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Retrospective Cost Adaptive Spacecraft Attitude Control using Control Moment Gyros

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Abstract— **We present a numerical investigation of the performance of retrospective cost adaptive control (RCAC) for spacecraft attitude control using control-moment-gyroscopes (CMG). The setup consists of three orthogonally mounted CMG's that are velocity commanded without the use of a steering law. RCAC is applied in a decentralized architecture; each CMG is commanded by an independent RCAC control laws. This architecture simplifies the required modeling information and treats the axis-coupling effects as unmodeled disturbances.A rotation-matrix parameterization of attitude is used to implement a dynamic compensator with state feedback. The adaptive controller is able to complete various slew and spin maneuvers using limited information about the mass-distribution of the spacecraft.**

I. Introduction

The traditional approach to spacecraft attitude control using CMG's is to implement a controller that specifies a desired torque. This torque is then realized by a steering law that commands the angular velocities of each CMG gimbal [1]. A difficulty of this approach is the fact that the CMG's may reach a singular configuration in which the torques that can be produced are confined to a plane. The singularity prevents the use of the matrix inverse required by many steering laws [2]. Therefore, various methods for constructing singularity-avoiding steering laws have been developed, [1], [3]–[5]. Singularities can also be avoided by implementing CMG's with variable-speed wheels at the cost of additional mechanical complexity [6].

In the present paper we consider spacecraft attitude control with CMG's using an adaptive control law that directly commands the gimbal velocities. Previous adaptive methods for single-gimbal CMG's include singularity avoiding steering laws as part of the control synthesis [7]. Other adaptive methods have focused on variable speed [8], or double gimbaled CMG's [9]. Instead of using steering laws and pseudo-inverses to deal with singularities we apply retrospective cost adaptive control. This adaptive controller requires minimal modeling of the spacecraft inertia for thruster [10], reaction-wheel [11], and magnetic-torque [12] actuation.

The primary objective of this paper is to assess the performance and robustness of RCAC in the presence of time-varying singularities in the input matrix for velocity-commanded CMG's. The setup consists of three orthogonally mounted, single-gimbal, fixed-speed CMG's in a decentralized architecture; each of the CMG's has an independent controller. This architecture simplifies the required modeling information, leaves axis-coupling effects to be compensated by RCAC, and avoids the need for matrix inversion.

Two classes of maneuvers are considered, namely, motion-to-rest maneuvers, where the goal is to bring the spacecraft attitude to rest with a specified inertial orientation, and motion-to-spin maneuvers, where the goal is to bring the spacecraft to a spin with a given rate about a specified body-fixed axis with a specified inertial attitude.

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II. Spacecraft Model with CMGs

We develop the dynamics for a rigid spacecraft actuated by three single-gimbal, fixed-speed CMGs. We begin by defining several frames: an inertial frame $\mathrm{F_I}$, a body frame $\mathrm{F_B}$, fixed to the spacecraft bus, a frame $\mathrm{F}_{\mathrm{G}_i}$, attached to each gimbal, and a frame $\mathrm{F}_{\mathrm{W}_i}$, fixed to each wheel . We specify the kinematics using Poisson's equation,

$$\overset{\mathrm{B}\bullet}{\vec{R}}_{\mathrm{B/I}} = \vec{R}_{\mathrm{B/I}} \vec{\omega}^{\times}_{\mathrm{B/I}}, \tag{1}$$

where $\mathrm{B}\bullet$ denotes a derivative with respect to $\mathrm{F_B}$. The proper orthogonal matrix $\vec{R}_{\mathrm{B/I}}$ is a rotation matrix that transforms the components of a vector resolved in frame B into the components of the same vector resolved in frame I, $\vec{\omega}_{\mathrm{B/I}}$ is the angular velocity of the body frame with respect to the inertial frame, and the superscript $\times$ denotes the skew-symmetric cross product matrix.

Consider a spacecraft composed of a bus b and control moment gyros. We can describe the spacecraft's dynamics using Euler's equation,

$$\overset{\mathrm{B}\bullet}{\vec{H}}_{\mathrm{sc/c/I}} = \vec{H}_{\mathrm{sc/c/I}} \times \vec{\omega}_{B/I} + \vec{\tau}_{\mathrm{d}}, \tag{2}$$
$$\vec{H}_{\mathrm{sc/c/I}} = \vec{J}_{\mathrm{b/c}} \vec{\omega}_{\mathrm{B/I}} + \sum_{i=1}^{n_{\mathrm{G}}} \vec{H}_{\mathrm{w}_i/\mathrm{c/I}},$$

where $\vec{J}_{\mathrm{b/c}}$ is the inertia tensor of the bus relative to the spacecraft's center of mass c, $\vec{\tau}_{\mathrm{d}}$ is the sum of external disturbance torques, and n_{G} is the number of CMG's. The angular momentum of gyro wheel i relative to c with respect to $\mathrm{F_I}$ is given by

$$\vec{H}_{\mathrm{w}_i/\mathrm{c/I}} = \vec{J}_{\mathrm{w}_i/\mathrm{c}} \vec{\omega}_{\mathrm{B/I}} + \vec{J}_{\mathrm{w}_i/\mathrm{c}_i} \vec{\omega}_{\mathrm{W}_i/\mathrm{B}}, \tag{3}$$

where the inertia of gyro wheel i relative to the spacecraft's center of mass is $\vec{J}_{\mathrm{w}_i/\mathrm{c}}$ and $\vec{J}_{\mathrm{w}_i/\mathrm{c}_i}$ is the inertia tensor of gyro wheel i relative to its respective center of mass c_i. The angular velocity of gyro wheel i relative to $\mathrm{F_B}$ is given by

$$\vec{\omega}_{\mathrm{W}_i/\mathrm{B}} = \vec{\omega}_{\mathrm{W}_i/\mathrm{G}_i} + \vec{\omega}_{\mathrm{G}_i/\mathrm{B}}, \tag{4}$$

where $\vec{\omega}_{\mathrm{W}_i/\mathrm{G}_i}$ is the angular velocity of gyro wheel i relative to gimbal i, and $\vec{\omega}_{\mathrm{G}_i/\mathrm{B}}$ is the angular velocity of gimbal i with respect to F_B.

We assume that the gimbal wheels are spherical with moment of inertia β,

$$\vec{J}_{\mathrm{W_i/c_i}} = \beta \vec{I},$$

where $\vec{I}$ is the physical identity matrix. This assumption results in a time-invariant spacecraft inertia and sidesteps a common assumption in the CMG literature in which the gyros are assumed to be much smaller than the bus. Furthermore, we assume that each wheel spins about its axis at a constant angular rate that is identical for all CMGs.

The above assumptions combined with (2), (3), and (4), yield the dynamic equation for a spacecraft with spherical, constant-speed CMG's,

$$\begin{aligned}\vec{J}_{\mathrm{sc/c}} \overset{\mathrm{B}\bullet}{\vec{\omega}}_{\mathrm{B/I}} &= \left[\vec{J}_{\mathrm{sc/c}}\vec{\omega}_{\mathrm{B/I}} + \beta \sum_{i=1}^{n_\mathrm{G}} \vec{\omega}_{\mathrm{W}_i/\mathrm{G}_i}\right]^\times \vec{\omega}_{\mathrm{B/I}} \\ &+ \beta \sum_{i=1}^{n_\mathrm{G}} \left[\vec{\omega}_{\mathrm{W}_i/\mathrm{G}_i} - \vec{\omega}_{\mathrm{B/I}}\right]^\times \vec{\omega}_{\mathrm{G}_i/\mathrm{B}} \\ &- \beta \sum_{i=1}^{n_\mathrm{G}} \overset{\mathrm{B}\bullet}{\vec{\omega}}_{\mathrm{G}_i/\mathrm{B}} + \vec{\tau}_\mathrm{d}. \end{aligned} \tag{5}$$

where $\vec{J}_{\mathrm{sc/c}}$ is the combined inertia of the bus and the CMG's relative to c. A more detailed derivation is given in [13].

To formulate the control problem we resolve the kinematics and dynamics in the body frame. To resolve (5) in F_B we first resolve $\vec{\omega}_{\mathrm{W}_i/\mathrm{G}_i}$ and $\vec{\omega}_{\mathrm{G}_i/\mathrm{B}}$ in their respective gyro frames, that is,

$$\vec{\omega}_{\mathrm{W}_i/\mathrm{G}_i}\Big|_{\mathrm{G}_i} = e_1 q, \qquad \vec{\omega}_{\mathrm{G}_i/\mathrm{B}}\Big|_{\mathrm{G}_i} = e_2 u_i$$

where e_i is the ith column of the identity matrix, u_i is the scalar commanded angular rate of gimbal i, and $q > 0$ is the angular rate of each wheel. Thus, in the body frame,

$$\vec{\omega}_{\mathrm{W}_i/\mathrm{G}_i}\Big|_{\mathrm{B}} = \mathcal{O}_i e_1 q, \qquad \vec{\omega}_{\mathrm{G}_i/\mathrm{B}}\Big|_{\mathrm{B}} = \mathcal{O}_i e_2 u_i$$

where $\mathcal{O}_i$ transforms vectors resolved in $\mathrm{F}_{\mathrm{G}_i}$ into F_B. We assume that $n_\mathrm{G} = 3$ and that the gimbals are mounted orthogonally along the body axes such that

$$\vec{\omega}_{\mathrm{G}_i/\mathrm{B}}\Big|_{\mathrm{B}} = e_i u_i, \qquad \overset{\mathrm{B}\bullet}{\vec{\omega}}_{\mathrm{G}_i/\mathrm{B}}\Big|_{\mathrm{B}} = e_i \dot{u}_i.$$

Note that, the control u_i also affects the alignment of the gyros, that is,

$$\dot{\mathcal{O}}_i = -u_i e_i^\times \mathcal{O}_i. \tag{6}$$

Finally, we define the notation

$$R \triangleq \vec{R}_{\mathrm{B/I}}\Big|_{\mathrm{B}}, \qquad J_\mathrm{sc} \triangleq \vec{J}_{\mathrm{sc/c}}\Big|_{\mathrm{B}},$$

$$\omega \triangleq \vec{\omega}_{\mathrm{B/I}}\Big|_{\mathrm{B}}, \qquad \dot{\omega} \triangleq \overset{\mathrm{B}\bullet}{\vec{\omega}}_{\mathrm{B/I}}\Big|_{\mathrm{B}}, \qquad \tau_\mathrm{d} \triangleq \vec{\tau}_\mathrm{d}\Big|_{\mathrm{B}}.$$

Therefore, the kinematics in (1) are given by,

$$\dot{R} = R\omega^\times, \tag{7}$$

and the dynamics in (5) are

$$J_\mathrm{sc}\dot{\omega} = \left[J_\mathrm{sc}\omega + \beta \sum_{i=1}^{n_\mathrm{G}} \mathcal{O}_i e_1 q\right]^\times \omega + B_\mathrm{sc} u - \beta \dot{u} + \tau_\mathrm{d}, \tag{8}$$

where

$$u \triangleq \left[\begin{array}{ccc} u_1 & u_2 & u_3 \end{array}\right]^\mathrm{T},$$

and the ith column of B_sc is given by

$$b_i \triangleq \beta \left(\mathcal{O}_i e_1 q - \omega\right)^\times e_i. \tag{9}$$

Note that B_sc is implicitly time-varying due to its dependence on the states. Furthermore, in practice $q >> \|\omega\|$. Therefore, the input matrix is dominated by the gimbal angles embedded in the orientation matrices $\mathcal{O}_i$:

$$B_\mathrm{sc} \approx q\beta \left[\begin{array}{ccc} (\mathcal{O}_1 e_1)^\times e_1 & (\mathcal{O}_2 e_1)^\times e_2 & (\mathcal{O}_3 e_1)^\times e_3 \end{array}\right]. \tag{10}$$

III. RCAC ALGORITHM

We review retrospective cost adaptive control, a more detailed development can be found in [14]. Consider a MIMO discrete-time system

$$x(k) = Ax(k-1) + Bu(k-1), \tag{11}$$

$$z(k) = Cx(k) - r(k), \tag{12}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^{l_u}$, $z(k) \in \mathbb{R}^{l_z}$ is the *performance variable*, $r(k) \in \mathbb{R}^{l_z}$ is the command, the matrices A, B, C have appropriate dimensions, and $k \geq 1$.

RCAC is a direct adaptive controller that minimizes the performance variable z. The controller uses the Markov parameters of the transfer function from u to z as modeling parameters. The ith Markov parameter is given by

$$H_i \triangleq E_1 A^{i-1} B, \ i \geq 1. \tag{13}$$

We define the controller $u(k)$ by the strictly proper time-series,

$$u(k) \triangleq \sum_{i=1}^{n_\mathrm{c}} M_i u(k-i) + \sum_{i=1}^{n_\mathrm{c}} N_i z(k-i) = \theta(k)\phi(k), \tag{14}$$

where

$$\theta(k) \triangleq [M_1(k) \cdots M_{n_\mathrm{c}}(k)\ N_1(k) \cdots N_{n_\mathrm{c}}(k)] \in \mathbb{R}^{l_u \times n_\mathrm{c}(l_u + l_z)}, \tag{15}$$

for $M_i(k) \in \mathbb{R}^{l_u \times l_u}$, $N_i(k) \in \mathbb{R}^{l_u \times l_z}$, and

$$\phi(k) \triangleq \left[\begin{array}{c} u(k-1) \\ \vdots \\ u(k-n_\mathrm{c}) \\ z(k-1) \\ \vdots \\ z(k-n_\mathrm{c}) \end{array}\right] \in \mathbb{R}^{n_\mathrm{c}(l_u+l_z)}. \tag{16}$$

The controller parameter $\theta(k)$ is updated using the Markov parameters to compute the *retrospective control*,

$$\hat{u}(k-1) \triangleq \left[H_1^{\mathrm{T}} H_1 + \eta(k) I(k)\right]^{-1} H_1^{\mathrm{T}} \left[H_1 u(k-1) - z(k)\right]. \quad (17)$$

The retrospective control is then used in a recursive least squares update given by,

$$\theta^{\mathrm{T}}(k) \triangleq \theta^{\mathrm{T}}(k-1) + \frac{P(k-1)\phi(k-2)[\theta(k-1)\phi(k-2) - \hat{u}(k-1)]^{\mathrm{T}}}{[1+\phi^{\mathrm{T}}(k-2)P(k-1)\phi(k-2)]}. \quad (18)$$

where the regressor matrix $P(k)$ is updated by

$$P(k) \triangleq P(k-1) - \frac{P(k-1)\phi(k-2)\phi^{\mathrm{T}}(k-2)P(k-1)}{[1+\phi^{\mathrm{T}}(k-2)P(k-1)\phi(k-2)]}. \quad (19)$$

IV. Attitude Control with RCAC using CMG's

The objective of the attitude control problem is to determine control inputs such that the spacecraft attitude given by R follows a commanded attitude trajectory given by a possibly time-varying C^1 rotation matrix $R_{\mathrm{d}}(t)$. For $t \geq 0$, $R_{\mathrm{d}}(t)$ is given by

$$\dot{R}_{\mathrm{d}}(t) = R_{\mathrm{d}}(t)\omega_{\mathrm{d}}(t)^{\times}, \quad (20)$$

where ω_{d} is the desired, possibly time-varying angular velocity. The error between $R(t)$ and $R_{\mathrm{d}}(t)$ is given in terms of the attitude-error rotation matrix

$$\tilde{R} \triangleq R_{\mathrm{d}}^{\mathrm{T}} R, \qquad \dot{\tilde{R}} = \tilde{R}\tilde{\omega}^{\times}. \quad (21)$$

The angular velocity error $\tilde{\omega}$ is defined by

$$\tilde{\omega} \triangleq \omega - \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}}. \quad (22)$$

We reformulate the attitude-error using the vector parameter presented [15]. Let $A_{\mathrm{att}} = \mathrm{diag}(a_1, a_2, a_3)$ be a diagonal, positive-definitie matrix, then

$$S_{\mathrm{od}} \triangleq \sum_{i=1}^{3} a_i(\tilde{R}^{\mathrm{T}} e_i) \times e_i = \begin{bmatrix} -a_2\tilde{R}_{23} + a_3\tilde{R}_{32} \\ a_1\tilde{R}_{13} - a_3\tilde{R}_{31} \\ -a_1\tilde{R}_{13} + a_3\tilde{R}_{21} \end{bmatrix}, \quad (23)$$

is a vector measure of attitude-error that utilizes the off-diagonal entries of the attitude error $\tilde{R}$.

Proposition 1. $S_{\mathrm{od}} = 0$ if and only if $\tilde{R} \in \{I, \mathrm{diag}(1,-1,-1), \mathrm{diag}(-1,1,-1), \mathrm{diag}(-1,-1,1)\}$. Thus, S_{od} has three spurious equilibria, to distinguish between these we define,

$$S_{\mathrm{d}} \triangleq \begin{bmatrix} 2 - \tilde{R}_{22} - \tilde{R}_{33} \\ 2 - \tilde{R}_{11} - \tilde{R}_{33} \\ 2 - \tilde{R}_{11} - \tilde{R}_{22} \end{bmatrix}. \quad (24)$$

Proposition 2. $S_{\mathrm{od}} = S_{\mathrm{d}} = 0$ if and only if $\tilde{R} = I$.

Thus, we define the performance variable z for the attitude control problem as

$$z \triangleq \begin{bmatrix} \tilde{\omega}^{\mathrm{T}} & S_{\mathrm{od}}^{\mathrm{T}} & S_{\mathrm{d}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^9, \quad (25)$$

Therfore, $z = 0$ corresponds to $\tilde{R} = I, \tilde{\omega} = 0$.

A. Markov Parameter

We use the linearized model described in [11] to obtain an inertia-free Markov parameter for attitude control,

$$H \triangleq CB \approx \begin{bmatrix} hB_{\mathrm{sc}} \\ \frac{1}{2}h^2 M_a R_a M_a^{\mathrm{T}} B_{\mathrm{sc}} \\ \frac{1}{2}h^2 M_a M_a^{\mathrm{T}} B_{\mathrm{sc}} \end{bmatrix} \in \mathbb{R}^{9\times 3}. \quad (26)$$

Where h is the controller time-step and

$$M_a \triangleq \begin{bmatrix} e_1^{\times} & e_2^{\times} & e_3^{\times} \end{bmatrix}, \qquad R_a \triangleq A_{\mathrm{att}} \otimes I_3.$$

Note that the Markov parameter corresponding to S_{d} was chosen heuristically.

B. Singularities and Decentralized Control

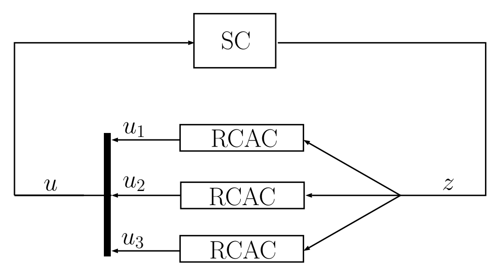


Fig. 1. Block diagram for decentralized control of CMGs using RCAC.

For the spacecraft described by (8), CMG singularities present themselves as a rank deficiency in the actuator matrix B_{sc} in (10). Singular CMG configurations result in a rank deficient H in (26) which may prevent the computation of (17). Therefore, we implement three multi-input single-output controllers; each RCAC subcontroller commands the shaft angular velocity of one of the gimbals. However, unlike the approach in [12], each controller utilizes the same performance variable as shown in Figure 1. The modeling information used by each controller is given by the ith column of (26), and thus the Markov parameter for the ith RCAC controller is given by

$$H_i' \triangleq He_i \in \mathbb{R}^9. \quad (27)$$

Thus, the retrospective controls in (17) are now scalar and defined even in singular CMG configurations,

$$\hat{u}_i(k-1) = \frac{{H_i'}^{\mathrm{T}}\left[H_i' u_i(k-1) - z(k)\right]}{{H_i'}^{\mathrm{T}} H_i' + \eta(k)}.$$

V. Numerical Examples

We consider two scenarios, namely, motion-to-rest (M2R) maneuvers and motion-to-spin (M2S) maneuvers, where rest and spin refer to motion relative to an inertial frame. A M2R maneuver may begin from rest or an arbitrary angular velocity. Thus, M2R includes slews, detumbling, and stabilization,the goal is to have the spacecraft come to rest with a specified attitude. M2S maneuvers require that the spacecraft spin about a specified body axis that is pointed in a specified inertial direction. If the M2R and M2S maneuvers begin from zero angular velocity, then we use the terminology rest-to-rest (R2R) and rest-to-spin (R2S), respectively.

A. Gimbal Angles and Singularities

The orientation matrix $\mathcal{O}_i$ depends on the angle ϕ_i rotated by gimbal i about its shaft. We define these angles as follows, $\phi_1 = 0$ when $\vec{\omega}_{\mathrm{W}_1/\mathrm{G}_1}\Big|_B = e_2 q$, $\phi_2 = 0$ when $\vec{\omega}_{\mathrm{W}_2/\mathrm{G}_2}\Big|_B = e_3 q$, and $\phi_3 = 0$ when $\vec{\omega}_{\mathrm{W}_3/\mathrm{G}_3}\Big|_B = e_1 q$. The input matrix B_{sc} is singular when the configuration of gimbal angles ϕ_i cannot provide instantaneous torque about every direction. When B_{sc} is singular, also known as gimbal lock, the instantaneous torque that the gimbals can provide is confined to a plane. If the required instantaneous torque lies in this plane, we use the term *non-obstructing singularity*, otherwise it is an *obstructing singularity*.

B. Baseline Spacecraft Parameters

The bus inertia matrix J_{b} is given by

$$J_{\mathrm{sc}} = \mathrm{diag}(3.2894, 7.8994, 10.7112)\ \text{kg-m}^2. \tag{28}$$

The gyro-wheel moment of inertia is given by

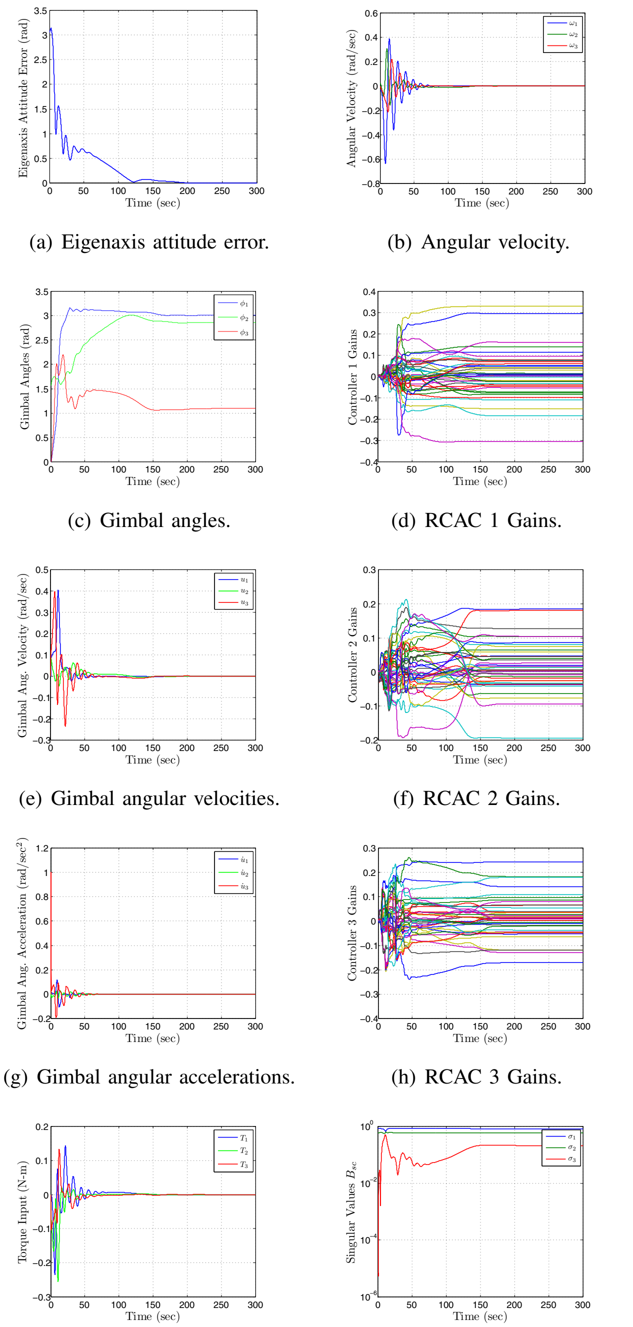
$$\beta = 0.001\ \text{kg-m}^2. \tag{29}$$

For all of the simulations, the initial attitude is given by $R_0 = I$ and the initial desired attitude, $R_{\mathrm{d}}(0)$, is a rotation about the body-fixed direction $n = [1\ 0\ 0]^{\mathrm{T}}$ with angle $\theta_{\mathrm{d}}(0)$. The gyro-wheels are assumed to be spinning at a constant angular velocity $q = 600$ rad/sec, and the initial shaft's angular velocity is $u_0 = [0\ \ 0\ \ 0]^{\mathrm{T}}$ rad/sec. Furthermore, each gimbal's angular velocity and angular acceleration are saturated at $u_{\max} = 1$ rad/sec and $\dot{u}_{\max} = 1$ rad/sec^2, respectively, this keeps the control input small and bounded regardless of the adaptation gains.

C. Motion-to-Rest Maneuvers

We command R2R and M2R maneuvers about principal axes using step and ramp commands. Figure 2 shows a M2R maneuver about a principal axis with a step command. The maneuver is initialized at an obstructing singularity as shown by the singular values of the matrix B_{sc} in Figure 2(j) and more singularities are encountered along the maneuver. The spacecraft is able to avoid these singularities and achieve the desired attitude and angular velocity.

Next, Figure 3 shows a R2R maneuver about a principal axis with a ramp command. The maneuver is initialized at a non-obstructing singularity. The spacecraft initially deviates



(a) Eigenaxis attitude error.

(b) Angular velocity.

(c) Gimbal angles.

(d) RCAC 1 Gains.

(e) Gimbal angular velocities.

(f) RCAC 2 Gains.

(g) Gimbal angular accelerations.

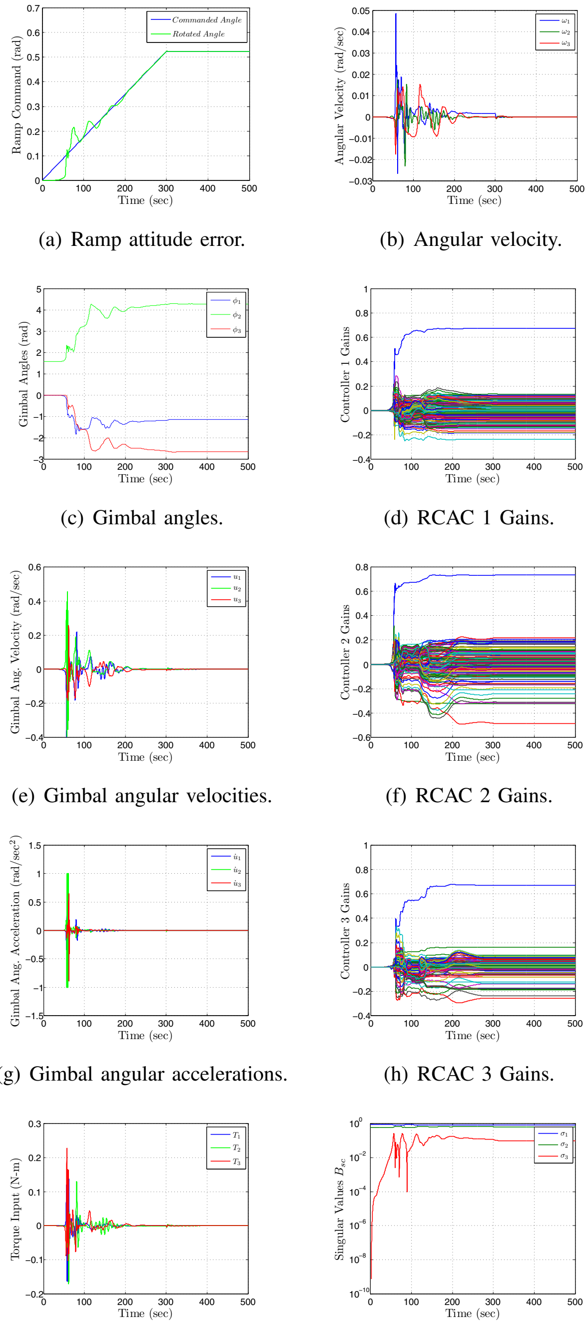
(h) RCAC 3 Gains.

(i) Torque inputs.

(j) Singular values of B_{sc}

Fig. 2. M2R for RCAC with the desired attitude $\theta_{\mathrm{d}}(0) = 170$ deg and the initial angular velocity $\omega_0 = [-0.03\ 0.02\ \ -0.01]^{\mathrm{T}}$ rad/sec. The controller weights are $R_z = 2 \cdot 10^{-1} I$ and $R_u = I$, the initial covariance is $P(0) = 10^2 I$, the initial controller parameter are $\theta(0) = 0$, and the controller order is $n_{\mathrm{c}} = 4$. The initial gimbal angles are $\phi_1 = 0$ deg, $\phi_2 = 90$ deg, and $\phi_3 = 0$ deg which is an obstructing singularity.

from the commanded ramp to avoid the singularity and returns to follow the command. Note that after 300 sec RCAC re-adapts as a response to the change in slope in the command.



(a) Ramp attitude error. (b) Angular velocity.

(c) Gimbal angles. (d) RCAC 1 Gains.

(e) Gimbal angular velocities. (f) RCAC 2 Gains.

(g) Gimbal angular accelerations. (h) RCAC 3 Gains.

(i) Torque inputs. (j) Singular values of $B_{\rm sc}$

Fig. 3. R2R for RCAC with the desired attitude $\theta_{\rm d}(0) = 30$ deg. The controller weights are $R_z = 10^{-1}I$ and $R_u = 7 \cdot 10^{-4}I$, the initial covariance is $P(0) = 10^1 I$, the initial controller parameter are $\theta(0) = 0$, and the controller order is $n_{\rm c} = 13$. The initial gimbal angles are $\phi_1 = 0$ deg, $\phi_2 = 90$ deg, and $\phi_3 = 0$ deg which is a non-obstructing singularity.

D. Motion-to-Spin Maneuvers

We command a step and a sinusoid desired angular velocity for motion-to-spin.

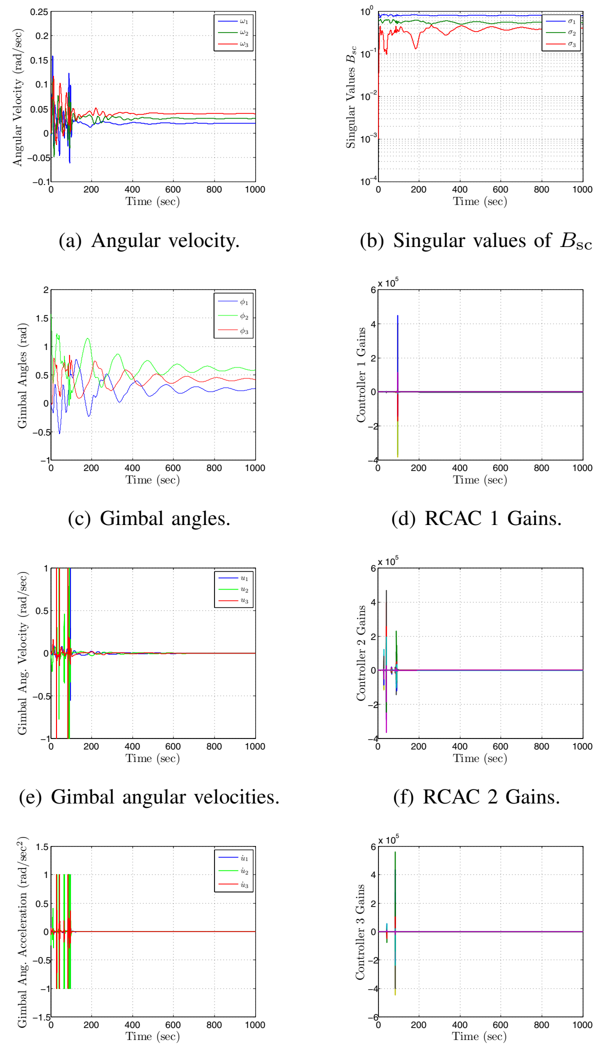
Figure 4 shows a R2S maneuver with a step command, where the final spin is about a non-principal axis. The maneuver is initialized with arbitrary initial angles, and the non-principal inertia is given by

$$J_{\rm sc} = \begin{bmatrix} 5.6096 & -1.0121 & 0.6143 \\ -1.0121 & 2.8718 & 0.3979 \\ 0.6143 & 0.3979 & 4.7187 \end{bmatrix} \text{kg-m}^2. \quad (30)$$

The spacecraft is able to avoid the singularities and achieve the desired attitude and angular velocity. Figure 5 shows angular velocity command following. The desired time-varying angular velocity is given by

$$\omega_{\rm d}(t) = $$
$$0.005 \begin{bmatrix} \sin(t) & \cos(0.5t + \frac{\pi}{3}) & \sin(0.1t + \frac{\pi}{4}) \end{bmatrix}^{\rm T} \text{rad/sec}. \quad (31)$$

The maneuver is initialized at rest and the spacecraft is able to follow the command.



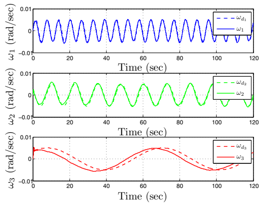
(a) Angular velocity. (b) Singular values of $B_{\rm sc}$

(c) Gimbal angles. (d) RCAC 1 Gains.

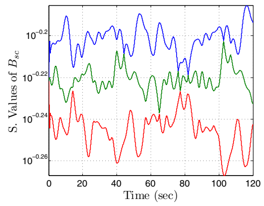
(e) Gimbal angular velocities. (f) RCAC 2 Gains.

(g) Gimbal angular accelerations. (h) RCAC 3 Gains.

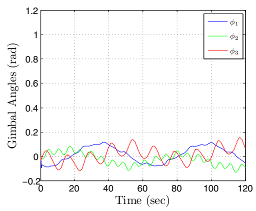
Fig. 4. R2S for RCAC using the non-principal inertia matrix in (30) with the desired attitude $\theta_{\rm d}(0) = 30$ deg, and the desired angular velocity $\omega_{\rm d} = [0.02\ 0.03\ 0.04]^{\rm T}$ rad/sec. The controller weights are $R_z = I$ and $R_u = 10^{-1}I$, the initial covariance is $P(0) = 10^{10}I$, the initial controller parameter is $\theta(0) = 0$, and the controller order is $n_{\rm c} = 4$. The initial gimbal angles are $\phi_1 = 0$ deg, $\phi_2 = 90$ deg, and $\phi_3 = 0$ deg, which is a non-obstructing singularity.



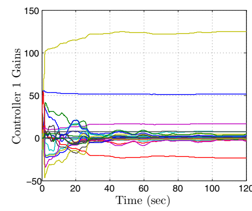
(a) Angular velocity command following.



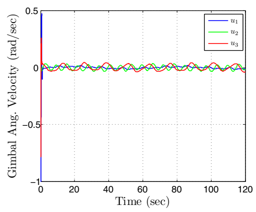
(b) Singular values of $B_{\rm sc}$



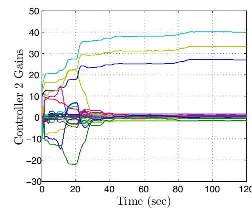
(c) Gimbal angles.



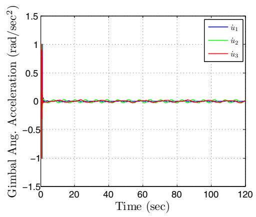
(d) RCAC 1 Gains.



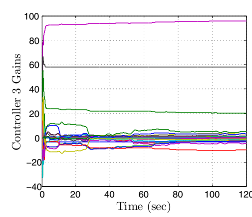
(e) Gimbal angular velocities.



(f) RCAC 2 Gains.



(g) Gimbal angular accelerations.



(h) RCAC 3 Gains.

Fig. 5. R2S for RCAC using the non-principal inertia matrix in (28) with the desired time-varying angular velocity given by (31). The controller weights are $R_z = 10^{-1}I$ and $R_u = 10^{-6}I$, the initial covariance is $P(0) = 10^6 I$, the initial controller parameter is $\theta(0) = 0$, and the controller order is $n_{\rm c} = 3$. The initial gimbal angles are $\phi_1 = 0$ deg, $\phi_2 = 0$ deg, and $\phi_3 = 0$ deg which is a non-obstructing singularity.

VI. Conclusions

We apply RCAC to spacecraft attitude control with CMG's actuation. The CMG's are assumed to be mounted in a known and linearly independent gimbal-axis configuration with an arbitrary and unknown orientation relative to the spacecraft principal axes. A decentralized architecture with one RCAC for each channel is used. The RCAC algorithm is able to bring the spacecraft to a desired attitude and angular velocity for both M2R and M2S scenarios.

For R2R maneuvers, we use two different methods to achieve the desired attitude. First we specify the final desired attitude rotation matrix (that is, step command). Second we specify the rotational path as a function of time needed to achieve the desired attitude (that is, ramp command).

To study singularity avoidance, we command a rotation about a body-fixed direction and initialize the CMG's shaft angles at a position for which a torque about the desired body-fixed direction cannot be initially commanded. When a step command is used, RCAC initially commands torque about some axis and then figures out how to achieve the desired attitude. When a ramp is commanded, RCAC deviates momentarily from the commanded rotation path to avoid the singularity and quickly comes back to follow the command. The error between the desired and commanded rotations is slight. In the case that no singularities appear, RCAC follows the command precisely. Furthermore, no saturated pseudo-inverse or similar methodologies to avoid singularities are used.

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