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Continuous-Time Retrospective Cost Adaptive Control for Nonminimum-Phase Systems

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Abstract: We present a continuous-time output-feedback direct adaptive control method to deal with command following or disturbance rejection problem for systems that are possibly nonminimum phase and exponentially unstable. The adaptive control algorithm requires knowledge of the nonminimum-phase zeros of the transfer function from the control to the output. However, the knowledge of the characteristics or measurement of the reference command and disturbance is not required. The closed-loop stability is analyzed and the convergence of tracking error is proved under assumptions. The proposed adaptive control method is an extension of discrete-time retrospective cost adaptive control to continuous-time. Interpretation of retrospective cost adaptive control is provided, which is applicable for both discrete-time and continuous-time version. Numerical examples show that the proposed adaptive control method is effective for command following problem with unmeasured reference command and, in addition, robust to errors in the nonminimum-phase zero estimates.

Key Words: Adaptive Control, Retrospective Cost, Nonminimum-phase, Unstable, Continuous-time

1 Introduction

Adaptive control is motivated by control applications where reliable model prior to operation is not available because the physical processes involved is either difficult to model or subjects to unpredictable changes. Adaptive controllers have the ability to modify the control law in response to the actual plant dynamics, commands, and disturbances and thus can be used to circumvent the loss of performance due to uncertainty.

However, the challenges of adaptive control, which are well documented in the literature [1–5], come with the benefits of adaptive control. The existence of nonminimum-phase (NMP) zeros is one of the major challenges in direct adaptive control. Many control processes are nonminimum-phase, for example, a tail-controlled missile is known to be NMP [6], and the flight-path angle dynamics of an air-breathing hypersonic vehicle is NMP [7]. In some cases, by relocating sensors and actuators or by using linear combinations of sensor measurements, NMP zero can be removed. For example, the use of a canard counteracting the NMP characteristic in hypersonic vehicle [8]. However, constraints on the design of the physical plant can make this approach infeasible. In addition, the nonminimum-phase zero cannot be cancelled by standard state or output feedback controllers [9]. Thus, special designs are needed for NMP systems.

Retrospective cost adaptive control (RCAC) is known as a discrete-time adaptive control technique that is applicable to stabilization, command following, and disturbance rejection, which was proposed in [10], and subsequently developed in [11], [12], [13]. As shown in [13], with limited modelling information, RCAC yields error convergence to zero asymptotically with output feedback for SISO discrete-time systems that are possibly unstable and NMP in the presence of unknown disturbance. The MIMO case is more complicated, see [14]. For a detailed summary and references about RCAC, see [15].

The key idea in discrete-time RCAC is to re-optimize the current controller using past data, which gave rise to the name of this control method. However, this intuitive interpretation makes the extension of RCAC to continuous-time hard to make because the procedure of re-optimization and replacement of the controller parameters are discrete in time.

In recent years, [13] provided a different interpretation of RCAC in the analysis of closed-loop stability of retrospective cost model reference adaptive control (RC-MRAC). The analysis implies that RCAC drives the adaptive controller to reshape the closed-loop transfer function from control perturbation to performance variable as a given target transfer function. By extending the interpretation of RCAC in [13] to continuous-time, the present paper proposes a continuous-time direct output-feedback adaptive control algorithm. To the best of the authors knowledge, there is no continuous-time retrospective cost adaptive control algorithm in the literature.

The main contribution of this paper is a continuous-time output-feedback direct adaptive control method to deal with command following or disturbance rejection problem for system that are possibly NMP and exponentially unstable with limited modelling information. The closed-loop stability is analyzed and the convergence of tracking error is proved under assumptions. Interpretation of continuous-time RCAC is provided, which is also applicable to discrete-time RCAC. In addition, the extension of retrospective cost adaptive control to continuous-time facilitates the understanding of RCAC for continuous-time adaptive control re-

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searchers. Numerical examples show that the proposed adaptive control method is effective for command following problem with unmeasured reference command and, in addition, robust to errors in the NMP zero estimates.

2 Problem Formulation

Consider the SISO continuous-time system

$$z(t) = \frac{N_{\rm p}(\mathbf{s})}{D_{\rm p}(\mathbf{s})}u(t) + w(t), \tag{1}$$

where $u(t) \in \mathbb{R}$ is the plant input, $w(t) \in \mathbb{R}$ is the exogenous signal representing a combination of disturbance to be rejected and reference command to be followed, $z(t) \in \mathbb{R}$ is the performance variable and tracking error, $D_{\rm p}(\mathbf{s})$ is a monic polynomial of order n, and $N_{\rm p}(\mathbf{s})$ is a polynomial with factorization

$$N_{\rm p}(\mathbf{s}) = K_{\rm p}N_{\rm p,u}(\mathbf{s})N_{\rm p,s}(\mathbf{s}), \tag{2}$$

where $N_{\rm p,u}(\mathbf{s})$ and $N_{\rm p,s}(\mathbf{s})$ are monic polynomials with degree $n_{\rm u}$ and $n_{\rm s}$ respectively; if $\zeta \geq 0$ and $N_{\rm p}(\zeta) = 0$, then $N_{\rm p,u}(\zeta) = 0$ and $N_{\rm p,s}(\zeta) \neq 0$. Define $G_{zu}(\mathbf{s}) \triangleq \frac{N_{\rm p}(\mathbf{s})}{D_{\rm p}(\mathbf{s})}$.

The goal is to develop an adaptive output feedback controller that minimizes z in the presence of exogenous w. The performance z is assumed to be measured, however the exogenous signal w is not. The following assumptions are made regarding (1).

Assumption 1. *If $\zeta > 0$ and $N_{\rm p}(\zeta) = 0$, then ζ and its multiplicity is known.*

Assumption 2. *$K_{\rm p}$ is known.*

Assumption 3. *The relative degree of G_{zu}, $d \triangleq \mathrm{rd}(G_{zu})$, is known.*

Assumption 1 implies that all of the NMP zeros of the open-loop system and their multiplicities are known. Assumption 1–3 are made in order to design the target closed-loop transfer function from control perturbation to tracking error. Further explanations about the reasons to introduce Assumption 1–3 can be found in subsection 4.3.

Assumption 4. *There exists a known integer $\bar{n}$ such that the system order $n \leq \bar{n}$.*

Assumption 5. *The signal $w(t)$ is bounded, and, for all $t > 0$, $w(t)$ satisfies*

$$D_w(\mathbf{s})w(t) = 0, \tag{3}$$

where $D_w(\mathbf{s})$ is a unknown nonzero monic polynomial whose roots do not coincide with the roots of $D_{\rm p}(\mathbf{s})$. There exists a known integer $\bar{n}_w$ such that $n_w \leq \bar{n}_w$.

Assumption 5 implies that there exists an internal model controller for the disturbance in system (1) and the upper bound of the order of the internal model is known and the internal model is otherwise unknown. Assumption 4 and 5 are introduced to prove that there exists an ideal fixed-gain controller that results in the designed target closed-loop system, that is, to prove Theorem 2 later in this paper.

3 Continuous-time Retrospective Cost Adaptive Control Algorithm

3.1 Controller

We use a continuous-time output-feedback controller of order $n_{\rm c}$

$$u = \theta_u^{\rm T}(t)\frac{\alpha(\mathbf{s})}{\Lambda_{\rm c}(\mathbf{s})}u + \theta_z^{\rm T}(t)\frac{\alpha(\mathbf{s})}{\Lambda_{\rm c}(\mathbf{s})}z, \tag{4}$$

where $\theta_u \in \mathbb{R}^{n_{\rm c}}$, $\theta_z \in \mathbb{R}^{n_{\rm c}}$, for an arbitrary $n \in \mathbb{N}$,

$$\alpha(\mathbf{s}) \triangleq [\mathbf{s}^{n_{\rm c}-1} \ \cdots \mathbf{s} \ 1]^{\rm T}, \tag{5}$$

$\Lambda_{\rm c}(\mathbf{s}) \triangleq \mathbf{s}^{n_{\rm c}} + \lambda_{n_{\rm c}-1}\mathbf{s}^{n_{\rm c}-1} + \cdots + \lambda_1\mathbf{s} + \lambda_0$ is a Hurwitz polynomial need to be designed. The state-space realization of (4) is given by

$$\dot{\omega}_u(t) = F\omega_u(t) + gu, \quad \omega_u(0) = 0_{n_{\rm c}\times 1}, \tag{6}$$
$$\dot{\omega}_z(t) = F\omega_z(t) + gy, \quad \omega_z(0) = 0_{n_{\rm c}\times 1}, \tag{7}$$
$$u = \phi^{\rm T}(t)\theta(t), \tag{8}$$

where $\phi(t) \triangleq [\omega_u^{\rm T} \ \omega_z^{\rm T}]^{\rm T}$, $\theta(t) \triangleq [\theta_u^{\rm T} \ \theta_z^{\rm T}]^{\rm T}$,

$$\omega_u(t) \triangleq \frac{\alpha_{n_{\rm c}-1}(\mathbf{s})}{\Lambda_{\rm c}(\mathbf{s})}u(t) \in \mathbb{R}^{n_{\rm c}}, \tag{9}$$

$$\omega_z(t) \triangleq \frac{\alpha_{n_{\rm c}-1}(\mathbf{s})}{\Lambda_{\rm c}(\mathbf{s})}y(t) \in \mathbb{R}^{n_{\rm c}}, \tag{10}$$

and

$$F \triangleq \begin{bmatrix} \lambda_{n_{\rm c}-1} & \lambda_{n_{\rm c}-2} & \lambda_{n_{\rm c}-3} & \cdots & \lambda_0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, g \triangleq \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{11}$$

3.2 Retrospective Cost Performance and Parametric Model

For $\hat{\theta}$ we define the *retrospective performance*

$$\hat{z}(\hat{\theta}, t) = z(t) + G_{\rm f}(\mathbf{s})[\phi^{\rm T}(t)\hat{\theta} - u(t)], \tag{12}$$

where $G_{\rm f}(\mathbf{s})$ is a transfer function need to be designed. $G_{\rm f}(\mathbf{s})$ can be interpreted as a target closed-loop transfer function from $\phi^{\rm T}(t)\tilde{\theta}(t)$ to $z(t)$, where $\tilde{\theta} \triangleq \theta - \hat{\theta}$. Further analysis and design method see Section 4

Letting $\hat{z}(\hat{\theta}, t) = 0$ yields the parametric model

$$-z(t) + G_{\rm f}(\mathbf{s})u(t) = G_{\rm f}(\mathbf{s})\phi^{\rm T}(t)\hat{\theta}, \tag{13}$$

which can be represented as

$$z_{\rm pm}(t) = \hat{\theta}^{\rm T}\phi_{\rm pm}(t),$$

where

$$\begin{aligned} z_{\rm pm}(t) &\triangleq -z(t) + G_{\rm f}(\mathbf{s})u(t), \\ \phi_{\rm pm}(t) &\triangleq G_{\rm f}(\mathbf{s})\phi(t). \end{aligned}$$

3.3 Normalized Retrospective Cost Optimization

The normalized retrospective cost function is defined by

$$\begin{aligned} J(\theta(t)) =& \frac{1}{2}\int_0^t \frac{\hat{z}(\theta(t),t)^2}{m^2(\tau)} d\tau \\ &+ \frac{1}{2}(\theta(t)-\theta_0)^{\rm T} R_\theta (\theta(t)-\theta_0), \end{aligned} \tag{14}$$

where $R_\theta \in \mathbb{R}^{2n_{\rm c}\times 2n_{\rm c}}$ is positive definite and $m^2(t) \triangleq 1+\eta_{\rm s}(t)^2$ and $\eta_{\rm s}(t)^2 \triangleq \phi_{\rm pm}^{\rm T}(t)\phi_{\rm pm}(t)$. For use below, we define the normalized estimation error

$$\varepsilon(t) \triangleq \frac{z_{\rm pm}(t) - \theta^{\rm T}(t)\phi_{\rm pm}(t)}{m^2}. \tag{15}$$

The next result follows from recursive-least-squares theory in [4, p. 195].

Theorem 1. *Let $P(0) = R_\theta^{-1}$ and $\theta(0) \in \mathbb{R}^{2n_{\rm c}}$. Then, the update law given by*

$$\dot{\theta}(t) = P(t)\varepsilon(t)\phi_{pm}(t), \tag{16}$$

$$\dot{P}(t) = -\frac{P(t)\phi_{pm}(t)\phi_{pm}^{\rm T}(t)P(t)}{m^2}, \tag{17}$$

guarantees that

(i) ε, $\varepsilon\eta_{\rm s}$, θ, $\dot{\theta}$, $P \in \mathcal{L}_\infty$.

(ii) ε, $\varepsilon\eta_{\rm s}$, $\dot{\theta} \in \mathcal{L}_2$.

(iii) $\lim_{t\to\infty}\theta(t) = \bar{\theta}$, *where* $\bar{\theta} \in \mathbb{R}^{2n_{\rm c}}$ *is a constant vector.*

(iv) If $\eta_{\rm s}$, $\phi_{\rm pm} \in \mathcal{L}_\infty$ *and* $\phi_{\rm pm}$ *is persistent excitation then* $\lim_{t\to\infty}\theta(t) = \theta_*$, *where, for all* $t > 0$, $\hat{z}(\theta_*, t) = 0$.

4 Closed-loop Analysis

4.1 Perturbed Ideal Closed-loop

Let $\theta_* \triangleq [\theta_{u,*}^{\rm T}\ \theta_{z,*}^{\rm T}]^{\rm T}$ be an ideal fixed gain controller, then the perturbed ideal fixed gain controller can be presented as

$$\frac{\Lambda_{\rm c} - \theta_{u,*}^{\rm T}\alpha(\mathbf{s})}{\Lambda_{\rm c}(\mathbf{s})}u(t) = \frac{\theta_{z,*}^{\rm T}\alpha(\mathbf{s})}{\Lambda_{\rm c}(\mathbf{s})}z(t) + v(t), \tag{18}$$

where $v(t)$ is the control perturbation. Thus

$$u(t) = \frac{N_{{\rm c},*}(\mathbf{s})}{D_{{\rm c},*}(\mathbf{s})}y(t) + \frac{\Lambda_{\rm c}(\mathbf{s})}{D_{{\rm c},*}(\mathbf{s})}v(t), \tag{19}$$

where

$$N_{{\rm c},*} = \theta_{z,*}^{\rm T}\alpha(\mathbf{s}), \tag{20}$$

$$D_{{\rm c},*} = \Lambda_{\rm c} - \theta_{u,*}^{\rm T}\alpha(\mathbf{s}). \tag{21}$$

Substitute (1) into (19) yields

$$\begin{aligned} z =& \frac{\Lambda_{\rm c}(\mathbf{s})N_{\rm p}(\mathbf{s})}{D_{{\rm c},*}(\mathbf{s})D_{\rm p}(\mathbf{s}) - N_{{\rm c},*}(\mathbf{s})N_{\rm p}(\mathbf{s})}v(t) \\ &+ \frac{D_{{\rm c},*}(\mathbf{s})D_{\rm p}(\mathbf{s})}{D_{{\rm c},*}(\mathbf{s})D_{\rm p}(\mathbf{s}) - N_{{\rm c},*}(\mathbf{s})N_{\rm p}(\mathbf{s})}w(t), \end{aligned} \tag{22}$$

Thus the ideal closed-loop transfer function from control perturbation v to tracking error z is

$$\tilde{G}_{zv,*}(\mathbf{s}) \triangleq \frac{\Lambda_{\rm c}(\mathbf{s})N_{\rm p}(\mathbf{s})}{D_{{\rm c},*}(\mathbf{s})D_{\rm p}(\mathbf{s}) - N_{{\rm c},*}(\mathbf{s})N_{\rm p}(\mathbf{s})}. \tag{23}$$

4.2 Existence of an Ideal Controller

The following theorem guarantee the existence of an ideal fixed gain controller consists of three parts connected serially :
1) a precompensator to cancel the minimum-phase zeros of the open-loop plant: $\frac{1}{N_{\rm p,s}(\mathbf{s})}$.
2) an internal model controller $\frac{1}{D_w(\mathbf{s})}$.
3) a controller to stabilize the closed-loop $\frac{N_{\rm c}'(\mathbf{s})}{D_{\rm c}'(\mathbf{s})}$, where $\deg N_{\rm c}' < \deg D_{\rm c}'$. Figure 1 shows the structure of the ideal controller and the perturbed closed-loop.

Theorem 2. *Let*

$$n_{\rm c} \geq \bar{n} + \bar{n}_w - n_{\rm u} - d \geq n + n_w - n_{\rm u} - d, \tag{24}$$

then there exists an ideal fixed-gain controller of order $n_{\rm c}$ *such that the following statements hold in the perturbed ideal closed-loop system consisting of* (1) *and* (19)*:*

$$z = \frac{N_{\rm f}(\mathbf{s})}{D_{\rm f}(\mathbf{s})}v(t), \tag{25}$$

where $N_{\rm f}(\mathbf{s}) = K_{\rm p}N_{\rm p,u}(\mathbf{s})$ *and* $D_{\rm f}(\mathbf{s})$ *is an arbitrary monic Hurwitz polynomial of order* $n_{\rm f} \triangleq n_{\rm u} + d$.

Proof. Since the ideal controller consists of three serially connected parts, $\frac{1}{N_{\rm p,s}(\mathbf{s})}$, $\frac{1}{D_w(\mathbf{s})}$, and $\frac{N_{\rm c}'(\mathbf{s})}{D_{\rm c}'(\mathbf{s})}$,

$$N_{{\rm c},*}(\mathbf{s}) = N_{\rm c}'(\mathbf{s}) \tag{26}$$

$$D_{{\rm c},*}(\mathbf{s}) = N_{\rm p,s}(\mathbf{s})D_w(\mathbf{s})D_{\rm c}'(\mathbf{s}) \tag{27}$$

Substituting (26) and (27) into (22) yields

$$\begin{aligned} z =& \frac{\Lambda_{\rm c}(\mathbf{s})K_{\rm p}N_{\rm p,u}(\mathbf{s})}{D_w(\mathbf{s})D_{\rm p}(\mathbf{s})D_{\rm c}'(\mathbf{s}) - N_{\rm p,u}(\mathbf{s})N_{\rm c}'(\mathbf{s})}v(t) \\ &+ \frac{N_{\rm p,s}(\mathbf{s})D_w(\mathbf{s})D_{\rm c}'(\mathbf{s})D_{\rm p}(\mathbf{s})}{D_{{\rm c},*}(\mathbf{s})D_{\rm p}(\mathbf{s}) - N_{{\rm c},*}(\mathbf{s})N_{\rm p}(\mathbf{s})}w(t), \end{aligned} \tag{28}$$

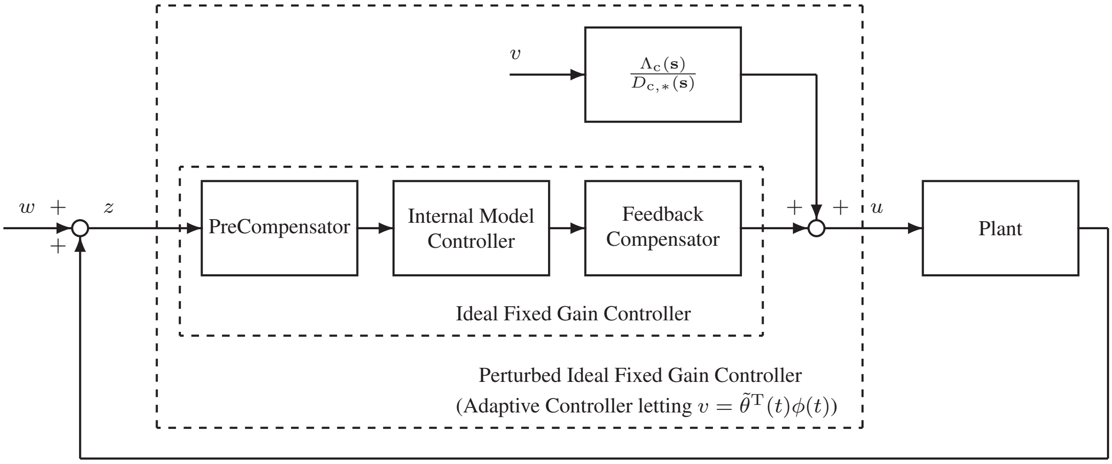


Fig. 1: Closed-loop system for command following with perturbed ideal fixed gain controller

Substituting the internal model equation (3) into (28) yields

$$z = \frac{\Lambda_c(s) K_p N_{p,u}(s)}{D_w(s) D_p(s) D_c'(s) - K_p N_{p,u}(s) N_c'(s)} v(t),. \quad (29)$$

where $\deg D_c'(s) = n_c - n_w - n_s \geq 0$. Thus $n_c \geq n_s + n_w = n + n_w - n_u - d$. Note that $\deg D_w(s) D_p(s) D_c'(s) = n_w + n_p + (n_c - n_w - n_s) = n_c + n_p - n_s = n_c + n_u + d$. Since, in addition, $X(s) \triangleq D_w(s) D_p(s)$ and $Y(s) \triangleq -K_p N_{p,u}(s)$ are coprime, it follows from the Diophantine equation (see [4, p. 41], Theorem 2.3.1) that the roots of $D_w(s) D_p(s) D_c'(s) - K_p N_{p,u}(s) N_c'(s)$ can be assigned arbitrarily by choosing $N_c'(s)$ and $D_c'(s)$. Therefore, there exist $N_c'(s)$ and $D_c'(s)$ such that $D_w(s) D_p(s) D_c'(s) - K_p N_{p,u}(s) N_c'(s) = \Lambda_c(s) D_f(s)$, and thus (29) implies that

$$z = \frac{\Lambda_c(s) K_p N_{p,u}(s)}{\Lambda_c(s) D_f(s)} v(t) = \frac{N_f(s)}{D_f(s)} v(t). \qquad \square$$

4.3 Interpretation of G_f

Comparing (25) and (22) yields

$$G_f(s) = \tilde{G}_{zv,*}(s) = \frac{\Lambda_c(s) N_p(s)}{D_{c,*}(s) D_p(s) - N_{c,*}(s) N_p(s)}, \quad (30)$$

which shows that G_f is the target closed-loop transfer function from control perturbation v to tracking error z. Since the poles of the ideal closed-loop (that is $\tilde{G}_{zv,*}(s)$) need to be stable, the NMP zeros of the open-loop system (unstable roots of $N_p(s)$) can not be cancelled by roots of the numerator of $\tilde{G}_{zv,*}(s)$. Thus, the target closed-loop transfer function G_f is required to contains the same NMP zeros as the open-loop system, which is the reason for Assumption 1 to be introduced.

In addition, note that the order of $N_{c,*}(s) N_p(s)$ is smaller than that of $D_{c,*}(s) D_p(s)$, which implies that $D_{c,*}(s) D_p(s) - N_{c,*}(s) N_p(s)$ is monic. Thus, $\Lambda_c(s)$ and $D_{c,*}(s) D_p(s) - N_{c,*}(s) N_p(s)$ are both monic, which implies the high frequency gain of $\tilde{G}_{zv,*}$ is the same as that of the open-loop transfer function G_{zu} for any possible controller in the form of (4). Thus Assumption 2 is introduced to make G_f have the same high frequency gain as $\tilde{G}_{zv,*}$.

Furthermore, $\deg[D_{c,*}(s) D_p(s) - N_{c,*}(s) N_p(s)] = \deg D_{c,*}(s) D_p(s) = n + n_c$ and $\deg \Lambda_c(s) N_p(s) = n + n_c - d$ imply that $\mathrm{rd}(G_f) = \mathrm{rd}(\tilde{G}_{zv,*}) = \mathrm{rd}(G_{zu}) = d$, which is the reason for Assumption 3 to be introduced.

4.4 Interpretation of Retrospective Performance

Substituting $\tilde{\theta} \triangleq \theta(t) - \theta_*$ into (4) yields

$$\begin{aligned} u(t) &= \theta^{\mathrm{T}}(t)\phi(t) \\ &= \frac{N_{c,*}(s)}{D_{c,*}(s)} y(t) + \frac{\Lambda_c(s)}{D_{c,*}(s)} \tilde{\theta}^{\mathrm{T}}(t)\phi(t), \end{aligned} \quad (31)$$

Note that the adaptive controller (31) is a special case of the perturbed ideal fixed gain controller (19) with $v(t) = \tilde{\theta}^{\mathrm{T}}(t)\phi(t)$, thus Theorem 2 with $v(t) = \tilde{\theta}^{\mathrm{T}}(t)\phi(t)$ implies that the close-loop system using the adaptive controller in Section 3 satisfies

$$\begin{aligned} z(t) &= \frac{N_f(s)}{D_f(s)} [\tilde{\theta}^{\mathrm{T}}(t)\phi(t)] \\ &= \frac{N_f(s)}{D_f(s)} [\phi^{\mathrm{T}}(t)\theta(t) - \phi^{\mathrm{T}}(t)\theta_*] \\ &= \frac{N_f(s)}{D_f(s)} [u(t) - \phi^{\mathrm{T}}(t)\theta_*], \end{aligned} \quad (32)$$

Letting $G_f(s) = \frac{N_f(s)}{D_f(s)}$ and substituting (32) into (12) yields

$$\begin{aligned} \hat{z}(\hat{\theta}, t) &= G_f[u(t) - \phi^{\mathrm{T}}(t)\theta_*] + G_f(s)\phi^{\mathrm{T}}(t)(\hat{\theta} - \theta_*) \\ &= G_f(s)\phi^{\mathrm{T}}(t)(\hat{\theta} - \theta_*). \end{aligned} \quad (33)$$

Thus $\hat{z}(\hat{\theta}, t)$ can be interpreted as a filtered estimation error $\hat{\theta} - \theta_*$, and $\hat{z}(\hat{\theta}, t) = 0$ is a necessary condition for $\hat{\theta} = \theta_*$, which implies $z = G_f(s) v(t)$.

4.5 Convergence of Tracking Error

Theorem 3. *Consider the plant* (1) *satisfying Assumptions 3–5, and the continuous-time retrospective cost adaptive controller* (6), (7), (8), (12), (16), *and* (17), *where* n_c *satisfies* (24) *and* $G_f(s) = \frac{N_f(s)}{D_f(s)}$. *If* η_s, $\phi_{pm} \in \mathcal{L}_\infty$ *and* ϕ_{pm} *is persistent excitation, then* $\lim_{t\to\infty} z(t) = 0$.

Proof. Since $\eta_{\rm s}$, $\phi_{\rm pm} \in \mathcal{L}_\infty$ and $\phi_{\rm pm}$ is persistent excitation, (iv) of Theorem 1 implies that $\lim_{t\to\infty} \tilde{\theta} = 0$. In addition, $\eta_{\rm s}(t)^2 \triangleq \phi_{\rm pm}^{\rm T}(t)\phi_{\rm pm}(t) \in \mathcal{L}_\infty$ implies that $\phi_{\rm pm}(t) = G_{\rm f}(\mathbf{s})\phi(t)$ is bounded. Since $G_{\rm f}(\mathbf{s})$ is asymptotically stable, it follows that $\phi(t)$ is bounded, and thus $\lim_{t\to\infty} \tilde{\theta}^{\rm T}(t)\phi(t) = 0$. Furthermore, (32) implies that $z(t) = G_{\rm f}(\mathbf{s})[\tilde{\theta}^{\rm T}(t)\phi(t)]$, and thus $\lim_{t\to\infty} z(t) = 0$. $\square$

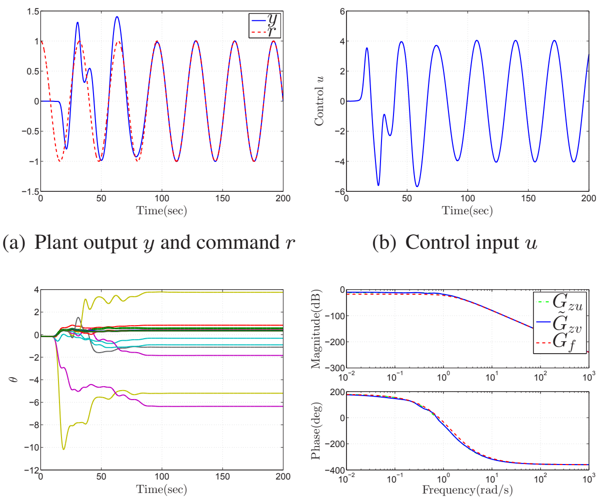
5 Numerical Examples

Example 1. Consider the asymptotically stable, NMP plant

$$z(t) = \frac{(\mathbf{s}-1)(\mathbf{s}+0.5)}{(\mathbf{s}+2)(\mathbf{s}+1)^5} u(t) + w(t), \tag{34}$$

which has relative degree 4. Let (34) have zero initial state and, for all $t > 0$, let $w(t) = -\sin(\frac{\pi}{16}t + \frac{\pi}{2})$. Defining $y(t) \triangleq z(t) - w(t)$ and $r(t) \triangleq -w(t)$, this example imitates the scenario where $y(t)$ needs to track the reference command $r(t)$. Note that w are not assumed to be measured.

We choose $n_{\rm c} = 13$, $G_{\rm f}(\mathbf{s}) = \frac{(\mathbf{s}-1)}{(\mathbf{s}+1.5)^5}$, $\Lambda_{\rm c}(\mathbf{s}) = (\mathbf{s}+1)^5$, $P(0) = 100 I_{26}$, and $\theta(0) = 0_{26\times 1}$. Figure 2 shows that the largest absolute value of tracking error in the last period is 0.001 and the largest absolute value of transient error is 1 which happens at $t = 0$. The absolute value of tracking error is smaller than 0.1 in 90 sec.



(a) Plant output y and command r (b) Control input u

(c) Controller coefficients θ (d) G_{zu}, $\tilde{G}_{zv}$, and $G_{\rm f}$

Fig. 2: Example 1 (Sinusoidal command following for asymptotically stable NMP system). (a) shows that the largest absolute value of tracking error in the last period is 0.001 and the largest absolute value of transient error is 1 which happens at $t = 0$. The absolute value of tracking error is smaller than 0.1 in 90 sec. (c) shows that the controller coefficients stop varying in 80 sec. (d) shows that the open-loop transfer function G_{zu}, the actual ($\tilde{G}_{zv}$) and ideal ($G_{\rm f}$) closed-loop transfer from control perturbation v to z are close to each other. This is reasonable because these three transfer functions are all asymptotically stable and with the same NMP zero.

Example 2. Consider the exponentially unstable, NMP plant

$$z(t) = \frac{(\mathbf{s}-1)}{(\mathbf{s}-0.1)(\mathbf{s}+0.5)} u(t) + w(t), \tag{35}$$

which has relative degree 1. Let (35) have zero initial state and, for all $t > 0$, let $w(t) = -1$. Similar to Example 2, we define $y(t) \triangleq z(t) - w(t)$ and $r(t) \triangleq -w(t)$ and this example imitates the scenario where $y(t)$ needs to track the reference command $r(t)$. w is not measured.

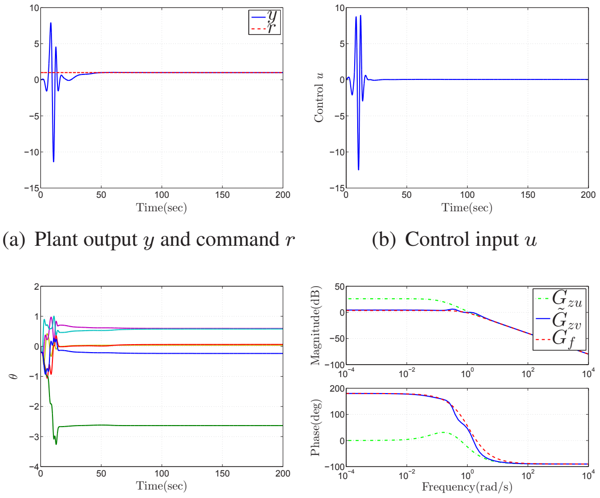
Different from Example 1, we choose $G_{\rm f}(\mathbf{s}) = \frac{(\mathbf{s}-2)}{(\mathbf{s}+1)^2}$, which does not have the same NMP zero as G_{zu}. In addition, we choose $n_{\rm c} = 3$, $\Lambda_{\rm c}(\mathbf{s}) = (\mathbf{s}+0.4)^3$, $P(0) = 100 I_6$, and $\theta(0) = 0_{6\times 1}$. Figure 3 shows that the absolute value of tracking error at $t = 200$ sec is 0.0008 and the largest absolute value of transient error is 12.37 which happens at $t = 10.68$. The absolute value of tracking error is smaller than 0.1 in 45 sec. The actual $\tilde{G}_{zv}$ with $\theta(t = 200)$ is

$$\frac{(\mathbf{s}-1)(\mathbf{s}+0.4)^3}{(\mathbf{s}+0.1288)(\mathbf{s}^2+0.259\mathbf{s}+0.128)(\mathbf{s}^2+1.448\mathbf{s}+2.339)},$$

which is asymptotically stable. It can be seen that $\tilde{G}_{zv}$ has the same NMP zero $s = 1$ as G_{zu}, despite that $G_{\rm f}$ has a different NMP zero $s = 2$. However, the adaptive controller drives the tracking error to zero despite of the NMP zero estimation error. Figure 3(d) shows that $\tilde{G}_{zv}$ and $G_{\rm f}$ are asymptotically stable and the Bode plot of the two are close to each other. Since G_{zu} is unstable, the Bode plot of G_{zu} is not close to the other two. This implies that the adaptive controller stabilize the closed-loop and reshape the closed-loop.

6 Conclusion

We proposed an continuous-time direct output-feedback adaptive control method to deal with command following or disturbance rejection problem for systems that are possibly NMP and exponentially unstable. The adaptive control algorithm requires knowledge of the nonminimum-phase zeros of the transfer function from the control to the output. However, the knowledge of the characteristics or measurement of the reference command and disturbance is not required. The closed-loop stability is analyzed and the convergence of tracking error is proved under assumptions. This adaptive control method is an extension of discrete-time retrospective cost adaptive control to continuous-time. Interpretation of retrospective cost adaptive control is provided, which is applicable for both discrete-time and continuous-time version. Numerical examples show that the proposed adaptive control method is effective for command following problem with unmeasured reference command and, in addition, robust to errors in the NMP zero estimates. Future work includes proof of error convergence with weaker assumptions, mechanisms to decrease transient error, and application on nonlinear plant.



(a) Plant output y and command r (b) Control input u

(c) Controller coefficients θ (d) G_{zu}, $\tilde{G}_{zv}$, and $G_{\rm f}$

Fig. 3: Example 2 (Constant command following for exponentially unstable NMP system). (a) shows that the absolute value of tracking error at $t = 200\,\text{sec}$ is 1.03×10^{-6} and the largest absolute value of transient error is 10 which happens at $t = 11.36$. The absolute value of tracking error is smaller than 0.1 in $45\,\text{sec}$. (c) shows that the controller coefficients stop varying in $40\,\text{sec}$. (d) shows that the actual ($\tilde{G}_{zv}$) and ideal ($G_{\rm f}$) closed-loop transfer from control perturbation v to z are close to each other. However, the the open-loop transfer function G_{zu} is not close to the other two. This is because $\tilde{G}_{zv}$ and $G_{\rm f}$ are asymptotically stable but G_{zu} is unstable. This implies that the adaptive controller stabilize the closed-loop and reshape the closed-loop as designed.

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