\left(\eta_{\rho i} - 1\right), \tag{19}$$

$$b_i \triangleq \frac{|A_u|}{\pi}\left(\frac{\pi}{2} - \sin^{-1}\eta_{\rho i} - \eta_{\rho i}\sqrt{1-\eta_{\rho i}^2}\ \right), \tag{20}$$

where

$$\eta_{\rho i} \triangleq \frac{2d_i}{|A_u|} - 1.$$

The describing function of the Prandtl-Ishlinskii hysteresis model is given approximately by

$$H(\Omega, |A_u|) \triangleq \sum_{i=1}^{n} \kappa_i \mathrm{Re}\big\{|F_i(|A_u|)|e^{\jmath(\angle F_i(|A_u|))}\big\}. \tag{21}$$

Then, the output of the Prandtl-Ishlinskii hysteresis model is thus given approximately by

$$v(k) \triangleq \sum_{i=1}^{n} \kappa_i \mathrm{Re}\big\{A_u|F_i(|A_u|)|e^{\jmath(\Omega k + \angle F_i(|A_u|))}\big\}. \tag{22}$$

Consequently, ignoring transient effects, the output of (1)-(2) is given approximately by

$$y(k) \cong$$

$$\sum_{i=1}^{n} \mathrm{Re}\big\{A_r|G(e^{\jmath\Omega})|F_i(|A_u|)e^{\jmath(\Omega k + \angle F_i(|A_u|) + \angle G(e^{\jmath\Omega}))}\big\}. \tag{23}$$

Example 3.1: We consider the command $u(k) = \sin(\Omega k)$, where $\Omega = \pi/5$ rad/sample, the Prandtl-Ishlinskii model $\mathcal{P}$ with $n = 3$, $d_1 = 0.1, d_2 = 0.2, d_3 = 0.3$, $\kappa_1 = 0.6$, $\kappa_2 = 0.5, \kappa_3 = 0.4$. Figure 5(a) compares the output of the Prandtl-Ishlinskii model and the describing function output (22). Figure 5(b) shows the magnitude of the discrete Fourier transform $|U(\Omega)|$ of the command signal. As shown in Figure 5(b), the magnitude of the discrete Fourier transform $|Y(\Omega)|$ of the output of the Prandtl-Ishlinskii model indicates the presence of harmonics at only odd multiples of the command frequency Ω. The presence of these harmonics is consistent with the fact that the hysteresis map of the Prandtl-Ishlinskii model is an odd set-valued map. ■

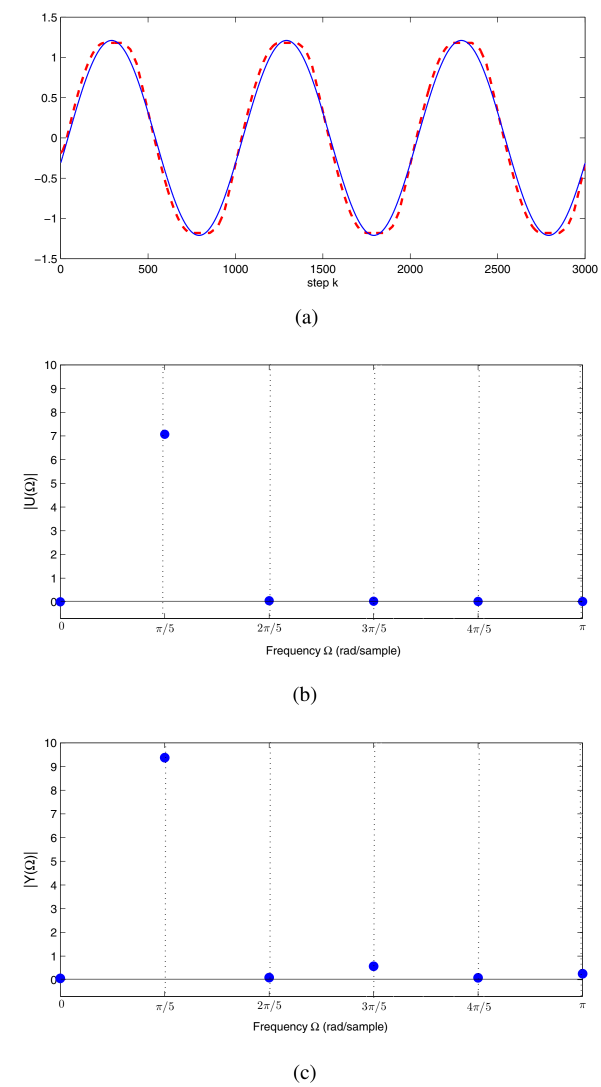

Fig. 5. (a) Shows the output (22) of the describing function (solid line) and the Prandtl-Ishlinskii model (10) (dashed line), (b) shows the magnitude of the discrete Fourier transform $|U(\Omega)|$ of the command signal, and (c) shows the magnitude of the discrete Fourier transform $|Y(\Omega)|$ of the output of the Prandtl-Ishlinskii model.

IV. Adaptive Control of Hammerstein Systems with Prandtl-Ishlinskii Hysteresis

Various techniques have been used to control systems with uncertain input nonlinearities and linear dynamics [1]-[9]. In this paper we focus on RCAC. Note that, unlike [1], RCAC does not attempt to estimate the hysteresis nonlinearity.

For the Hammerstein command-following problem, we assume that G is unknown except for an estimate of a single nonzero Markov parameter and nonminimum-phase zeros, if any are present. The input hysteresis nonlinearity $\mathcal{P}$ is also unknown.

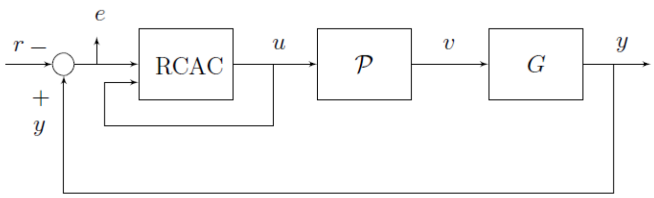

Fig. 6. Hammerstein command-following problem with the RCAC adaptive controller. The Hammerstein system consists of the input nonlinearity $\mathcal{P}$ cascaded with the linear plant G, where u is the control signal. Measurements of $y(k)$ are available for feedback; however, measurements of $v(k) = \mathcal{P}(u(k))$ are not available.

A. Control Law

In this section we present the adaptive RCAC controller used to formulate G_{ur}. Consider the controller of order n_c given by

$$u(k) = \sum_{i=1}^{n_c} M_i(k)u(k-i) + \sum_{i=1}^{n_c} N_i(k)e(k-i), \quad (24)$$

where, for all $i = 1, \dots, n_c$, $M_i(k) \in \mathbb{R}$ and $N_i(k) \in \mathbb{R}$. The control (24) can be expressed as

$$u(k) = \theta(k)\phi(k-1),$$

where

$$\theta(k) \triangleq \left[M_1(k) \cdots M_{n_c}(k) \quad N_1(k) \cdots N_{n_c}(k)\right]$$

is the matrix of controller coefficients, and the regressor vector $\phi(k)$ is given by

$$\phi(k-1) \triangleq [u(k-1) \cdots u(k-n_c) \quad e(k-1) \cdots e(k-n_c)]^{\mathrm{T}}.$$

The transfer function matrix $G_{c,k}(\mathbf{q})$ from e to u at time step k can be represented by

$$\frac{N_1(k)\mathbf{q}^{n_c-1} + N_2(k)\mathbf{q}^{n_c-2} + \cdots + N_{n_c}(k)}{\mathbf{q}^{n_c} - \left(M_1(k)\mathbf{q}^{n_c-1} + \cdots + M_{n_c-1}(k)\mathbf{q} + M_{n_c}(k)\right)}. \quad (25)$$

B. Retrospective Cost Adaptive Control

For $i \geq 1$, define the Markov parameter

$$H_i \triangleq CA^{i-1}B.$$

For example,

$$H_1 = CB$$

and

$$H_2 = CAB.$$

Let ℓ be a positive integer. Then, for all $k \geq \ell$,

$$x(k) = A^{\ell}x(k-\ell) + \sum_{i=1}^{\ell} A^{i-1}B\mathcal{P}(u(k-i)), \quad (26)$$

and thus

$$e(k) = CA^{\ell}x(k-\ell) - r(k) + \bar{H}\bar{U}(k-1), \quad (27)$$

where

$$\bar{H} \triangleq \left[\begin{array}{ccc} H_1 & \cdots & H_{\ell} \end{array}\right] \in \mathbb{R}^{1\times\ell}$$

and

$$\bar{U}(k-1) \triangleq \left[\begin{array}{c} \mathcal{P}(u(k-1)) \\ \vdots \\ \mathcal{P}(u(k-\ell)) \end{array}\right].$$

Next, we rearrange the columns of $\bar{H}$ and the components of $\bar{U}(k-1)$ and partition the resulting matrix and vector so that

$$\bar{H}\bar{U}(k-1) = \mathcal{H}'U'(k-1) + \mathcal{H}U(k-1), \quad (28)$$

where $\mathcal{H}' \in \mathbb{R}^{1\times(\ell-l_U)}$, $\mathcal{H} \in \mathbb{R}^{1\times l_U}$, $U'(k-1) \in \mathbb{R}^{\ell-l_U}$, and $U(k-1) \in \mathbb{R}^{l_U}$. Then, we can rewrite (27) as

$$e(k) = \mathcal{S}(k) + \mathcal{H}U(k-1), \quad (29)$$

where

$$\mathcal{S}(k) \triangleq CA^{\ell}x(k-\ell) - r(k) + \mathcal{H}'U'(k-1). \quad (30)$$

Next, we define the *retrospective performance*

$$\hat{e}(k) = e(k) - \mathcal{H}U(k-1) + \mathcal{H}\hat{U}(k-1). \quad (31)$$

Finally, we define the *retrospective cost function*

$$J(\hat{U}(k-1), k) \triangleq \hat{e}^2(k). \quad (32)$$

The goal is to determine refined controls $\hat{U}(k-1)$ that would have provided better performance than the controls $U(k)$ that were applied to the system. The refined control values $\hat{U}(k-1)$ are subsequently used to update the controller. Next, to ensure that (32) has a global minimizer, we consider the regularized cost

$$\bar{J}(\hat{U}(k-1), k) \triangleq \hat{e}^2(k) + \eta(k)\hat{U}^{\mathrm{T}}(k-1)\hat{U}(k-1), \quad (33)$$

where $\eta(k) \geq 0$. Substituting (31) into (33) yields

$$\bar{J}(\hat{U}(k-1), k) = \hat{U}(k-1)^{\mathrm{T}}\mathcal{A}(k)\hat{U}(k-1) + \mathcal{B}(k)\hat{U}(k-1) + \mathcal{C}(k),$$

where

$$\begin{aligned} \mathcal{A}(k) &\triangleq \mathcal{H}^{\mathrm{T}}\mathcal{H} + \eta(k)I_{l_U}, \\ \mathcal{B}(k) &\triangleq 2\mathcal{H}^{\mathrm{T}}[e(k) - \mathcal{H}U(k-1)], \\ \mathcal{C}(k) &\triangleq e^2(k) - 2e(k)\mathcal{H}U(k-1) + U^{\mathrm{T}}(k-1) + \\ &\quad \mathcal{H}^{\mathrm{T}}\mathcal{H}U(k-1). \end{aligned} \quad (34)$$

If either $\mathcal{H}$ has full column rank or $\eta(k) > 0$, then $\mathcal{A}(k)$ is positive definite. In this case, $\bar{J}(\hat{U}(k-1), k)$ has the unique global minimizer

$$\hat{U}(k-1) = -\frac{1}{2}\mathcal{A}^{-1}(k)\mathcal{B}(k). \quad (35)$$

Next, define the cumulative cost function

$$J(\theta,k) \triangleq \sum_{i=1}^{k} \lambda^{k-i} \|\phi^{\mathrm{T}}(i-1)\theta^{\mathrm{T}}(k) - \hat{u}^{\mathrm{T}}(i)\|^2 + \lambda^k(\theta(k)-\theta(0))P^{-1}(0)(\theta(k)-\theta(0))^{\mathrm{T}}, \quad (36)$$

where $\|\cdot\|$ is the Euclidean norm, and $\lambda \in (0,1]$ is the forgetting factor. Minimizing (36) yields

$$\theta^{\mathrm{T}}(k) = \theta^{\mathrm{T}}(k-1) + P(k-1)\phi(k-1) \cdot [\phi^{\mathrm{T}}(k)P(k-1)\phi(k-1) + \lambda(k)]^{-1} \cdot [\phi^{\mathrm{T}}(k-1)\theta^{\mathrm{T}}(k-1) - \hat{u}^{\mathrm{T}}(k)]. \quad (37)$$

The error covariance is updated by

$$P(k) = \lambda^{-1}P(k-1) - \lambda^{-1}P(k-1)\phi(k-1) \cdot [\phi^{\mathrm{T}}(k-1)P(k-1)\phi(k) + \lambda]^{-1} \cdot \phi^{\mathrm{T}}(k-1)P(k-1). \quad (38)$$

We initialize the error covariance matrix as $P(0) = \alpha I_{2n_c}$, where $\alpha > 0$.

V. Numerical Examples

In this section we present simulation results for adaptive control of the Hammerstein system presented in Figure 3. The objective is to determine whether RCAC can achieve internal model control in the presence of the unknown input hysteresis nonlinearity.

A. The Prandtl-Ishlinskii Hysteresis Model

In this section we consider the Prandtl-Ishlinskii hysteresis nonlinearity. To investigate this question, we examine the magnitude and phase of

$$\tilde{G}_{ur}(e^{\jmath\Omega}) \triangleq \frac{G_{c,2000}(e^{\jmath\Omega})}{1 + H(\Omega,|A_u|)G(e^{\jmath\Omega})G_{c,2000}(e^{\jmath\Omega})}. \quad (39)$$

The magnitude $|\tilde{G}_{ur}(e^{\jmath\Omega})|$ reveals whether the controller $G_{c,2000}(e^{\jmath\Omega})$ provides high magnitude at the command frequencies and the harmonics introduced by the Hammerstein system in Figure 3. The phase $\angle\tilde{G}_{ur}(e^{\jmath\Omega})$ shows whether $G_{c,2000}(e^{\jmath\Omega})$ compensates the phase shift provided by the Hammerstein system presented in Figure 3 at the command frequencies and their harmonics.

Example 5.1: Consider the command $r(k) = \sin(\frac{\pi}{5}k)$, the Prandtl-Ishlinskii hysteresis model $\mathcal{P}$ with $n=4$, $d_1 = 0, d_2 = 0.1, d_3 = 0.2, d_4 = 0.3, \kappa_1 = 0.8, \kappa_2 = 0.6, \kappa_3 = 0.4, \kappa_4 = 0.3$, and the asymptotically stable linear plant $G(z) = \frac{z-0.5}{(z-0.8)(z-0.6)}$. We use RCAC with $n_c = 14$, $\lambda = 1$, and $\alpha = 1$. Figure 7 shows the closed-loop response. RCAC minimizes the command-following error e when the input hysteresis nonlinearity shown in Figure 7(b) is considered. Figure 7(e) shows that $1/\tilde{G}_{ur}$ and HG coincide at the frequencies $\pi/5$, $3\pi/5$, π rad/sample. ■

Example 5.2: Consider the command $r(k) = \sin(\frac{\pi}{5}k)$, the Prandtl-Ishlinskii model $\mathcal{P}$ with $n=4$, $d_1 = 0$, $d_2 = 0.1$, $d_3 = 0.2$, $d_4 = 0.3$, $\kappa_1 = 0.8$, $\kappa_2 = 0.6$, $\kappa_3 = 0.4$, $\kappa_4 = 0.3$, with the unstable plant $G(z) = \frac{1}{z-1.1}$. We use RCAC with $n_c = 14$, $\lambda = 1$, and $\alpha = 1$. Figure 8 shows the closed-loop response. As shown in Example 5.1, Figure 8(e) shows that $1/\tilde{G}_{ur}$ and HG coincide at the frequencies $\pi/5$, $3\pi/5$, π rad/sample. ■

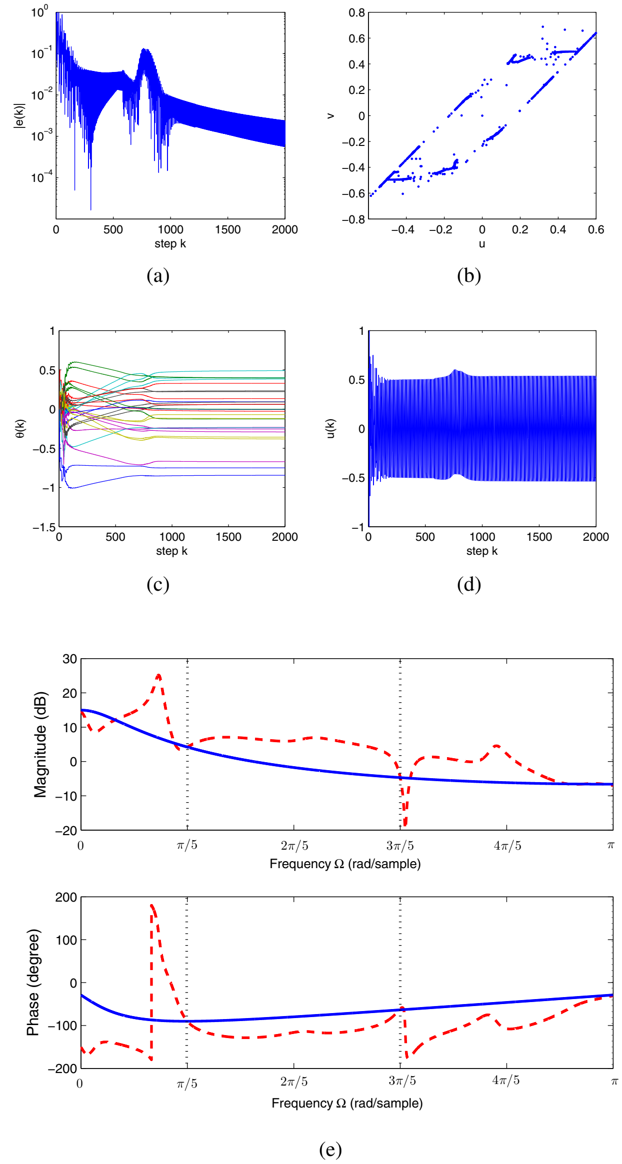

Fig. 7. Example 5.1: (a) shows the command-following error e for the asymptotically stable linear plant $G(z) = \frac{z-0.5}{(z-0.8)(z-0.6)}$ with the Prandtl-Ishlinskii model $\mathcal{P}$ whose input and output are shown in (b) for the closed-loop system with RCAC. (c) shows the evolution of the controller θ and the command following error e for the asymptotically stable linear plant $G(z) = \frac{z-0.5}{(z-0.8)(z-0.6)}$, (e) shows the frequency response of $1/\tilde{G}_{ur}$ (dashed line) and HG (solid line). Note that $1/\tilde{G}_{ur}$ and HG coincide at the frequencies $\pi/5$, $3\pi/5$, π rad/sample.

Consistent with Example 3.1, the output of the Prandtl-Ishlinskii hysteresis model shows harmonics at odd multiples of the command frequency Ω. Examples 5.1 and 5.2 show that $\tilde{G}_{ur}$ constructed with RCAC inverts the magnitude and phase of the Hammerstein system. That is, the magnitude and phase of $\tilde{G}_{ur}(e^{\jmath\Omega})$ approximately satisfy

$$|\tilde{G}_{ur}(e^{\jmath\Omega})| = \frac{1}{\sum_{i=0}^{n} \mathrm{Re}\{A_r|G(e^{\jmath\Omega})|F_i(|A_u|)\}}, \quad (40)$$

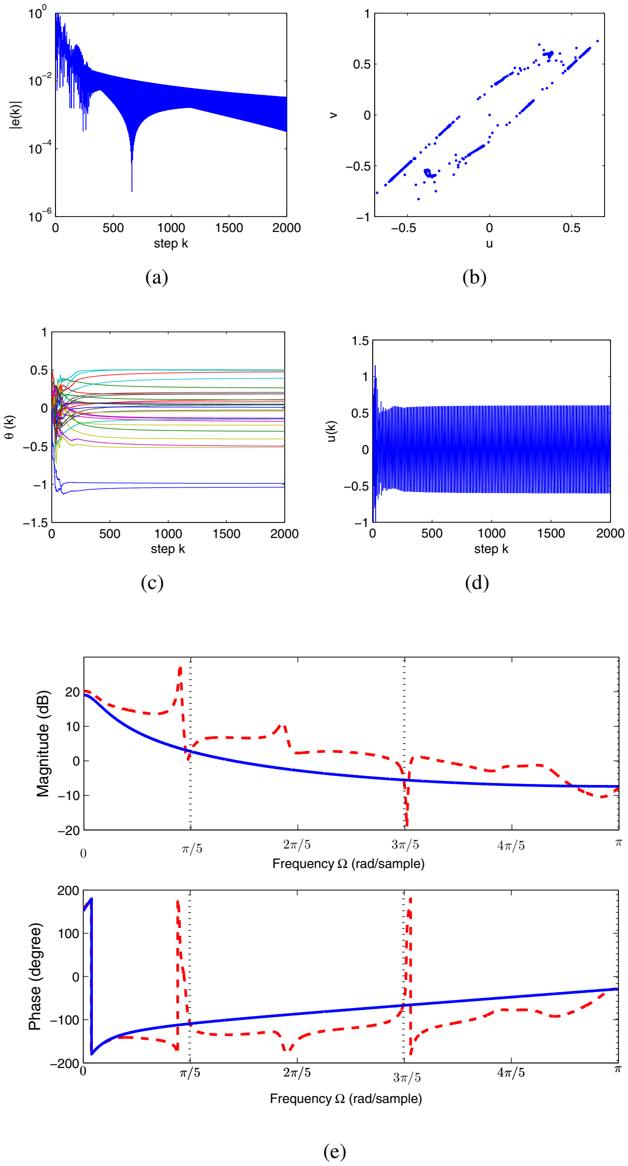

Fig. 8. Example 5.2: (a) shows the command-following error e for the unstable linear plant $G(z) = \frac{1}{z-1.1}$ with the Prandtl-Ishlinskii model $\mathcal{P}$, whose input and output are shown in (b) for the closed-loop system with RCAC. (c) shows the evolution of the controller θ and the command-following error e for the unstable linear plant $G(z) = \frac{1}{z-1.1}$, (e) shows the frequency response for $1/\tilde{G}_{ur}$ (dashed line) and HG (solid line). Note that $1/\tilde{G}_{ur}$ and HG coincide at the frequencies $\pi/5$, $3\pi/5$, π rad/sample.

$$\angle\tilde{G}_{ur}(e^{\jmath\Omega}) = -\angle G(e^{\jmath\Omega}) - \sum_{i=0}^{n} \angle F_i(|A_u|). \tag{41}$$

VI. Conclusions

The numerical investigation in the paper shows that RCAC can achieve internal model control of Hammerstein systems with an unknown Prandtl-Ishlinskii input hysteresis. A describing function was used to show that RCAC inverts the Hammerstein system at the command frequency of the harmonic command input.

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