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Forward-Integration Riccati-Based Feedback Control of Magnetically Actuated Spacecraft

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We apply the forward-integration Riccati-based feedback controller developed in prior work to a magnetically actuated spacecraft for the cases of both inertial and nadir pointing. The spacecraft is assumed to be in low-Earth orbit and actuated by only three orthogonal electromagnetic actuators. We assume no advance knowledge of the magnetic field, and thus make no periodicity assumptions, instead relying only on measurements that are available at the current time. We simulate the spacecraft attitude with actuator saturation, noisy magnetic measurements, and without rate feedback. The simulations are based on the International Geomagnetic Reference Field model of the magnetic field.

I. Introduction

Satellites in low-Earth orbit (LEO) can take advantage of the Earth's magnetic field for attitude control.[1,2] In particular, magnetic actuation is an elegant way to change the total angular momentum of a spacecraft without using mass ejection, such as thrusters. Consequently, magnetic actuation can reduce or remove the need for fuel. For small spacecraft, the benefits of magnetic actuation include cost, power, weight, and spatial efficiency.

The challenging aspect of magnetic actuation is that the torque produced on the spacecraft lies in the plane that is perpendicular to the local direction of Earth's geomagnetic field. The spacecraft is thus, at each moment in time, underactuated. Nevertheless, Earth's geomagnetic field is sufficiently varying in time and space that, for orbits not coinciding with Earth's magnetic equator (when using a nonrotating dipole model of the geomagnetic field), the spacecraft is fully controllable.[3]

Magnetic attitude control has been studied,[4] and various techniques have been developed for both linear and nonlinear problem formulations. Periodic approximations of the time-variation of the geomagnetic field are considered in refs.,[5–7] a model predictive controller is developed in ref.,[4] and Lyapunov methods are applied in ref.[8]

An additional challenge in magnetic actuation, is the fact that the magnitude and direction of the local geomagnetic field may be uncertain. Although the geomagnetic field is modeled and updated periodically,[9] these models have limited accuracy, and forecasts of the geomagnetic field may be erroneous due to unmodeled effects and unpredictable disturbances.[10] Consequently, it is desirable to develop control techniques for magnetic actuation that rely solely on current, on-board measurements of the geomagnetic field.

To address this need, we apply the forward-integrating Riccati-based (FIR) linear time-varying feedback controller developed in ref.[11] to spacecraft magnetic actuation. FIR control is a technique for stabilizing linear time-varying systems without the need for knowing the dynamics in advance. As in ref.,[5] the controller uses a linear time-varying model of the dynamics, but makes no periodicity assumptions, which are accurate only to first order. Since FIR feedback requires knowledge of only the current magnetic field, this approach obviates the need for advance knowledge of the geomagnetic field and thus does not rely on either geomagnetic approximations or forecasts. The controller given in ref.[8] also has this feature, and similarly works based on a measurement of the geomagnetic field at the current time.

In the present paper we show through simulation that the FIR controller is robust to realistic error sources, including the nonlinearities of attitude control that are not captured by the linearized model, actuator

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saturation, and noisy magnetic measurements. Finally, we consider an output feedback configuration, where angular rate measurements are not available.

The paper is organized as follows. Section II describes the FIR controller. In section III, we present a model for a spacecraft with three magnetic actuators. In section IV we give simulation results that highlight the effectiveness of the forward Riccati controller. Finally, in section V we discuss future refinements of our method.

II. Forward-Integrating Riccati-based (FIR) Control

In ref.,[11] we analyzed a Riccati-based controller for stabilizing a class of linear time-varying systems. Unlike standard, backward-integrating Riccati-based controllers, the approach of ref.[11] integrates a Riccati equation forward in time. As such, the controller does not require advance knowledge of the system dynamics, and thus is applicable to magnetically actuated spacecraft, where the local magnetic field can be measured on board but is not known in advance.

The forward-integrating Riccati (FIR) controller acts on the linear system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \tag{1}$$

and takes the form

$$u(t) = -R_2^{-1}B^{\mathrm{T}}(t)P_{\mathrm{f}}(t)x(t), \tag{2}$$

where $P_{\mathrm{f}}(t)$ is the solution to the *forward-in-time* control Riccati differential equation

$$\dot{P}_{\mathrm{f}}(t) = A^{\mathrm{T}}(t)P_{\mathrm{f}}(t) + P_{\mathrm{f}}(t)A(t) - P_{\mathrm{f}}(t)B(t)R_2^{-1}B^{\mathrm{T}}(t)P_{\mathrm{f}}(t) + R_1, \tag{3}$$

with the *initial-time* boundary condition $P_{\mathrm{f}}(t_0) \geq 0$.

In ref.[11] it is shown that, if the closed-loop dynamics matrix is symmetric, then the FIR controller is asymptotically stabilizing. We also showed, using averaging theory, that, in the case of periodically time-varying systems, and under suitable assumptions, there exists a period below which the dynamics of the closed-loop system are asymptotically stable. In other words, closed-loop stability is guaranteed for systems with time-varying dynamics of sufficiently high frequency. Note that tuning the FIR controller is similar to tuning LQR, namely, by adjusting the relative weighting matrices R_1 and R_2.

In this paper, we apply the FIR controller (2)-(3) to the spacecraft attitude control problem with magnetic actuation. The system model is time-varying due to the time-varying nature of the magnetic field that the spacecraft experiences as it moves through an orbit. The FIR controller enables us to control the spacecraft without knowing the magnetic field in advance. Even though the spacecraft model may not satisfy the sufficient conditions of ref.,[11] we show through numerical experiments that the controller is stabilizing and provides good performance.

III. Spacecraft Model, Assumptions, and Control Objectives

As a spacecraft model, we consider a single rigid body controlled by three magnetic torque devices, and without on-board momentum storage. We assume that a body-fixed frame is defined for the spacecraft, whose origin is chosen to be the center of mass, and that an inertial frame is specified for determining the attitude of the spacecraft. The spacecraft equations of motion are given by Euler's equation and Poisson's equation,

$$J\dot{\omega}(t) = J\omega(t) \times \omega(t) + T_{\mathrm{m}}(t), \tag{4}$$

$$\dot{R}(t) = -\omega^{\times}(t)R(t), \tag{5}$$

where $\omega(t) \in \mathbb{R}^3$ is the angular velocity of the spacecraft frame with respect to the inertial frame resolved in the spacecraft frame, $\omega^{\times}(t)$ is the cross-product matrix of $\omega(t)$, $J \in \mathbb{R}^{3\times 3}$ is the constant, positive-definite inertia matrix of the spacecraft, that is, the inertia dyadic of the spacecraft relative to the spacecraft center of mass resolved in the spacecraft frame, and $R(t) \in \mathbb{R}^{3\times 3}$ is the rotation dyadic that transforms the spacecraft frame into the inertial frame resolved in the spacecraft frame. Therefore, $R(t)$ is the proper orthogonal matrix (that is, the rotation matrix) that transforms the components of a vector resolved in the inertial frame into the components of the same vector resolved in the spacecraft frame.

The vector $T_{\rm m}(t) \in \mathbb{R}^3$ represents the torque on the spacecraft generated by the magnetic actuators, and can be written as[12]

$$T_{\rm m}(t) = u(t) \times b(t) = -b^{\times}(t)u(t), \tag{6}$$

where $b(t) = [b_x(t) \quad b_y(t) \quad b_z(t)]^{\rm T}$ is Earth's geomagnetic field measured in teslas (T) and resolved in the body frame, and $u(t)$ is the magnetic dipole moment generated by the currents in the magnetic actuators measured in ampere-square meters (A-m^2). For a discussion on generating magnetic dipole moments from magnetic torquer rods see ref.[16]

Both inertial-rate and attitude measurements are assumed to be available. Gyro measurements provide measurements of the angular velocity resolved in the spacecraft frame. Attitude is measured indirectly using sensors such as magnetometers or star trackers. When attitude measurements are given in terms of an alternative representation, such as quaternions, Rodrigues's formula can be used to determine the corresponding rotation matrix. Attitude estimation on SO(3) is considered in ref.[14]

The general objective of the attitude control problem is to determine control inputs such that the spacecraft attitude given by R follows a commanded attitude trajectory given by a possibly time-varying continuously differentiable rotation matrix $R_{\rm d}(t)$. For $t \geq 0$, $R_{\rm d}(t)$ is given by

$$\dot{R}_{\rm d}(t) = -\omega_{\rm d}(t)^{\times} R_{\rm d}(t), \tag{7}$$

$$R_{\rm d}(0) = R_{\rm d0}, \tag{8}$$

where $\omega_{\rm d}$ is the desired, possibly time-varying angular velocity. For the case of magnetic attitude control, we consider both inertial pointing with fixed $R_{\rm d}$, and nadir pointing on circular orbits with a time-varying $R_{\rm d}(t)$ that corresponds to the local vertical/local horizontal (LVLH) frame.

The error between $R(t)$ and $R_{\rm d}(t)$ is given in terms of the attitude-error rotation matrix

$$\tilde{R} \triangleq R^{\rm T} R_{\rm d}. \tag{9}$$

A scalar measure of attitude error is given by the rotation angle $\theta(t)$ about the eigenaxis needed to rotate the spacecraft from its attitude $R(t)$ to the desired attitude $R_{\rm d}(t)$. This angle, called the eigenaxis attitude error, is given by[15]

$$\theta(t) = \cos^{-1}(\tfrac{1}{2}[\operatorname{tr} \tilde{R}(t) - 1]). \tag{10}$$

In order to use the FIR controller (2)-(3), we linearize the equations of motion (4)-(5) about an equilibrium that, depending on the control objective, corresponds to either inertial pointing or Earth (nadir) pointing. These linearizations yield the system

$$\dot{x}(t) = Ax(t) + B(t)u(t), \tag{11}$$

where $x(t) = \left[\begin{array}{cc} \zeta^{\rm T}(t) & \delta\omega^{\rm T}(t) \end{array}\right]^{\rm T}$, $\zeta = \left[\begin{array}{ccc} \phi & \theta & \psi \end{array}\right]^{\rm T} \in \mathbb{R}^3$ represents the spacecraft's 3-2-1 Euler angles relative to the inertial frame for inertial pointing and relative to the LVLH frame for nadir pointing; $\delta\omega \in \mathbb{R}^3$ is the angular velocity of the spacecraft relative to the inertial frame for inertial pointing and relative to the LVLH frame for nadir pointing, that is, for the inertial pointing linearization, $\delta\omega = \omega$ since the equilibrium point has zero angular velocity, and, for the nadir pointing linearization, $\delta\omega$ is a perturbation about the nominal Earth-pointing angular velocity. Furthermore,

$$B(t) = \left[\begin{array}{c} 0 \\ -J^{-1}b^{\times}(t) \end{array}\right] \in \mathbb{R}^{6\times 3},$$

$A = A_{\rm inertial}$ for the inertial pointing linearization, and is given by the upper block-triangular matrix

$$A_{\rm inertial} = \left[\begin{array}{cc} 0 & I_3 \\ 0 & 0 \end{array}\right] \in \mathbb{R}^{6\times 6},$$

$A = A_{\text{nadir}}$ for the nadir pointing linearization, and is given by

$$A_{\text{nadir}} = \begin{bmatrix} n_v^{\times} & I_3 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{6\times 6},$$

where $n_v = \begin{bmatrix} 0 & n & 0 \end{bmatrix}$, and n is the mean motion, that is, the angular rate of the circular orbit.

III.A. Euler Angles from a Rotation Matrix

In order to implement the FIR controller (2)-(3) in a nonlinear simulation (4)-(5), we convert the attitude-error rotation matrix $\tilde{R}$ into Euler angles. Algorithm 1 is a method to resolve the singularities that arise from this mapping, and is adapted from ref.[17] for the case of 3-2-1 Euler angles.

Algorithm 1 Pseudocode for calculating 3-2-1 Euler angles from the attitude-error rotation matrix.

if $\tilde{R}_{13} \neq \pm 1$ **then**
  $\theta_1 = -\arcsin(\tilde{R}_{13})$
  $\psi_1 = \text{atan2}(\frac{\tilde{R}_{12}}{\cos(\theta_1)}, \frac{\tilde{R}_{11}}{\cos(\theta_1)})$
  $\phi_1 = \text{atan2}(\frac{\tilde{R}_{23}}{\cos(\theta_1)}, \frac{\tilde{R}_{33}}{\cos(\theta_1)})$

  # Comment: second set of Euler angles
  $\theta_2 = \pi - \theta_1$
  $\psi_2 = \text{atan2}(\frac{\tilde{R}_{12}}{\cos(\theta_2)}, \frac{\tilde{R}_{11}}{\cos(\theta_2)})$
  $\phi_2 = \text{atan2}(\frac{\tilde{R}_{23}}{\cos(\theta_2)}, \frac{\tilde{R}_{33}}{\cos(\theta_2)})$
else
  $\phi =$ anything; can set to 0
  if $\tilde{R}_{13} = -1$ **then**
    $\theta = \frac{\pi}{2}$
    $\psi = \phi + \text{atan2}(\tilde{R}_{32}, \tilde{R}_{31})$
  else
    $\theta = -\frac{\pi}{2}$
    $\psi = -\phi + \text{atan2}(-\tilde{R}_{32}, -\tilde{R}_{31})$
  end if
end if

Note that there exist multiple solutions for the sequence of Euler angle rotations that represent a given attitude orientation. In our simulations we set $\zeta(t) = \begin{bmatrix} \phi_1 & \theta_1 & \psi_1 \end{bmatrix}^{\mathrm{T}}$ if $\tilde{R}_{13}(t) \neq \pm 1$; otherwise we set $\phi = 0$ and proceed according to Algorithm 1.

IV. Numerical Studies

We consider a spacecraft in a 450-km circular orbit above the Earth with an inclination of 87 degrees. The International Geomagnetic Reference Field (IGRF) model is used to simulate Earth's geomagnetic field as a function of orbital position.[9] The spacecraft inertia matrix J is given by

$$J = \begin{bmatrix} 5 & -0.1 & -0.5 \\ -0.1 & 2 & 1 \\ -0.5 & 1 & 3.5 \end{bmatrix} \text{kg-m}^2, \tag{12}$$

with principal moments of inertia equal to 1.4947, 3.7997, and 5.2056 kg-m^2. We stress that, although the FIR controller uses a linearized model, all closed-loop simulations are fully nonlinear.

IV.A. Rest-to-Rest Maneuver

We use the FIR controller (2)-(3) for a rest-to-rest (slew) maneuver, where the objective is to bring the spacecraft from the initial attitude

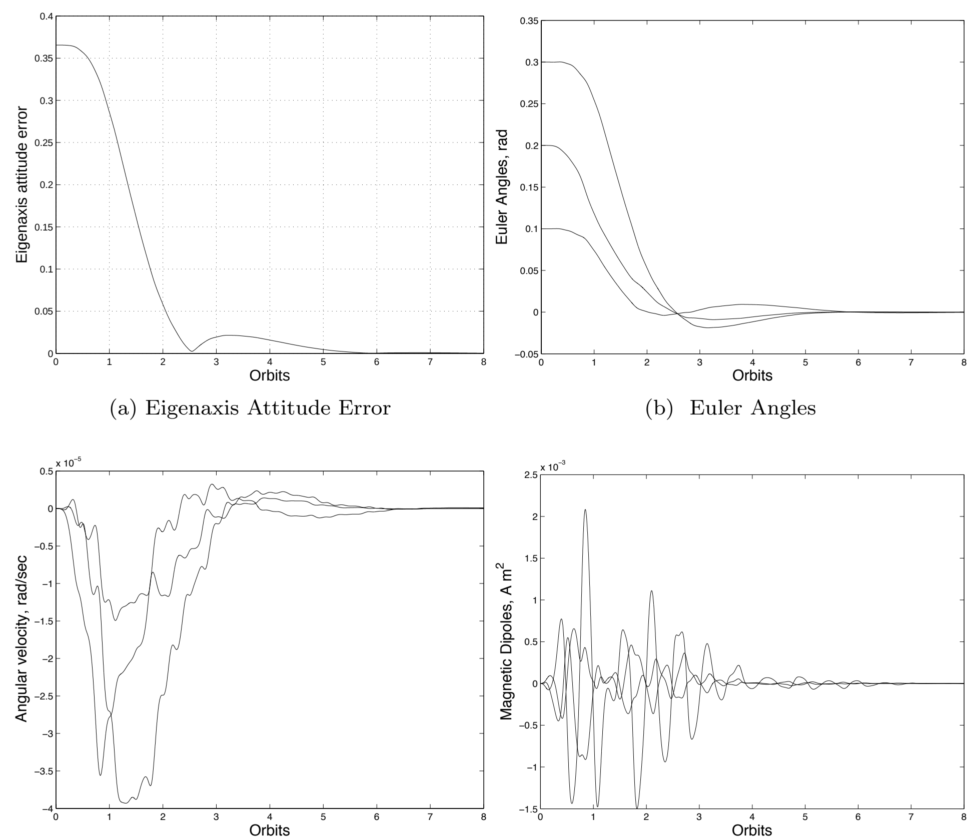
$$R(0) = \begin{bmatrix} 0.097 & 0.349 & -0.932 \\ 0.973 & -0.230 & 0.015 \\ -0.209 & -0.908 & -0.362 \end{bmatrix}, \tag{13}$$

which corresponds to the 3-2-1 Euler angles

$$\zeta(0) = \begin{bmatrix} 0.1 & 0.2 & 0.3 \end{bmatrix}^{\mathrm{T}} \text{ rad},$$

with zero initial angular velocity $\omega(0) = \delta\omega(0) = 0$, to rest at the desired final orientation $R_{\mathrm{d}} = I_3$, $\zeta = 0$. Let the parameters of the FIR controller (2)-(3) be given by $R_1 = I_6$, $R_2^{-1} = 0.0001$, and $P_f(0) = I_6$. These values were tuned to give nominal magnetic dipole moments around 2×10^{-3} A-m^2, which is about an order of magnitude larger than the residual dipole moment of a typical nanosatellite,[13] and a settling time of around 8 orbits. We test the controller in a nonlinear simulation of (4)-(5).

Figure 1 shows the eigenaxis attitude error, Euler angles, angular velocity, and magnetic dipole moment for the simulation described above. The spacecraft comes to rest at the commanded attitude within 7 orbits. The maximum magnetic dipole moment generated is less than 3×10^{-3} A-m^2. This quantity can be further tuned by modifying the weights R_1 and R_2.



(a) Eigenaxis Attitude Error (b) Euler Angles

(c) Angular Velocity (d) Magnetic Dipole Moments

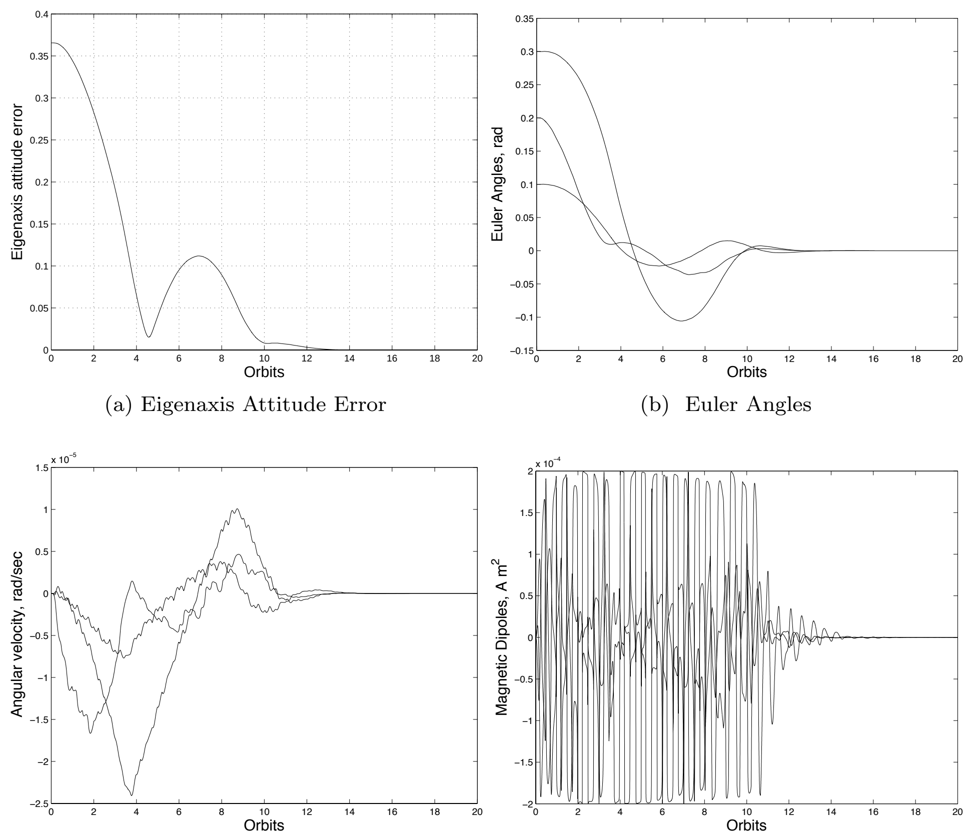
Figure 1: Full-state feedback for the rest-to-rest maneuver. (a) Eigenaxis Attitude Error, (b) Euler Angles, (c) Angular Velocity, (d) Magnetic Dipole Moments. The spacecraft comes to rest at the commanded attitude within 7 orbits, and the maximum magnetic dipole moment required by the controller is less than 3×10^{-3} A-m^2.

IV.A.1. Actuator Saturation

We now illustrate actuator-saturation handling. Let $u_{\max} = 2 \times 10^{-4}$ A-m^2 be the saturation limit on the magnetic dipole moments, which is about an order of magnitude less than the nominal controller tuning. If the controller specifies a magnetic dipole moment larger than $u_{\max}$, we apply the saturation as the vector scaling

$$u_{\text{sat}}(t) = u_{\max} \frac{u(t)}{||u(t)||}. \tag{14}$$

Figure 2 shows the eigenaxis attitude error, Euler angles, angular velocity, and magnetic dipole moment. The spacecraft comes to rest at the commanded attitude within 12 orbits. The magnetic dipole moment is saturated at 2×10^{-4} A-m^2.



(a) Eigenaxis Attitude Error (b) Euler Angles

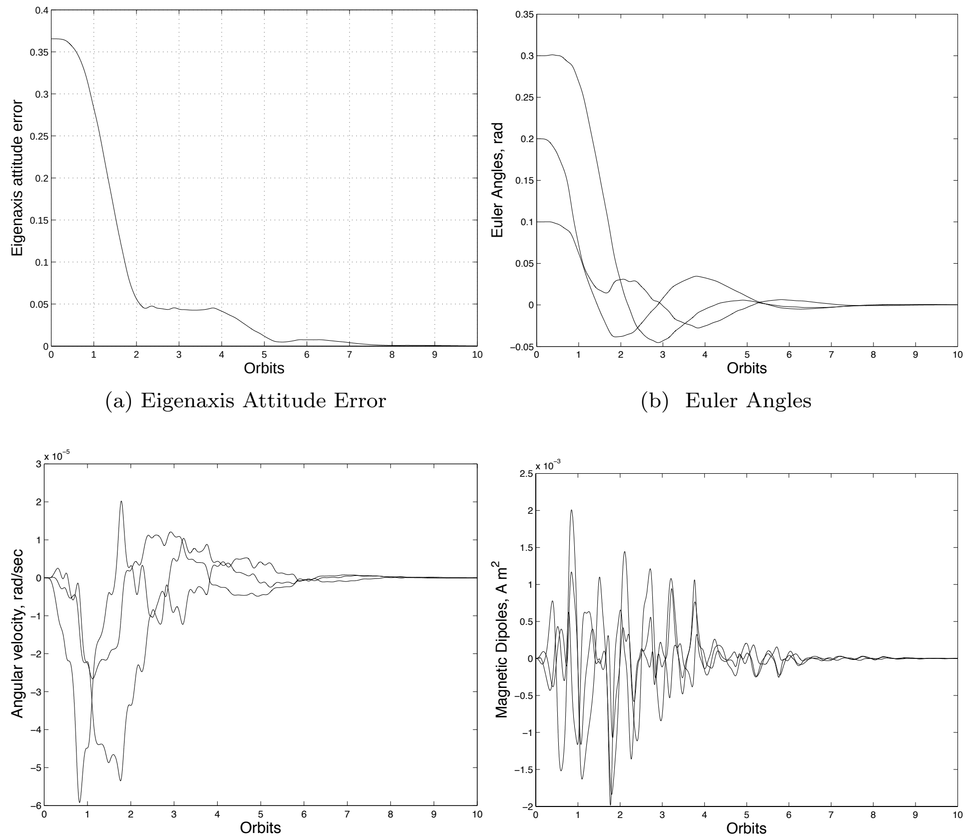
(c) Angular Velocity (d) Magnetic Dipole Moments

Figure 2: Full-state feedback with magnetic dipole moment saturation of 2×10^{-4} A-m^2 for the rest-to-rest maneuver. (a) Eigenaxis Attitude Error (b) Euler Angles, (c) Angular Velocity, (d) Magnetic Dipole Moments. The spacecraft comes to rest at the commanded attitude within 16 orbits, and the maximum magnetic dipole moment is less than 2×10^{-4} A-m^2.

IV.A.2. Noisy Magnetic Field Measurement

We now consider the effects of noisy and biased magnetometer measurements. In the controller (2)-(3), we replace $-b^{\times}(t)$ with $-(R_n(\alpha)b(t) + m)^{\times}$, where $R_n(\alpha) = e^{\alpha n^{\times}}$ is a rotation matrix that rotates the magnetic field measurement by an angle α around axis n, and m is random additive noise. Let $\alpha = 45°$, let $n = [-0.868 \quad 0.420 \quad 0.266]^{\mathrm{T}}$, and let m be normally distributed with zero mean and standard deviation 10^{-5} T, which is roughly one order of magnitude less than the nominal magnetic field strength. For a detailed discussion on magnetometer bias determination and calibration see ref.[18]

Figure 3 shows the eigenaxis attitude error, Euler angles, angular velocity, and magnetic dipole moment. The spacecraft comes to rest at the commanded attitude within 9 orbits, demonstrating that the controller is forgiving to large errors in the magnetic-field measurement.



(a) Eigenaxis Attitude Error (b) Euler Angles

(c) Angular Velocity (d) Magnetic Dipole Moments

Figure 3: Full-state feedback with noisy magnetic field measurements for the rest-to-rest maneuver. The measurements are off by 45° and corrupted by gaussian noise. (a) Eigenaxis Attitude Error (b) Euler Angles, (c) Angular Velocity, (d) Magnetic Dipole Moments. The spacecraft comes to rest at the commanded attitude within 9 orbits, and the maximum magnetic dipole moment is less than 3×10^{-3} A-m^2.

IV.A.3. Output Feedback

We now consider the situation where the full-state measurement is not available. In particular, we assume that we have measurements of only the attitude, that is,

$$C(t) = \begin{bmatrix} I_3 & 0 \end{bmatrix} \in \mathbb{R}^{3\times 6}.$$

We consider the observer-based dynamic compensator[11]

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t) + B(t)u(t) + F(t)\Big(y(t) - C(t)\hat{x}(t)\Big), \tag{15}$$

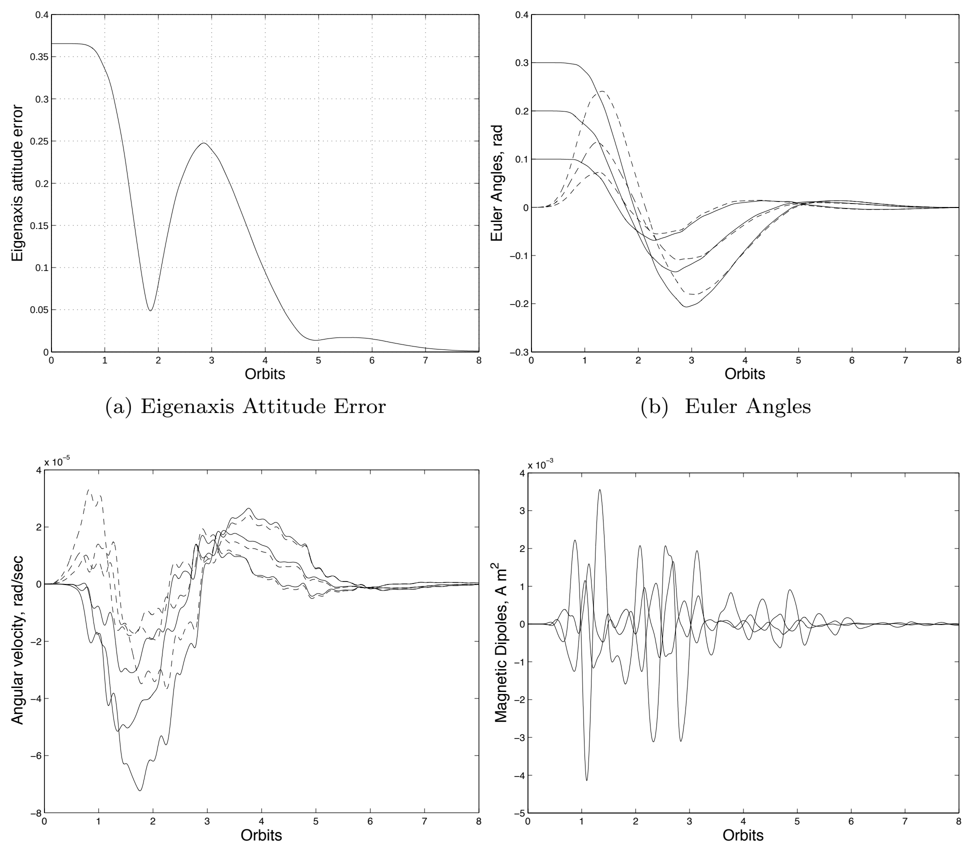
$$u(t) = -R_2^{-1}B^{\mathrm{T}}(t)P_{\mathrm{f}}(t)\hat{x}(t), \tag{16}$$

where $F(t) = Q(t)C^{\mathrm{T}}(t)V_2^{-1}$, and $Q(t)$ is produced using the estimator Riccati equation

$$\dot{Q}(t) = A(t)Q(t) + Q(t)A^{\mathrm{T}}(t) - Q(t)C^{\mathrm{T}}(t)V_2^{-1}C(t)Q(t) + V_1. \tag{17}$$

We let $V_1 = I_6$, and $V_2^{-1} = 10^{-14}$ in order to slow down the convergence of the estimated states so that they are visible in the simulation. Note that, unlike the standard LQG problem, the entire system of differential equations is solved forward-in-time, and therefore (15), (16) can be implemented on a time-varying system without full-state feedback and without knowing the dynamics $A(t)$, $B(t)$, and $C(t)$ in advance.

Figure 4 shows the eigenaxis attitude error, Euler angles, angular velocity, and magnetic dipole moment. The estimated states converge to the true state values, and the spacecraft comes to rest at the commanded attitude within 8 orbits. Note that if the convergence of the estimated states is not slowed down, the spacecraft comes to rest faster.



(a) Eigenaxis Attitude Error (b) Euler Angles

(c) Angular Velocity (d) Magnetic Dipole Moments

Figure 4: Output feedback without angular velocity measurements for the rest-to-rest maneuver. (a) Eigenaxis Attitude Error (b) Euler Angles, solid, and estimates, dashed, (c) Angular Velocity, solid, and estimates, dashed, (c) Magnetic Dipole Moments. The estimated states converge to the true values, the spacecraft comes to rest at the commanded attitude within 8 orbits, and the maximum magnetic dipole moment is less than 4×10^{-3} A-m^2.

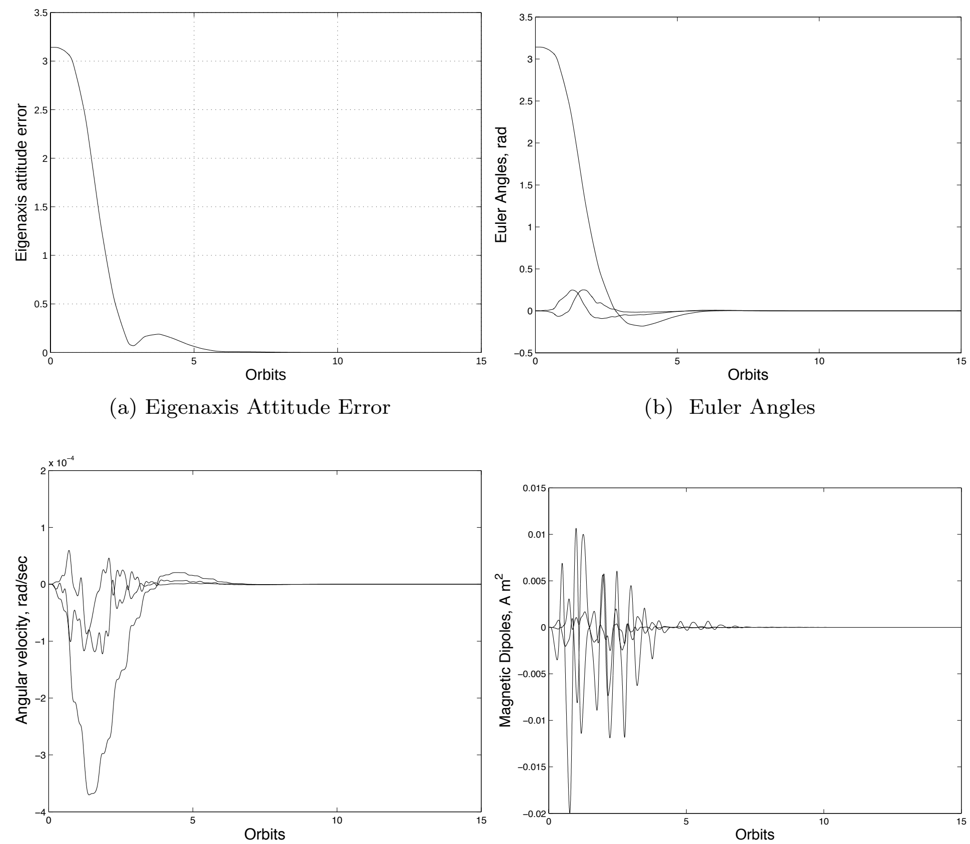
IV.A.4. Large-Angle Maneuver

We use the FIR controller (2)-(3) for a large slew maneuver, rotating 180 degrees about the x-axis. The objective is to bring the spacecraft from the initial attitude $R(0) = \mathrm{diag}(1, -1, -1)$, which corresponds to the 3-2-1 Euler angles

$$\zeta(0) = \begin{bmatrix} \pi & 0 & 0 \end{bmatrix}^{\mathrm{T}} \text{rad},$$

with zero initial angular velocity, $\omega(0) = \delta\omega(0) = 0$, to rest at the desired final orientation, $R_{\mathrm{d}} = I_3$, $\zeta = 0$.

Figure 5 shows the eigenaxis attitude error, Euler angles, angular velocity, and magnetic dipole moment for the simulation described above. The spacecraft comes to rest at the commanded attitude within 10 orbits. The maximum magnetic dipole moment generated is less than 2×10^{-2} A-m^2.



(a) Eigenaxis Attitude Error (b) Euler Angles

(c) Angular Velocity (d) Magnetic Dipole Moments

Figure 5: Full-state feedback for the large-angle maneuver. (a) Eigenaxis Attitude Error (b) Euler Angles, (c) Angular Velocity, (d) Magnetic Dipole Moments. The spacecraft comes to rest at the commanded attitude within 10 orbits, and the maximum magnetic dipole moment is less than 2×10^{-2} A-m^2.

IV.B. Motion-to-Rest Maneuver

We now give the spacecraft the non-zero initial angular velocity

$$\omega(0) = \delta\omega(0) = \begin{bmatrix} 0.025 & 0.025 & -0.03 \end{bmatrix}^{\mathrm{T}} \text{ rad/sec.}$$

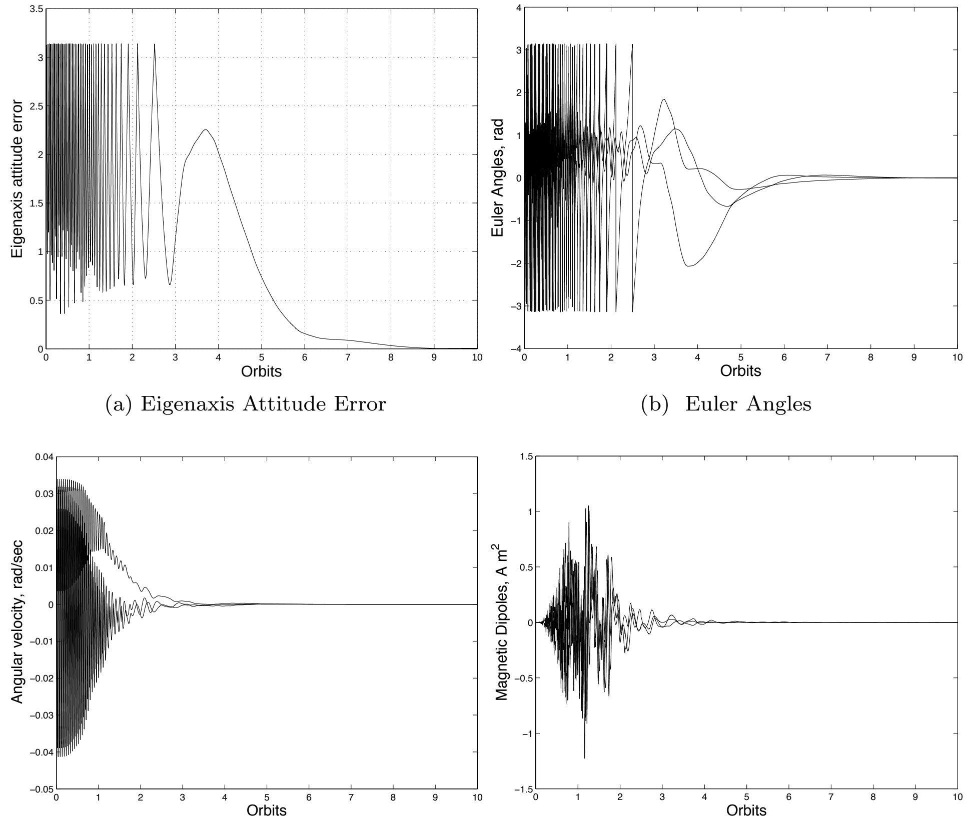
The parameters of the FIR controller are as given in the previous section.

Figure 6 shows the eigenaxis attitude error, Euler angles, angular velocity, and magnetic dipole moment for the motion-to-rest maneuver. The spacecraft now tumbles before the magnetic actuators are able to regulate the attitude. Note that, as in the previous simulation, the controller is stabilizing for maneuvers outside the expected region of validity of the linearized model. The spacecraft comes to rest at the commanded attitude within 10 orbits. The maximum magnetic dipole moment generated is less than 1.5 A-m^2.

Note that the parameters of the controller were tuned for a small rest-to-rest maneuver and are now being applied to a motion-to-rest maneuver. If the spacecraft cannot generate the requested magnetic dipole moments, they could either be saturated, or one could retune the weight matrices R_1 and R_2 for the motion-to-rest maneuver. Also note that, since Algorithm 1 maps the Euler angles to the range $(-\pi, \pi)$, there are discontinuous jumps in Figure 6b.

IV.C. Rest-to-Spin Maneuver (Nadir Pointing)

We now use the FIR controller (2)-(3) for a rest-to-spin maneuver, where the objective is to bring the spacecraft from rest, with initial attitude $R(0) = I_3$, which corresponds to the 3-2-1 Euler angles $\zeta(0) = 0$,



(a) Eigenaxis Attitude Error (b) Euler Angles

(c) Angular Velocity (d) Magnetic Dipole Moments

Figure 6: Full-state feedback for the motion-to-rest maneuver. (a) Eigenaxis Attitude Error (b) Euler Angles, (c) Angular Velocity, (d) Magnetic Dipole Moments. The spacecraft comes to rest at the commanded attitude within 10 orbits, and the maximum magnetic dipole moment is less than 1.5 A-m^2.

and zero initial angular velocity $\omega(0) = 0$, which corresponds to $\delta\omega(0) = \begin{bmatrix} 0 & n & 0 \end{bmatrix}^{\mathrm{T}}$ rad/sec, to a nadir pointing configuration, with

$$\omega_{\mathrm{d}} = \begin{bmatrix} 0 & -n & 0 \end{bmatrix}^{\mathrm{T}} \text{rad/sec}, \quad (\delta\omega_{\mathrm{d}} = 0)$$

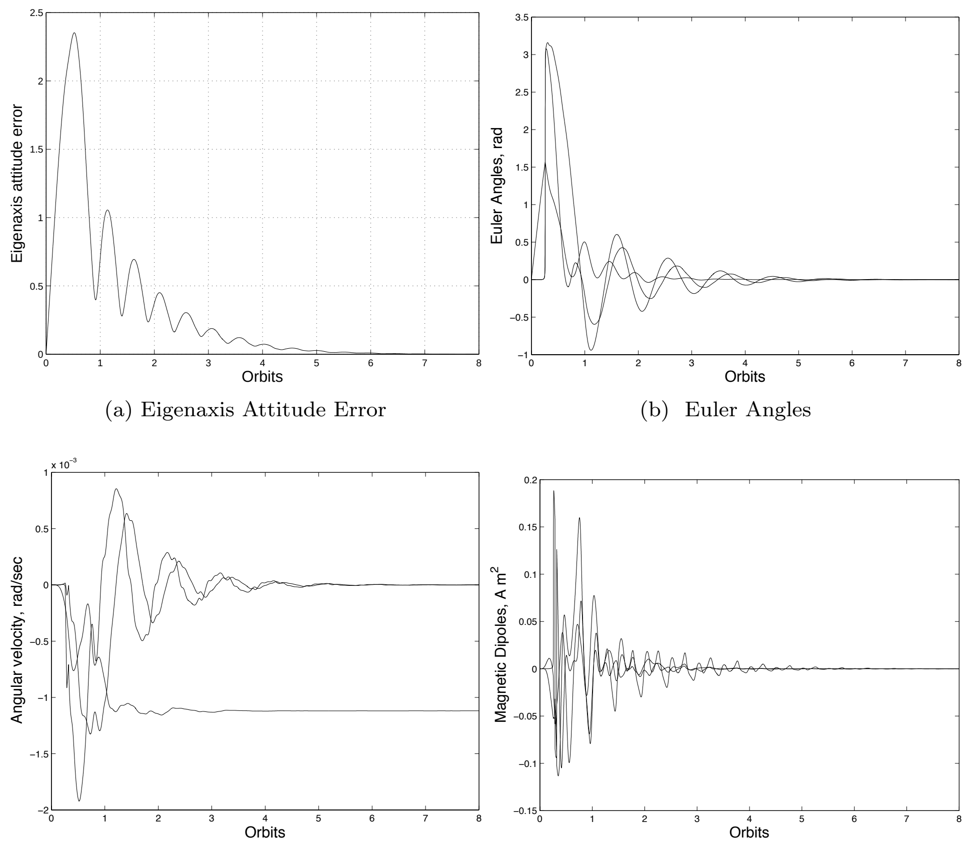
where $n = 0.0011$, and $R_{\mathrm{d}}(0) = I_3$.

We rotate the spacecraft frame so that the inertia matrix J is now given by $J = \mathrm{diag}(1.4947, 5.2056, 3.7997)$ kg-m^2. This choice ensures that the spacecraft spins about its major axis as it points at the Earth. Alternatively, we could have specified ω_{d} to align with the spacecraft's major axis in the original coordinates. Note that we use $A = A_{\mathrm{nadir}}$.

Figure 7 shows the eigenaxis attitude error, Euler angles, angular velocity, and magnetic dipole moment for the nadir pointing maneuver. The spacecraft comes to rest at the commanded attitude within 8 orbits. The maximum magnetic dipole moment generated is less than 0.2 A-m^2.

V. Conclusion

The forward-integrating Riccati-based controller was applied to the problem of spacecraft attitude regulation using only magnetic actuation. In this paper we have shown that the FIR controller is stabilizing for inertially pointing the spacecraft under actuator saturation, noisy magnetic-field measurements, and without using rate feedback. Additionally, we demonstrated nadir-pointing capabilities by spinning the spacecraft up from rest. The above results have been demonstrated with simulations on a fully nonlinear model. Future work will consider nadir pointing on elliptic orbits, uncertain spacecraft inertia, and mixed actuation architectures such as magnetic torquers combined with reaction wheels.



(a) Eigenaxis Attitude Error (b) Euler Angles

(c) Angular Velocity (d) Magnetic Dipole Moments

Figure 7: Full-state feedback for the nadir-pointing maneuver. (a) Eigenaxis Attitude Error (b) Euler Angles, (c) Angular Velocity, (d) Magnetic Dipole Moments. The spacecraft converges to the commanded spin within 8 orbits, and the maximum magnetic dipole moment is less than 0.2 A-m^2.

The FIR controller is advantageous for general linear time-varying systems, does not require future knowledge of model parameters, is tuned similarly to conventional LQR, and has some stability guarantees presented in ref.[11] Future work includes extending the theoretical stability guarantees beyond the results given in ref.[11] We consider the application of orbital stabilization on elliptic orbits using the FIR controller in ref.[19]

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