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Hybrid Retrospective-Cost-Based Adaptive Control Using Concurrent Parameter Estimation

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Abstract— **We present an adaptive control methodology that requires no plant modeling information. The method is based on a cumulative retrospective cost adaptive control algorithm, which is a direct adaptive control algorithm for stabilization, disturbance rejection, and command following when partial plant modeling information is available, specifically, the first nonzero Markov parameter, the relative degree, and estimates of nonminimum-phase zeros. The same adaptive algorithm is used online to estimate the required modeling information. By merging these processes into a single architecture, the resulting hybrid adaptive control algorithm requires no prior modeling information. The method is demonstrated on several illustrative disturbance rejection and command following problems, where the plant can be either minimum or nonminimum phase, and stable or unstable.**

I. Introduction

Although model-free control is possible in theory [1], practical considerations regarding transient response and the effect of noise generally require that some modeling information be known. If the adaptation procedure updates the controller gains directly based on model information that is known beforehand, then the adaptive control law is direct; if model information is learned online and the controller gains are updated based on the current model estimate, then the adaptive control law is indirect; and, finally, if online learning is used in support of adaptation, then the adaptive control law is hybrid. As expressed in [2], hybrid adaptive control entails the "deeper question", namely, "how much needs to be known (in order that an acceptable level of performance can be secured, during the learning phase and at the conclusion of learning)?"

In the present paper, we develop and illustrate a hybrid adaptive control law based on cumulative retrospective cost optimization. Direct adaptive control based on retrospective cost optimization [3–6] is a discrete-time approach to adaptive control based on identified Markov parameters. As shown in [4, 5] the Markov parameters capture the relative degree, sign of the high frequency gain, and nonminimum-phase zeros outside of the spectral radius of the plant. This approach does not depend on matching conditions and does not require any information about the poles of the system or the disturbance signal.

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To extend retrospective-cost-based adaptive control, Markov parameters can be learned online. This approach is demonstrated in [7], where a recursive-least-squares algorithm is used to update the Markov parameters based on closed-loop identification. Examples in [7] illustrate the ability to adapt to plant modifications in which a minimum-phase zero changes to a nonminimum-phase zero.

In the present paper, we develop an improved approach to hybrid retrospective-cost-based adaptive control in which the online learning is based on retrospective cost optimization. In particular, it is demonstrated in [8–10] that retrospective cost optimization provides a technique for updating a subsystem model, thereby providing the means for online model refinement. The updated subsystem can represent an unknown component of the overall system, or it can represent the entire system, where the latter case provides online model identification either with or without prior modeling information.

In the present paper, we use retrospective-cost model identification concurrently with direct retrospective-cost adaptive control. At each step, the direct retrospective-cost adaptive control algorithm uses estimates of the numerator polynomial needed for the controller update law. Simultaneously, the retrospective-cost model identification procedure uses data from the plant to estimate the numerator polynomial needed for the controller update law.

The resulting hybrid retrospective-cost-based adaptive control is based on an extended retrospective performance measure consisting of a cumulative sum of retrospective costs, as described in [6]. This extended measure, which provides improved transient response compared to [4, 5], is minimized by a recursive-least-squares algorithm, which may involve a forgetting factor. When abrupt plant changes occur, covariance resetting is used to restart the recursive minimization and thus the model updating.

II. Disturbance Rejection and Command Following

Consider the MIMO discrete-time system

$$x(k+1) = Ax(k) + Bu(k) + D_1 w(k), \tag{1}$$

$$y(k) = Cx(k) + D_2 w(k), \tag{2}$$

$$z(k) = E_1 x(k) + E_0 w(k), \tag{3}$$

where $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^{l_y}$, $z(k) \in \mathbb{R}^{l_z}$, $u(k) \in \mathbb{R}^{l_u}$, $w(k) \in \mathbb{R}^{l_w}$, and $k \geq 0$. Our goal is to develop an adaptive output feedback controller under which the performance variable z is minimized in the presence of the exogenous signal w. The block diagram for (1)-(3) is shown in Figure 1. Note that w can represent either a command signal to be followed, an external disturbance to be rejected, or both.

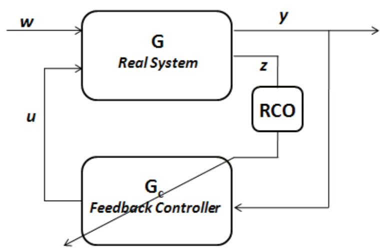

Fig. 1. Disturbance rejection and command following architecture.

For example, if $D_1 = 0$ and $E_0 \neq 0$, then the objective is to have the output $E_1 x$ follow the command signal $-E_0 w$. On the other hand, if $D_1 \neq 0$ and $E_0 = 0$, then the objective is to reject the disturbance w from the performance measurement $E_1 x$. The combined command following and disturbance rejection problem is addressed when D_1 and E_0 are block matrices. More precisely, if $D_1 = \begin{bmatrix} \hat{D}_1 & 0 \end{bmatrix}$, $E_0 = \begin{bmatrix} 0 & \hat{E}_0 \end{bmatrix}$, and $w(k) = \begin{bmatrix} w_1(k)^{\mathrm{T}} & w_2(k)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, then the objective is to have $E_1 x$ follow the command $-\hat{E}_0 w_2$ while rejecting the disturbance w_1. Lastly, if D_1 and E_0 are empty matrices, then the objective is output stabilization, that is, convergence of z to zero.

III. Cumulative Retrospective Cost Adaptive Controller

In this section, we review the cumulative retrospective cost adaptive control algorithm developed in [6]. Consider a strictly proper time-series controller of order n_{c}, such that the control $u(k)$ is given by

$$u(k) = \sum_{i=1}^{n_{\mathrm{c}}} M_i(k) u(k-i) + \sum_{i=1}^{n_{\mathrm{c}}} N_i(k) y(k-i), \tag{4}$$

where, for all $i = 1, \ldots, n_{\mathrm{c}}$, $M_i : \mathbb{N} \to \mathbb{R}^{l_u \times l_u}$ and $N_i : \mathbb{N} \to \mathbb{R}^{l_u \times l_y}$ are determined by the adaptive control law presented below. The control (4) can be expressed as

$$u(k) = \theta(k)\phi(k), \tag{5}$$

where

$$\theta(k) \triangleq \begin{bmatrix} N_1(k) & \cdots & N_{n_{\mathrm{c}}}(k) & M_1(k) & \cdots & M_{n_{\mathrm{c}}}(k) \end{bmatrix},$$

and

$$\begin{aligned}\phi(k) \triangleq \big[& \; y^{\mathrm{T}}(k-1) \quad \cdots \quad y^{\mathrm{T}}(k-n_{\mathrm{c}}) \\ & u^{\mathrm{T}}(k-1) \quad \cdots \quad u^{\mathrm{T}}(k-n_{\mathrm{c}}) \; \big]^{\mathrm{T}} \in \mathbb{R}^{n_{\mathrm{c}}(l_u+l_y)}.\end{aligned}$$

Next, we represent (1) and (3) as the time-series model from u and w to z given by

$$z(k) = \sum_{i=1}^{n} -\alpha_i z(k-i) + \sum_{i=d}^{n} \beta_i u(k-i) + \sum_{i=0}^{n} \gamma_i w(k-i), \tag{6}$$

where $\alpha_1, \ldots, \alpha_n \in \mathbb{R}$, $\beta_d, \ldots, \beta_n \in \mathbb{R}^{l_z \times l_u}$, $\gamma_0, \ldots, \gamma_n \in \mathbb{R}^{l_z \times l_w}$, and the relative degree d is the smallest non-negative integer i such that the ith Markov parameter, either $H_0 \triangleq E_2$ if $i = 0$ or $H_i \triangleq E_1 A^{i-1} B$ if $i > 0$, is nonzero. Note that $\beta_d = H_d$.

Next, we define the *retrospective performance*

$$\hat{z}(\hat{\theta}, k) \triangleq z(k) + \sum_{i=d}^{\nu} \bar{\beta}_i \left[\hat{\theta} - \theta(k-i)\right] \phi(k-i), \tag{7}$$

where $\nu \geq d$, $\hat{\theta} \in \mathbb{R}^{l_u \times (n_c(l_y+l_u))}$ is an optimization variable used to derive the adaptive law, and $\bar{\beta}_d, \ldots, \bar{\beta}_\nu \in \mathbb{R}^{l_z \times l_u}$. The parameters ν and $\bar{\beta}_d, \ldots, \bar{\beta}_\nu$ must capture the information included in the first nonzero Markov parameter and the nonminimum-phase zeros from u to z [6]. In this paper, we let $\bar{\beta}_d, \ldots, \bar{\beta}_\nu$ be the coefficients of the numerator polynomial matrix of the transfer function from u to z, that is, $\nu = n$ and, for $i = d, \ldots, n$, $\bar{\beta}_i \triangleq \beta_i$. For other choices of the parameters ν and $\bar{\beta}_d, \ldots, \bar{\beta}_\nu$, see [6].

Defining $\hat{\Theta} \triangleq \operatorname{vec} \hat{\theta} \in \mathbb{R}^{n_c l_u(l_y+l_u)}$ and $\Theta(k) \triangleq \operatorname{vec} \theta(k) \in \mathbb{R}^{n_c l_u(l_y+l_u)}$, it follows that

$$\begin{aligned}\hat{z}(\hat{\Theta}, k) &= z(k) + \sum_{i=d}^{\nu} \Phi_i^{\mathrm{T}}(k) \left[\hat{\Theta} - \Theta(k-i)\right] \\ &= z(k) - \sum_{i=d}^{\nu} \Phi_i^{\mathrm{T}}(k)\Theta(k-i) + \Psi^{\mathrm{T}}(k)\hat{\Theta},\end{aligned} \tag{8}$$

where, for $i = d, \ldots, \nu$, $\Phi_i(k) \triangleq \phi(k-i) \otimes \bar{\beta}_i^{\mathrm{T}} \in \mathbb{R}^{(n_c l_u(l_y+l_u)) \times l_z}$, where $\otimes$ represents the Kronecker product, and $\Psi(k) \triangleq \sum_{i=d}^{\nu} \Phi_i(k)$.

Now, define the *cumulative retrospective cost function*

$$\begin{aligned}J(\hat{\Theta}, k) \triangleq & \sum_{i=0}^{k} \lambda^{k-i} \hat{z}^{\mathrm{T}}(\hat{\Theta}, i) R \hat{z}(\hat{\Theta}, i) \\ & + \lambda^k \left(\hat{\Theta} - \Theta(0)\right)^{\mathrm{T}} Q \left(\hat{\Theta} - \Theta(0)\right),\end{aligned} \tag{9}$$

where $\lambda \in (0, 1]$, and $R \in \mathbb{R}^{l_z \times l_z}$ and $Q \in \mathbb{R}^{(n_c l_u(l_y+l_u)) \times (n_c l_u(l_y+l_u))}$ are positive definite. Note that λ serves as a forgetting factor, which allows more recent data to be weighted more heavily than past data.

The cumulative retrospective cost function (9) is minimized by a recursive least-squares (RLS) algorithm with a forgetting factor [11–13]. Therefore, $J(\hat{\Theta}, k)$ is minimized by the adaptive law

$$\Theta(k+1) = \Theta(k) - P(k)\Psi(k)\Omega(k)^{-1} z_{\mathrm{R}}(k), \tag{10}$$

$$P(k+1) = \frac{1}{\lambda} P(k) - \frac{1}{\lambda} P(k)\Psi(k)\Omega(k)^{-1}\Psi^{\mathrm{T}}(k)P(k), \tag{11}$$

where $\Omega(k) \triangleq \lambda R^{-1} + \Psi^{\mathrm{T}}(k)P(k)\Psi(k)$, $P(0) = Q^{-1}$, $\Theta(0) \in \mathbb{R}^{n_c l_u(l_y+l_u)}$, and the *retrospective performance measurement* $z_{\mathrm{R}}(k) \triangleq \hat{z}(\Theta(k), k)$. Note that the retrospective performance measurement is computable from (8) using measured signals z, y, u, θ, and the matrix coefficients $\bar{\beta}_d, \ldots, \bar{\beta}_\nu$. The cumulative retrospective cost adaptive control law is thus given by (10), (11), and

$$u(k) = \theta(k)\phi(k) = \operatorname{vec}^{-1}(\Theta(k))\phi(k). \tag{12}$$

The key feature of the adaptive control algorithm is the use of the retrospective performance (8), which modifies the performance variable $z(k)$ based on the difference between the actual past control inputs $u(k-d),\ldots,u(k-n)$ and the recomputed past control inputs $\hat{u}(\hat{\Theta},k-d) \stackrel{\triangle}{=} \text{vec}^{-1}(\hat{\Theta})\phi(k-d),\ldots,\hat{u}(\hat{\Theta},k-n) \stackrel{\triangle}{=} \text{vec}^{-1}(\hat{\Theta})\phi(k-n)$, assuming that the current controller $\hat{\Theta}$ had been used in the past.

Note, that the direct retrospective cost adaptive controller presented in this section requires knowledge of the coefficients $\beta_d,\ldots,\beta_n$. In the next section, we show how the algorithm presented in this section can be used for model identification as well as direct adaptive control.

IV. Retrospective-Cost Model Identification

To implement the direct adaptive control law presented in Section III, we require the sign of the high frequency gain, the relative degree, and the nonminimum phase zeros, which are captured by the numerator polynomial from u to z, given by

$$\beta(\mathbf{q}) \stackrel{\triangle}{=} \mathbf{q}^{n-d}\beta_d + \mathbf{q}^{n-d-1}\beta_{d+1} + \ldots + \mathbf{q}\beta_{n-1} + \beta_n. \tag{13}$$

These values can be obtained through system identification before implementing the control or from analytical models of the system such as discretized differential equations. In this section, we use the basic algorithm presented in Section III to estimate (13) from an identified model of

$$G_{zu}(\mathbf{q}) \stackrel{\triangle}{=} E_1[\mathbf{q}I - A]^{-1}B = \frac{1}{\alpha(\mathbf{q})}\beta(\mathbf{q}), \tag{14}$$

where $\alpha(\mathbf{q}) = \mathbf{q}^n + \alpha_1\mathbf{q}^{n-1} + \cdots + \alpha_{n-1}\mathbf{q} + \alpha_n$.

We seek to identify a model of (14) using a known initial model $G_0(\mathbf{q}) = \dfrac{1}{\alpha_0(\mathbf{q})}\beta_0(\mathbf{q})$, where $\beta_0(\mathbf{q}) \stackrel{\triangle}{=} \mathbf{q}^{n_0}\beta_{0,0} + \ldots + \mathbf{q}\beta_{0,n_0-1} + \beta_{0,n_0}$, and $\beta_{0,0},\ldots,\beta_{0,n_0} \in \mathbb{R}^{l_z\times l_{u_\Delta}}$, furthermore, $\alpha_0(\mathbf{q})$ is a monic polynomial of degree n_0. In general l_{u_Δ} is chosen to be equal to l_u.

The initial model is connected in feedback with an unknown but structured model of the uncertainty $[\Delta_z(\mathbf{q}) \quad \Delta_u(\mathbf{q})]$. The objective is to determine $[\Delta_z(\mathbf{q}) \quad \Delta_u(\mathbf{q})]$ such that the output of the closed-loop model $\hat{G}_{zu}(\mathbf{q}) \stackrel{\triangle}{=} [I - G_0(\mathbf{q})\Delta_z(\mathbf{q})]^{-1}[G_0(\mathbf{q})\Delta_u(\mathbf{q})]$, given by z_Δ is as close as possible to z. More precisely our objective is to minimize $e_z \stackrel{\triangle}{=} z - z_\Delta$.

To identify $[\Delta_z(\mathbf{q})\ \Delta_u(\mathbf{q})]$, we use the architecture shown in Figure 2, where we minimize the identification performance variable e_z, using the cumulative retrospective-cost-based direct adaptive control algorithm given in Section III.

First, let $\Delta_z(\mathbf{q},k)$ and $\Delta_u(\mathbf{q},k)$ be estimates of $\Delta_z(\mathbf{q})$ and $\Delta_u(\mathbf{q})$, respectively, attained at each time step k. Next we write $\Delta_z(\mathbf{q},k) = \alpha_\Delta^{-1}(\mathbf{q},k)\beta_z(\mathbf{q},k)$, $\Delta_u(\mathbf{q},k) = \alpha_\Delta^{-1}(\mathbf{q},k)\beta_u(\mathbf{q},k)$, where $\alpha_\Delta(\mathbf{q}) = \mathbf{q}^{n_\Delta} - \alpha_{\Delta,1}(k)\mathbf{q}^{n_\Delta-1} - \alpha_{\Delta,n_\Delta-1}(k)\mathbf{q} - \alpha_{\Delta,n_\Delta}(k)$, $\beta_z(\mathbf{q}) = \beta_{z,1}(k)\mathbf{q}^{n_\Delta-1} + \beta_{z,2}(k)\mathbf{q}^{n_\Delta-2} + \beta_{z,n_\Delta-1}(k)\mathbf{q} + \beta_{\Delta,n_\Delta}(k)$, $\beta_u(\mathbf{q}) = \beta_{u,1}(k)\mathbf{q}^{n_\Delta-1} + \beta_{u,2}(k)\mathbf{q}^{n_\Delta-2} + \beta_{u,n_\Delta-1}(k)\mathbf{q} + \beta_{\Delta,n_\Delta}(k)$, where n_Δ is the order of the identified model, $\alpha_{\Delta,1},\ldots,\alpha_{\Delta,n_\Delta} \in \mathbb{R}^{l_{u_\Delta}\times l_{u_\Delta}}$, $\beta_{z,1},\ldots,\beta_{z,n_\Delta} \in \mathbb{R}^{l_{u_\Delta}\times l_z}$, $\beta_{u,1},\ldots,\beta_{u,n_\Delta} \in \mathbb{R}^{l_{u_\Delta}\times l_u}$.

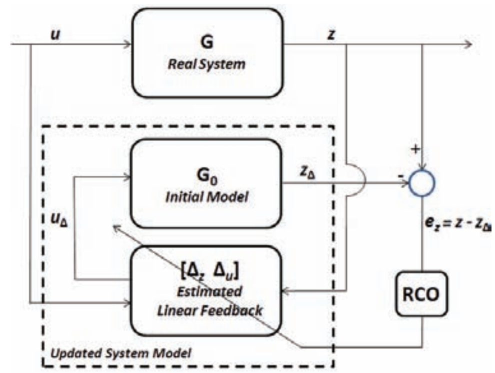

Fig. 2. Retrospective-cost model identification. The identified model resides in the dashed box. The diagonal arrow represents data-driven adaptation.

Next, consider the time-series representation of $[\Delta_z(\mathbf{q},k) \quad \Delta_u(\mathbf{q},k)]$ given by

$$u_\Delta(k) = \sum_{i=1}^{n_\Delta}\alpha_{\Delta,i}u_\Delta(k-i) + \sum_{i=1}^{n_\Delta}[\beta_{z,i} \quad \beta_{u,i}]\begin{bmatrix} z(k-i) \\ u(k-i) \end{bmatrix} \tag{15}$$

which can be expressed as $u_\Delta(k) = \theta_\Delta(k)\phi_\Delta(k)$, where

$$\begin{aligned}\theta_\Delta(k) \stackrel{\triangle}{=} [&\beta_{z,1}(k)\ldots\beta_{z,n_\Delta}(k) \quad \beta_{u,1}(k)\ldots \\ &\beta_{u,n_\Delta}(k) \quad \alpha_{\Delta,1}(k)\ldots\alpha_{\Delta,n_\Delta}(k)],\end{aligned}$$

and

$$\phi(k) \stackrel{\triangle}{=} \begin{bmatrix} z^{\mathrm{T}}(k-1) & \cdots & z^{\mathrm{T}}(k-n_\Delta) \\ u^{\mathrm{T}}(k-1) & \cdots & u^{\mathrm{T}}(k-n_\Delta) \\ u_\Delta^{\mathrm{T}}(k-1) & \cdots & u_\Delta^{\mathrm{T}}(k-n_\Delta) \end{bmatrix}^{\mathrm{T}},$$

where $\phi(k) \in \mathbb{R}^{n_\Delta(l_{u_\Delta}+l_u+l_z)}$. Next, we define the retrospective performance for model identification

$$\begin{aligned}\hat{e}_z(\hat{\theta}_\Delta,k) &\stackrel{\triangle}{=} e_z(k) + \sum_{i=1}^{n_0}\beta_{0,i}\left[\hat{\theta}_\Delta - \theta_\Delta(k-i)\right]\phi_\Delta(k-i) \\ &= e_z(k) - \sum_{i=1}^{n_0}\Phi_{\Delta,i}^{\mathrm{T}}(k)\Theta_\Delta(k-i) + \Psi_\Delta^{\mathrm{T}}(k)\hat{\Theta}_\Delta,\end{aligned}$$

where, for $i = 0,\ldots,n_0$, $\Phi_{\Delta,i}(k) \stackrel{\triangle}{=} \phi_\Delta(k-i)\otimes\beta_{0,i}^{\mathrm{T}}$, $\Psi_\Delta(k) \stackrel{\triangle}{=} \sum_{i=0}^{n_0}\Phi_{\Delta,i}(k)$, $\hat{\Theta}_\Delta \stackrel{\triangle}{=} \text{vec}(\hat{\theta}_\Delta)$, and $\Theta_\Delta(k) \stackrel{\triangle}{=} \text{vec}(\theta_\Delta(k))$.

Now, define the retrospective cost function for model identification by

$$\begin{aligned}J(\hat{\Theta}_\Delta,k) \stackrel{\triangle}{=} &\sum_{i=0}^{k}\lambda_\Delta^{k-i}\hat{e}_z^{\mathrm{T}}(\hat{\Theta}_\Delta,i)R_\Delta\hat{e}_z(\hat{\Theta}_\Delta,i) \\ &+\lambda_\Delta^k\left(\hat{\Theta}_\Delta - \Theta_\Delta(0)\right)^{\mathrm{T}}Q_\Delta\left(\hat{\Theta}_\Delta - \Theta_\Delta(0)\right),\end{aligned} \tag{16}$$

which is minimized by the recursive-least-squares algorithm

$$\Theta_\Delta(k+1) = \Theta_\Delta(k) - P_\Delta(k)\Psi_\Delta(k)\Omega_\Delta(k)^{-1}e_{\rm R}(k), \quad (17)$$

$$P_\Delta(k+1) = \frac{1}{\lambda_\Delta}P_\Delta(k) - \frac{1}{\lambda_\Delta}P_\Delta(k)\Psi_\Delta(k)\Omega_\Delta(k)^{-1}\Psi_\Delta^{\rm T}(k)P_\Delta(k), \quad (18)$$

where $\Omega_\Delta(k) \triangleq \lambda_\Delta R_\Delta^{-1} + \Psi_\Delta^{\rm T}(k)P_\Delta(k)\Psi_\Delta(k)$, $P_\Delta(0) = Q_\Delta^{-1}$, $\Theta_\Delta(0) \in \mathbb{R}^{n_\Delta l_{u_\Delta}(l_z+l_u+l_{u_\Delta})}$, and $e_{\rm R}(k) \triangleq \hat{e}_z(\Theta_\Delta(k),k)$.

Therefore, the retrospective cost model identification algorithm (17) and (18), yields at each time step, $\hat{G}_{zu}(\mathbf{q},k)$, which is an estimate of $G_{zu}(\mathbf{q})$ given by $\hat{G}_{zu}(\mathbf{q},k) \triangleq \hat{\alpha}^{-1}(\mathbf{q},k)\hat{\beta}(\mathbf{q},k)$, where $\hat{\alpha}(\mathbf{q},k) \triangleq \alpha_0(\mathbf{q})\alpha_\Delta(\mathbf{q},k) - \beta_0(\mathbf{q})\beta_z(\mathbf{q},k)$ and $\hat{\beta}(\mathbf{q},k) \triangleq \beta_0(\mathbf{q})\beta_u(\mathbf{q},k)$.

V. Hybrid Retrospective Adaptive Control

In Section III, we presented a direct adaptive control method to achieve disturbance rejection and command following when $\beta(\mathbf{q})$ is known. In Section IV, we presented a recursive model identification technique, which uses an initial known model $G_0(\mathbf{q})$ to identify the model $\hat{G}_{zu}(\mathbf{q},k)$, which estimates $G_{zu}(\mathbf{q})$, and thus provides, $\hat{\beta}(\mathbf{q},k)$ an estimate of $\beta(\mathbf{q})$.

In this section, we augment the disturbance rejection and command following architecture shown in Figure 1 with the model identification architecture presented in Figure 2. Thus, the plant parameters $\beta_d,\ldots,\beta_n$ can be estimated online while simultaneously implementing the control required to achieve disturbance rejection and command following. The augmented architecture is shown in Figure 3.

At each step, the hybrid method implements $\hat{\beta}(\mathbf{q},k)$, which is an estimate of $\beta(\mathbf{q})$. A control $u(k)$ is determined based on the adaptive law (10)-(12), while $u(k)$ and $z(k)$ are simultaneously used to identify $\hat{G}_{zu}(\mathbf{q},k)$.

Using the hybrid architecture in Figure 3, we weaken the requirement for prior estimates of nonminimum-phase zeros, high-frequency gain and relative degree. Note that the hybrid retrospective-cost adaptive control performs well as long as the retrospective-cost model identification algorithm converges more quickly than the direct retrospective-cost adaptive control algorithm. We can enforce this condition by choosing $P_\Delta(0)$ large and $P(0)$ small.

VI. Disturbance Rejection Examples

The goal in the following examples is to reject $w(k) \triangleq [w_1(k) \quad w_2(k)]^{\rm T}$, where, for $i=1,2$, $w_i(k) \triangleq A_i\sin(2\pi\omega_i T_{\rm s}k)$, where the amplitudes are $A_1 = 1$ and $A_2 = 5$; the frequencies are $\omega_1 = 5$ and $\omega_2 = 10$. The sample time $T_{\rm s}$ is 0.01. The disturbances enter the plant through D_1, which is randomly generated.

Example VI.I, $3^{\rm rd}$ Order, Stable, Minimum Phase:

In this example, we choose G to have poles $-0.8, 0.5, -0.02$ and a zero 0.3, which is stable and minimum phase. We assume that the initial model is

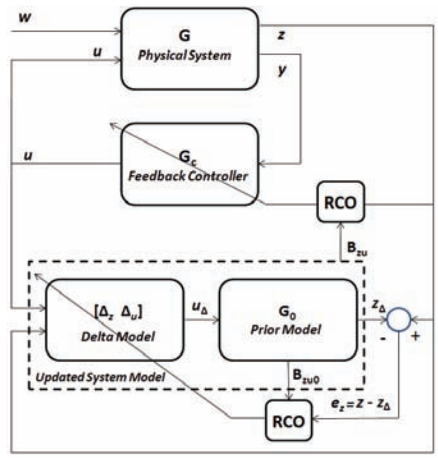

Fig. 3. The hybrid architecture is created by combining the direct retrospective-cost adaptive control and retrospective-cost model identification architectures.

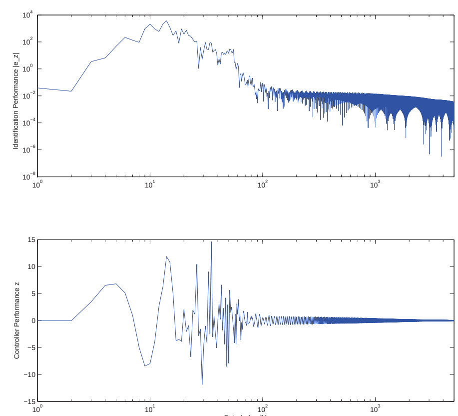

Fig. 4. Performance comparison. The upper plot shows the identification performance e_z. The lower plot shows the controller performance z.

$G_0 = \frac{1}{z}$, and we let $n_c = 15$, $P(0) = 0.01I_{30}$, $n_\Delta = 20$, and $P_\Delta(0) = 100I_{60}$. Figure 4 shows the performance of the identification loop and the controller loop. As shown in Figure 4, the identification performance e_z approaches zero and the controller performance z approaches zero. Figure 5 shows a frequency response comparison of the true system and the identified system after 1000 time steps. We note the peaks in the estimated frequency response, which are at the disturbance frequencies.

Example VI.II, $8^{\rm th}$ Order, Stable, Nonminimum Phase:

In this example, we choose G to have poles $-0.9, 0.9, -0.5 \pm 0.5\jmath, 0.5 \pm 0.5\jmath, \pm 0.7\jmath$ and zeros $1.5, 0.1, -0.7 \pm 0.3\jmath, 0.3 \pm 0.7\jmath$, which is stable and nonminimum phase. We assume that the initial model is $G_0 = \frac{1}{z}$, and we let $n_c = 15$, $P(0) = 0.01I_{30}$, $n_\Delta = 15$, and $P_\Delta(0) = 0.1I_{45}$. Figure 6 shows the performance of the identification loop and the controller loop. As shown in Figure 6, the identification performance e_z approaches zero and the controller performance z approaches zero.

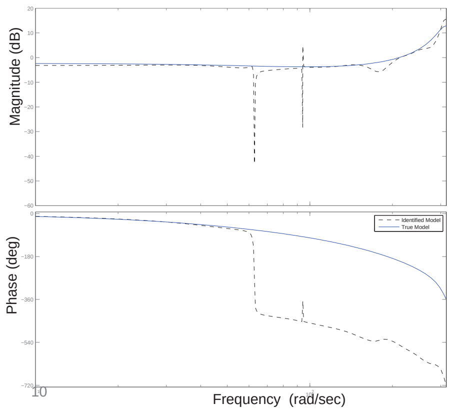

Fig. 5. Frequency response comparison of the true system G_{zu} and the estimated system $\hat{G}_{zu}(k)$, where $k = 1000$.

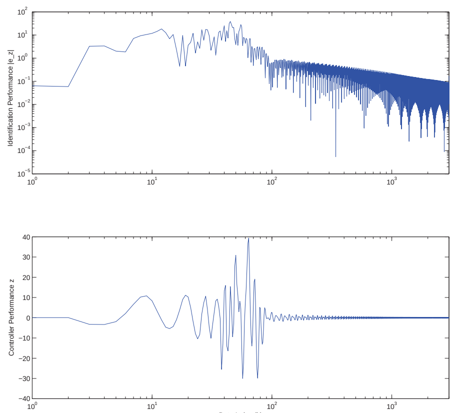

Fig. 6. Performance comparison. The upper plot shows the identification performance e_z. The lower plot shows the controller performance z.

Example VI.III, 8^{th} Order, Unstable, Nonminimum Phase:

In this example, we choose G to have poles $-1.04, 1.04, 0.1 \pm 1.0251\jmath, -0.5 \pm 0.5\jmath, 0.5 \pm 0.5\jmath$, and zeros $1.5, 0.1, -0.7 \pm 0.3\jmath, 0.3 \pm 0.7\jmath$, which is unstable and nonminimum phase. We assume that the initial model is $G_0 = \frac{1}{z}$, and we let $n_c = 15$, $P(0) = 0.01 I_{30}$, $n_\Delta = 15$, and $P_\Delta(0) = 0.1 I_{45}$. Figure 7 shows the performance of the identification loop and the controller loop. As shown in Figure 7, the identification performance e_z approaches zero and the controller performance z approaches zero.

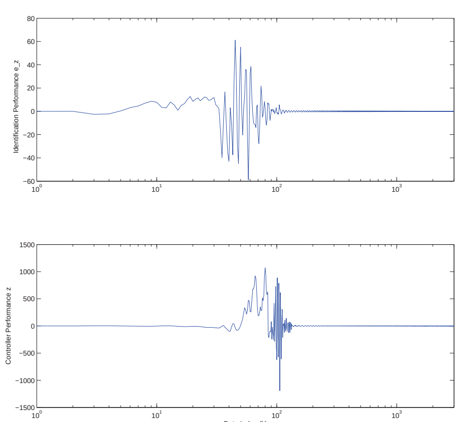

Fig. 7. Performance comparison. The upper plot shows the identification performance e_z. The lower plot shows the controller performance z.

VII. Command Following Examples

For the following examples $w(k) \triangleq [w_1(k) \quad w_2(k)]^{\mathrm{T}}$, where, for $i = 1, 2$, $w_1(k)$ is a command signal to be followed, where $E_0 = [1 \quad 0]$, and $w_2(k)$ is a disturbance to be rejected, specifically, $w_2(k) \triangleq 2\sin(2\pi 2 T_{\mathrm{s}} k)$, unless otherwise noted. The sample time T_{s} is 0.01. The disturbance enters the plant through $D_1 = \begin{bmatrix} 0_{n\times 1} & \hat{D}_1 \end{bmatrix}$, where $\hat{D}_1$ is randomly generated.

Example VII.I, 8^{th} Order, Stable, Nonminimum Phase:

In this example, we choose G to have poles $-0.9, 0.9, -0.5 \pm 0.5\jmath, 0.5 \pm 0.5\jmath, \pm 0.7\jmath$ and zeros $1.5, 0.1, -0.7 \pm 0.3\jmath, 0.3 \pm 0.7\jmath$, which is stable and nonminimum phase. The goal is to have the output y follow $w_1(k)$ which is a step command at $k = 50$. For this examples $w_2(k) = 0$.

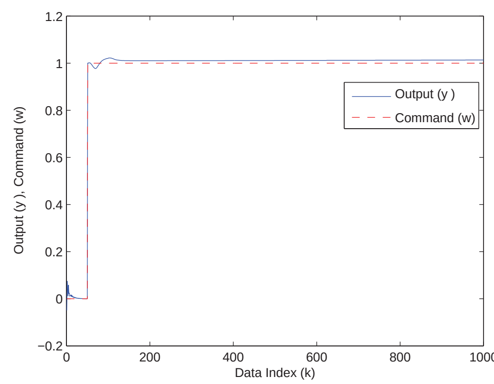

Fig. 8. Plant output for command following. The goal is to have the system trajectory follow a step command.

We assume that the initial model is $G_0 = \frac{1}{z}$, and we let $n_c = 15$, $P(0) = 0.1 I_{30}$, $n_\Delta = 20$, and $P_\Delta(0) = 10 I_{60}$. Figure 8 is a plot of the output y and the step command to be followed w. From Figure 8, the step is followed with a small transient.

Example VII.II, 8^{th} Order, Stable, Nonminimum Phase:

In this example, we choose G to have poles $-0.9, 0.9, -0.5 \pm 0.5\jmath, 0.5 \pm 0.5\jmath, \pm 0.7\jmath$ and zeros $1.5, 0.1, -0.7 \pm 0.3\jmath, 0.3 \pm 0.7\jmath$, which is stable and nonminimum phase. The goal is to have the output y following a step command at $k = 50$, while simultaneously rejecting a disturbance with amplitude 2 and frequency of 2 Hz.

We assume that the initial model is $G_0 = \frac{1}{z}$, and we let $n_c = 15$, $P(0) = 0.1 I_{30}$, $n_\Delta = 20$, and $P_\Delta(0) = 10 I_{60}$. Figure 9 is a plot of the y and the step command to be followed. Note that the adaptive controller is also rejecting the sinusoidal disturbance w_2.

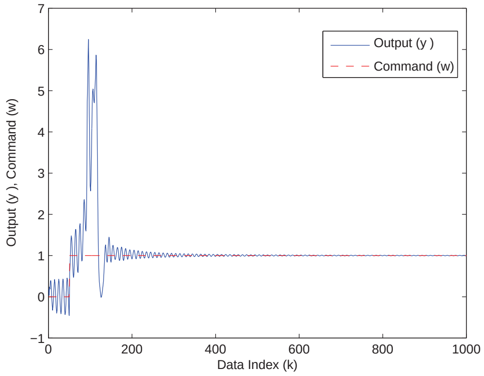

Fig. 9. Plant output for command following. The goal is to have the system trajectory follow a step command, while also rejecting a periodic disturbance.

Example VII.III, 8^{th} *Order, Stable, Nonminimum Phase*

In this example, we choose G to have poles $-0.9, 0.9, -0.5 \pm 0.5\jmath, 0.5 \pm 0.5\jmath, \pm 0.7\jmath$ and zeros $1.5, 0.1, -0.7 \pm 0.3\jmath, 0.3 \pm 0.7\jmath$, which is stable and nonminimum phase. The goal is to have the output y follow w_1 which is a sinusoidal signal with amplitude 1 and frequency 0.6 Hz. Furthermore, y must follow a step command at $k = 50$ that is

$$w_1(k) = \begin{cases} \sin(\pi 0.012k), & k < 50; \\ \sin(\pi 0.012k) + 1, & k \geq 50. \end{cases} \tag{19}$$

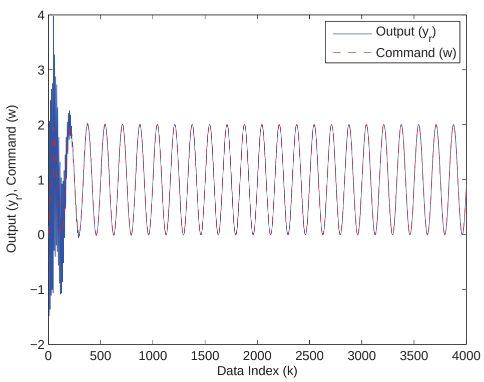

Fig. 10. Plant output for command following. The goal is to have the system trajectory follow a step command and a periodic signal, while also rejecting a periodic disturbance.

We assume that the initial model is $G_0 = \frac{1}{z}$, and we let $n_c = 15$, $P(0) = 0.1I_{30}$, $n_\Delta = 20$, and $P_\Delta(0) = 10I_{60}$. Figure 10 is a plot of the y and the command to be followed. Figure 11 shows the performance of the identification loop and the controller loop. As shown in Figure 4, the identification performance e_z approaches zero and the controller performance z approaches zero, indicating that the command has been effectively followed and the disturbances rejected.

VIII. Conclusions

In this paper, we presented an adaptive control architecture that requires no prior model information. We achieved this model-free control architecture by combining a cumulative retrospective cost direct adaptive control algorithm, with an online model estimation technique that also uses a cumulative retrospective cost algorithm. More specifically, the online retrospective cost identification estimates the numerator polynomial of the plant from the control to the performance. These estimates are then used by the direct adaptive control algorithm. The method was demonstrated on several illustrative disturbance rejection and command following problems, where the plant was either minimum or nonminimum phase, and stable or unstable. Future work includes convergence analysis of the hybrid algorithm.

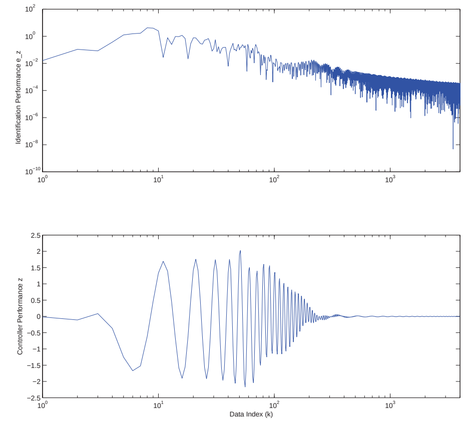

Fig. 11. Performance comparison. The upper plot shows the identification performance e_z. The lower plot shows the controller performance z.

References

[1] A. Ilchmann, *Non-Identifier-Based High-Gain Adaptive Controls*. Springer, 1993.

[2] B. D. O. Anderson, “Topical problems of adaptive control,” in *Proc. European Contr. Conf.*, Kos, Greece, July 2007, pp. 4997–4998.

[3] R. Venugopal and D. S. Bernstein, “Adaptive disturbance rejection using ARMARKOV/Toeplitz models,” *IEEE Trans. Contr. Sys. Tech.*, vol. 8, pp. 257–269, 2000.

[4] M. A. Santillo and D. S. Bernstein, “Adaptive control based on retrospective cost optimization,” *AIAA J. Guid. Contr. Dyn.*, vol. 33, pp. 289–304, 2010.

[5] M. S. Holzel, M. A. Santillo, J. B. Hoagg, and D. S. Bernstein, “Adaptive control of the nasa generic transport model using retrospective cost optimization,” in *Proc. AIAA Guid. Nav. Contr. Conf.*, Chicago, IL, August 2009, pp. AIAA–2009–5616.

[6] J. B. Hoagg and D. S. Bernstein, “Cumulative retrospective cost adaptive control with rls-based optimization,” in *Proc. Amer. Contr. Conf.*, Baltimore, MD, June 2010.

[7] M. A. Santillo, M. S. Holzel, J. B. Hoagg, and D. S. Bernstein, “Adaptive control using retrospective cost optimization with rls-based estimation for concurrent markov–parameter updating,” in *Proc. Conf. Dec. Contr.*, Shanghai, China, August 2009, pp. 3466–3471.

[8] H. J. Palanthandalam-Madapusi, E. L. Renk, and D. S. Bernstein, “Data-based model refinement for linear and hammerstein systems using subspace identification and adaptive disturbance rejection,” in *Proc. Conf. Cont. App.*, Toronto, Canada, August 2005, pp. 1630–1635.

[9] M. A. Santillo, A. M. D’Amato, and D. S. Bernstein, “System identification using a retrospective correction filter for adaptive feedback model updating,” in *Proc. Amer. Contr. Conf.*, St. Louis, MO, June 2009, pp. 4392–4397.

[10] A. M. D’Amato, A. Brzezinski, M. S. Holzel, J. Ni, and D. S. Bernstein, “Sensor–only noncausal blind identification of pseudo transfer functions,” in *Proc. SYSID*, Saint–Malo, July 2009, pp. 1698–1703.

[11] G. C. Goodwin and K. S. Sin, *Adaptive Filtering, Prediction, and Control*. Prentice Hall, 1984.

[12] K. J. Åström and B. Wittenmark, *Adaptive Control*, 2nd ed. Addison-Wesley, 1995.

[13] G. Tao, *Adaptive Control Design and Analysis*. Wiley, 2003.