Adaptive Error Correction for Discrete-Time Systems with Amplitude and Rate Saturation

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Abstract: We present an add-on scheme for multi-input, multi-output systems with a nominal controller in order to avoid problems caused by amplitude and rate saturation such as integrator windup and phase lag. No modelling information is needed except the number of input and output. The adaptive error correction estimates the effect of control saturation on the command following error and directly modify the command following error that is input to the nominal controller in order to prevent further saturation. A retrospective cost optimization algorithm is applied to obtain the correction on-line based on measurements of the command following error and the amount of control saturation. Different from anti-windup scheme, which compensates the control command to enlarge the convergence region, this scheme intends to adaptively contain the error inside the convergence region provided by the nominal controller. Numerical examples show that, together with fixed-gain proportional-integral type controller, the adaptive error correction scheme can prevent integrator windup and phase lag for asymptotically stable plant and critically stable plant in the presence of amplitude and rate saturation.

Key Words: Adaptive Control, Saturation, Input Constrain, Discrete-Time, Retrospective Cost Optimization

1 Introduction

Input saturation reflects the hard physical limit of the actuators or the economic margin, and thus is ubiquitous in control applications [1]. As pointed out by [2], linear control laws that are designed without regard to saturation effects can result in closed-loop instability even for open-loop asymptotically stable system. In addition, integral control for command following or disturbance rejection can experience integrator windup, which may cause instability [3]. Consequently, an extensive literature has been devoted to analyzing the domain of attraction in the presence of input constraints [4, 5], some of which appear as anti-windup scheme. An alternative approach to addressing the effects of saturation is to design a command filter to avoid violation of saturation bound [6, 7]. However, most of the methods above assume that the plant dynamics is known. Thus it is difficult to apply the above methods on plants with large modelling error.

An alternative approach to address the input saturation on system with uncertainty is to use adaptive control. Considering the tracking error caused by input error as a disturbance signal to reject, various adaptive mechanisms are designed [8]. Without intending to reject the tracking error caused by the input error, an indirect adaptive compensation method is designed to hedge the adaptation mechanism from saturation characteristics of the plant [9]. With similar intuitive reason, the saturated control is applied directly in the regressor to avoid effects of saturation on the retrospective cost adaption [10].

In this paper, we present a different approach to tackle the input saturation problem, which is called adaptive error correction. As an add-on scheme, adaptive error correction modifies the command following error in order to compensate for the saturation. Different from anti-windup methods which directly modify the control input, this method modifies the command following error instead. Since a saturated system is very hard to be globally stabilized, it is preferable to correct the command following error to be inside the convergence region. Different from the error governor in [7, 11], this method uses a correction signal instead of a command filter or an error filter. Intuitively speaking, the adaptive error correction revises the command following error to an amount which can be attenuated by the nominal controller without causing saturation.

The main contribution of this paper is a technique to adaptively correct the command following error without modeling information in order to avoid problems caused by saturation such as integrator windup. This technique is an add-on scheme working with a nominal controller that is well designed for system without saturation. In addition,

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the adaptive error correction does not need modelling information of the system dynamics, instead, adaptive error correction re-optimizes the current corrector online based on past data, which known as retrospective cost optimization (see [10, 12, 13])

2 Problem Formulation

Consider the MIMO discrete-time system

$$x(k+1) = Ax(k) + Bu(k) + D_1 w(k), \tag{1}$$

$$y(k) = Cx(k) + D_2 w(k), \tag{2}$$

$$z(k) = E_1 x(k) + E_0 w(k), \tag{3}$$

where $x(k) \in \mathbb{R}^{l_x}$ is the state, $y(k) \in \mathbb{R}^{l_y}$ is the output, $u(k) \in \mathbb{R}^{l_u}$ is the actual input, $w(k) \in \mathbb{R}^{l_w}$ is the exogenous signal, and $z(k) \in \mathbb{R}^{l_z}$ is the performance variable and command following error. The goal is to develop an adaptive output feedback controller that minimizes z in the presence of input constraint on u with exogenous signal w and limited modeling information about (1)–(3). Depending on the choice of D_1, D_2, and E_0, the components of w can represent either command signals to be followed, external disturbances to be rejected, or both. This formulation defines the signals that play a role in adaptive error correction. However, no assumptions are made concerning the state space realization since adaptive error correction uses an input-output model rather than a detailed state space realization.

We assume that there is a well designed nominal controller that can stabilize the closed-loop and accomplish satisfactory command following performance without saturation. However, we do not assume any knowledge about (1)–(3) except for the number of input l_u and the number of output l_y, i.e. l_u and l_y are known and A, B, C, D_1, D_2, E_0, E_1 are otherwise unknown. We assume that measurements of the actual input u and the error z are available for feedback while a direct measurement of the plant output y and exogenous signal w is not available.

Let $u_{\rm cmd}$ be the commanded control signal, that is, the output of the nominal controller. In this paper, we consider the case where there is no saturation (i.e., $u = u_{\rm cmd}$) or the case where the commanded control $u_{\rm cmd}$ is either amplitude saturated, rate saturated, or both (i.e., $u_{\rm cmd} \neq u$).

3 Adaptive Error Correction

Since the nominal controller is designed without considering saturation, it is preferable that the error caused by saturation be invisible to the nominal controller. Thus the goal of adaptive error correction is to eliminate the effect of saturation in the corrected error $\tilde{z} \stackrel{\triangle}{=} z + z_{\rm crt}$ in order to prevent the nominal controller from causing further saturation.

3.1 Error Corrector

We use a strictly proper time-series error corrector of order $n_{\rm crt}$ of the form

$$z_{\rm crt}(k) = \sum_{i=1}^{n_{\rm crt}} M_i(k) z_{\rm crt}(k-i) + \sum_{i=1}^{n_{\rm crt}} N_i(k) \Delta u(k-i), \tag{4}$$

where

$$\Delta u(k) = u(k) - u_{\rm cmd}(k), \tag{5}$$

and, for all $i = 1, \ldots, n_{\rm crt}$, $M_i(k) \in \mathbb{R}^{l_z \times l_z}$ and $N_i(k) \in \mathbb{R}^{l_z \times l_u}$. We rewrite (4) as

$$z_{\rm crt}(k) = \Phi(k)\theta(k), \tag{6}$$

where the regressor matrix $\Phi(k)$ is defined by

$$\begin{aligned}\Phi(k) \stackrel{\triangle}{=} \big[& z_{\rm crt}^{\rm T}(k-1) \ \ldots \ z_{\rm crt}^{\rm T}(k-n_{\rm crt}) \\ & \Delta u^{\rm T}(k-1) \ \ldots \ \Delta u^{\rm T}(k-n_{\rm crt}) \ \big] \otimes I_{l_z} \\ & \in \mathbb{R}^{l_z \times l_\theta},\end{aligned} \tag{7}$$

and

$$\theta(k) \stackrel{\triangle}{=} \text{vec} \left[\ M_1(k) \cdots M_{n_{\rm crt}}(k) \ N_1(k) \cdots N_{n_{\rm crt}}(k) \ \right] \in \mathbb{R}^{l_\theta}, \tag{8}$$

where $l_\theta \stackrel{\triangle}{=} n_{\rm crt} l_z (l_z + l_u)$, "$\otimes$" is the Kronecker product, and "vec" is the column-stacking operator.

3.2 Cumulative Retrospective Cost Optimization

We define the retrospective corrected error as

$$\hat{\tilde{z}}(k) \stackrel{\triangle}{=} z(k) + \Phi(k)\hat{\theta}, \tag{9}$$

where $\hat{\theta} \in \mathbb{R}^{l_\theta}$ is determined by optimization below.

Using the retrospective corrected error $\hat{\tilde{z}}(k)$, we define the cumulative retrospective cost function

$$\begin{aligned} J(\hat{\theta}, k) \stackrel{\triangle}{=} & \sum_{j=0}^{k} \lambda^{k-i} \hat{\tilde{z}}^{\rm T}(j) R_z \hat{\tilde{z}}(j) \\ & + \lambda^k (\hat{\theta} - \theta(0))^{\rm T} R_\theta (\hat{\theta} - \theta(0)), \end{aligned} \tag{10}$$

where $R_z \in \mathbb{R}^{l_z \times l_z}$, $R_\theta \in \mathbb{R}^{l_\theta \times l_\theta}$ is positive definite, and $\lambda \in (0, 1]$ is the forgetting factor.

The minimization of J can be interpreted as the minimization of $|\hat{\tilde{z}}(k)|$ with the cost of $\hat{\theta}$. Note that $\hat{\tilde{z}} = 0$ implies that $z(k) = -\Phi(k)\hat{\theta}$, where $\Phi(k)$ consists of the history data of Δu and $z_{\rm crt}$. Thus, defining $z_{\Delta u}$ as the error part that is caused by Δu, $-\Phi(k)\hat{\theta}$ can be interpreted as an approximation of $z_{\Delta u}(k)$ in an IIR filter form for a minimized $J(\hat{\theta}, k)$. In addition, the optimization process can

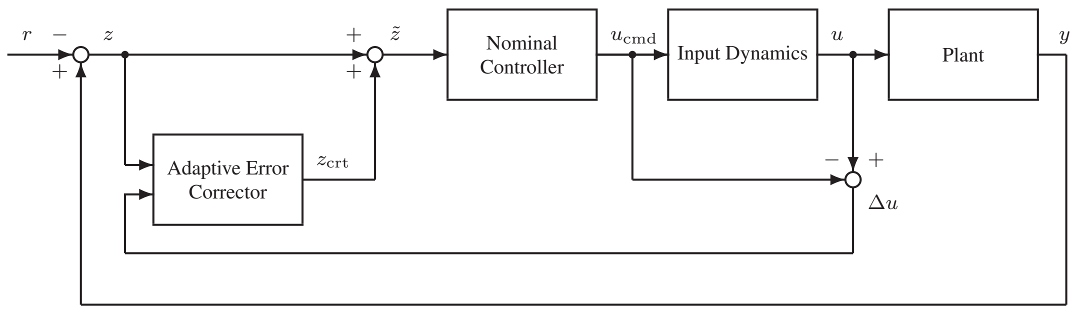


Figure 1: Closed-loop system for command following with retrospective cost adaptive error correction.

be interpreted as evaluating $\hat{\theta}$ with respect to the approximation error assuming that $\hat{\theta}$ was used in the past. Intuitively speaking, $z_{\Delta u}$ cannot be attenuated by the nominal controller, thus the nominal controller should not take $z_{\Delta u}$ as an error, otherwise nominal controller may "try harder" and result in deeper saturation. Therefore, $z_{\rm crt}$ serves as a correction for $z(k)$ such that the corrected error $\tilde{z}(k) = z(k) + z_{\rm crt}(k) = z(k) + \Phi(k)\hat{\theta}$ ideally do not contain $z_{\Delta u}$.

The next result follows from standard recursive-least-squares theory [14].

Proposition: Let $P(0) = R_\theta^{-1}$ and $\theta(0) \in \mathbb{R}^{l_\theta \times l_\theta}$. Then, for all $k \geq 1$, the unique global minimizer of (10) is given by $\hat{\theta} = \theta(k)$, where

$$\theta(k+1) = \theta(k) - P(k)\Phi^{\rm T}(k)\Gamma^{-1}(k) \cdot [z(k) + \Phi(k)\theta(k)], \tag{11}$$

$$P(k+1) = \frac{1}{\lambda}P(k) - \frac{1}{\lambda}P(k)\Phi^{\rm T}(k)\Gamma^{-1}(k)\Phi(k)P(k). \tag{12}$$

where $\Gamma(k) \triangleq \lambda R_z^{-1} + \Phi(k)P(k)\Phi^{\rm T}(k)$.

The adaptive error correction algorithm is given by (6), (7), (8), (11), (12) with design of λ, $\theta(0)$, $n_{\rm crt}$, R_z, R_θ. The forgetting factor λ can be chosen as 1. The initial error corrector coefficient $\theta(0)$ can be chosen arbitrarily and is chosen as $0_{l_\theta \times 1}$ normally.

4 Command Following Using PI Control and Adaptive Error Correction

In this section we intend to compare the performance of fixed gain controller with and without adaptive error correction in different saturation levels and system types. The numerical examples are constructed as follows:

1. We assume that exogenous commands $w = r$, where r is the reference signal. The reference signal $r(k)$ are either square waves or triangle waves. Let $T_{\rm r} = 2000$ and $A_{\rm r} = 1$, the square wave reference signal is given by

$$r_{\rm s}(k) \triangleq \begin{cases} A_{\rm r}, & T_{\rm r}N \leq k < T_{\rm r}N + T_{\rm r}/2, \\ -A_{\rm r}, & T_{\rm r}N + T_{\rm r}/2 \leq k < T_{\rm r}N, \end{cases} \tag{13}$$

and the triangle wave reference signal is given by

$$r_{\rm t}(k) \triangleq \begin{cases} +A_{\rm r}(2k/T_{\rm r} - N), \\ \quad T_{\rm r}N - T_{\rm r}/4 \leq k < T_{\rm r}N + T_{\rm r}/4, \\ -A_{\rm r}(2k/T_{\rm r} - N - 1), \\ \quad T_{\rm r}N + T_{\rm r}/4 \leq k < T_{\rm r}N + 3T_{\rm r}/4. \end{cases} \tag{14}$$

2. (A, B, C) is a controllable canonical realization of the transfer function from u to z, $C = E_1$, $D_2 = 0$, $D_1 = 0_{l_x \times 1}$, and $E_0 = -1$. Therefore, (1)–(3) becomes $x(k+1) = Ax(k) + Bu(k)$ and $z(k) = Cx(k) - r(k)$, where objective is to have $y = Cx$ follow r.

The closed-loop with both adaptive error correction and feedback controller is shown in Figure 1.

Example 1 (Square wave command following for an asymptotically stable system with amplitude saturation using PI controller and adaptive error correction).

Consider the transfer function

$$G_{zu}(z) = \frac{(z + 0.2 + 0.5\jmath)(z + 0.2 - 0.5\jmath)}{(z + 0.5 + 0.5\jmath)(z + 0.5 - 0.5\jmath)(z - 0.9)} \tag{15}$$

The control objective is to have y follow the square wave $r_{\rm s}$ given by (13) with zero initial state in the presence of amplitude saturation. The amplitude saturated control is defined as

$$u(k) \triangleq \begin{cases} u_{\rm cmd}(k), & |u_{\rm cmd}(k)| < \bar{u}, \\ {\rm sgn}(u_{\rm cmd}(k))\bar{u}, & |u_{\rm cmd}(k)| > \bar{u}, \end{cases} \tag{16}$$

where $\bar{u} > 0$ is the amplitude saturation level. We consider the PI feedback control $u_{\rm cmd} = G_{\rm PI}(z)\tilde{z}$, where

$$G_{\rm PI}(z) \triangleq -0.2 - \frac{0.02}{z-1}. \tag{17}$$

Rewrite (17) in time domain yields

$$u_{\rm cmd}(k) \triangleq u_{\rm cmd}(k-1) - 0.2\,\tilde{z}(k) + 0.18\,\tilde{z}(k-1). \tag{18}$$

Let $u_{\rm ss}$ be the steady-state command value required to achieve zero steady-state tracking error. For the plant (15), $u_{\rm ss} \approx 0.15$. The adaptive error correction parameters are chosen as $n_{\rm crt} = 2$, $R_z = 1$, $R_\theta = 0.1 I_4$, $\theta(0) = 0_{4\times 1}$.

Figure 2 shows the time history of $r_{\rm s}$, y, $u_{\rm cmd}$, and u for the square wave command following problem for (15) with various methods and saturation level. For the case without amplitude saturation, which is shown on the first row of Figure 2, y follows r with zero steady-state error. Since $\Delta u \equiv 0$ in this case, there is no difference in turning on or off the adaptive error correction. For the cases with a PI controller alone (i.e., $\tilde{z} = z$) in the presence of amplitude saturation, which is shown on the second and third row of Figure 2, y is prevented by amplitude saturation to follow r with zero steady-state error, and the unsaturated control signal $u_{\rm cmd}$ exhibits windup, which causes a phase lag in y relative to r. For the cases with a PI controller and adaptive error correction (i.e., $\tilde{z} = z + z_\Delta$) in the presence of amplitude saturation, which is shown on the fourth and fifth row of Figure 2, y is unable to follow r with zero steady-state error as the case without adaptive error correction. However, $u_{\rm cmd}$ does not exhibit integrator windup in this case, and y is able to shift direction to match the direction of r without phase lag.

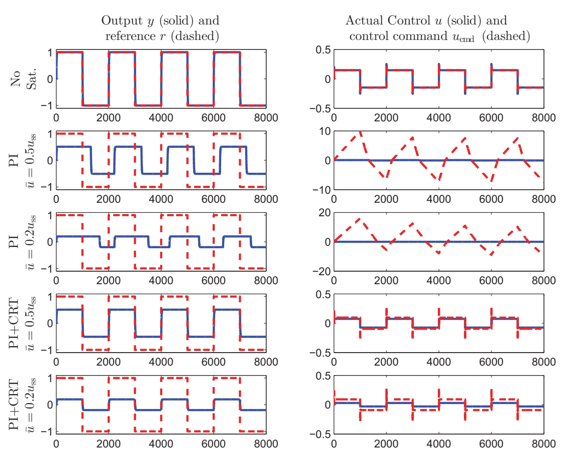


Figure 2: Example 1 (Square wave command following for an asymptotically stable system with amplitude saturation using PI controller and adaptive error correction).

Example 2 (Triangle wave command following for an asymptotically stable system with rate saturation using PI controller and adaptive error correction).

We consider triangle wave command following problem for (15) with reference given by $r_{\rm t}$ in (14) with the same controller and adaptive error correction parameters as in Example 1.

The rate saturated control is defined as

$$u(k) \triangleq \begin{cases} u_{\rm cmd}(k), & |\delta u(k)| \le \delta\bar{u}, \\ u(k-1) + {\rm sgn}(\delta u(k))\delta\bar{u}, & |\delta u(k)| > \delta\bar{u}, \end{cases} \tag{19}$$

where $\delta u(k) \triangleq u_{\rm cmd}(k) - u(k-1)$ and $\delta\bar{u} > 0$ is the rate saturation level. Let $\delta u_{\rm ss}$ be the steady-state rate required by the command to achieve zero steady-state tracking error. For the plant (15), $\delta u_{\rm ss} \approx 0.0003$.

Figure 3 shows the time history of $r_{\rm t}$, y, $u_{\rm cmd}$, and u for the triangle wave command following problem for (15) with various methods and saturation level. For the case without amplitude saturation, which is shown on the first row of Figure 3, y follows r with zero steady-state error. Since $\Delta u \equiv 0$ in this case, there is no difference in turning on or off the adaptive error correction. For the cases with a PI controller alone (i.e., $\tilde{z} = z$) in the presence of rate saturation, which is shown on the second and third row of Figure 3, y is prevented by rate saturation to follow r with zero steady-state error, and the control command $u_{\rm cmd}$ exhibits windup, which causes that y has larger time period than r. For the cases with a PI controller and adaptive error correction (i.e., $\tilde{z} = z + z_\Delta$) in the presence of rate saturation, which is shown on the fourth and fifth row of Figure 3, y is unable to follow r with zero steady-state error as the case without adaptive error correction. However, $u_{\rm cmd}$ does not exhibit integrator windup in this case, and thus y has the same time period as r.

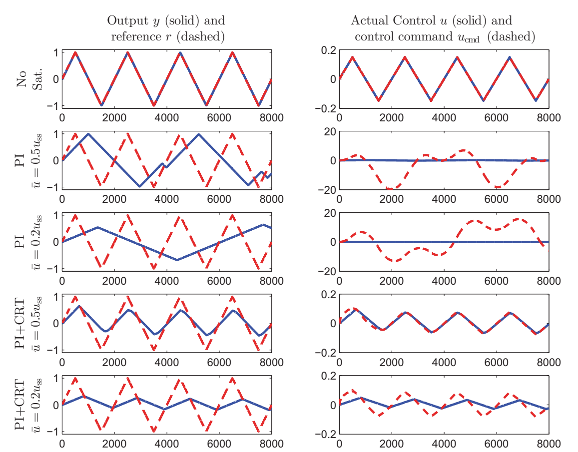


Figure 3: Example 2 (Triangle wave command following for an asymptotically stable system with rate saturation using PI controller and adaptive error correction).

Example 3 (Triangle wave command following for a critically stable system with amplitude saturation using PI controller and adaptive error correction).

Consider the transfer function

$$G_{zu}(z) = \frac{(z + 0.2 + 0.5\jmath)(z + 0.2 - 0.5\jmath)}{(z + 0.5 + 0.5\jmath)(z + 0.5 - 0.5\jmath)(z - 1)} \tag{20}$$

The control objective is to have y follow the triangle wave r_t with zero initial state in the presence of amplitude saturation. We consider the same PI controller and the same adaptive error correction parameters as in Example 1 and Example 2. Let u_{ss} be the steady-state command value required to achieve zero steady-state tracking error. For the plant (20), $u_{ss} \approx 0.0003$.

Figure 4 shows the time history of r_t, y, u_{cmd}, and u for the square wave command following problem for (20) with various methods and saturation level. For the case without amplitude saturation, which is shown on the first row of Figure 4, y follows r with zero steady-state error. Since $\Delta u \equiv 0$ in this case, there is no difference in turning on or off the adaptive error correction. For the cases with a PI controller alone (i.e., $\tilde{z} = z$) in the presence of amplitude saturation, which is shown on the second and third row of Figure 4, y is prevented by amplitude saturation to follow r with zero steady-state error, and the unsaturated control signal u_{cmd} exhibits windup, which causes a phase lag in y relative to r. For the cases with a PI controller and adaptive error correction (i.e., $\tilde{z} = z + z_\Delta$) in the presence of amplitude saturation, which is shown on the fourth and fifth row of Figure 4, y is unable to follow r with zero steady-state error the same as the case without adaptive error correction. However, u_{cmd} does not exhibit integrator windup in this case, and y is able to shift direction to match the direction of r without phase lag.

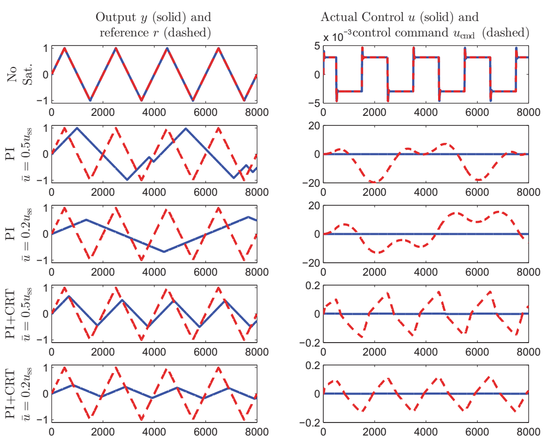


Figure 4: Example 3 (Triangle wave command following for a critically stable system with amplitude saturation using PI controller and adaptive error correction).

Example 4 (Square wave command following for an asymptotically stable system with both amplitude and rate saturation using PI controller and adaptive error correction).

We consider square wave command following problem for (15) with reference given by r_s in (13) with the same controller and adaptive error correction parameters as in Example 1. Different from previous examples, we consider both amplitude and rate saturation with $\bar{u} = 0.03 \approx 20\% u_{ss}$ and $\delta\bar{u} = 0.00015$ and show more details for adaptive error correction.

Figure 5(a) shows that without the adaptive error correction, plant output y suffers from phase lag. Furthermore, the phase lag increases with the number of reference command cycle. Figure 5(b) exhibits the controller windup without the adaptive error correction. The control command keep increasing until the reference command r change sign and the sign of control command does invert, which lead to the phase lag in the output. On the contrary, Figure 5(c) shows that with the adaptive error correction, there is no phase lag for y and Figure 5(d) shows that the control command invert sign the same instant of the reversal of the reference command.

The direct reason for the adaptive error correction to prevent phase lag is that the corrected error $\tilde{z}$ becomes approximately zero when the control is saturated, which is shown in 5(e). In addition, Figure 5(e) shows that the corrected error $\tilde{z}$ keep small in most of the time. From step 2000 to 2400 when the absolute value of error $|z|$ keeps decreasing and the control is not saturated, $\tilde{z}$ keeps a constant -0.003, which enable the integrator in the controller to work on the error. From step 2400 to 3000, when the error z is a constant and the control is saturated, $|\tilde{z}|$ is smaller than 1×10^{-6} which prevent the integrator in the controller from generating larger control command.

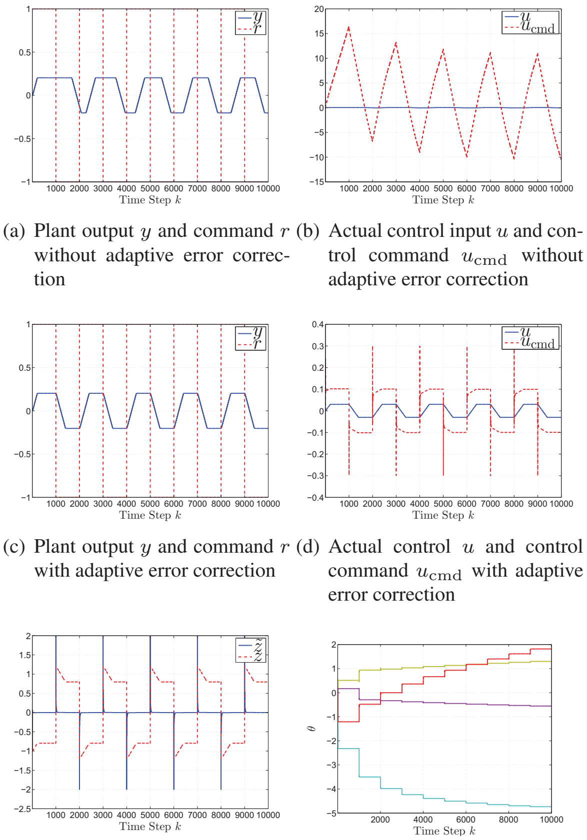
Since adaptive error correction tend to eliminate the command following error that caused by saturation and the corrected error $\tilde{z}$ ideally do not contain the part of error that is caused by Δu. Thus, $\tilde{z} = 0$ implies that all the actual command following error is caused by saturation. Applying the final value theorem of Z-transform on the output signal of (15) with the constant input 0.03 yields

$$\lim_{z\to 1} \frac{(z+0.2+0.5\jmath)(z+0.2-0.5\jmath)}{(z+0.5+0.5\jmath)(z+0.5-0.5\jmath)(z-0.9)} \frac{0.03z}{z-1}(z-1) = 0.2028. \quad (21)$$

Note that in each reference cycle in Figure 5(c) when the reference command equals to 1, the output of the plant with adaptive error correction converges to 0.2028 and $\tilde{z}$ converges to 0. The fact that 0.2028 is the closest possible output to the reference command 1 with input smaller or equals to 0.03 is in accordance with the inference that $\tilde{z} = 0$ implies that all the actual command following error is caused by saturation.

5 Conclusion

We proposed an adaptive error correction method as an add-on scheme that can cooperate with nominal controller to deal with amplitude or rate saturation. Numerical examples show that adaptive error correction prevents PI controller from windup and prevents phase-lag in command



(a) Plant output y and command r without adaptive error correction

(b) Actual control input u and control command $u_{\rm cmd}$ without adaptive error correction

(c) Plant output y and command r with adaptive error correction

(d) Actual control u and control command $u_{\rm cmd}$ with adaptive error correction

(e) Corrected command following error $\tilde{z}$ and actual command following error z with adaptive error correction

(f) Error corrector coefficients θ

Figure 5: Example 4 (Square wave command following for asymptotically stable system with both amplitude and rate saturation using PI controller and adaptive error correction). $\delta u = 0.03$ and $\delta \bar{u} = 0.00015$.

following. Future work includes stability analysis, extension to nonminimum-phase system and unstable system, and application on nonlinear plant.

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