IDENTIFICATION OF THE INERTIA MATRIX OF A ROTATING BODY BASED ON ERRORS-IN-VARIABLES MODELS

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Abstract: This paper proposes a procedure for identifying the inertia matrix of a rotating body. The procedure based on Euler equation governing rotational motion assumes errors-in-variables models in which all measurements, torque as well as angular velocities, are corrupted by noises. In order for consistent estimation, we introduce an extended linear regression model by augmenting the regressors with constants and the parameters with noise-contributed terms. A transformation, based on low-pass filtering, of the extended model cancels out angular acceleration terms in the regressors. Applying the method of least correlation to the extended and transformed model identifies the elements of inertia matrix. Analysis shows that the estimates converge to the true parameters as the number of samples increases to infinity. Monte Carlo simulations demonstrate the performance of the algorithm and support the analytical consistency.

Keywords: Nonlinear system identification, Errors-in-variables nonlinear model, Least-correlation estimate

1. INTRODUCTION

It is a trend to use smaller, lighter and cheaper instruments for systems, which usually means that measurements are more corrupted by noise. This is true for uninhabited air or space vehicles, where there is a premium on size and weight. Control requirements, however, may be stricter than those of conventional vehicles in order to meet the needs for clustering or formation flight (Giulietti *et al.*, 2000; Zetocha *et al.*, 2000). Many existing algorithms for identifying inertial parameters in space use the method of least squares (Hahn and Niebergall, 2001; Kim *et al.*, 2003; Lee and Wertz, 2002; Peck, 2000; Tanygin and Williams, 1997). Least-squares estimation, developed in the 18th century, is still the most popular approach to obtain the best fit to a given structure, but it exhibits high sensitivity to errors in regressors (van Huffel and Vandewalle, 1991). The regressors in identification models are composed of measurements, such as angular velocity, angular acceleration and attitude, which are not free from noise (Hahn and Niebergall, 2001; Kim *et al.*, 2003; Lee and Wertz, 2002; Peck, 2000; Tanygin and Williams, 1997). The models, where input as well as output measurements are contaminated by noise, are known as errors-in-variables(EIV) models (Soderstrom *et al.*, 2002; van Huffel and Lemmerling, 2002). It is known that the least-squares method tends to generate error-prone estimates for EIV models (van Huffel and Vandewalle, 1991). Making the problem worse is that the regressors in the estimation models are not

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linear in the measurements whether the models are based on Euler equation (Hahn and Niebergall, 2001; Kim *et al.*, 2003; Tanygin and Williams, 1997) or derived from angular-momentum conservation (Lee and Wertz, 2002; Peck, 2000).

The method of least correlation has a capability to cope with the noisy measurements of all variables provided that the regressors are linear in the variables (Jun and Bernstein, 2006). An extension of the method of least correlation provides consistent estimates for a type of nonlinear systems which are described by polynomials in the variables (Jun and Bernstein, 2007). This paper describes an application of the least-correlation methods for identifying the inertia matrix of a rotating body. In this work we assume that the external torque and angular velocity are measured with noise, but the angular acceleration is not available. The estimation model based on Euler equation of motion is formulated via two steps - extending the linear regression model by augmenting the regressors with constants and the parameters with noise-contributed terms, and transforming the extended model to an equivalent form without angular acceleration terms. Applying the method of least correlation to the extended and transformed model gives an algorithm identifying the inertia matrix of a rotating body with consistency.

The estimation method introduced in this work can be applied to various kinds of systems such as spacecraft (Lee and Wertz, 2002; Peck, 2000; Bergmann *et al.*, 1987; Tanygin and Williams, 1997), robots (Hahn and Niebergall, 2001) and other rotating structures (Kim *et al.*, 2003; Schwartz *et al.*, 2003). Analysis shows that the procedure gives consistent estimates, that is, the estimates converge to the true parameters as the number of samples increase to infinity. Simulation results for an example confirm the performance of the estimation method numerically.

2. PROBLEM AND ASSUMPTIONS

The rotational motion of a rigid body is governed by the Euler equation

$$J\dot{\omega}_*(t) + \omega_*(t) \times J\omega_*(t) = M_*(t), \tag{1}$$

where $J \in \mathbb{R}^{3\times3}$ denotes the inertia matrix which is constant, symmetric and positive definite, $\omega_*(t) \in \mathbb{R}^3$ is the angular velocity vector, $M_*(t) \in \mathbb{R}^3$ is the external moment (or torque) acting on the body about its mass center. Let $\omega(t) \in \mathbb{R}^3$ and $M(t) \in \mathbb{R}^3$ denote the measurements of $\omega_*(t)$ and $M_*(t)$, respectively, that is,

$$\omega(t) \triangleq \omega_*(t) + \zeta(t), \tag{2}$$
$$M(t) \triangleq M_*(t) + \eta(t), \tag{3}$$

where $\zeta(t) \in \mathbb{R}^3$ and $\eta(t) \in \mathbb{R}^3$ are measurement noises. Assuming that $\omega(t)$ is measured, but $\dot{\omega}(t)$ is not. Our goal is to identify all components of J by using $\omega(t)$ and $M(t)$.

Measurements are frequently described as stochastic processes with deterministic components. For a common framework for deterministic and stochastic signals (Ljung, 1999, pp.33–34), we assume that all measurements are quasi-stationary and employ the notation

$$\bar{E}[f(kh)] \triangleq \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} E[f(kh)] \tag{4}$$

for discrete-time signal $f(kh), k = 1, 2, \ldots, N$ with sampling interval h, where E denotes the usual mathematical expectation. We implicitly assume that the limit in (4) exists.

We introduce the following assumptions.
A1. Measurements $\omega(kh)$ and $M(kh)$ are quasi-stationary and jointly quasi-stationary (Ljung, 1999, p.34).
A2. Noises $\zeta(kh)$ and $\eta(kh)$ are zero-mean and finitely cross-correlated with $\omega(kh)$, that is, there exists $\tau > 0$ such that

$$\bar{E}\left[\omega(kh)\zeta^T(kh - sh)\right] = 0 \ \text{ for all } |s| \geq \tau, \tag{5}$$
$$\bar{E}\left[\omega(kh)\eta^T(kh - sh)\right] = 0 \ \text{ for all } |s| \geq \tau. \tag{6}$$

A3. For τ given by A2, $\omega(kh)$ satisfies

$$\operatorname{rank}\left\{\bar{R}_{\omega\omega}(k, k', N) + \bar{R}_{\omega\omega}(k', k, N)\right\} = 3, \tag{7}$$

where $k' = k - \tau$, N denotes the number of samples and the empirical correlation $\bar{R}_{\omega\omega}(k_1, k_2, N)$ is defined by

$$\bar{R}_{\omega\omega}(k_1, k_2, N) \triangleq \frac{1}{N_\tau} \sum_{k=1+\tau}^{N} \omega(k_1 h)\omega^T(k_2 h) \tag{8}$$

where $N_\tau = N - \tau$ and $k_1 = k, k_2 = k - \tau$ or $k_1 = k - \tau, k_2 = k$.

3. ESTIMATION OF INERTIA MATRIX

By using (2) and (3), (1) is written as

$$J\left(\dot{\omega} - \dot{\zeta}\right) + (\omega - \zeta) \times J(\omega - \zeta) = M - \eta \tag{9}$$

where the time t is omitted for convenience. For an arbitrary vector $x \triangleq [\, x_1 \ x_2 \ x_3 \,]^T$, let us define two operators (Ahmed *et al.*, 1998)

$$L(x) \triangleq \begin{bmatrix} x_1 & 0 & 0 & 0 & x_3 & x_2 \\ 0 & x_2 & 0 & x_3 & 0 & x_1 \\ 0 & 0 & x_3 & x_2 & x_1 & 0 \end{bmatrix}, \tag{10}$$

$$x^{\times} \triangleq \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}, \tag{11}$$

and a parameter vector

$$\theta \triangleq \left[\, J_{11} \;\; J_{22} \;\; J_{33} \;\; J_{23} \;\; J_{13} \;\; J_{12} \,\right]^T, \tag{12}$$

so that $Jx = L(x)\theta$. Then with the regressor matrix $\phi(t) \in \mathbb{R}^{6\times 3}$ defined by

$$\phi^T(t) \triangleq L\left(\dot{\omega} - \dot{\zeta}\right) + (\omega - \zeta)^{\times} L(\omega - \zeta), \tag{13}$$

(9) is equivalent to the linear regression equation

$$z(t) = \phi^T(t)\theta, \tag{14}$$

where $z(t) \triangleq M_*(t) = M(t) - \eta(t)$.

Let us split $\phi(t)$ into three parts as

$$\phi(t) = \psi(t) - \delta(t) - \xi(t), \tag{15}$$

where

$$\psi^T(t) \triangleq L\left(\dot{\omega}\right) + \omega^{\times} L(\omega), \tag{16}$$

$$\delta^T(t) \triangleq L\left(\dot{\zeta}\right) - \zeta^{\times} L(\zeta), \tag{17}$$

$$\xi^T(t) \triangleq \omega^{\times} L(\zeta) + \zeta^{\times} L(\omega). \tag{18}$$

Letting

$$\omega(t) \triangleq \left[\, p \;\; q \;\; r \,\right]^T, \;\; \zeta(t) \triangleq \left[\, \zeta_p \;\; \zeta_q \;\; \zeta_r \,\right]^T$$

gives the expressions of all elements consisting of ψ, δ, ξ in (16)-(18). Substituting (15) into (14) yields an EIV model

$$y(t) = \psi^T(t)\theta + e(t), \tag{19}$$

$$e(t) = \eta(t) - \delta^T(t)\theta - \xi^T(t)\theta, \tag{20}$$

where $y(t) \triangleq M(t)$.

Let us consider how to treat the noise included in $\psi(t)$. Assume for the moment that $\dot{\omega}(t)$ is given. Employing the method of least correlation for (19)-(20) yields estimates with biases even if the noises satisfy A2. The bias comes from $\omega(t), \zeta(t)$ since the second terms of the right-hand side of (16),(17) contain some quadratic components of p, q, r and $\zeta_p, \zeta_q, \zeta_r$, respectively. According to the extended least-correlation estimates (Jun and Bernstein, 2007), the augmented regressor matrix $\psi_a(t) \in \mathbb{R}^{9\times 3}$ and the extended parameter vector $\theta_a(t) \in \mathbb{R}^9$ are defined as follows:

$$\psi_a(t) = \left[\, \psi^T(t) \;\; -I_3 \,\right]^T, \tag{21}$$

$$\theta_a(t) = \left[\, \theta^T \;\; \theta_7(t) \;\; \theta_8(t) \;\; \theta_9(t) \,\right]^T, \tag{22}$$

where $I_3 \in \mathbb{R}^{3\times 3}$ is the identity matrix and $\theta_7, \theta_8, \theta_9$ are defined by

$$\begin{aligned} \theta_7 \triangleq{}& -J_{22}\zeta_q\zeta_r + J_{33}\zeta_q\zeta_r + J_{23}\left(\zeta_q^2 - \zeta_r^2\right) \\ &+ J_{13}\zeta_p\zeta_q - J_{12}\zeta_r\zeta_p, \end{aligned} \tag{23}$$

$$\begin{aligned} \theta_8 \triangleq{}& J_{11}\zeta_r\zeta_p - J_{33}\zeta_r\zeta_p - J_{23}\zeta_p\zeta_q \\ &+ J_{13}\left(\zeta_r^2 - \zeta_p^2\right) + J_{12}\zeta_q\zeta_r, \end{aligned} \tag{24}$$

$$\begin{aligned} \theta_9 \triangleq{}& -J_{11}\zeta_p\zeta_q + J_{22}\zeta_p\zeta_q + J_{23}\zeta_r\zeta_p \\ &- J_{13}\zeta_q\zeta_r + J_{12}\left(\zeta_p^2 - \zeta_q^2\right). \end{aligned} \tag{25}$$

Using (21)-(22) gives an extended EIV model

$$y(t) = \psi_a^T(t)\theta_a(t) + e_a(t), \tag{26}$$

$$e_a(t) = \eta(t) - L(\dot{\zeta})\theta - \xi^T(t)\theta. \tag{27}$$

Let us recall that $\psi_a(t)$ in (26) is still not available because $\dot{\omega}(t)$ is not measured. With a differential operator $p \triangleq d/dt$ and a constant $\gamma > 0$, we introduce a low-pass filter (Johansson, 1993, p.284)

$$\lambda = \frac{1}{1 + \gamma p} \tag{28}$$

in order to get rid of $\dot{\omega}(t)$ from (26)-(27). Applying the operator (28) to (26)-(27) yields

$$y_f(t) = \psi_{a_f}^T(t)\theta_{a_f}(t) + e_{a_f}(t), \tag{29}$$

$$e_{a_f}(t) = \eta_f(t) - L\left(\frac{\zeta - \zeta_f}{\gamma}\right)\theta - \xi_f^T(t)\theta \tag{30}$$

since $\left[\lambda\left(\psi_a^T(t)\theta_a(t)\right)\right](t) = \psi_{a_f}^T(t)\theta_{a_f}(t)$, where $(\cdot)_f(t) \triangleq [\lambda(\cdot)](t)$, $(\cdot)_f(0) \triangleq 0$ and $\psi_{a_f}(t)$, $\theta_{a_f}(t)$ are defined as, respectively,

$$\psi_{a_f}(t) \triangleq \left[\, \psi_f^T(t) \;\; -I_3 \,\right]^T, \tag{31}$$

$$\theta_{a_f}(t) \triangleq \left[\, \theta^T \;\; \theta_{[7-9]_f}^T(t) \,\right]^T, \tag{32}$$

$$\psi_f^T(t) \triangleq L\left(\frac{\omega - \omega_f}{\gamma}\right) + \lambda\left(\omega^{\times} L(\omega)\right) \tag{33}$$

$$\theta_{[7-9]_f}(t) \triangleq \left[\, \theta_{7_f}(t) \;\; \theta_{8_f}(t) \;\; \theta_{9_f}(t) \,\right]^T. \tag{34}$$

It is noted that $\psi_{a_f}(t)$ and $e_{a_f}(t)$ do not contain $\dot{\omega}(t)$ and $\dot{\zeta}$, respectively.

Now let us work with sampled measurements. Given an arbitrary estimate $\bar{\theta}_{a_f}$, consider the criterion

$$J^2(\bar{\theta}_{a_f}, \tau, N) = \left(\frac{1}{N_\tau}\left(Y_0 - \Psi_0\bar{\theta}_{a_f}\right)^T\left(Y_\tau - \Psi_\tau\bar{\theta}_{a_f}\right)\right)^2, \tag{35}$$

where $Y_0, Y_\tau \in \mathbb{R}^{3N_\tau}$ and $\Psi_0, \Psi_\tau \in \mathbb{R}^{3N_\tau \times 9}$ are defined by

$$Y_0 \triangleq \begin{bmatrix} y_f(Nh) \\ y_f(N_1 h) \\ \vdots \\ y_f(h + \tau h) \end{bmatrix}, \;\; Y_\tau \triangleq \begin{bmatrix} y_f(N_\tau h) \\ y_f(N_{\tau+1} h) \\ \vdots \\ y_f(h) \end{bmatrix}, \tag{36}$$

$$\Psi_0 \triangleq \begin{bmatrix} \psi^T_{a_f}(Nh) \\ \psi^T_{a_f}(N_1 h) \\ \vdots \\ \psi^T_{a_f}(h+\tau h) \end{bmatrix}, \Psi_\tau \triangleq \begin{bmatrix} \psi^T_{a_f}(N_\tau h) \\ \psi^T_{a_f}(N_{\tau+1} h) \\ \vdots \\ \psi^T_{a_f}(h) \end{bmatrix}. \quad (37)$$

In the criterion J^2, J is an empirical correlation between the residuals of the estimate $\bar{\theta}_{a_f}$. We note that J turns to the criterion of the least-squares estimate when $\tau = 0$.

Minimizing (35) with respect to $\bar{\theta}_{a_f}$ gives

$$\hat{\theta}_{a_f}(\tau, N) = \left(\Psi^T_{0/\tau}\Psi_{\tau/0}\right)^{-1} \Psi^T_{0/\tau} Y_{\tau/0}, \quad (38)$$

where

$$\Psi_{0/\tau} \triangleq \begin{bmatrix} \Psi_0 \\ \Psi_\tau \end{bmatrix}, \ \Psi_{\tau/0} \triangleq \begin{bmatrix} \Psi_\tau \\ \Psi_0 \end{bmatrix}, \ Y_{\tau/0} \triangleq \begin{bmatrix} Y_\tau \\ Y_0 \end{bmatrix}. (39)$$

The matrix $\Psi^T_{0/\tau}\Psi_{\tau/0} \in \mathbb{R}^{9\times 9}$ in (38), given as

$$\Psi^T_{0/\tau}\Psi_{\tau/0} = \sum_{k=1+\tau}^{N} \begin{bmatrix} \psi_f(kh)\psi^T_f((k-\tau)h) & -\psi_f(kh) \\ -\psi^T_f((k-\tau)h) & I_3 \end{bmatrix} + \sum_{k=1+\tau}^{N} \begin{bmatrix} \psi_f((k-\tau)h)\psi^T_f(kh) & -\psi_f((k-\tau)h) \\ -\psi^T_f(kh) & I_3 \end{bmatrix},$$

is nonsingular owing to A3. The estimate (38) has following property.

Theorem 1. Suppose that A1-A3 are satisfied. Then as N goes to infinity, the least-correlation estimate (38) for the model (29)-(30) converges to the expectation of $\theta_{a_f}(kh)$, that is,

$$\lim_{N\to\infty} \hat{\theta}_{a_f}(\tau, N) = E\left[\theta_{a_f}(kh)\right] \quad (40)$$

for all k.

Proof. The proof is sketched in Appendix A. □

Note that (38) is a consistent estimate of θ, which is clear from the componentwise expression of (40) written as

$$\lim_{N\to\infty} \begin{bmatrix} \hat{\theta}(\tau, N) \\ \hat{\theta}_{[7-9]_f}(\tau, N) \end{bmatrix} = \begin{bmatrix} \theta \\ E\left[\theta_{[7-9]_f}(kh)\right] \end{bmatrix} \quad (41)$$

for all k.

If each component of $\zeta(kh)$ is independent, identically distributed (i.i.d.) and has the same variance, then the step, augmenting the regressor matrix and extending the parameter vector, is not necessary since $\bar{E}\left[\theta_i(kh)\right] = 0, i = 7, 8, 9$ for all k from (23)-(25). That is, the estimate

$$\hat{\theta}_f(\tau, N) = \left(\Psi^{\circ T}_{0/\tau}\Psi^{\circ}_{\tau/0}\right)^{-1} \Psi^{\circ T}_{0/\tau} Y_{\tau/0} \quad (42)$$

is consistent, where the relevant matrices are defined by

$$\Psi^\circ_{0/\tau} \triangleq \begin{bmatrix} \Psi^\circ_0 \\ \Psi^\circ_\tau \end{bmatrix}, \quad \Psi^\circ_{\tau/0} \triangleq \begin{bmatrix} \Psi^\circ_\tau \\ \Psi^\circ_0 \end{bmatrix}, \quad (43)$$

$$\Psi^\circ_0 \triangleq \begin{bmatrix} \psi^T_f(Nh) \\ \psi^T_f(N_1 h) \\ \vdots \\ \psi^T_f(h+\tau h) \end{bmatrix}, \Psi^\circ_\tau \triangleq \begin{bmatrix} \psi^T_f(N_\tau h) \\ \psi^T_f(N_{\tau+1} h) \\ \vdots \\ \psi^T_f(h) \end{bmatrix}. \quad (44)$$

4. NUMERICAL EXAMPLE

As an example we consider a body with inertia

$$J = \begin{bmatrix} 1.20 & 0.11 & -0.18 \\ 0.11 & 1.70 & 0.16 \\ -0.18 & 0.16 & 2.13 \end{bmatrix}. \quad (45)$$

The system (1) with (45) is driven by $M_*(t)$ in Figure 1. We assume that $\eta(t) = 0$ since it does not contribute to the bias errors of identification.

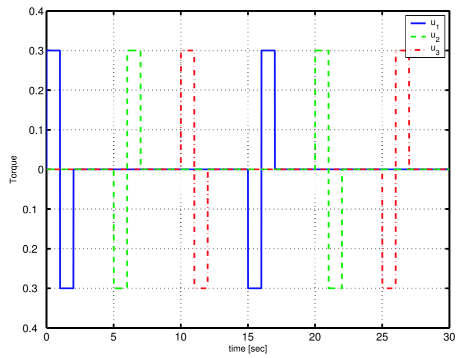

Fig. 1. Moment $M(t)$ applied to the body

In each simulation, we sample $M(t)$ and $\omega(t)$ at 100Hz rate. The measurement noise $\zeta(t)$ is Gaussian with zero mean and covariance

$$\mathrm{Cov}\left[\zeta(t)\zeta^T(t)\right] = \begin{bmatrix} \sigma_p^2 & 0 & 0 \\ 0 & \sigma_q^2 & 0 \\ 0 & 0 & \sigma_r^2 \end{bmatrix}.$$

Since $\zeta(kh)$ is uncorrelated with $\omega((k-\tau)h)$ if $|\tau| \geq 1$, we choose $\tau = 1$ or $\tau h = 0.01$ sec. And $\gamma = 10$ is chosen for a small bandwidth of the low-pass filter (28).

Table 1 and Table 2 summarize the simulation results from 100 Monte Carlo runs. Each table shows the estimation errors defined by

$$\breve{\theta}_i = \frac{1}{|\theta_i|}\left\{\bar{\tilde{\theta}}_i \pm \bar{\sigma}(\tilde{\theta}_i)\right\}, \ i = 1, \cdots, 6, \quad (46)$$

Table 1. Identification errors in case of finitely correlated measurements

$(\sigma_p, \sigma_q, \sigma_r)$	Alg.	$\breve{J}_{11}$ [%]	$\breve{J}_{22}$ [%]	$\breve{J}_{33}$ [%]
(0.3,0.2,0.1)	LS	-29 $\pm$ 0.5	-30 $\pm$ 0.4	-6.7 $\pm$ 0.2
deg/sec.	LC	-1.3 $\pm$ 1.8	-0.8 $\pm$ 2.0	0.5 $\pm$ 0.7
	ELC	0.4 $\pm$ 1.8	0.4 $\pm$ 2.1	0.5 $\pm$ 0.8
(0.1,0.2,0.3)	LS	-3.3 $\pm$ 0.2	-24 $\pm$ 0.3	-52 $\pm$ 0.5
deg/sec.	LC	1.2 $\pm$ 0.4	1.7 $\pm$ 2.2	-1.0 $\pm$ 5.2
	ELC	0.3 $\pm$ 0.3	1.2 $\pm$ 2.2	0.6 $\pm$ 5.7
$(\sigma_p, \sigma_q, \sigma_r)$	Alg.	$\breve{J}_{23}$ [%]	$\breve{J}_{13}$ [%]	$\breve{J}_{12}$ [%]
(0.3,0.2,0.1)	LS	-148 $\pm$ 2.4	92 $\pm$ 1.5	-325 $\pm$ 3.6
deg/sec.	LC	-17 $\pm$ 10	7.9 $\pm$ 5.6	17 $\pm$ 18
	ELC	-4.3 $\pm$ 12	0.6 $\pm$ 6.7	2.6 $\pm$ 20
(0.1,0.2,0.3)	LS	-404 $\pm$ 3.9	41 $\pm$ 1.8	-67 $\pm$ 2.2
deg/sec.	LC	-7.4 $\pm$ 34	1.9 $\pm$ 12	-2.7 $\pm$ 13
	ELC	3.1 $\pm$ 36	-0.6 $\pm$ 12	-3.3 $\pm$ 13

where $\bar{\tilde{\theta}}_i$ and $\bar{\sigma}(\tilde{\theta}_i)$ denote the empirical mean and the empirical standard deviation of ith parameter error $\tilde{\theta}_i$. In the tables, 'LC' stands for the least-correlation estimate (42), 'LS' denotes the least-squares estimate which is obtained from (42) by setting $\tau = 0$, and 'ELC' denotes the extended least-correlation estimate in (38). Table 1 shows that both the LC and ELC outperform the LS. Moreover, the ELC yields more accurate estimates than the LC.

Table 2. Identification errors in case of infinitely correlated measurements, $(\sigma_p, \sigma_q, \sigma_r) = (0.4, 0.3, 0.2)$ deg/sec.

Alg.	τ	$\breve{J}_{11}$ [%]	$\breve{J}_{22}$ [%]	$\breve{J}_{33}$ [%]
LS	0	-9.7 $\pm$ 1.1	-15 $\pm$ 1.2	-6.4 $\pm$ 0.9
ELC	4	-4.0 $\pm$ 1.7	-6.4 $\pm$ 1.8	-2.6 $\pm$ 1.3
	8	-1.5 $\pm$ 1.7	-2.4 $\pm$ 1.9	-1.0 $\pm$ 1.3
	16	-0.1 $\pm$ 2.1	-0.0 $\pm$ 2.6	-0.2 $\pm$ 1.8
Alg.	τ	$\breve{J}_{23}$ [%]	$\breve{J}_{13}$ [%]	$\breve{J}_{12}$ [%]
LS	0	-109 $\pm$ 8.2	54 $\pm$ 4.8	-134 $\pm$ 8.7
ELC	4	-48 $\pm$ 13	27 $\pm$ 7.7	-59 $\pm$ 17
	8	-19 $\pm$ 15	12 $\pm$ 8.5	-24 $\pm$ 18
	16	-4.3 $\pm$ 19	5.2 $\pm$ 11	-2.9 $\pm$ 22

According to Theorem 1, (38) gives consistent estimates provided that the measurement noise in regressor matrix is at most finitely correlated. We, however, try to show numerically that the estimates can be applied to problems with infinitely correlated noise violating A2. For this case we use the angular velocity measurements given by

$$\omega(t) \triangleq \omega_*(t) + \frac{1}{1+\beta p}\zeta(t) \tag{47}$$

instead of (2), where we choose $\beta = 0.04$ sec. Table 2 shows that the ELC works well on the infinitely correlated noise provided that A3 is satisfied with a large value of τ. According to Table 2 when $\tau h = 0.16$ sec. ($\tau = 16$) is four times of $\beta = 0.04$, the ELC algorithm generates the best results.

5. CONCLUDING REMARKS

This paper introduces a procedure for identifying the inertia matrix of a rotating body. The estimation algorithm is based on the Euler equation governing rotational motion and assumes errors-in-variables models in which all variables are corrupted by noise. The main idea is composed of three steps - extending regressors and parameters, filtering out angular acceleration terms, and employing the method of least correlation. In the first step, the regressor matrix is augmented by the identity matrix with proper size and the parameter vector is augmented by terms contributed by noisy measurements. The second step is to transform the errors-in-variables model of the Euler equation, which contains angular acceleration terms, to a model which does not contain the components. The last step, employing the method of least correlation to the extended and transformed model, gives an estimate of the inertia matrix. Analysis shows that the estimates are consistent in the sense that the estimates converge to the true values as the number of samples increases to infinity. Monte Carlo simulations demonstrate the performance and support the analytical results.

Given the measurements of translational accelerations, attitude angles and external forces of a translating and rotating rigid body, the proposed algorithm can be extended to the problem (Hahn and Niebergall, 2001) which identifies all inertial parameters including mass and center of mass as well as inertia matrix. We expect that this extension gives reasonable results even though the estimates are not free from bias. If the attitude angles are measured almost free from noise, then the proposed procedure gives good estimates of the complete set of inertial parameters.

Appendix A. PROOF OF THEOREM 1

Equation (38) is equivalent to

$$\hat{\theta}_{a_f}(\tau, N) = \Big(\bar{R}_{\psi_{a_f}\psi_{a_f}}(k_1,k_2) + \bar{R}_{\psi_{a_f}\psi_{a_f}}(k_2,k_1)\Big)^{-1} \times \Big(\bar{r}_{\psi_{a_f}y_f}(k_1,k_2) + \bar{r}_{\psi_{a_f}y_f}(k_2,k_1)\Big), \tag{A.1}$$

where the empirical correlations are defined by

$$\bar{R}_{\psi_{a_f}\psi_{a_f}}(k_1,k_2) \triangleq \frac{1}{N_\tau}\sum_{k=1+\tau}^{N} \psi_{a_f}(k_1 h)\psi_{a_f}^T(k_2 h),$$

$$\bar{r}_{\psi_{a_f}y_f}(k_1,k_2) \triangleq \frac{1}{N_\tau}\sum_{k=1+\tau}^{N} \psi_{a_f}(k_1 h) y_f(k_2 h)$$

with either $k_1 = k, k_2 = k - \tau$ or $k_1 = k - \tau, k_2 = k$. Using the discrete-time equivalence of (29) to $\bar{r}_{\psi_{a_f}y_f}(k_1,k_2)$ gives

$$\bar{r}_{\psi_{a_f} y_f}(k_1,k_2) = \bar{\mathbf{t}}_{\psi_{a_f}\psi_{a_f}\theta_{a_f}}(k_1,k_2,k_2) + \bar{r}_{\psi_a e_{a_f}}(k_1,k_2), \quad \text{(A.2)}$$

where $\bar{\mathbf{t}}_{\psi_{a_f}\psi_{a_f}\theta_{a_f}}$, the empirical bicorrelation (Koh and Powers, 1985) and $\bar{r}_{\psi_{a_f} e_{a_f}}$ are defined by

$$\bar{\mathbf{t}}_{\psi_{a_f}\psi_{a_f}\theta_{a_f}} \triangleq \frac{1}{N_\tau}\sum_{k=1+\tau}^{N} \psi_{a_f}(k_1 h)\psi_{a_f}^T(k_2 h)\theta_{a_f}(k_2 h),$$

$$\bar{r}_{\psi_{a_f} e_{a_f}} \triangleq \frac{1}{N_\tau}\sum_{k=1+\tau}^{N} \psi_{a_f}(k_1 h) e_{a_f}(k_2 h),$$

respectively.

When N goes to infinity, $\bar{R}_{\psi_{a_f}\psi_{a_f}}(k_1,k_2)$ converges to $R_{\psi_{a_f}\psi_{a_f}}(\tau)$ due to the ergodic theory (Ljung, 1999, Theorem 2.3 in p.43) and A1, i.e.,

$$\lim_{N\to\infty} \bar{R}_{\psi_{a_f}\psi_{a_f}}(k_1,k_2) = R_{\psi_{a_f}\psi_{a_f}}(\tau). \quad \text{(A.3)}$$

Applying above approach to (A.2) yields

$$r_{\psi_{a_f} y_f}(\tau) = \mathbf{t}_{\psi_{a_f}\psi_{a_f}\theta_{a_f}}(\tau,\tau) + r_{\psi_a e_{a_f}}(\tau), \quad \text{(A.4)}$$

where each term is evaluated as follows:

$$\mathbf{t}_{\psi_{a_f}\psi_{a_f}\theta_{a_f}}(\tau,\tau) = R_{\psi_{a_f}\psi_{a_f}}(\tau) E[\theta_{a_f}(kh)], \quad \text{(A.5)}$$

$$r_{\psi_{a_f} e_{a_f}}(\tau) = 0. \quad \text{(A.6)}$$

Using (A.4)-(A.6), $\bar{r}_{\psi_{a_f} y_f}(k_1,k_2)$ at $N \to \infty$ is expressed as

$$\lim_{N\to\infty} \bar{r}_{\psi_{a_f} y_f}(k_1,k_2) = R_{\psi_{a_f}\psi_{a_f}}(\tau) E[\theta_{a_f}(kh)] \quad \text{(A.7)}$$

for all k. Applying (A.3) and (A.7) to

$$\lim_{N\to\infty} \hat{\theta}_{a_f}(\tau,N) = R^{-1}_{\psi_{a_f}\psi_{a_f}}(\tau) r_{\psi_{a_f} y_f}(\tau), \quad \text{(A.8)}$$

which is an expression of (A.1) at $N \to \infty$, yields (40).

REFERENCES

Ahmed, J., V. T. Coppola and D. S. Bernstein (1998). Adaptive asymptotic tracking of spacecraft attitude motion with inertia matrix identification. *Journal of Guidance, Control, and Dynamics* **21**, 684–691.

Bergmann, E. V., B. K. Walker and D. R. Levy (1987). Mass property estimation for control of asymmetrical satellites. *Journal of Guidance, Control, and Dynamics* **10**, 483–491.

Giulietti, F., L. Pollini and M. Innocenti (2000). Autonomous formation flight. *IEEE Control Systems Magazine* **20**, 34–44.

Hahn, H. and M. Niebergall (2001). Development of a measurement robot for identifying all inertia parameters of a rigid body in a single experiment. *IEEE Trans. Control Systems Technology* **9**, 416–423.

Johansson, R. (1993). *System Modeling and Identification*. Prentice-Hall. Englewood Cliffs, NJ.

Jun, B. E. and D. S. Bernstein (2006). Least-correlations estimates for errors-in-variables models. *Int. J. Adaptive Control and Signal Processing* **20**, 337–351.

Jun, B. E. and D. S. Bernstein (2007). Extended least-correlations estimates for errors-in-variables nonlinear models. *Int. J. Control* **80**, 256–267.

Kim, B., E. Velenis, P. Kriengsiri and P. Tsiotras (2003). Designing a low-cost spacecraft simulator. *IEEE Control System Magazine* **23**, 26–37.

Koh, T. and E. J. Powers (1985). Second-order volterra filtering and its application to nonlinear system identification. *IEEE Trans. Acoustics, Speech, and Signal Processing* **33**, 1445–1455.

Lee, A. Y. and J. A. Wertz (2002). In-flight estimation of the cassini spacecraft's inertia tensor. *J. Spacecraft* **39**, 153–155.

Ljung, L. (1999). *System Identification - Theory for the User, 2nd Edition*. Prentice-Hall. Englewood Cliffs, NJ.

Peck, M. A. (2000). Mass-properties estimation for spacecraft with powerful damping. *Advances in the Astronautical Sciences* **103**, 2005–2024.

Schwartz, J. L., M. A. Peck and C. D. Hall (2003). Historical review of air-bearing spacecraft simulators. *Journal of Guidance, Control, and Dynamics* **26**, 513–522.

Soderstrom, T., U. Soverini and K. Mahata (2002). Perspectives on errors-in-variables estimation for dynamic systems. *Signal Processing* **82**, 1139–1154.

Tanygin, S. and T. Williams (1997). Mass property estimation using coasting maneuvers. *Journal of Guidance, Control, and Dynamics* **20**, 625–632.

van Huffel, S. and J. Vandewalle (1991). *The Total Least Squares Problem: Computational Aspects and Analysis*. SIAM. Philadelphia.

van Huffel, S. and Lemmerling, P., Eds. (2002). *Total Least Squares and Errors-in-Variables Modeling: Analysis, Algorithms and Applications*. Kluwer Academic Publishers. Dordrecht, TheNetherlands.

Zetocha, P., L. Self, R. Wainwright and R. Burns (2000). Commanding and controlling satellite clusters. *IEEE Intelligent Systems* **15**, 8–13.