Adaptive Spacecraft Attitude Control with Reaction Wheel Actuation

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\textbf{Abstract}—We apply retrospective cost adaptive control (RCAC) to spacecraft attitude control using reaction-wheel actuators. The controller is applied to the motion-to-rest (M2R) maneuver, where the spacecraft has an arbitrary initial attitude and angular-velocity and the objective is to bring the body to rest at a specified inertial attitude. First, we show that RCAC can complete the M2R maneuver using full information of the mass properties of the spacecraft. Then, we remove the spacecraft and reaction-wheel inertia information from the controller and examine the closed-loop settling time for off-nominal spacecraft inertias. Thus, the controller uses limited knowledge of the system. Finally, the algorithm is tested in the presence of disturbances, unknown actuator misalignment, and noise and bias in the angular-velocity measurement.

I. INTRODUCTION

Spacecraft attitude control is a nonlinear problem involving the special orthogonal group SO(3) of $3 \times 3$ rotation matrices. Thus, adaptive control is useful when a sufficiently accurate model of the spacecraft is not available for fixed-gain controller synthesis. We take a different approach to the problem of attitude control for reaction-wheel-actuated spacecraft motivated by [1], which uses linear adaptive controllers based on parameterizations of the equations of motion.

Retrospective cost adaptive control (RCAC) is a multi-input, multi-output direct adaptive controller that requires limited modeling information. The algorithm has been applied to nonlinear plants such as multiple linkages [5], [6]. In this paper we apply RCAC to spacecraft attitude control. We extend the thruster control approach in [7] by utilizing reaction wheels with momentum-storage capabilities as actuators. The momentum stored in the wheels introduces additional nonlinearities in Euler’s equation as shown in [8]. These nonlinearities present an additional challenge for the linear compensator produced by RCAC. The goal of this work is to control the spacecraft without knowledge of the spacecraft or reaction-wheel inertias. Specifically, we evaluate the closed-loop performance for motion-to-rest (M2R) maneuvers. For these maneuvers the spacecraft has an arbitrary initial attitude and angular-velocity, and the objective is to bring the spacecraft to rest at a specified inertial attitude. We examine the relation between the spacecraft inertia, the reaction-wheel inertia, and the closed-loop settling time. Furthermore, we investigate the effect of inertially constant disturbance torques as well as noise and bias on the angular-velocity measurement.

II. NONLINEAR EQUATIONS OF MOTION

For the spacecraft model, we consider a single rigid body controlled by torque actuators, in this case reaction wheels, which provide on-board momentum storage. We consider only the rotational motion of the spacecraft and not the translational motion of the spacecraft’s center of mass. We assume that a body-fixed frame is defined for the spacecraft, whose origin is chosen to be the center of mass, and that an inertial frame is specified for determining the attitude of the spacecraft. The equations of motion for a spacecraft actuated by reaction wheels are given by Euler’s equation, Poisson’s equation, and the reaction-wheel dynamics, which are given, respectively, by

\begin{align*}
J \dot{\omega} &= (J \omega - B_{sc} \nu) \times \omega + B_{sc} u + z_{\text{dist}}, \quad (1) \\
\dot{R} &= R \omega^x, \quad (2) \\
\nu_i &= R_{W_i} e_{W_i} u, \quad \text{for } i = 1, 2, \ldots n_w, \quad (3)
\end{align*}

where $\omega \in \mathbb{R}^3$ is the angular-velocity of the spacecraft frame with respect to the inertial frame resolved in the spacecraft frame, $J \in \mathbb{R}^{3 \times 3}$ is the constant, positive-definite inertia matrix of the spacecraft, the columns of $B_{sc} \in \mathbb{R}^{3 \times n_w}$ are given by $b_i = -\alpha_i R_{W_i} e_{W_i}, \quad \text{for } i = 1, 2, \ldots n_w, \quad (4)$

\begin{align*}
\nu \in \mathbb{R}^{n_w} & \text{ is a vector composed of the angular-rate of each reaction-wheel with respect to the spacecraft frame,} \\
u \in \mathbb{R}^{n_w} & \text{ is a vector composed of the control input to each wheel, which has units of angular acceleration, and} \\
z_{\text{dist}} & \in \mathbb{R}^3 \text{ represents all internal and external disturbance torques applied to the spacecraft aside from control torques.} \\
R & \in \mathbb{R}^{3 \times 3} \text{ is the proper orthogonal matrix (that is, the rotation matrix) that transforms the components of a vector resolved in the spacecraft frame into the components of the same vector resolved in the inertial frame and } \omega^x \text{ is the skew-symmetric cross-product matrix of } \omega. \\
\alpha_i & \text{ is the positive moment of inertia about the rotation axis of the } i^{th} \text{ reaction-wheel, } R_{W_i} \in \mathbb{R}^{3 \times 3} \text{ is the rotation matrix that transforms the components of a vector resolved in the } i^{th} \text{ reaction-wheel frame into the components of the same vector resolved in the spacecraft body frame, and } e_{W_i} \in \mathbb{R}^3 \text{ is the unit vector that represents the direction of the } i^{th} \text{ reaction-wheel’s spin axis resolved in the } i^{th} \text{ wheel frame.}
\end{align*}

A. Error Dynamics

The objective of the attitude control problem is to determine control inputs such that the spacecraft attitude given

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by R follows a commanded attitude trajectory given by a possibly time-varying C1 rotation matrix \( R_d(t) \). For \( t \geq 0 \), \( R_d(t) \) is given by

\[
\begin{align*}
\dot{R}_d(t) &= R_d(t)\omega_d(t), \\
R_d(0) &= R_{d0},
\end{align*}
\]

where \( \omega_d \) is the desired, possibly time-varying angular velocity of the spacecraft relative to the inertial frame. The attitude error, that is, the rotation between \( R(t) \) and \( R_d(t) \), is given by

\[
\Delta R = R_d^T R,
\]

which satisfies Poisson’s equation

\[
\dot{\Delta R} = \Delta R \omega_d \times,
\]

where the angular-velocity error \( \Delta \omega \) is defined by

\[
\Delta \omega = \omega - \Delta R \omega_d.
\]

We rewrite (1) in terms of the angular-velocity error as

\[
\begin{align*}
J \dot{\Delta \omega} &= [J(\Delta \omega + \Delta R \omega_d) + B_{\text{sc}} c] \times (\Delta \omega + \Delta R \omega_d) \\
&+ J(\Delta \omega \times \Delta R \omega_d - \Delta R \Delta \omega_d) + B_{\text{sc}} c + z_{\text{dist}}.
\end{align*}
\]

### III. The RCAC Algorithm

RCAC is a discrete-time controller that minimizes the command-following error corresponding to the performance variable \( z \). The algorithm does not require detailed plant information; instead, RCAC uses knowledge of the Markov parameters of the transfer function \( G_{zu} \) which relates the input \( u \) to the performance \( z \).

Consider the MIMO discrete-time system

\[
\begin{align*}
x(k) &= Ax(k-1) + Bu(k-1), \\
y_0(k) &= E_1 x(k), \\
z(k) &= y_0(k) - E_0 r(k),
\end{align*}
\]

where \( x(k) \in \mathbb{R}^{l_x}, y_0(k) \in \mathbb{R}^{l_s}, z(k) \in \mathbb{R}^{l_z}, u(k) \in \mathbb{R}^{l_u}, \) and \( r(k) \in \mathbb{R}^{l_r} \), and \( k \geq 0 \). For \( i \geq 1 \), define the Markov parameter of \( G_{zu} \)

\[
\mathcal{H}_i \triangleq E_1 A^{i-1} B.
\]

Let \( n \) be a positive integer. Then, for all \( k \geq n \),

\[
x(k) = A^n x(k-n) + \sum_{i=1}^{n} A^{i-1} Bu(k-i)
\]

thus

\[
z(k) = E_1 A^n x(k-n) + HU(k-1) - E_0 r(k),
\]

where

\[
H \triangleq \begin{bmatrix} \mathcal{H}_1 & \cdots & \mathcal{H}_n \end{bmatrix} \in \mathbb{R}^{l_z \times n l_u}
\]

and

\[
U(k-1) \triangleq \begin{bmatrix} u(k-1) \\
\vdots \\
u(k-n) \end{bmatrix} \in \mathbb{R}^{n l_u}.
\]

We divide the matrices \( H \) and \( U(k-1) \) into known and unknown parts so that

\[
HU(k-1) = H''U''(k-1) + H'U'(k-1).
\]

where \( H' \in \mathbb{R}^{l_z \times l_u} \) and \( H'' \in \mathbb{R}^{l_z \times (l_u + l_s)} \) are the known and unknown Markov parameters respectively and \( U' \in \mathbb{R}^{l_1} \) and \( U'' \in \mathbb{R}^{l_1 \times (l_u + l_s)} \) are the corresponding control inputs. Next, we collect the unknown parts of the system into

\[
S(k) \overset{\triangleq}{=} E_1 A^n x(k-n) - E_0 r(k) + H''U''(k-1).
\]

and rewrite (16) using (17) as

\[
z(k) = S(k) + H'U'(k-1).
\]

#### A. Retrospective Cost

We replace the controls in \( U' \) in (19) with the retrospective controls \( \hat{U}' \) and define the retrospective performance,

\[
\dot{z}(k) \overset{\triangleq}{=} S(k) + H'\hat{U}'(k-1),
\]

we substract (19) from (20) and solve for \( \dot{z}(k) \) which yields

\[
\dot{z}(k) = z(k) - H'U'(k-1) + H'\hat{U}'(k-1).
\]

Then, we define the retrospective cost function

\[
J_z(\hat{U}'(k-1), k) \overset{\triangleq}{=} \dot{z}(k) R(k) \dot{z}(k),
\]

where \( R(k) \in \mathbb{R}^{l_z \times l_z} \) is a positive-definite performance weighting. The optimized control values \( \hat{U}'(k-1) \) are then used to compute the next control input \( u(k) \). Given a full column rank \( H' \), the global minimizer for the quadratic cost function in (22) is given by

\[
\hat{U}'(k-1) = \left[H'^T R(k) H'\right]^{-1} H'^T R(k) [H'U'(k-1) - z(k)].
\]

#### B. Controller Construction

We design a strictly proper time-series controller of order \( n_c \) which can be written as

\[
u(k) = \theta(k) \phi(k-1),
\]

where

\[
\theta(k) \overset{\triangleq}{=} [M_1(k) \cdots M_{n_c}(k) N_1(k) \cdots N_{n_c}(k)] \in \mathbb{R}^{l_u \times n_c(l_u + l_z)},
\]

for \( M_i(k) \in \mathbb{R}^{l_u \times l_u}, N_i(k) \in \mathbb{R}^{l_u \times l_z}, \) and

\[
\phi(k-1) \overset{\triangleq}{=} \begin{bmatrix} u(k-1) \\
\vdots \\
u(k-n_c) \\
z(k-1) \\
\vdots \\
z(k-n_c) \end{bmatrix} \in \mathbb{R}^{n_c(l_u + l_z)}.
\]
Next, we define the Recursive Least Squares cumulative cost function
\[ J_R(\theta(k)) \triangleq \left[ (\theta(k) - \theta(0)) P^{-1}(0) (\theta(k) - \theta(0)) \right]^T \]
\[ + \sum_{i=1}^{k} \| \phi^T(i - 2) \theta^T(k) - \hat{u}^T(i - 1) \|^2, \]  
(27)
where \( P(0) \in \mathbb{R}^{n_c(\ell_a + \ell_c) \times n_c(\ell_a + \ell_c)} \) is the initial covariance matrix. The minimizer for (27) is given by
\[ \theta^T(k) \triangleq \frac{\phi^T(k - 1) + P(k - 1) \phi(k - 2) \phi^T(k - 2) P(k - 1)}{[1 + \phi^T(k - 1) P(k - 1) \phi(k - 2)]}, \]  
(28)
where the error covariance \( P(k) \) is updated by
\[ P(k) \triangleq P(k - 1) - \frac{P(k - 1) \phi(k - 2) \phi^T(k - 2) P(k - 1)}{[1 + \phi^T(k - 1) P(k - 1) \phi(k - 2)]}. \]  
(29)

IV. RCAC Parameters for Attitude Control

RCAC requires a vector parameter variable, thus, the attitude-error \( \hat{\theta} \) given by Poisson’s equation (8) cannot be used directly. As in [7], we reformulate the attitude-error dynamics using the vector parameter \( \tilde{\theta} \) to compute the output matrices.

We rewrite Euler’s equation in (9) as
\[ J \ddot{\omega} = J^{-1} \left[ -\left( \dot{\omega}_e + D(\omega d) \dot{\tilde{r}}_e \right)^X J \right] \]
\[ + \left( J \left( \dot{\omega}_e + D(\omega d) \dot{\tilde{r}}_e \right) + B_{sc} \nu \right)^X D(\omega d) \]
\[ + \omega_e^X D(\omega d) - D(\omega d), \]  
(36)
\[ \frac{\partial \tilde{r}}{\partial \nu} = J^{-1} \left[ \dot{\omega}_e + D(\omega d) \dot{\tilde{r}}_e \right]^X B_{sc}. \]  
(38)

We linearize the system by computing a Jacobian about the equilibrium \( x_e^T = [ \dot{\omega}_e^T \dot{\tilde{r}}_e^T \nu_e^T ]^T \). The derivative of (34) with respect to \( \dot{\omega}, \dot{\tilde{r}}, \) and \( \nu \) are
\[ \frac{\partial \dot{r}_i}{\partial \omega} = J^{-1} \left[ -\dot{\omega}_e + D(\omega d) \dot{\tilde{r}}_e \right]^X J \]
\[ + \left( J \left( \dot{\omega}_e + D(\omega d) \dot{\tilde{r}}_e \right) + B_{sc} \nu \right)^X D(\omega d) \]
\[ + \dot{\omega}_e^X D(\omega d) - D(\omega d), \]  
(37)
\[ \frac{\partial \dot{r}_i}{\partial \nu} = J^{-1} \left[ \dot{\omega}_e + D(\omega d) \dot{\tilde{r}}_e \right]^X B_{sc}. \]  
(39)

We rewrite the derivative for each row \( \dot{r}_i \) of \( \tilde{\theta} \) as
\[ \dot{r}_i = (\dot{\tilde{r}}_i \times \dot{\omega})^T. \]  
(39)

Then, the derivatives of (39) with respect to \( \dot{\omega}, \dot{\tilde{r}}, \) and \( \nu \) are,
\[ \frac{\partial \dot{r}_i}{\partial \omega} = -\dot{\omega}_e^X \delta_{e,i}, \]  
(40)
\[ \frac{\partial \dot{r}_i}{\partial \nu} = -\dot{\omega}_e^X \delta_{e,i} + \dot{\tilde{r}}_i \frac{\partial \omega}{\partial \nu}, \]  
(41)
\[ \frac{\partial \dot{r}_i}{\partial \nu} = 0, \]  
(42)
where
\[ \frac{\partial \dot{r}_i}{\partial \omega} = \left\{ \begin{array}{ll} I_3, & i = j, \\ 0_{3 \times 3}, & i \neq j. \end{array} \right. \]  
(43)
\[ \frac{\partial \dot{r}_i}{\partial \nu} = \left\{ \begin{array}{ll} D(\omega d) \left[ \begin{array}{ccc} I_3 & 0 & 0 \end{array} \right]^T, & j = 1, \\ D(\omega d) \left[ \begin{array}{ccc} 0 & I_3 & 0 \end{array} \right]^T, & j = 2, \\ D(\omega d) \left[ \begin{array}{ccc} 0 & 0 & I_3 \end{array} \right]^T, & j = 3. \end{array} \right. \]  
(44)

Finally, the Jacobian for the reaction-wheel dynamics (3) is
\[ \frac{\partial \nu}{\partial x} = 0. \]  
(45)

We construct a state vector by stacking the angular-velocity error \( \dot{\omega} \), the attitude-error \( \dot{\tilde{r}} \), and the reaction-wheel angular-rates \( \nu \). We define our system using this state vector and the rewritten error dynamics in (33) and (34) combined with the reaction wheel dynamics in (3).

We rewrite Euler’s equation in (9) as
\[ J \ddot{\omega} = J^{-1} \left[ -\left( \dot{\omega}_e + D(\omega d) \dot{\tilde{r}}_e \right)^X J \right] \]
\[ + \left( J \left( \dot{\omega}_e + D(\omega d) \dot{\tilde{r}}_e \right) + B_{sc} \nu \right)^X D(\omega d) \]
\[ + \dot{\omega}_e^X D(\omega d) - D(\omega d), \]  
(36)
\[ \frac{\partial \dot{r}_i}{\partial \nu} = J^{-1} \left[ \dot{\omega}_e + D(\omega d) \dot{\tilde{r}}_e \right]^X B_{sc}. \]  
(38)

We rewrite the attitude parameter \( S \) in terms of the attitude state \( \dot{\tilde{r}} \) to compute the output matrices.
\[ S = -M_a R_a \dot{\tilde{r}}, \]  
(45)
where
\[ M_a \triangleq \begin{bmatrix} e_1^x & e_2^x & e_3^x \end{bmatrix}, \quad R_a \triangleq \begin{bmatrix} a_1 I_3 & a_2 I_3 & a_3 I_3 \end{bmatrix}. \]

Thus,
\[ C = \begin{bmatrix} I_3 & 0_{3\times9} & 0_{3\times nw} \\ 0_{3\times3} & M_a & 0_{3\times nw} \end{bmatrix}. \quad (46) \]

We obtain the discrete-time dynamics matrix from
\[ A = e^{A_c h}, \quad (47) \]
where \( A_c \) is the Jacobian constructed from the derivatives in (36) - (43) and \( h \) is the controller time step. The discrete-time input matrix is given by
\[ B = \int_0^h e^{A_c t} dTB_c. \quad (48) \]

The discrete-time output matrices are \( E_1 = C \) and \( D_d = 0 \).

V. NUMERICAL EXAMPLES

Consider a rigid body whose inertia matrix relative to its center of mass resolved in the body frame is
\[ J_0 = \begin{bmatrix} 5 & -0.1 & -0.5 \\ -0.1 & 2 & 1 \\ -0.5 & 1 & 3.5 \end{bmatrix} \text{ kg-m}^2. \quad (49) \]

We use \( n_{ww} = 3 \) reaction wheels whose rotational-axis moments of inertia are \( \alpha_0 = 0.1 \text{ kg-m}^2 \) for \( i = 1, 2, 3 \). The wheel spin axes are chosen such that \( e_{Wi} = e_1 \), for \( i = 1, 2, 3 \), are aligned orthogonally. Thus, the actuator matrix \( B_{sc} \) in (10) becomes
\[ B_{sc} = -0.1 I_3. \quad (50) \]

The spacecraft’s initial motion is described by
\[ \omega(0) = \frac{0.1}{\sqrt{3}} \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^T \text{ rad/sec} \quad (51) \]
with the reaction-wheels initially at rest so that
\[ \nu_i(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \text{ rad/sec, for } i = 1, 2, 3. \quad (52) \]

The spacecraft’s initial attitude is given by \( R(0) = I_3 \). The goal of the controller is to bring the spacecraft to rest with an attitude corresponding to an eigenaxis rotation of \( 40^\circ \) about the body direction \( [1 \quad 1 \quad 1]^T \).

The controller order is set to \( n_c = 3 \), the recursive cost weighting \( R = I \), the initial controller coefficients and the initial covariance are set to \( \theta(0) = 0 \) and \( P(0) = 100 I \), and the attitude parameter weights for \( S \) in (30) are given by \( a_1 = 1, a_2 = 2, \) and \( a_3 = 3 \). To compute the baseline Markov parameter, we compute the Jacobian about the equilibrium \( \omega_e = 0, \dot{R}_e = I_3, \nu_e = 0 \). Then, the first Markov parameter for the linearized system is
\[ \hat{\mathcal{H}}_1 \triangleq E_1 B = \begin{bmatrix} h J^{-1} B_{sc} \end{bmatrix}. \quad (53) \]

As in [7], we saturate the control input proportionally, we limit the magnitude of the angular acceleration of each wheel to 1 rad/sec^2.

A. Robustnes to Inertia Uncertainty

To evaluate the robustness of RCAC to inertia uncertainty, we examine the relationship between the Markov parameter and the spacecraft and reaction wheel inertias. We introduce two inertia-free Markov parameters \( \hat{\mathcal{H}}_1 \) and \( \hat{\mathcal{H}}_1' \). The first inertia-free Markov parameter \( \hat{\mathcal{H}}_1 \), does not have information about the spacecraft inertia \( J \), that is
\[ \hat{\mathcal{H}}_1 \triangleq \begin{bmatrix} h B_{sc} \\ \frac{1}{2} h^2 M_a R_a M_a^T B_{sc} \end{bmatrix}. \quad (54) \]

The second inertia-free Markov parameter, \( \hat{\mathcal{H}}_1' \) does not contain information about either the spacecraft inertia \( J \) or the reaction wheel inertias \( \alpha_0 \), thus
\[ \hat{\mathcal{H}}_1' \triangleq \begin{bmatrix} h B_{sc} \\ \frac{1}{2} h^2 M_a R_a M_a^T \tilde{B}_{sc} \end{bmatrix}, \quad (55) \]
where the columns of the inertia-free \( \tilde{B}_{sc} \) are given by
\[ \tilde{b}_i = -R_W e_{Wi}. \quad (56) \]

Thus, the Markov parameter \( \hat{\mathcal{H}}_1' \) is inertia-free but uses the actuator alignment information in \( \tilde{B}_{sc} \).

Figure 1 compares the eigenaxis attitude error,
\[ \theta_{eig} \triangleq \cos^{-1} \frac{1}{2} \left( \text{tr} \hat{R} - 1 \right), \quad (57) \]
using the baseline and inertia-free Markov parameters \( \mathcal{H}_1 \), \( \hat{\mathcal{H}}_1 \), and \( \hat{\mathcal{H}}_1' \). Figure 1 shows that the removal of the spacecraft inertia information affects the steady state error. When the reaction wheel inertia is also removed, the settling time approximately doubles.

Next, we vary the scaling of the inertia matrix and compare the settling time for various spacecraft inertias \( J = \beta J_0 \) for \( \beta > 0 \). To evaluate the closed-loop performance, we define the settling time \( t_{ss} \) as the time needed to bring the eigenaxis attitude-error within a bound \( \theta_{ss} \), that is
\[ \theta_{eig}(t) \leq \theta_{ss}, \quad \text{for all } t \geq t_{ss}. \]
For the following examples $\theta_{ss} = 1^\circ$ and we use the inertia-free Markov parameter $\hat{H}'_1$.

Figure 2 shows that, for the Markov parameter $\hat{H}'_1$, the settling time increases as the inertia of the spacecraft increases. This result is expected and is due to the decrease of control authority of the fixed-size reaction wheels. As the spacecraft inertia increases, the reaction wheels have a smaller effect on the angular acceleration $\dot{\omega}$. However, the steady state error is less than $1 \times 10^{-7}$ radians and the settling time is within 200 seconds which is acceptable for many spacecraft applications.

### B. Frame Rotation and Actuator Misalignment

We evaluate the robustness of RCAC to unknown rotations for the M2R maneuver. First, we examine the M2R settling time as the actuator and sensor axes are rotated according to

$$J_{sc} = R J_0 R^T,$$

where $J_0$ is the nominal inertia matrix and $R(\phi, \hat{n}) \in \mathbb{R}^{3 \times 3}$ is the rotation matrix given by Rodrigues’s formula

$$R(\phi, \hat{n}) = \cos(\phi) I_3 + [1 - \cos(\phi)] \hat{n} \hat{n}^T + \sin(\phi) \hat{n} \times,$$

where $\phi$ is the misalignment angle and $\hat{n}$ is the misalignment axis. Figure 3 shows that RCAC can complete the M2R maneuver for all single-axis rotations of the actuator and sensor axes.

Next, we misalign the reaction-wheels by applying an unknown rotation $\hat{R}$ according to (59) so that the columns of the actuator matrix $\hat{B}_{sc}$ become

$$b_i = -\alpha_i \hat{R} W_i e W_i.$$  

We vary the angle $\phi$ and consider the misalignment axes

$$\hat{n}_1 = e_1, \quad \hat{n}_2 = e_2, \quad \hat{n}_3 = e_3.$$  

We use the Markov parameter $\hat{H}'_1$, which does not assume knowledge of the misalignment information $\hat{R}$. Figure 4 shows that, as the misalignment angle increases, RCAC requires more time to settle, which reflects the lack of alignment information in the Markov parameter $\hat{H}'_1$.

### C. Constant Disturbance Torque

We examine the performance of RCAC under the presence of a disturbance torque that is inertially constant such as solar radiation pressure. We test three different cases using the constant magnitude disturbance

$$z_{dist} = R^T d_0 \hat{n}_d,$$

where $d_0$ is the disturbance magnitude and $\hat{n}$ is the disturbance vector direction resolved in the inertial frame. Figure 5 shows the eigenaxis attitude error for a disturbance of magnitude $d_0 = 1 \times 10^{-4}$ N-m with various inertial directions.

### D. Noise and Bias

We introduce noise and bias into the angular-velocity measurement and examine the impact on the settling time. Let the measured angular velocity be given by

$$\dot{y}_\omega = \omega + b + \nu_\omega,$$

where $b \in \mathbb{R}^3$ is a constant bias and $\nu_\omega$ is white Gaussian noise. Figure 6 shows the eigenaxis attitude error when the
angular velocity measurement is corrupted by a constant biases $b = b_0 \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ and Gaussian white noise with variance $\sigma_b^2 = 10^{-12} \text{rad/sec}$. Since the angular-velocity measurement is included in the performance variable, the steady-state error increases as the bias magnitude increases.

VI. CONCLUSION

RCAC was shown to achieve attitude command following for motion-to-rest maneuvers using reaction-wheel actuators. Dependence on the inertia of the spacecraft and reaction wheels was removed from the nominal Markov parameter. Thus, an inertia-free controller was utilized to achieve command following for spacecraft under various off-nominal conditions. The spacecraft and reaction wheel inertia free Markov parameter enabled the controller to reach the desired attitude with an acceptable steady state error and settling time. We demonstrated through simulations robustness to scaling of the spacecraft inertia demonstrated as well as to unknown single-axis misalignments of the reaction wheel spin axes. Furthermore, the algorithm can handle unknown inertially constant disturbances. Finally, we introduced Gaussian white noise and bias in the angular-velocity measurement and showed that the controller is stable can reach the desired attitude. Extensions of the reaction wheel case will focus on additional wheel configurations such as nonorthogonal wheel arrangements and the four wheel pyramid commonly used for redundancy. We are also interested in the problem of momentum dumping using a combination of thrusters and reaction wheels. Future applications will examine RCAC’s ability to control spacecraft systems with alternative actuators such as control moment gyros and magnetic torquers. These actuators have a time-varying actuator matrix which, in the magnetic case, is singular. Also of interest are nonrigid spacecraft with time-varying inertia properties such as to actuated solar arrays, as well as spacecraft with flexible bodies such as gravity gradient booms and deployable antennas.

A mathematical proof of the algorithm’s stability is still under development. The version of RCAC used in this paper can be extended to utilize a time history of control input and performances. We will examine the effect of using more measurements on the steady state error and settling time. Further work will examine other spacecraft inertias as well as thorough testing of initial conditions and set points. However, numerical simulations indicate that RCAC can effectively control the nonlinear rigid body spacecraft with reaction wheels and is robust to common off-nominal conditions.

REFERENCES