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Adaptive Control of Uncertain Hammerstein Systems with Monotonic Input Nonlinearities Using Auxiliary Nonlinearities

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***Abstract*— We extend retrospective cost adaptive control (RCAC) to command following for uncertain Hammerstein systems. We assume that only one Markov parameter of the linear plant is known and that the input nonlinearity is monotonic but otherwise unknown. Auxiliary nonlinearities are used within RCAC to account for the effect of the input nonlinearity.**

I. INTRODUCTION

In many practical applications, an input nonlinearity precedes the linear plant dynamics; systems with this structure are called *Hammerstein systems* [1–3]. The input nonlinearity may represent properties of an actuator, such as saturation to reflect magnitude restrictions on the control input, deadzone to represent actuator stiction, and a signum nonlinearity to represent on-off behavior.

Adaptive control of Hammerstein systems with uncertain input nonlinearities and linear dynamics is considered in [4–6]. Unlike [4–6], however, we make no attempt to identify and invert the input nonlinearity. Instead, we apply retrospective-cost adaptive control (RCAC), which can be used for plants that are possibly MIMO, nonminimum phase (NMP), and unstable [7–13]. This approach relies on knowledge of Markov parameters and, for NMP open-loop-unstable plants, estimates of the NMP zeros. This information can be obtained from either analytical modeling or system identification [14].

In the present paper we consider a command-following problem for SISO Hammerstein plants where limited modeling information is available concerning the input nonlinearity and the linear dynamics. For the linear dynamics, we assume that one nonzero Markov parameter is known. In addition, we consider plants that are open-loop asymptotically stable and thus, as shown in [12, 13], knowledge of the NMP zeros is not needed. We also assume that the input nonlinearity is monotonic but not necessarily continuous.

The novel contribution of the present paper is the augmentation of RCAC with two auxiliary nonlinearities that account for the presence of the uncertain input nonlinearity $\mathcal{N}$. The auxiliary nonlinearity $\mathcal{N}_1$ is a saturation nonlinearity, which is chosen to tune the transient response of the closed-loop system and which may depend on estimates of the range of the input nonlinearity $\mathcal{N}$ and the gain of the linear dynamics. In contrast, the auxiliary nonlinearity $\mathcal{N}_2$ is chosen so that the composite nonlinear function $\mathcal{N} \circ \mathcal{N}_2$ is nondecreasing. Therefore, if $\mathcal{N}$ is nondecreasing, then $\mathcal{N}_2$ is not needed. If, however, $\mathcal{N}$ is nonincreasing, then $\mathcal{N}_2$ can be chosen such that $\mathcal{N} \circ \mathcal{N}_2$ is nondecreasing. Note that $\mathcal{N}$ need not be one-to-one or onto. This approach extends the technique used in [15] for Hammerstein systems with amplitude and rate saturation.

In [4–6], the input nonlinearities are assumed to be piecewise linear. The present paper does not impose this restriction. Numerical examples involving cubic, deadzone, saturation, and on-off input nonlinearities are presented.

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II. HAMMERSTEIN COMMAND-FOLLOWING PROBLEM

Consider the SISO discrete-time Hammerstein system

$$x(k+1) = Ax(k) + B\mathcal{N}(u(k)) + D_1 w(k), \quad (1)$$

$$y(k) = Cx(k), \quad (2)$$

where $x(k) \in \mathbb{R}^n$, $u(k)$, $y(k) \in \mathbb{R}$, $w(k) \in \mathbb{R}^d$, $\mathcal{N} : \mathbb{R} \to \mathbb{R}$, and $k \geq 0$. We consider the Hammerstein command-following problem with the performance variable

$$z(k) = y(k) - r(k), \quad (3)$$

where $z(k)$, $r(k) \in \mathbb{R}$. The goal is to develop an adaptive output feedback controller that minimizes the command-following error z with minimal modeling information about the dynamics, disturbance w, and input nonlinearity $\mathcal{N}$. We assume that measurements of $z(k)$ are available for feedback; however, measurements of $v(k) = \mathcal{N}(u(k))$ are not available. A block diagram for (1)-(3) is shown in Figure 1.

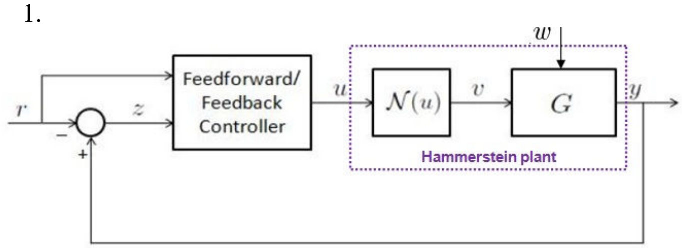


Fig. 1. Adaptive command-following problem for a Hammerstein plant. We assume that measurements of $z(k)$ are available for feedback; however, measurements of $v(k) = \mathcal{N}(u(k))$ and $w(k)$ are not available. The feedforward path is optional.

III. ADAPTIVE CONTROL FOR THE HAMMERSTEIN COMMAND-FOLLOWING PROBLEM

For the Hammerstein command-following problem, we assume that G is uncertain except for an estimate of a single nonzero Markov parameter. The input nonlinearity $\mathcal{N}$ is also uncertain.

To account for the presence of the input nonlinearity $\mathcal{N}$, the RCAC controller in Figure 2 uses two auxiliary nonlinearities. The auxiliary nonlinearity $\mathcal{N}_1$ modifies $u_{\rm c}$ to obtain the regressor input $u_{\rm r}$, while the auxiliary nonlinearity $\mathcal{N}_2$ modifies $u_{\rm r}$ to produce the Hammerstein plant input u. The auxiliary nonlinearities $\mathcal{N}_1$ and $\mathcal{N}_2$ are chosen based on limited knowledge of the input nonlinearity $\mathcal{N}$, as described below.

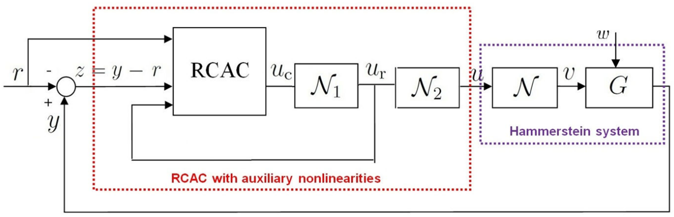


Fig. 2. Hammerstein command-following problem with the RCAC adaptive controller and auxiliary nonlinearities $\mathcal{N}_1$ and $\mathcal{N}_2$.

A. Auxiliary Nonlinearity $\mathcal{N}_1$

Define the saturation function $\mathrm{sat}_{p,q}$ by

$$\mathcal{N}_1(u_{\rm c}) = \mathrm{sat}_{p,q}(u_{\rm c}) = \begin{cases} p, & \text{if} \quad u_{\rm c} < p, \\ u_{\rm c}, & \text{if} \quad p \le u_{\rm c} \le q, \\ q, & \text{if} \quad u_{\rm c} > q, \end{cases} \tag{4}$$

where $p \in \mathbb{R}$ and $q \in \mathbb{R}$ are the lower and upper saturation levels, respectively. For minimum-phase plants, the auxiliary nonlinearity $\mathcal{N}_1$ is not needed, and thus the saturation levels p and q are chosen to be large negative and positive numbers, respectively. For NMP plants, the saturation levels are used to tune the transient behavior. In addition, the saturation levels are chosen to provide the magnitude of the control input in order to follow the command r. These values depend on the range of the input nonlinearity $\mathcal{N}$ as well as the gain of the linear system G at frequencies in the spectra of r and w.

B. Auxiliary Nonlinearity $\mathcal{N}_2$

If $\mathcal{N}$ is nondecreasing, then $\mathcal{N}_2$ is not needed. We thus consider the case in which $\mathcal{N}$ is monotonically nonincreasing on the finite interval $I = [p, q]$. Since the range of $\mathcal{N}_1$ is $[p, q]$, we need to consider only $u_{\rm r} \in [p, q]$. If $\mathcal{N}$ is nonincreasing on I, then we define $\mathcal{N}_2(u_{\rm r}) \stackrel{\triangle}{=} p + q - u_{\rm r} \in I$ for all $u_{\rm r} \in I$. Thus, $\mathcal{N}_2$ is a piecewise-linear function that replaces $\mathcal{N}$ by its mirror image, which is nondecreasing in I. Let $\mathcal{R}_I(f)$ denote the range of f with arguments in I.

Proposition 3.1: Assume that $\mathcal{N}_2$ is constructed by the above rule. Then the following statements hold:

i) $\mathcal{N} \circ \mathcal{N}_2$ is nondecreasing.
ii) $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_2) = \mathcal{R}_I(\mathcal{N})$.

Proof. If $\mathcal{N}$ is nondecreasing on I, then $\mathcal{N}_2$ is the identity function and thus $i)$ holds. Now, assume that $\mathcal{N}$ is nonincreasing on I, and let $u_{\rm r,1}, u_{\rm r,2} \in I$, where $u_{\rm r,1} \le u_{\rm r,2}$. Then,

$$u_2 \stackrel{\triangle}{=} p + q - u_{\rm r,2} \le u_1 \stackrel{\triangle}{=} p + q - u_{\rm r,1}.$$

Therefore, since $\mathcal{N}$ is nonincreasing on I and $u_2 \le u_1$, it follows that $\mathcal{N}(\mathcal{N}_2(u_{\rm r,1})) = \mathcal{N}(u_1) \le \mathcal{N}(u_2) = \mathcal{N}(\mathcal{N}_2(u_{\rm r,2}))$. Thus, $i)$ holds.

To prove $ii)$, assume that $\mathcal{N}$ is nondecreasing on I. Since $\mathcal{N}_2(u_{\rm r}) = u_{\rm r}$ for all $u_{\rm r} \in I$, it follows that $\mathcal{N}_2(I) = I$, that is, $\mathcal{N}_2 : I \to I$ is onto. Alternatively, assume that $\mathcal{N}$ is nonincreasing on I so that $\mathcal{N}_2(u_{\rm r}) = p + q - u_{\rm r}$. Note that $\mathcal{N}_2(p_i) = q_i$, $\mathcal{N}_2(q) = p$, and $\mathcal{N}_2$ is continuous and decreasing on I. Therefore, $\mathcal{N}_2(I_i) = I_i$, and thus $\mathcal{N}_2 : I \to I$ is onto. Hence, $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_2) = \mathcal{R}_I(\mathcal{N})$. □

As an example, consider the nonincreasing input nonlinearity $\mathcal{N}(u) = -\mathrm{sat}_{0.5,0.5}(u - 0.5)$. Let $\mathcal{N}_1(u_{\rm c}) = \mathrm{sat}_{p,q}(u_{\rm c})$, where $p = -2, q = 2$, and $\mathcal{N}_2(u_{\rm r}) = -u_{\rm r} + 1$ for all $u_{\rm r} \in [-2, 2]$ according to Proposition 3.1. Figure 3(c) shows that the composite nonlinearity $\mathcal{N} \circ \mathcal{N}_2$ is nondecreasing on $[-2, 2]$. Note that $\mathcal{R}_I(\mathcal{N} \circ \mathcal{N}_2) = \mathcal{R}_I(\mathcal{N}) = [-0.5, 0.5]$.

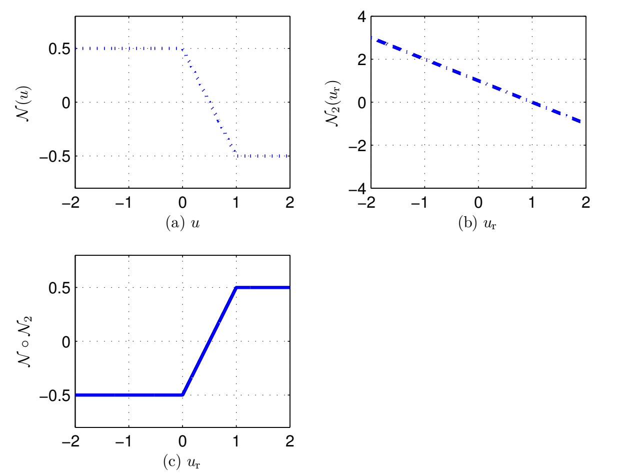


Fig. 3. (a) Input nonlinearity $\mathcal{N}(u) = -\mathrm{sat}_{0.5,0.5}(u - 0.5)$. (b) Auxiliary nonlinearity $\mathcal{N}_2(u_{\rm r}) = -u_{\rm r} + 1$ for $u_{\rm r} \in [-2, 2]$. (c) Composite nonlinearity $\mathcal{N} \circ \mathcal{N}_2$. Note that $\mathcal{N} \circ \mathcal{N}_2$ is nondecreasing and $\mathcal{R}(\mathcal{N} \circ \mathcal{N}_2) = \mathcal{R}(\mathcal{N}) = [-0.5, 0.5]$.

Knowledge of only the monotonicity of $\mathcal{N}$ and the interval I are needed to modify the controller output $u_{\rm r}$ so that $\mathcal{N} \circ \mathcal{N}_2$ is nondecreasing. It thus follows that $\mathcal{N} \circ \mathcal{N}_2$ preserves the signs of the Markov parameters of the linearized Hammerstein system. For details, see [13].

IV. Retrospective-Cost Adaptive Control

For $i \ge 1$, define the Markov parameter

$$H_i \stackrel{\triangle}{=} E_1 A^{i-1} B.$$

For example, $H_1 = E_1 B$ and $H_2 = E_1 AB$. Let ℓ be a positive integer. Then, for all $k \ge \ell$,

$$x(k) = A^\ell x(k - \ell) + \sum_{i=1}^{\ell} A^{i-1} B \mathcal{N}(\mathcal{N}_2(\mathcal{N}_1(u_{\rm c}(k - i)))),$$

and thus

$$z(k) = E_1 A^\ell x(k - \ell) - E_0 r(k) + \bar{H} \bar{U}(k - 1), \tag{5}$$

where

$$\bar{H} \triangleq \begin{bmatrix} H_1 & \cdots & H_\ell \end{bmatrix} \in \mathbb{R}^{1\times\ell}$$

and

$$\bar{U}(k-1) \triangleq \begin{bmatrix} \mathcal{N}(\mathcal{N}_2(\mathcal{N}_1(u_{\rm c}(k-1)))) \\ \vdots \\ \mathcal{N}(\mathcal{N}_2(\mathcal{N}_1(u_{\rm c}(k-\ell)))) \end{bmatrix}.$$

Next, we rearrange the columns of $\bar{H}$ and the components of $\bar{U}(k-1)$ and partition the resulting matrix and vector so that

$$\bar{H}\bar{U}(k-1) = \mathcal{H}'U'(k-1) + \mathcal{H}U(k-1), \tag{6}$$

where $\mathcal{H}' \in \mathbb{R}^{1\times(\ell-l_U)}$, $\mathcal{H} \in \mathbb{R}^{1\times l_U}$, $U'(k-1) \in \mathbb{R}^{\ell-l_U}$, and $U(k-1) \in \mathbb{R}^{l_U}$. Then, we can rewrite (5) as

$$z(k) = \mathcal{S}(k) + \mathcal{H}U(k-1), \tag{7}$$

where

$$\mathcal{S}(k) \triangleq E_1A^\ell x(k-\ell) - E_0r(k) + \mathcal{H}'U'(k-1). \tag{8}$$

Next, for $j = 1,\ldots,s$, we rewrite (7) with a delay of k_j time steps, where $0 \le k_1 \le k_2 \le \cdots \le k_s$, in the form

$$z(k-k_j) = \mathcal{S}_j(k-k_j) + \mathcal{H}_jU_j(k-k_j-1), \tag{9}$$

where (8) becomes

$$\mathcal{S}_j(k-k_j) \triangleq E_1A^\ell x(k-k_j-\ell) + \mathcal{H}_j'U_j'(k-k_j-1)$$

and (6) becomes

$$\bar{H}\bar{U}(k-k_j-1) = \mathcal{H}_j'U_j'(k-k_j-1) + \mathcal{H}_jU_j(k-k_j-1),$$

where $\mathcal{H}_j' \in \mathbb{R}^{1\times(\ell-l_{U_j})}$, $\mathcal{H}_j \in \mathbb{R}^{1\times l_{U_j}}$, $U_j'(k-k_j-1) \in \mathbb{R}^{\ell-l_{U_j}}$, and $U_j(k-k_j-1) \in \mathbb{R}^{l_{U_j}}$. Now, by stacking $z(k-k_1),\ldots,z(k-k_s)$, we define the *extended performance*

$$Z(k) \triangleq \begin{bmatrix} z(k-k_1) \\ \vdots \\ z(k-k_s) \end{bmatrix} \in \mathbb{R}^s. \tag{10}$$

Therefore,

$$Z(k) \triangleq \tilde{\mathcal{S}}(k) + \tilde{\mathcal{H}}\tilde{U}(k-1), \tag{11}$$

where

$$\tilde{\mathcal{S}}(k) \triangleq \begin{bmatrix} \mathcal{S}_1(k-k_1) \\ \vdots \\ \mathcal{S}_s(k-k_s) \end{bmatrix} \in \mathbb{R}^s,$$

$\tilde{U}(k-1)$ has the form

$$\tilde{U}(k-1) \triangleq \begin{bmatrix} \mathcal{N}(\mathcal{N}_2(\mathcal{N}_1(u_{\rm c}(k-q_1)))) \\ \vdots \\ \mathcal{N}(\mathcal{N}_2(\mathcal{N}_1(u_{\rm c}(k-q_{l_{\tilde{U}}})))) \end{bmatrix} \in \mathbb{R}^{l_{\tilde{U}}},$$

where, for $i = 1,\ldots,l_{\tilde{U}}$, $k_1 \le q_i \le k_s+\ell$, and $\tilde{\mathcal{H}} \in \mathbb{R}^{s\times l_{\tilde{U}}}$ is constructed according to the structure of $\tilde{U}(k-1)$. The vector $\tilde{U}(k-1)$ is formed by stacking $U_1(k-k_1-1),\ldots,U_s(k-k_s-1)$ and removing copies of repeated components.

Next, for $j = 1,\ldots,s$, we define the *retrospective performance*

$$\hat{z}_j(k-k_j) \triangleq \mathcal{S}_j(k-k_j) + \mathcal{H}_j\hat{U}_j(k-k_j-1), \tag{12}$$

where the past controls $U_j(k-k_j-1)$ in (9) are replaced by the retrospective controls $\hat{U}_j(k-k_j-1)$. In analogy with (10), the *extended retrospective performance* for (12) is defined as

$$\hat{Z}(k) \triangleq \begin{bmatrix} \hat{z}_1(k-k_1) \\ \vdots \\ \hat{z}_s(k-k_s) \end{bmatrix} \in \mathbb{R}^s$$

and thus is given by

$$\hat{Z}(k) = \tilde{\mathcal{S}}(k) + \tilde{\mathcal{H}}\hat{\tilde{U}}(k-1), \tag{13}$$

where the components of $\hat{\tilde{U}}(k-1) \in \mathbb{R}^{l_{\tilde{U}}}$ are the components of $\hat{U}_1(k-k_1-1),\ldots,\hat{U}_s(k-k_s-1)$ ordered in the same way as the components of $\tilde{U}(k-1)$. Subtracting (11) from (13) yields

$$\hat{Z}(k) = Z(k) - \tilde{\mathcal{H}}\tilde{U}(k-1) + \tilde{\mathcal{H}}\hat{\tilde{U}}(k-1). \tag{14}$$

Finally, we define the *retrospective cost function*

$$J(\hat{\tilde{U}}(k-1),k) \triangleq \hat{Z}^{\rm T}(k)R(k)\hat{Z}(k), \tag{15}$$

where $R(k) \in \mathbb{R}^{s\times s}$ is a positive-definite performance weighting. The goal is to determine refined controls $\hat{\tilde{U}}(k-1)$ that would have provided better performance than the controls $U(k)$ that were applied to the system. The refined control values $\hat{\tilde{U}}(k-1)$ are subsequently used to update the controller.

Next, to ensure that (15) has a global minimizer, we consider the regularized cost

$$\begin{aligned}\bar{J}(\hat{\tilde{U}}(k-1),k) \triangleq{}& \hat{Z}^{\rm T}(k)R(k)\hat{Z}(k) \\ &+ \eta(k)\hat{\tilde{U}}^{\rm T}(k-1)\hat{\tilde{U}}(k-1),\end{aligned} \tag{16}$$

where $\eta(k) \ge 0$. Substituting (14) into (16) yields

$$\begin{aligned}\bar{J}(\hat{\tilde{U}}(k-1),k) ={}& \hat{\tilde{U}}(k-1)^{\rm T}\mathcal{A}(k)\hat{\tilde{U}}(k-1) \\ &+ \mathcal{B}(k)\hat{\tilde{U}}(k-1) + \mathcal{C}(k),\end{aligned}$$

where

$$\begin{aligned}\mathcal{A}(k) &\triangleq \tilde{\mathcal{H}}^{\rm T}R(k)\tilde{\mathcal{H}} + \eta(k)I_{l_{\tilde{U}}}, \\ \mathcal{B}(k) &\triangleq 2\tilde{\mathcal{H}}^{\rm T}R(k)[Z(k) - \tilde{\mathcal{H}}\tilde{U}(k-1)], \\ \mathcal{C}(k) &\triangleq Z^{\rm T}(k)R(k)Z(k) - 2Z^{\rm T}(k)R(k)\tilde{\mathcal{H}}\tilde{U}(k-1) \\ &\quad + \tilde{U}^{\rm T}(k-1)\tilde{\mathcal{H}}^{\rm T}R(k)\tilde{\mathcal{H}}\tilde{U}(k-1).\end{aligned}$$

If either $\tilde{\mathcal{H}}$ has full column rank or $\eta(k) > 0$, then $\mathcal{A}(k)$ is positive definite. In this case, $\bar{J}(\hat{\tilde{U}}(k-1),k)$ has the unique global minimizer

$$\hat{\tilde{U}}(k-1) = -\frac{1}{2}\mathcal{A}^{-1}(k)\mathcal{B}(k). \tag{17}$$

A. Controller Construction

The control $u(k)$ is given by the strictly proper time-series controller of order n_c given by

$$u(k) = \sum_{i=1}^{n_c} M_i(k)u(k-i) + \sum_{i=1}^{n_c} N_i(k)z(k-i) + \sum_{i=1}^{n_c} Q_i(k)r(k-i), \qquad (18)$$

where, for all $i = 1, \ldots, n_c$, $M_i(k) \in \mathbb{R}$, $N_i(k) \in \mathbb{R}$, and $Q_i(k) \in \mathbb{R}$. The control (18) can be expressed as

$$u(k) = \theta(k)\phi(k-1),$$

where

$$\theta(k) \stackrel{\triangle}{=} \left[\, M_1(k) \ \cdots \ M_{n_c}(k) \ N_1(k) \ \cdots \ N_{n_c}(k) \ Q_1(k) \ \cdots \ Q_{n_c}(k) \,\right] \in \mathbb{R}^{l_u \times 3n_c}$$

and

$$\phi(k-1) \stackrel{\triangle}{=} \left[\, u(k-1) \ \cdots \ u(k-n_c) \ z(k-1) \ \cdots \ z(k-n_c) \ r(k-1) \ \cdots \ r(k-n_c) \,\right]^{\mathrm{T}} \in \mathbb{R}^{3n_c}.$$

Next, let d be a positive integer such that $\tilde{U}(k-1)$ contains $u(k-d)$ and define the cumulative cost function

$$J_{\mathrm{R}}(\theta, k) \stackrel{\triangle}{=} \sum_{i=d+1}^{k} \lambda^{k-i} \|\phi^{\mathrm{T}}(i-d-1)\theta^{\mathrm{T}}(k) - \hat{u}^{\mathrm{T}}(i-d)\|^2 + \lambda^k(\theta(k)-\theta_0)P_0^{-1}(\theta(k)-\theta_0)^{\mathrm{T}}, \qquad (19)$$

where $\|\cdot\|$ is the Euclidean norm, and $\lambda \in (0,1]$ is the forgetting factor. Minimizing (19) yields

$$\begin{aligned}\theta^{\mathrm{T}}(k) = \theta^{\mathrm{T}}(k-1) &+ \beta(k)P(k-1)\phi(k-d-1) \\ &\cdot [\phi^{\mathrm{T}}(k-d)P(k-1)\phi(k-d-1) + \lambda(k)]^{-1} \\ &\cdot [\phi^{\mathrm{T}}(k-d-1)\theta^{\mathrm{T}}(k-1) - \hat{u}^{\mathrm{T}}(k-d)],\end{aligned}$$

where $\beta(k)$ is either zero or one. The error covariance is updated by

$$\begin{aligned}P(k) = \beta(k)\lambda^{-1}P(k-1) &+ [1-\beta(k)]P(k-1) \\ &- \beta(k)\lambda^{-1}P(k-1)\phi(k-d-1) \\ &\cdot [\phi^{\mathrm{T}}(k-d-1)P(k-1)\phi(k-d) + \lambda]^{-1} \\ &\cdot \phi^{\mathrm{T}}(k-d-1)P(k-1).\end{aligned}$$

We initialize the error covariance matrix as $P(0) = \alpha I_{3n_c}$, where $\alpha > 0$. Note that when $\beta(k) = 0$, $\theta(k) = \theta(k-1)$ and $P(k) = P(k-1)$. Therefore, setting $\beta(k) = 0$ switches off the controller adaptation, and thus freezes the control gains. When $\beta(k) = 1$, the controller is allowed to adapt.

V. NUMERICAL EXAMPLES

In all examples, we assume that at least one nonzero Markov parameter of G is known. For convenience, each example is constructed such that the first nonzero Markov parameter $H_d = 1$, where d is the relative degree of G. RCAC generates a control signal $u_c(k)$ that attempts to minimize the performance $z(k)$ in the presence of the input nonlinearity $\mathcal{N}$. In all cases, we initialize the adaptive controller to be zero, that is, $\theta(0) = 0$. We let $\lambda = 1$ for all examples.

Example 5.1: We consider the asymptotically stable, minimum-phase plant

$$G(z) = \frac{(z-0.5)(z-0.9)}{(z-0.7)(z-0.5-\jmath 0.5)(z-0.5+\jmath 0.5)}, \qquad (20)$$

with the cubic input nonlinearity

$$\mathcal{N}(u) = -u^3 - 2, \qquad (21)$$

which is nonincreasing, one-to-one, and onto and has the offset $\mathcal{N}(0) = -2$. Note that $d = 1$ and $H_d = 1$. We consider the sinusoidal command $r(k) = \sin(\theta_1 k)$, where $\theta_1 = \pi/5$ rad/sample. To illustrate the effect of the nonlinearities on the closed-loop command-following performance, we first remove the input nonlinearity $\mathcal{N}(u)$ and simulate the open-loop system for the first 100 time steps. Then, at $k = 100$, we turn the adaptation on and let RCAC adapt to the linear system for 300 time steps. Next, at $k = 400$, we stop the adaptation and introduce the input nonlinearity. Consequently, from $k = 400$ to $k = 700$, we use the frozen gain matrix $\theta(400)$ as the feedback gain without adaptation in order to demonstrate the performance degradation due to the input nonlinearity. Finally, at $k = 700$, we restart the adaptation and let RCAC adapt to the Hammerstein system.

As shown in Figure 4(a), we choose $\mathcal{N}_1(u_c) = \mathrm{sat}_{p,q}(u_c)$, where $p = -10^6$ and $q = 10^6$ in (4). Since $\mathcal{N}$ is decreasing for all $u \in [-10^6, 10^6]$, we let $\mathcal{N}_2(u_r) = -u_r$. Note that knowledge of only the monotonicity of $\mathcal{N}$ is used to choose $\mathcal{N}_2$. We let $n_c = 10$, $P_0 = 0.01 I_{3n_c}$, $\eta_0 = 0$, and $\tilde{\mathcal{H}} = H_1$. Figure 4(b) shows the resulting time history of the command-following performance z, while Figure 4(c) shows the time history of the control u and linear plant input v. Finally, Figure 4(d) shows the time history of the controller gain vector θ. ■

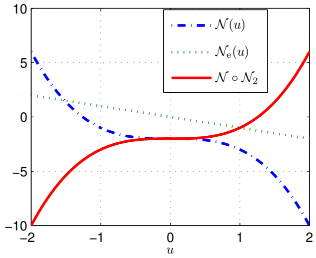
Example 5.2: We consider the asymptotically stable, NMP plant

$$G(z) = \frac{z-1.5}{(z-0.8)(z-0.6)}, \qquad (22)$$

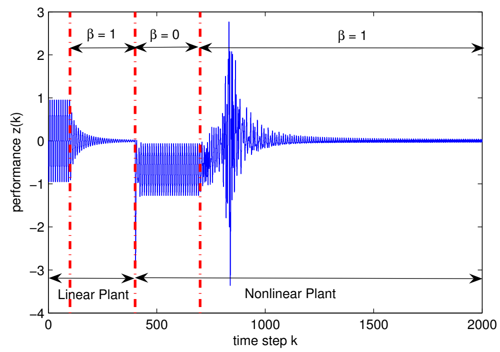
with the deadzone input nonlinearity

$$\mathcal{N}(u) = \begin{cases} u+0.5, & \text{if } u < -0.5, \\ 0, & \text{if } -0.5 \le u \le 0.5, \\ u-0.5, & \text{if } u > 0.5, \end{cases} \qquad (23)$$

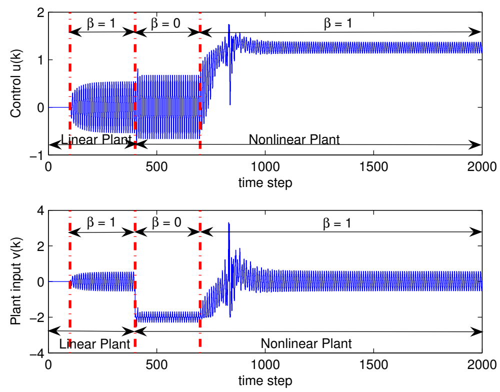
which is not one-to-one but onto and satisfies $\mathcal{N}(0) = 0$. Note that $d = 1$ and $H_d = 1$. We consider the two-tone sinusoidal command $r(k) = \sin(\theta_1 k) + 0.5\sin(\theta_2 k)$, where $\theta_1 = \pi/4$ rad/sample, and $\theta_2 = \pi/10$ rad/sample. As shown in Figure 5(a), since $\mathcal{N}(u)$ is nondecreasing for all $u \in \mathbb{R}$, we choose $\mathcal{N}_1(u_c) = \mathrm{sat}_{p,q}(u_c)$, where $p = -a$, $q = a$, and $\mathcal{N}_2(u_r) = u_r$. We let $n_c = 10$, $P_0 = 0.1 I_{3n_c}$, $\eta_0 = 0.2$, and $\tilde{\mathcal{H}} = H_1$, and we vary the saturation level a for the NMP plant (22). Figure 5(b.i) shows the time history of the performance z with $a = 10$, where the transient behavior is



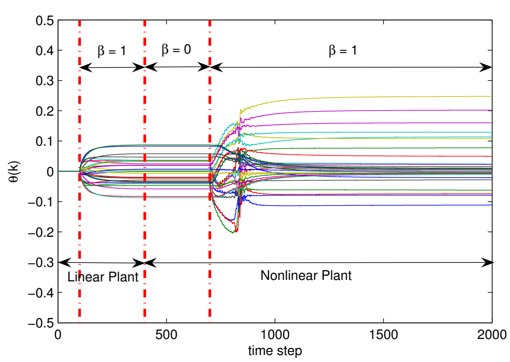
(a)



(b)



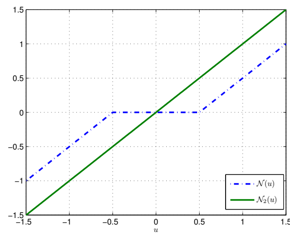
(c)



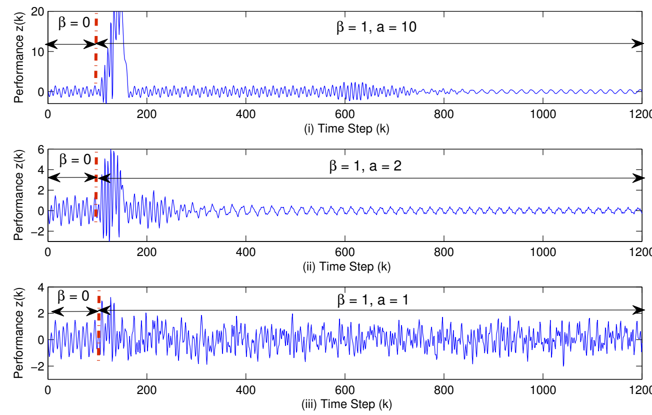
(d)

Fig. 4. Example 5.1. (a) shows the input nonlinearity $\mathcal{N}$ given by (21). (b) shows the closed-loop response to the sinusoidal command $r(k) = \sin(0.2\pi k)$ of the asymptotically stable minimum-phase plant G given by (20). The value of β indicates whether the controller is frozen or adapting. (c) shows the time history of the control u and the plant input v with and without the input nonlinearity $\mathcal{N}$ present. (d) shows the time history of the controller gain vector θ with and without $\mathcal{N}$ present.

poor. Figure 5(b.ii) shows the time history of the performance z with $a = 2$, where the transient performance is improved and z reaches steady state in about 300 time steps. Finally, we further reduce the saturation level. Figure 5(b.iii) shows the time history of the performance z with $a = 1$; in this case, RCAC cannot follow the command due to fact that $a = 1$ is not large enough to provide the control output $u_{\rm c}$ needed to drive z to a small value. ■



(a)



(b)

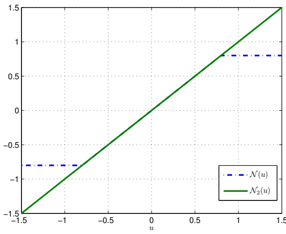
Fig. 5. Example 5.2. (a) shows the deadzone input nonlinearity $\mathcal{N}(u)$ given by (23). (b) shows the closed-loop response of the asymptotically stable NMP plant G given by (22) with the two-tone sinusoidal command $r(k) = \sin(\theta_1 k) + 0.5\sin(\theta_2 k)$, where $\theta_1 = \pi/4$ rad/sample, and $\theta_2 = \pi/10$ rad/sample. Figure 5(b.i) shows the time history of the performance z with $a = 10$, where the transient behavior is poor. Figure 5(b.ii) shows the time history of the performance z with $a = 2$. Note that the transient performance is improved and z reaches steady state in about 300 time steps. Finally, we further reduce the saturation level. Figure 5(b.iii) shows the time history of the performance z with $a = 1$; in this case, RCAC cannot follow the command due to the fact that $a = 1$ is not large enough to provide the control output $u_{\rm c}$ needed to drive z to a small value.

Example 5.3: We consider the asymptotically stable, NMP plant (22) with the saturation input nonlinearity

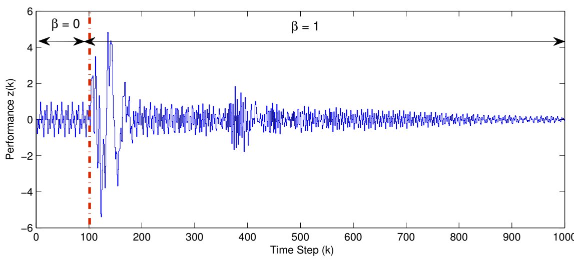
$$\mathcal{N}(u) = \begin{cases} -0.8, & \text{if } u < -1, \\ u, & \text{if } -1 \le u \le 1, \\ 0.8, & \text{if } u > 1, \end{cases} \tag{24}$$

which is nondecreasing and one-to-one but not onto, and satisfies $\mathcal{N}(0) = 0$. We consider the two-tone sinusoidal command $r(k) = 0.5\sin(\theta_1 k) + 0.5\sin(\theta_2 k)$, where $\theta_1 = \pi/5$ rad/sample and $\theta_2 = \pi/2$ rad/sample for the Hammerstein system with the input nonlinearity $\mathcal{N}$. As shown in Figure 6(a), since $\mathcal{N}(u)$ is nondecreasing for all $u \in \mathbb{R}$, we choose $\mathcal{N}_1(u_{\rm c}) = {\rm sat}_{p,q}(u_{\rm c})$, where $p = -2$ and $q = 2$ in (4), and $\mathcal{N}_2(u_{\rm r}) = u_{\rm r}$. We let $n_{\rm c} = 10$, $P_0 = 0.1 I_{3n_{\rm c}}$, $\eta_0 = 2$, and $\tilde{\mathcal{H}} = H_1$. The Hammerstein system runs open-loop for 100 time steps, and RCAC is turned on at $k = 100$. Figure 6(b) shows the time history of the performance z

with the input nonlinearity present. Note that z does not converge to zero due to the distortion introduced by the input nonlinearity $\mathcal{N}$. ■



(a)



(b)

Fig. 6. Example 5.3. (a) shows the saturating input nonlinearity $\mathcal{N}(u)$ given by (24). (b) shows the closed-loop response of the stable NMP plant G given by (22) with the two-tone sinusoidal command $r(k) = 0.5\sin(\theta_1 k) + 0.5\sin(\theta_2 k)$, $\theta_1 = \pi/5$ rad/sample, and $\theta_2 = \pi/2$ rad/sample.

Example 5.4: We consider the unstable double integrator plant

$$G(z) = \frac{z}{(z-1)^2} \tag{25}$$

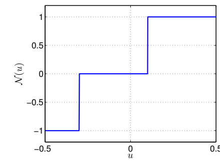
with the piecewise-constant input nonlinearity

$$\mathcal{N}(u) = \frac{1}{2}[\text{sign}(u-0.2) + \text{sign}(u+0.2)]. \tag{26}$$

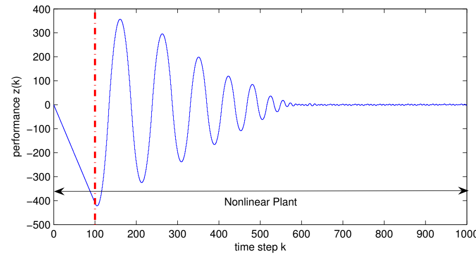
Note that $\mathcal{N}(u)$ can assume only the values -1, 0, and 1. Note that $d = 1$ and $H_d = 1$. We let the command $r(k)$ be zero, and consider stabilization using RCAC with the input relay nonlinearity given by (26). As shown in Figure 7(a), the relay nonlinearity is monotonically nondecreasing for all $u \in \mathbb{R}$, and we thus choose $\mathcal{N}_1(u_\text{c}) = \text{sat}_{p,q}(u_\text{c})$, where $p = -3$, $q = 3$, and $\mathcal{N}_2(u_\text{r}) = u_\text{r}$. We let $n_\text{c} = 2$, $P_0 = I_{3n_\text{c}}$, $\eta_0 = 0$, and $\tilde{\mathcal{H}} = H_1$. The closed-loop performance approaches ± 4 in about 500 time steps. Figure 7 shows the time history of z with the initial condition $x_0 = \left[\begin{array}{cc} -5.2 & -1.1 \end{array}\right]^\text{T}$. ■

VI. CONCLUSIONS

Retrospective cost adaptive control (RCAC) was applied to a command-following problem for Hammerstein systems with unknown disturbances. RCAC was used with limited modeling information. In particular, the input nonlinearity is assumed to be monotonic but is otherwise unknown, and RCAC uses knowledge of only the first nonzero Markov parameter of the linear dynamics. To handle the effect of the input nonlinearity, RCAC was augmented by auxiliary nonlinearities chosen based on the monotonicity of the input nonlinearity.



(a)



(b)

Fig. 7. Example 5.4. Closed-loop response of the plant G given by (25) with the initial condition $x_0 = [-5.2, -1.1]^\text{T}$. The system runs open loop for 100 time steps, and the adaptive controller is turned on at $k = 100$ with the input relay nonlinearity given by (26). The closed-loop performance z approaches ± 4 in about 500 time steps.

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