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INERTIA-FREE ATTITUDE CONTROL OF SPACECRAFT WITH UNKNOWN TIME-VARYING MASS DISTRIBUTION

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We derive a continuous, inertia-free control law for spacecraft attitude tracking that is applicable to non-rigid spacecraft with translating on-board components. This control law is simulated for slew and spin maneuvers.

I. INTRODUCTION

Attitude control of spacecraft remains a challenging nonlinear control problem of intense practical and intellectual importance. Since rotational motion evolves on the set of proper orthogonal matrices, continuous control must account for the presence of multiple equilibria, whereas discontinuous control laws based on quaternions and alternative parameterizations of the rotation matrices lead to additional complications[1]. Challenges also arise depending on the properties of the actuation hardware, for example, thrusters, reaction wheels, control-moment gyros, and magnetic torquers, as well as sensing hardware, for example, gyros, magnetometers, and star trackers. Finally, this problem is exacerbated by uncertainty involving the mass distribution of the spacecraft[2].

The present paper addresses an additional complication in spacecraft control, namely, the situation in which the mass distribution of the spacecraft is not only uncertain but also time-varying. Many spacecraft are built to deploy on orbit, for example, by expanding solar panels or a magnetometer boom. Furthermore, a spacecraft may have moving components, such as a reflector or antenna that rotates relative to the spacecraft bus in order to track a ground station. In these cases, the mass distribution changes as a function of time, which, in turn, gives rise to a time-varying inertia matrix.

In the present paper we address the problem of spacecraft attitude control with time-varying inertia. The approach that we take is an extension of the approach of ref.[2], where continuous control laws are developed based on rotation matrices. For motion-to-rest (that is, slew) maneuvers in the absence of disturbance torques, no knowledge of the inertia matrix is needed, and no estimates of the inertia matrix are constructed. For motion-to-specified-motion (for example, spin) maneuvers in the presence of harmonic (possibly constant) disturbances with known spectral content, the control law is based on an estimate of the inertia matrix; however, this estimate need not converge to the actual inertia matrix.

The contribution of the present paper is an extension of the results of ref.[2] to the case in which the mass distribution of the spacecraft is both uncertain and time-varying. For motion-to-rest maneuvers, we show that the corresponding control law of ref.[2] is effective under a special choice of the control-law parameters. This requirement can be ignored when the inertia matrix is increasing, for example, during deployment. For motion-to-specified-motion maneuvers, we make the additional assumption that the time-variation of the time-varying component of the inertia is known, whereas its spatial distribution is unknown. Under these assumptions, we extend the motion-to-specified-motion control law of ref.[2] to the case of time-varying inertia.

The contents of the paper are as follows. In Section II we develop a model of a spacecraft with time-varying inertia. In Section III we describe the attitude control objectives. Section IV deals with motion-to-rest maneuvers, while Section V treats motion-to-specified-motion maneuvers.

II. SPACECRAFT MODEL

Let the spacecraft be denoted by sc, and let c denote its center of mass. We assume that the spacecraft is composed of a rigid bus and additional moving components. These components are assumed to not rotate relative to the spacecraft; for example, they may move linearly in a body-fixed direction. We assume a bus-fixed frame $\mathrm{F_B}$

and an Earth-centered inertial frame $\mathrm{F_E}$. We begin with Newton's second law for rotation, which states that the derivative of the angular momentum of a body relative to its center of mass with respect to an inertial frame is equal to the sum of the moments applied to that body about its center of mass. We thus have

$$\begin{aligned}\vec{M}_{\mathrm{sc/c}} &= \overset{\mathrm{E}\bullet}{\vec{H}}_{\mathrm{sc/c/E}} \\ &= \overbrace{\vec{I}_{\mathrm{sc/c}}\vec{\omega}_{\mathrm{B/E}}}^{\mathrm{E}\bullet} \\ &= \overbrace{\vec{I}_{\mathrm{sc/c}}\vec{\omega}_{\mathrm{B/E}}}^{\mathrm{B}\bullet} + \vec{\omega}_{\mathrm{B/E}} \times \vec{I}_{\mathrm{sc/c}}\vec{\omega}_{\mathrm{B/E}} \\ &= \overset{\mathrm{B}\bullet}{\vec{I}}_{\mathrm{sc/c}}\,\vec{\omega}_{\mathrm{B/E}} + \vec{I}_{\mathrm{sc/c}}\,\overset{\mathrm{B}\bullet}{\vec{\omega}}_{\mathrm{B/E}} \\ &\quad + \vec{\omega}_{\mathrm{B/E}} \times \vec{I}_{\mathrm{sc/c}}\vec{\omega}_{\mathrm{B/E}}, \end{aligned} \tag{1}$$

where $\vec{I}_{\mathrm{sc/c}}$ is the positive-definite inertia tensor of the spacecraft relative to its center of mass, and $\vec{\omega}_{\mathrm{B/E}}$ is the angular velocity of $\mathrm{F_B}$ with respect to $\mathrm{F_E}$. We separate the moments on the spacecraft $\vec{M}_{\mathrm{sc/c}}$ into disturbance moments $\vec{M}_{\mathrm{dist}}$ and control moments $\vec{M}_{\mathrm{control}}$.

We now resolve [1] in $\mathrm{F_B}$ using the notation

$$J \triangleq \vec{I}_{\mathrm{b/c}}\Big|_{\mathrm{B}}, \qquad \dot{J} \triangleq \overset{\mathrm{B}\bullet}{\vec{I}}_{\mathrm{w}_i/\mathrm{c}}\Bigg|_{\mathrm{B}},$$

$$\omega \triangleq \vec{\omega}_{\mathrm{B/E}}\Big|_{\mathrm{B}}, \qquad \dot{\omega} \triangleq \overset{\mathrm{B}\bullet}{\vec{\omega}}_{\mathrm{B/E}}\Bigg|_{\mathrm{B}},$$

$$\tau_{\mathrm{dist}} \triangleq \vec{M}_{\mathrm{dist}}\Big|_{\mathrm{B}}, \qquad Bu \triangleq \vec{M}_{\mathrm{control}}\Big|_{\mathrm{B}},$$

where the components of the vector $u \in \mathbb{R}^3$ represent three independent torque inputs, while the rows of the matrix $B \in \mathbb{R}^{3\times 3}$ determine the applied torque about each axis of the spacecraft frame due to u as given by the product Bu. We let the vector τ_{dist} represent disturbance torques, that is, all internal and external torques applied to the spacecraft aside from control torques. Disturbance torques may be due to onboard components, gravity gradients, solar pressure, atmospheric drag, or the ambient magnetic field.

Resolving [1] in $\mathrm{F_B}$ and rearranging yields

$$J\dot{\omega} = J\omega \times \omega + Bu - \dot{J}\omega + \tau_{\mathrm{dist}}. \tag{2}$$

The kinematics of the spacecraft model are given by Poisson's equation

$$\dot{R} = R\omega^{\times}, \tag{3}$$

which complements [2]. In [3], $\omega^{\times}$ denotes the skew-symmetric matrix of ω, and $R = \mathcal{O}_{\mathrm{E/B}} \in \mathbb{R}^{3\times 3}$ is the rotation tensor that transforms $\mathrm{F_E}$ into $\mathrm{F_B}$ resolved in either $\mathrm{F_E}$ or $\mathrm{F_B}$. Therefore, R is the proper orthogonal matrix (that is, the rotation matrix) that transforms the components of a vector resolved in the bus-fixed frame into the components of the same vector resolved in the inertial frame.

Compared to the rigid body case treated in ref.[2], the time-varying inertia complicates the dynamic equations due to the term $-\dot{J}\omega$ added to [2]. Note that this term affects only the attitude of the spacecraft when the spacecraft has nonzero angular velocity. The kinematic relation [3] remains unchanged.

Both rate (inertial) and attitude (noninertial) measurements are assumed to be available. Gyro measurements $y_{\mathrm{rate}} \in \mathbb{R}^3$ are assumed to provide measurements of the angular velocity resolved in the spacecraft frame, that is,

$$y_{\mathrm{rate}} = \omega + v_{\mathrm{rate}}, \tag{4}$$

where $v_{\mathrm{rate}} \in \mathbb{R}^3$ represents the presence of noise in the gyro measurements. Attitude is measured indirectly using sensors such as magnetometers or star trackers. The attitude is determined to be

$$y_{\mathrm{attitude}} = R. \tag{5}$$

When attitude measurements are given in terms of an alternative attitude representation, such as quaternions, Rodrigues's formula can be used to determine the corresponding rotation matrix. Attitude estimation on $\mathrm{SO}(3)$ is considered in ref.[3].

III. OBJECTIVES FOR CONTROL DESIGN

The objective of the attitude control problem is to determine control inputs such that the spacecraft attitude given by R follows a commanded attitude trajectory given by a possibly time-varying C^1 rotation matrix $R_{\mathrm{d}}(t)$. For $t \geq 0$, $R_{\mathrm{d}}(t)$ is given by

$$\dot{R}_{\mathrm{d}}(t) = R_{\mathrm{d}}(t)\omega_{\mathrm{d}}(t)^{\times}, \tag{6}$$

$$R_{\mathrm{d}}(0) = R_{\mathrm{d0}}, \tag{7}$$

where ω_{d} is the desired, possibly time-varying angular velocity. The error between $R(t)$ and $R_{\mathrm{d}}(t)$ is given in terms of the attitude-error rotation matrix

$$\tilde{R} \triangleq R_{\mathrm{d}}^{\mathrm{T}} R,$$

which satisfies the differential equation

$$\dot{\tilde{R}} = \tilde{R}\tilde{\omega}^{\times}, \qquad [8]$$

where the angular velocity error $\tilde{\omega}$ is defined by

$$\tilde{\omega} \triangleq \omega - \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}}.$$

We rewrite [2] in terms of the angular-velocity error as

$$\begin{aligned} J\dot{\tilde{\omega}} = &\\ &[J(\tilde{\omega} + \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}})] \times (\tilde{\omega} + \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}}) + Bu + \tau_{\mathrm{dist}} \\ &+ J(\tilde{\omega} \times \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}} - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}}) - \dot{J}(\tilde{\omega} + \tilde{R}^{\mathrm{T}}\omega_{\mathrm{d}}). \qquad [9] \end{aligned}$$

A scalar measure of attitude error is given by the rotation angle $\theta(t)$ about an eigenaxis needed to rotate the spacecraft from its attitude $R(t)$ to the desired attitude $R_{\mathrm{d}}(t)$, which is given by[4]

$$\theta(t) = \cos^{-1}(\tfrac{1}{2}[\mathrm{tr}\,\tilde{R}(t) - 1]). \qquad [10]$$

IV. MOTION-TO-REST CONTROL

Two controllers are presented in ref.[2]. When no disturbances are present, the inertia-free control law given by (38) of ref.[2] achieves almost global stabilization of a constant desired configuration R_{d}, that is, a slew maneuver that brings the spacecraft to rest. As in ref.[2], define the Lyapunov candidate

$$V(\omega, \tilde{R}) \triangleq \tfrac{1}{2}\omega^{\mathrm{T}}J\omega + K_{\mathrm{p}}\mathrm{tr}(A - A\tilde{R}), \qquad [11]$$

where K_{p} is a positive number and $A \in \mathbb{R}^{3\times3}$ is a diagonal positive-definite matrix given by $A = \mathrm{diag}(a_1, a_2, a_3)$. Let u be given by (38) of ref.[2], that is,

$$u = -B^{-1}(K_{\mathrm{p}}S + K_{\mathrm{v}}\omega), \qquad [12]$$

where $K_{\mathrm{v}} \in \mathbb{R}^{3\times3}$ is positive definite, and S is defined as

$$S \triangleq \sum_{i=1}^{3} a_i(\tilde{R}^{\mathrm{T}}e_i) \times e_i, \qquad [13]$$

where, for $i = 1, 2, 3$, e_i denotes the ith column of the 3×3 identity matrix. Taking the derivative of (11) along the trajectories of (2) yields

$$\begin{aligned} \dot{V}(\omega, \tilde{R}) &= \omega^{\mathrm{T}}J\dot{\omega} + \tfrac{1}{2}\omega^{\mathrm{T}}\dot{J}\omega + K_{\mathrm{p}}\omega^{\mathrm{T}}S \\ &= \omega^{\mathrm{T}}\left(J\omega \times \omega + Bu - \dot{J}\omega + \tfrac{1}{2}\dot{J}\omega\right) \\ &\quad + K_{\mathrm{p}}\omega^{\mathrm{T}}S \\ &= \omega^{\mathrm{T}}\left(-K_{\mathrm{p}}S - K_{\mathrm{v}}\omega - \tfrac{1}{2}\dot{J}\omega\right) + K_{\mathrm{p}}\omega^{\mathrm{T}}S \\ &= -\omega^{\mathrm{T}}\left(K_{\mathrm{v}} + \tfrac{1}{2}\dot{J}\right)\omega, \qquad [14] \end{aligned}$$

where the derivative of $K_{\mathrm{p}}\mathrm{tr}(A - A\tilde{R})$ is given by $K_{\mathrm{p}}\omega^{\mathrm{T}}S$ as shown in Section V.

Selecting $K_{\mathrm{v}} > -\frac{1}{2}\dot{J} + \varepsilon I$ for some $\varepsilon > 0$ ensures that (11) decays as in ref.[2] but otherwise, the controller requires no modification for the case of time-varying inertia. This condition is automatically satisfied when the inertia matrix is increasing, that is, $J(t_1) \le J(t_2)$, for all $t_1 \le t_2$, which implies that $\dot{J} \ge 0$. Thus, for every positive-definite choice of K_{v}, it follows that $K_{\mathrm{v}} > -\frac{1}{2}\dot{J} + \varepsilon I$ for some $\varepsilon > 0$. This is the case, for example, during solar panel or magnetometer boom deployment. During retraction, a bound on $\dot{J}$ must be known in order to properly select K_{v}.

For simulation, we assume that the inertia of the spacecraft takes the form

$$J = J(t) = J_0 + J_1(t),$$

where J_0 is constant and represents the rigid part of the spacecraft, and $J_1(t)$ is time-varying and represents a moving part of the spacecraft. We simulate a point mass moving linearly in time outward along the spacecraft's y-axis, representing solar panel deployment. The inertia matrix $J_1(t)$ is thus given by

$$J_1(t) = \begin{bmatrix} \min(t^2, t_{\mathrm{d}}^2) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \min(t^2, t_{\mathrm{d}}^2) \end{bmatrix} \text{kg-m}^2,$$

where t_{d} is the time it takes to deploy.

The following parameters are used. The inertia matrix J_0 is given by

$$J_0 = \begin{bmatrix} 5 & -0.1 & -0.5 \\ -0.1 & 2 & 1 \\ -0.5 & 1 & 3.5 \end{bmatrix} \text{kg-m}^2,$$

with principal moments of inertia 1.4947, 3.7997, and 5.2056 kg-m^2. Let $t_{\mathrm{d}} = 10$, and set $K_{\mathrm{p}} = 15$ and $K_{\mathrm{v}} = 15\,I_3$. Since the inertia is increasing, any positive definite K_{v} is acceptable.

We use controller (12) for an aggressive slew maneuver, where the objective is to bring the spacecraft from the initial attitude $R_0 = I_3$ and initial angular velocity

$$\omega(0) = \begin{bmatrix} 1 & -1 & 0.5 \end{bmatrix}^{\mathrm{T}} \text{rad/sec}$$

to rest ($\omega_{\mathrm{d}} = 0$) at the desired final orientation $R_{\mathrm{d}} = \mathrm{diag}(1, -1, -1)$, which represents a rotation of 180 degrees about the x-axis.

Figures 1-3 show, respectively, the attitude error, angular velocity components, and control torque components. The spacecraft attitude and angular velocity components are brought close to the desired values in about 5 sec, before the solar panel deployment is complete, and are maintained throughout the remainder of the deployment.

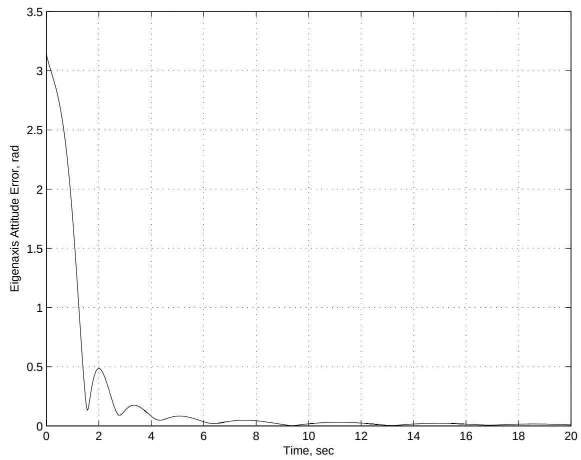

Fig. 1: Eigenaxis attitude error using the control law [12] for a slew maneuver during translational motion of an internal mass.

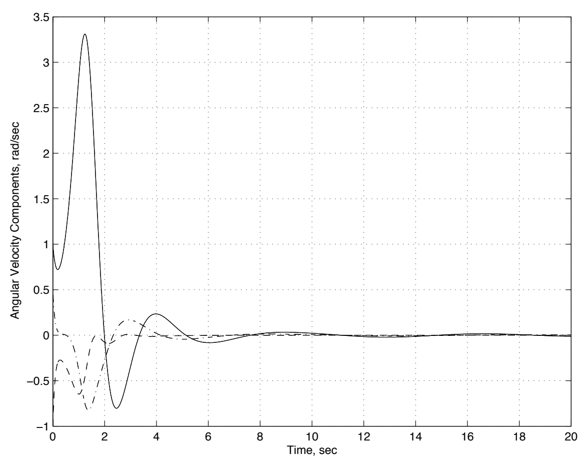

Fig. 2: Spacecraft angular-velocity components using the control law [12] for a slew maneuver during translational motion of an internal mass.

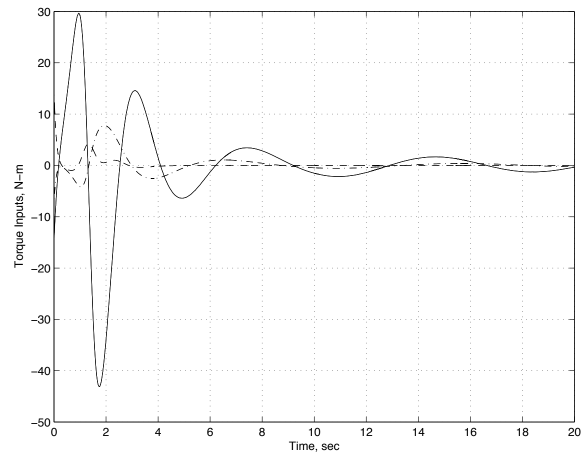

Fig. 3: Control torque components using the control law [12] for a slew maneuver during translational motion of an internal mass.

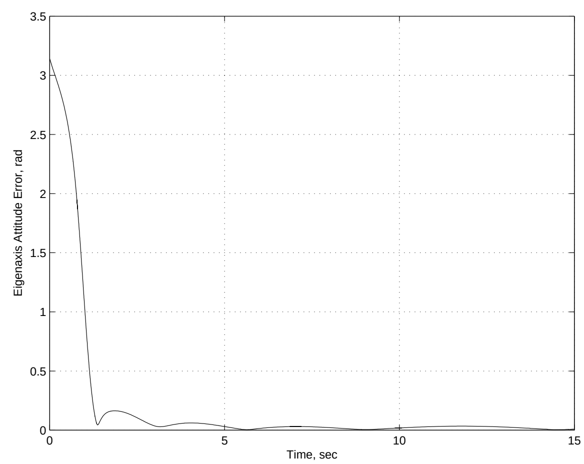

Fig. 4: Eigenaxis attitude error using the control law [21] of ref.[2] for a slew maneuver during translational motion of an internal mass.

V. MOTION-TO-SPECIFIED-MOTION CONTROL

A more general control law that tracks a desired attitude trajectory for rigid spacecraft in the presence of disturbances is given by [21] of ref.[2]. We apply this controller to the non-rigid spacecraft presented in the previous section. Additionally, we assume a constant nonzero disturbance torque, $\tau_{\rm dist} = [0.7\ -0.3\quad 0]^{\rm T}$. The parameters of the controller are chosen to be $K_1 = D = I_3$, $A = \mathrm{diag}(1, 2, 3)$, $K_{\rm p} = 6$, $K_{\rm v} = 6\, I_3$, and $Q = I_6$.

We first consider the slew maneuver. Figures 4-6 show, respectively, the attitude error, angular velocity components, and control torque components. The spacecraft attitude and angular velocity components are brought close to the desired values in under 5 sec. The persistent nonzero control torque seen in Figure 6 is due to the constant nonzero disturbance torque. While the controller [21] of ref.[2] assumes that the spacecraft is rigid, it successfully completes the slew maneuver by treating the term $-\dot{J}\omega$ as a disturbance that gradually disappears as the spacecraft is brought to rest ($\omega = 0$). This suggests that it might not succeed at spin maneuvers where $\omega \neq 0$ as $t \to \infty$.

Before simulating a spin maneuver, we modify $J_1(t)$ so that it is persistent throughout the simulation, rather

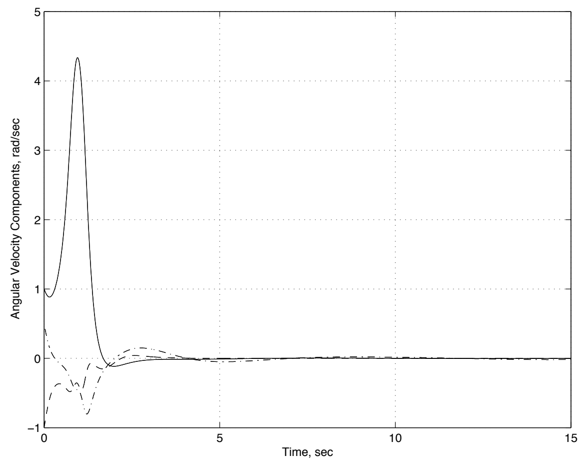

Fig. 5: Spacecraft angular-velocity components using the control law [21] of ref.[2] for a slew maneuver during translational motion of an internal mass.

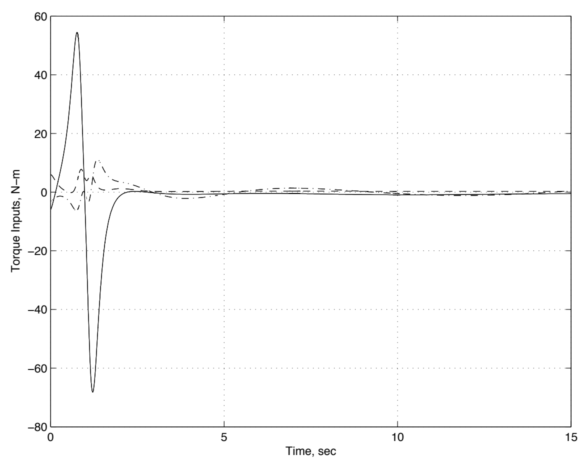

Fig. 6: Control torque components using the control law [21] of ref.[2] for a slew maneuver during translational motion of an internal mass.

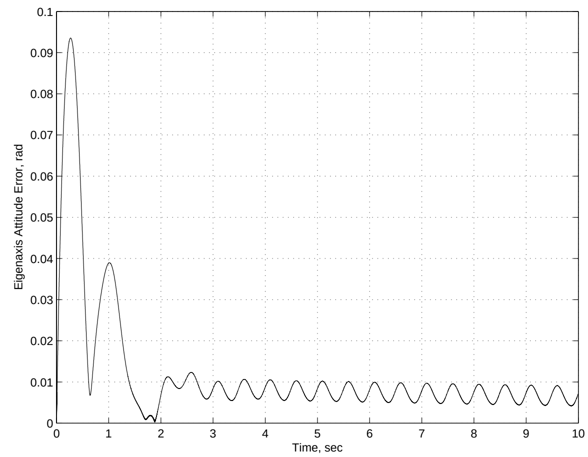

Fig. 7: Eigenaxis attitude error using the control law [21] of ref.[2] for a spin maneuver during accordion-like motion.

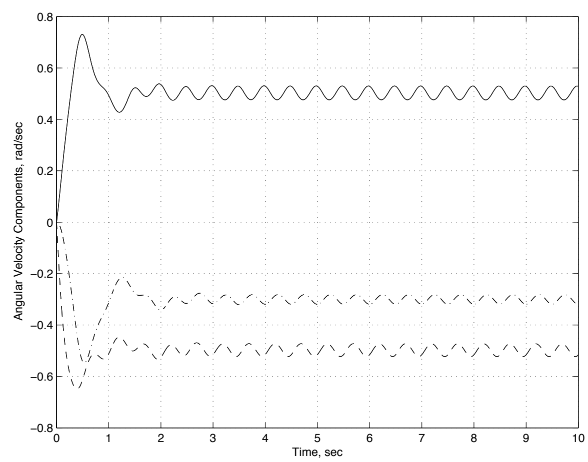

Fig. 8: Spacecraft angular-velocity components using the control law [21] of ref.[2] for a spin maneuver during accordion-like motion.

than coming to rest at 10 sec., as before. We let $J_1(t) = \frac{1}{10}\sin^2(2\pi t)J_0$, which represents an accordion like motion, while still preserving the required inertia inequalities

$$J_a \leq J_b + J_c, \quad J_b \leq J_a + J_c, \quad J_c \leq J_a + J_b,$$

where J_a, J_b, J_c are the principal moments of inertia.

We now consider a spin maneuver with the spacecraft initially at rest and $R(0) = I_3$. The specified attitude is given by $R_\mathrm{d}(0) = I_3$ with the desired constant angular velocity

$$\omega_\mathrm{d} = \begin{bmatrix} 0.5 & -0.5 & -0.3 \end{bmatrix}^\mathrm{T} \text{ rad/sec.}$$

Figures 7-9 show, respectively, the attitude error, angular velocity components, and control torque components. The spacecraft attitude and angular velocity components are brought close to the desired values in about 2 sec. but do not settle, as was expected. The non-rigidity of the spacecraft acts as a disturbance torque that cannot be rejected by the control.

V.I. Extended Control Law

We now extend controller [21] of ref.[2] to the case of a non-rigid spacecraft whose inertia matrix has the form

$$J(t) = J_0 + f(t)J_1,$$

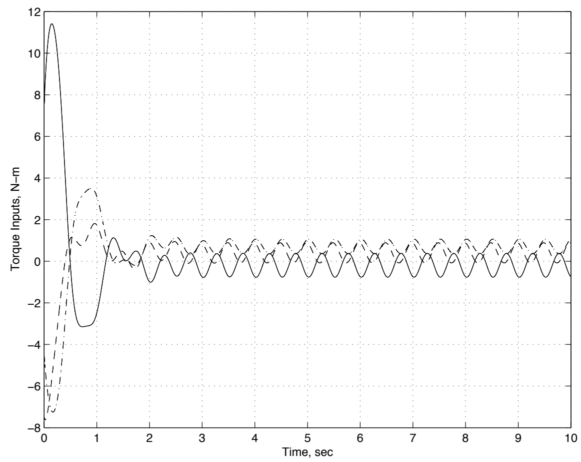

Fig. 9: Control torque components using the control law [21] of ref.[2] for a spin maneuver during accordion-like motion.

where $f(t)$ is known but J_0 and J_1 are unknown. The following preliminary results are needed.

Let I denote the identity matrix, whose dimensions are determined by context, and let M_{ij} denote the i, j entry of the matrix M. The following result is given in ref.[2].

Lemma 1. Let $A \in \mathbb{R}^{3\times 3}$ be a diagonal positive-definite matrix. Then the following statements hold for a proper orthogonal matrix R:

i) For all $i, j = 1, 2, 3$, $R_{ij} \in [-1, 1]$.

ii) $\operatorname{tr}(A - AR) \geq 0$.

iii) $\operatorname{tr}(A - AR) = 0$ if and only if $R = I$.

For convenience we note that, if R is a rotation matrix and $x, y \in \mathbb{R}^3$, then

$$(Rx)^\times = Rx^\times R^{\mathrm{T}},$$

and, therefore,

$$R(x \times y) = (Rx) \times Ry.$$

Next we introduce the notation

$$J_0\omega = L(\omega)\gamma,$$

where $\gamma \in \mathbb{R}^6$ is defined by

$$\gamma \triangleq \begin{bmatrix} J_{0_{11}} & J_{0_{22}} & J_{0_{33}} & J_{0_{23}} & J_{0_{13}} & J_{0_{12}} \end{bmatrix}^{\mathrm{T}}$$

and

$$L(\omega) \triangleq \begin{bmatrix} \omega_1 & 0 & 0 & 0 & \omega_3 & \omega_2 \\ 0 & \omega_2 & 0 & \omega_3 & 0 & \omega_1 \\ 0 & 0 & \omega_3 & \omega_2 & \omega_1 & 0 \end{bmatrix}.$$

Similarly, we let

$$J_1\omega = L(\omega)\zeta \qquad [15]$$

where $\zeta \in \mathbb{R}^6$ is defined by

$$\zeta \triangleq \begin{bmatrix} J_{1_{11}} & J_{1_{22}} & J_{1_{33}} & J_{1_{23}} & J_{1_{13}} & J_{1_{12}} \end{bmatrix}^{\mathrm{T}}.$$

Next, let $\hat{J}_0 \in \mathbb{R}^{3\times 3}$ denote an estimate of J_0, $\hat{J}_1 \in \mathbb{R}^{3\times 3}$ denote an estimate of J_1 and define the inertia-estimation errors

$$\tilde{J}_0 \triangleq J_0 - \hat{J}_0,$$

and

$$\tilde{J}_1 \triangleq J_1 - \hat{J}_1.$$

Letting $\hat{\gamma}, \tilde{\gamma} \in \mathbb{R}^6$ represent $\hat{J}_0, \tilde{J}_0$, respectively, and $\hat{\zeta}, \tilde{\zeta} \in \mathbb{R}^6$ represent $\hat{J}_1, \tilde{J}_1$, respectively, it follows that

$$\tilde{\gamma} = \gamma - \hat{\gamma},$$

and

$$\tilde{\zeta} = \zeta - \hat{\zeta}.$$

Likewise, let $\hat{\tau}_{\mathrm{dist}} \in \mathbb{R}^3$ denote an estimate of τ_{dist}, and define the disturbance-estimation error

$$\tilde{\tau}_{\mathrm{dist}} \triangleq \tau_{\mathrm{dist}} - \hat{\tau}_{\mathrm{dist}}.$$

We now state the assumptions upon which the following development is based:

Assumption 1. J_0 and J_1 are constant and unknown.

Assumption 2. $f(t)$ is time-varying and known.

Assumption 3. Each component of τ_{dist} is a linear combination of constant and harmonic signals, whose frequencies are known but whose amplitudes and phases are unknown.

Assumption 3 implies that τ_{dist} can be modeled as the output of an autonomous system of the form

$$\dot{d} = A_d d, \qquad [16]$$

$$\tau_{\mathrm{dist}} = C_d d, \qquad [17]$$

where $A_d \in \mathbb{R}^{n_d \times n_d}$ and $C_d \in \mathbb{R}^{3\times n_d}$ are known matrices and A_d is a Lyapunov-stable matrix. In this model, $d(0)$ is unknown, which is equivalent to the assumption that the amplitude and phase of each harmonic component of the disturbance is unknown. The matrix A_d is chosen to include eigenvalues of all frequency components that may be present in the disturbance signal, where the zero eigenvalue corresponds to a constant disturbance. In effect, the controller provides infinite gain at the disturbance frequency, which results in asymptotic rejection of harmonic disturbance components. In particular, an integral controller provides infinite gain at DC in order to reject constant disturbances. In the case of orbit-dependent disturbances, the frequencies can be estimated from the orbital parameters. Likewise, in the case of disturbances originating from on-board devices, the spectral content of the disturbances may be known. In other cases, it may be possible to estimate the spectrum of the disturbances through signal processing. Assumption 3 implies that A_d can be chosen to be skew symmetric, which we do henceforth. Let $\hat{d} \in \mathbb{R}^{n_d}$ denote an estimate of d, and define the disturbance-state estimation error

$$\tilde{d} \stackrel{\triangle}{=} d - \hat{d}.$$

Theorem 1. *Let K_p be a positive number, let $K_1 \in \mathbb{R}^{3\times 3}$, let $Q, Z \in \mathbb{R}^{6\times 6}$ and $D \in \mathbb{R}^{n_d\times n_d}$ be positive definite matrices, let $A = \mathrm{diag}(a_1, a_2, a_3)$ be a diagonal positive-definite matrix, and define*

$$S \stackrel{\triangle}{=} \sum_{i=1}^{3} a_i(\tilde{R}^\mathrm{T} e_i) \times e_i.$$

Then the function

$$\begin{aligned} V(\tilde{\omega}, \tilde{R}, \tilde{\gamma}, \tilde{d}) \stackrel{\triangle}{=} {} & \tfrac{1}{2}(\tilde{\omega} + K_1 S)^\mathrm{T} J(\tilde{\omega} + K_1 S) \\ & + K_\mathrm{p}\mathrm{tr}\,(A - A\tilde{R}) + \tfrac{1}{2}\tilde{\gamma}^\mathrm{T} Q\tilde{\gamma} \\ & + \tfrac{1}{2}\tilde{\zeta}^\mathrm{T} Z\tilde{\zeta} + \tfrac{1}{2}\tilde{d}^\mathrm{T} D\tilde{d} \end{aligned} \qquad [18]$$

is positive definite, that is, V is nonnegative, and $V = 0$ if and only if $\tilde{\omega} = 0$, $\tilde{R} = I$, $\tilde{\gamma} = 0$, and $\tilde{d} = 0$.

Proof. It follows from statement 2 of Lemma 1 that $\mathrm{tr}\,(A - A\tilde{R})$ is nonnegative. Hence V is nonnegative. Now suppose that $V = 0$. Then, $\tilde{\omega} + K_1 S = 0$, $\tilde{\gamma} = 0$, and $\tilde{d} = 0$, and it follows from statement 3 of Lemma 1 that $\tilde{R} = I$, and thus $S = 0$. Therefore, $\tilde{\omega} = 0$. □

Theorem 2. *Let K_p be a positive number, let $K_\mathrm{v} \in \mathbb{R}^{3\times 3}$, $K_1 \in \mathbb{R}^{3\times 3}$, $Q, Z \in \mathbb{R}^{6\times 6}$, and $D \in \mathbb{R}^{n_d\times n_d}$ be positive definite matrices, assume that $A_d^\mathrm{T} D + DA_d$ is negative semidefinite, let $A = \mathrm{diag}(a_1, a_2, a_3)$ be a diagonal positive-definite matrix, define S and V as in Theorem 1, and let $\hat{\gamma}$, $\hat{\zeta}$, and $\hat{d}$ satisfy*

$$\begin{aligned} \dot{\hat{\gamma}} = {} & Q^{-1}[L^\mathrm{T}(\omega)\omega^\times + L^\mathrm{T}(K_1\dot{S} + \tilde{\omega}\times\omega \\ & - \tilde{R}^\mathrm{T}\dot{\omega}_\mathrm{d})](\tilde{\omega} + K_1 S), \end{aligned} \qquad [19]$$

$$\begin{aligned} \dot{\hat{\zeta}} = {} & Z^{-1}[f(t)L^\mathrm{T}(\omega)\omega^\times + f(t)L^\mathrm{T}(K_1\dot{S} \\ & + \tilde{\omega}\times\omega - \tilde{R}^\mathrm{T}\dot{\omega}_\mathrm{d}) + \dot{f}(t)L^\mathrm{T}(\tfrac{1}{2}(\tilde{\omega} + K_1\dot{S}) \\ & - \omega)](\tilde{\omega} + K_1 S), \end{aligned} \qquad [20]$$

where

$$\dot{S} = \sum_{i=1}^{3} a_i[(\tilde{R}^\mathrm{T} e_i)\times\tilde{\omega}]\times e_i \qquad [21]$$

and

$$\dot{\hat{d}} = A_d\hat{d} + D^{-1}C_d^\mathrm{T}(\tilde{\omega} + K_1 S), \qquad [22]$$

$$\hat{\tau}_\mathrm{dist} = C_d\hat{d}, \qquad [23]$$

so that $\hat{\tau}_\mathrm{dist}$ is the disturbance torque estimate. Furthermore, consider the control law

$$u = -B^{-1}(v_1 + v_2 + v_3), \qquad [24]$$

where

$$\begin{aligned} v_1 \stackrel{\triangle}{=} {} & -(\hat{J}_0 + f(t)\hat{J}_1)\,\omega\times\omega \\ & - \dot{f}(t)\hat{J}_1(\tfrac{1}{2}(\tilde{\omega} + K_1\dot{S}) - \omega) \\ & - (\hat{J}_0 + f(t)\hat{J}_1)(K_1\dot{S} + \tilde{\omega}\times\omega - \tilde{R}^\mathrm{T}\dot{\omega}_\mathrm{d}), \end{aligned} \qquad [25]$$

$$v_2 \stackrel{\triangle}{=} -\hat{\tau}_\mathrm{dist}, \qquad [26]$$

and

$$v_3 \stackrel{\triangle}{=} -K_\mathrm{v}(\tilde{\omega} + K_1 S) - K_\mathrm{p} S. \qquad [27]$$

Then,

$$\begin{aligned} \dot{V}(\tilde{\omega}, \tilde{R}, \tilde{\gamma}, \tilde{d}) = {} & -(\tilde{\omega} + K_1 S)^\mathrm{T} K_\mathrm{v}(\tilde{\omega} + K_1 S) \\ & - K_\mathrm{p} S^\mathrm{T} K_1 S \\ & + \tfrac{1}{2}\tilde{d}^\mathrm{T}(A_d^\mathrm{T} D + DA_d)\tilde{d} \end{aligned} \qquad [28]$$

is negative semidefinite.

Proof. Noting that

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\mathrm{tr}\,(A - A\tilde{R}) &= -\mathrm{tr}\,A\dot{\tilde{R}} \\ &= -\mathrm{tr}\,A(\tilde{R}\omega^\times - \omega_\mathrm{d}^\times\tilde{R}) \\ &= -\sum_{i=1}^{3} a_i e_i^\mathrm{T}(\tilde{R}\omega^\times - \omega_\mathrm{d}^\times\tilde{R})e_i \\ &= -\sum_{i=1}^{3} a_i e_i^\mathrm{T}\tilde{R}(\omega^\times - \tilde{R}^\mathrm{T}\omega_\mathrm{d}^\times\tilde{R})e_i \\ &= -\sum_{i=1}^{3} a_i e_i^\mathrm{T}\tilde{R}(\omega - \tilde{R}^\mathrm{T}\omega_\mathrm{d})^\times e_i \\ &= \sum_{i=1}^{3} a_i e_i^\mathrm{T}\tilde{R}e_i^\times\tilde{\omega} \\ &= [-\sum_{i=1}^{3} a_i e_i\times\tilde{R}^\mathrm{T} e_i]^\mathrm{T}\tilde{\omega} \\ &= [\sum_{i=1}^{3} a_i(\tilde{R}^\mathrm{T} e_i)\times e_i]^\mathrm{T}\tilde{\omega} \\ &= \tilde{\omega}^\mathrm{T} S, \end{aligned}$$

we have

$$
\begin{aligned}
&\dot{V}(\tilde{\omega}, \tilde{R}, \tilde{\gamma}, \tilde{d})\\
&\quad = (\tilde{\omega} + K_1 S)^{\mathrm{T}}(J\dot{\tilde{\omega}} + JK_1\dot{S}) - K_{\mathrm{p}}\mathrm{tr}\, A\dot{\tilde{R}} - \tilde{\gamma}^{\mathrm{T}} Q\dot{\tilde{\gamma}}\\
&\qquad + \tfrac{1}{2}(\tilde{\omega} + K_1 S)^{\mathrm{T}}\dot{J}(\tilde{\omega} + K_1 S) - \tilde{\zeta}^{\mathrm{T}} T\dot{\tilde{\zeta}} + \tilde{d}^{\mathrm{T}} D\dot{\tilde{d}}\\
&\quad = (\tilde{\omega} + K_1 S)^{\mathrm{T}}[J\omega\times\omega + J(\tilde{\omega}\times\omega - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}}) - Bu\\
&\qquad - \dot{J}\omega + \tau_{\mathrm{dist}} + JK_1\dot{S} + \tfrac{1}{2}\dot{J}(\tilde{\omega} + K_1\dot{S})]\\
&\qquad + K_{\mathrm{p}}\tilde{\omega}^{\mathrm{T}} S - \tilde{\gamma}^{\mathrm{T}} Q\dot{\tilde{\gamma}} - \tilde{\zeta}^{\mathrm{T}} T\dot{\tilde{\zeta}} + \tilde{d}^{\mathrm{T}} D\dot{\tilde{d}}\\
&\quad = (\tilde{\omega} + K_1 S)^{\mathrm{T}}[J\omega\times\omega + J(K_1\dot{S} + \tilde{\omega}\times\omega - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}})\\
&\qquad + v_1 + v_2 + v_3 + \tau_{\mathrm{dist}} + \dot{J}(\tfrac{1}{2}(\tilde{\omega} + K_1\dot{S}) - \omega)]\\
&\qquad + K_{\mathrm{p}}\tilde{\omega}^{\mathrm{T}} S - \tilde{\gamma}^{\mathrm{T}} Q\dot{\tilde{\gamma}} - \tilde{\zeta}^{\mathrm{T}} T\dot{\tilde{\zeta}} + \tilde{d}^{\mathrm{T}} D\dot{\tilde{d}}\\
&\quad = (\tilde{\omega} + K_1 S)^{\mathrm{T}}[(\tilde{J}_0 + f(t)\tilde{J}_1)\omega\times\omega\\
&\qquad + (\tilde{J}_0 + f(t)\tilde{J}_1)(K_1\dot{S} + \tilde{\omega}\times\omega - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}})\\
&\qquad + \dot{f}(t)\tilde{J}_1(\tfrac{1}{2}(\tilde{\omega} + K_1\dot{S}) - \omega)] + (\tilde{\omega} + K_1 S)^{\mathrm{T}}\tilde{\tau}_{\mathrm{dist}}\\
&\qquad - (\tilde{\omega} + K_1 S)^{\mathrm{T}} K_{\mathrm{v}}(\tilde{\omega} + K_1 S) - K_{\mathrm{p}}(\tilde{\omega} + K_1 S)^{\mathrm{T}} S\\
&\qquad + K_{\mathrm{p}}\tilde{\omega}^{\mathrm{T}} S - \tilde{\gamma}^{\mathrm{T}} Q\dot{\tilde{\gamma}} - \tilde{\zeta}^{\mathrm{T}} T\dot{\tilde{\zeta}} + \tilde{d}^{\mathrm{T}} D\dot{\tilde{d}}\\
&\quad = (\tilde{\omega} + K_1 S)^{\mathrm{T}}[L(\omega)\tilde{\gamma}\times\omega + L(K_1\dot{S} + \tilde{\omega}\times\omega\\
&\qquad - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}})\tilde{\gamma}] + (\tilde{\omega} + K_1 S)^{\mathrm{T}}[f(t)L(\omega)\tilde{\zeta}\times\omega\\
&\qquad + f(t)L(K_1\dot{S} + \tilde{\omega}\times\omega - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}})\tilde{\zeta}\\
&\qquad + \dot{f}(t)L(\tfrac{1}{2}(\tilde{\omega} + K_1 S) - \omega)\tilde{\zeta}]\\
&\qquad - (\tilde{\omega} + K_1 S)^{\mathrm{T}} K_{\mathrm{v}}(\tilde{\omega} + K_1 S) - K_{\mathrm{p}} S^{\mathrm{T}} K_1 S\\
&\qquad - \tilde{\gamma}^{\mathrm{T}} Q\dot{\tilde{\gamma}} - \tilde{\zeta}^{\mathrm{T}} T\dot{\tilde{\zeta}} + \tilde{d}^{\mathrm{T}} C_d^{\mathrm{T}}(\tilde{\omega} + K_1 S)\\
&\qquad + \tilde{d}^{\mathrm{T}} D[A_d\tilde{d} - D^{-1} C_d^{\mathrm{T}}(\tilde{\omega} + K_1 S)]\\
&\quad = -(\tilde{\omega} + K_1 S)^{\mathrm{T}} K_{\mathrm{v}}(\tilde{\omega} + K_1 S) - K_{\mathrm{p}} S^{\mathrm{T}} K_1 S\\
&\qquad - \tilde{\gamma}^{\mathrm{T}} Q\dot{\tilde{\gamma}} + (\tilde{\omega} + K_1 S)^{\mathrm{T}}[-\omega^{\times} L(\omega)\\
&\qquad + L(K_1\dot{S} + \tilde{\omega}\times\omega - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}})]\tilde{\gamma} - \tilde{\zeta}^{\mathrm{T}} Z\dot{\tilde{\zeta}}\\
&\qquad + (\tilde{\omega} + K_1 S)^{\mathrm{T}}[-f(t)\omega^{\times} L(\omega) + f(t)L(K_1\dot{S}\\
&\qquad + \tilde{\omega}\times\omega - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}}) + \dot{f}(t)L(\tfrac{1}{2}(\tilde{\omega} + K_1 S)]\tilde{\zeta}\\
&\qquad + \tfrac{1}{2}\tilde{d}^{\mathrm{T}}(A_d^{\mathrm{T}} D + DA_d)\tilde{d}\\
&\quad = -(\tilde{\omega} + K_1 S)^{\mathrm{T}} K_{\mathrm{v}}(\tilde{\omega} + K_1 S) - K_{\mathrm{p}} S^{\mathrm{T}} K_1 S\\
&\qquad + \tilde{\gamma}^{\mathrm{T}}[-Q\dot{\tilde{\gamma}} + (L^{\mathrm{T}}(\omega)\omega^{\times} + L^{\mathrm{T}}(K_1\dot{S} + \tilde{\omega}\times\omega\\
&\qquad - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}}))(\tilde{\omega} + K_1 S)] + \tilde{\zeta}^{\mathrm{T}}[-Z\dot{\tilde{\zeta}}\\
&\qquad + (f(t)L^{\mathrm{T}}(\omega)\omega^{\times} + f(t)L^{\mathrm{T}}(K_1\dot{S} + \tilde{\omega}\times\omega\\
&\qquad - \tilde{R}^{\mathrm{T}}\dot{\omega}_{\mathrm{d}}))(\tilde{\omega} + K_1 S) + \dot{f}(t)L^{\mathrm{T}}(\tfrac{1}{2}(\tilde{\omega} + K_1 S)]\\
&\qquad + \tfrac{1}{2}\tilde{d}^{\mathrm{T}}(A_d^{\mathrm{T}} D + DA_d)\tilde{d}\\
&\quad = -(\tilde{\omega} + K_1 S)^{\mathrm{T}} K_{\mathrm{v}}(\tilde{\omega} + K_1 S) - K_{\mathrm{p}} S^{\mathrm{T}} K_1 S\\
&\qquad + \tfrac{1}{2}\tilde{d}^{\mathrm{T}}(A_d^{\mathrm{T}} D + DA_d)\tilde{d}.
\end{aligned}
$$

□

Note that a bound on $\dot{J}$ need not be known as in the case of for the motion-to-rest controller. This is due to the J_1 estimator [20] accounting for the extra term that appears in the derivative of [18] due to the spacecraft having a time-varying inertia matrix.

We simulate the spin maneuver using controller [24] on the non-rigid spacecraft with $J_1(t) = \frac{1}{10}\sin^2(2\pi t)J_0$, as before. We assume a constant nonzero disturbance torque, $\tau_{\mathrm{dist}} = [0.7 \quad -0.3 \quad 0]^{\mathrm{T}}$. The parameters of the controller are chosen to be $K_1 = I_3$, $A = \mathrm{diag}(1, 2, 3)$, $K_{\mathrm{p}} = \frac{1}{5}$, $K_{\mathrm{v}} = 10\, I_3$, $D = I_3$, and $Q = I_6$.

Figures 10-12 show, respectively, the attitude error, angular velocity components, and control torque components. The spacecraft attitude and angular velocity components are brought close to the desired values in under 10 sec. The modified controller [24] is able to reject the persistent disturbance caused by the non-rigidity of the spacecraft.

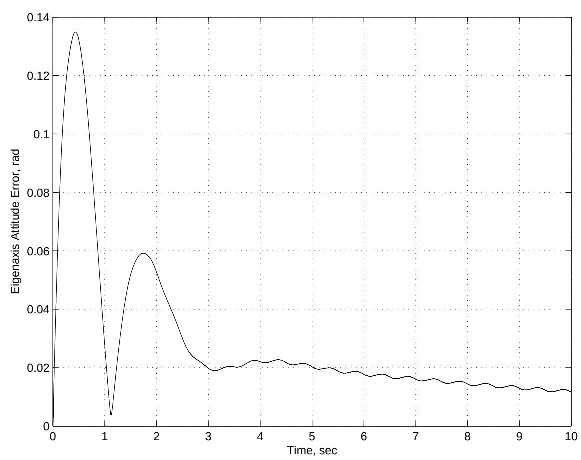

Fig. 10: Eigenaxis attitude error using the control law [24] for a spin maneuver during accordion-like motion.

VI. CONCLUSIONS

We extended the control laws of ref.[2] to the case of non-rigid spacecraft, without requiring knowledge of the spacecraft's time-varying inertia. These results have practical advantages relative to previous controllers that 1) require exact or approximate inertia information or 2) are based on attitude parameterizations such as quaternions that require discontinuous control laws. We simulated these controllers for various slew and spin maneuvers, demonstrating their effectiveness.

Future work will complete the proof for almost global stabilization (that is, Lyapunov stability with almost global convergence) of non-rigid spacecraft atti-

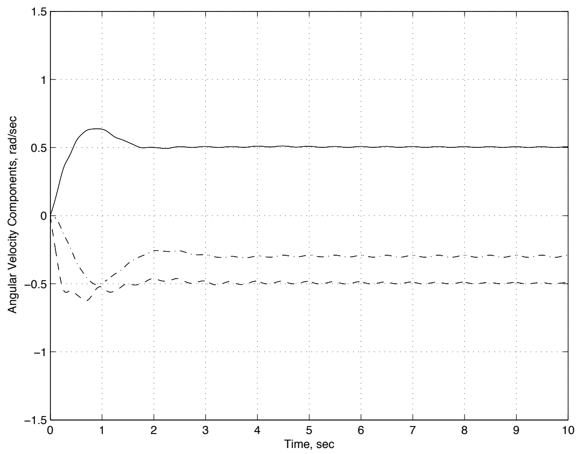

Fig. 11: Spacecraft angular-velocity components using the control law [24] for a spin maneuver during accordion-like motion.

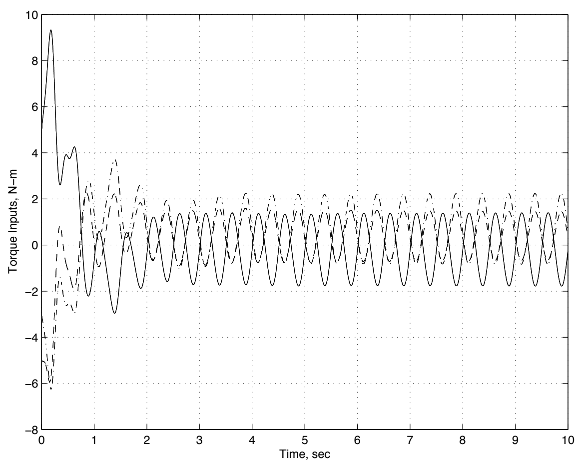

Fig. 12: Control torque components using the control law [24] for a spin maneuver during accordion-like motion.

tude tracking using controller [24]. Additionally, extensions of this method may be applicable to non-rigid spacecraft with moving components that are not necessarily translating relative to the spacecraft frame.

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