ARMARKOV Least-Squares Identification

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Abstract

In recent work, ARMARKOV representations have been proposed as an extension of ARMA representations of finite-dimensional linear time-invariant systems. ARMARKOV representations have the same form as ARMA representations, but explicitly involve Markov parameters. This paper generalizes ARMA least-squares time-domain identification to ARMARKOV representations. The ARMARKOV/least-squares identification algorithm is used to estimate the Markov parameters of a linear time-invariant system from measurements of the inputs and outputs. The eigensystem realization algorithm is then used to construct a minimal realization. A numerical example involving a second-order lightly damped system illustrates the decreased sensitivity of the eigenvalues to the Markov parameters of a perturbed ARMARKOV representation compared to the Markov parameters of a perturbed ARMA representation. Finally, using experimental data, the dynamics of an acoustic duct are identified using the ARMA/least-squares identification algorithm to obtain a transfer function representation and the ARMARKOV/least-squares identification algorithm with ERA to obtain a minimal realization.

1. Introduction

Time-domain identification of linear time-invariant finite-dimensional discrete-time systems using a least-squares algorithm has traditionally used an ARMA representation [7, pp. 176-178] [8, pp. 60-63]. We refer to this algorithm as the *ARMA/LS identification algorithm*. In this paper we propose to use the least-squares algorithm with the ARMARKOV representation first introduced by Hyland [2]. We refer to this algorithm as the *ARMARKOV/LS identification algorithm*. The ARMARKOV representation relates the current output of a system to past outputs as well as current and past inputs. While the ARMARKOV representation has the same form as an ARMA representation, the ARMARKOV representation explicitly contains Markov parameters of the system. For details on ARMARKOV representations see [1]. When the ARMARKOV representation contains more than one Markov parameter, the ARMARKOV representation can be viewed as an overparameterized and structurally constrained ARMA representation.

A widely used identification technique is the eigensystem realization algorithm (ERA) [4] [5, pp. 133-137] which uses Markov parameters to obtain a state space realization of the system. Estimates of the Markov parameters are used to construct a block-Hankel matrix whose singular value decomposition provides an estimate of the system order and which is used to construct state space realizations.

The ARMARKOV/LS identification algorithm uses vectors comprised of input-output data with a least-squares criterion to estimate a weight matrix containing a specified number of Markov parameters of the system. This algorithm provides the first-step in a two-step identification algorithm, where ERA is used as the second-step to construct a minimal realization from the estimated Markov parameters. The estimated Markov parameters are extracted from the estimated weight matrix and used to construct a Markov block Hankel matrix which in turn is used within ERA to obtain a minimal realization. We refer to this two-step algorithm as the *ARMARKOV/LS/ERA identification algorithm.*

The ARMARKOV/LS/ERA identification algorithm has three advantages compared to the ARMA/LS identification algorithm. First, as shown in Section 5, the eigenvalues are less sensitive to the Markov parameters of a perturbed ARMARKOV representation than to the Markov parameters of a perturbed ARMA representation. Second, the singular value decomposition of a block Hankel matrix constructed from the estimated Markov parameters provides an efficient model order indicator [4] [5, p. 139]. Third, the ARMA representation requires selection of the relative degree which is not required with the ARMARKOV representation. Note that only the Markov parameters obtained from the ARMARKOV/LS identification algorithm are used by ERA to construct a state space realization. Therefore, errors in the remaining parameters of the ARMARKOV representation have no effect on the identified model.

While the ARMARKOV/LS/ERA identification algorithm in this paper is developed for single-input

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single-output (SISO) systems, it can be used to identify multi-input multi-output (MIMO) systems. For MIMO systems the ARMARKOV/LS identification algorithm is used to estimate the Markov parameters of every input-output pair. These SISO Markov parameters are then assembled into the MIMO Markov parameters which are used within ERA to construct a MIMO state space realization.

Section 2 gives a brief review of ARMARKOV representations of linear finite-dimensional discrete-time systems which include ARMA representations as a special case. Section 3 introduces the ARMARKOV/LS identification algorithm. Section 4 provides a brief review of the ERA algorithm. In Section 5 the sensitivity of the eigenvalues of a second-order SISO system to Markov parameters of perturbed ARMARKOV representations and perturbed ARMA representations is illustrated. In section 6 the ARMA/LS identification algorithm and the ARMARKOV/LS/ERA identification algorithm are used with experimental data to identify the dynamics of an acoustic duct. Concluding remarks are given in Section 7.

2. ARMARKOV Representations

For a detailed description of ARMARKOV representations see [1]. Consider the discrete-time finite-dimensional linear time-invariant SISO system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), &(2.1)\\ y(k) &= Cx(k) + Du(k), &(2.2)\end{aligned}$$

where $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times 1}$, $C \in \mathcal{R}^{1\times n}$, and $D \in \mathcal{R}$. Let $G(z) \sim \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ denote the transfer function corresponding to the state space realization (2.1) and (2.2). The notation " $\overset{\min}{\sim}$ " denotes a minimal realization. The *Markov parameters* H_j are defined by

$$\begin{aligned} H_j &\triangleq D, \qquad j=-1, &(2.3)\\ &\triangleq CA^jB, \qquad j \ge 0, \end{aligned}$$

and satisfy

$$G(z) \triangleq C(zI-A)^{-1}B + D = \sum_{j=-1}^{\infty} H_j z^{-(j+1)}. \qquad (2.4)$$

We refer to $G(z) = \sum_{j=-1}^{\infty} H_j z^{-(j+1)}$ as the *Markov parameter representation of* $G(z)$.

The ARMA transfer function representation of $G(z)$ is given by

$$G(z) = \frac{b_0 z^n + b_1 z^{n-1} + \cdots + b_n}{z^n + a_1 z^{n-1} + \cdots + a_n}, \qquad (2.5)$$

where $\det(zI-A) = z^n + a_1 z^{n-1} + \cdots + a_n$ and $b_i \in \mathcal{R}$, $i = 0,\ldots,n$. The ARMA time-domain representation of $G(z)$ corresponding to (2.5) is given by

$$\begin{aligned} y(k) &= -a_1 y(k-1) - \cdots - a_n y(k-n) + b_0 u(k) \\ &\quad + \cdots + b_n u(k-n)\ , \quad k \ge 0. &(2.6)\end{aligned}$$

The coefficients of the ARMA representation and Markov parameters of $G(z)$ satisfy

$$\begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} H_{-1} & 0 & \ldots & 0 \\ H_0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ H_{n-1} & \ldots & H_0 & H_{-1} \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} \qquad (2.7)$$

and

$$H_{n+j} = -\sum_{i=1}^{n} a_i H_{n+j-i}\ , \quad j \ge 0. \qquad (2.8)$$

The *ARMARKOV transfer function representation* of $G(z)$ with μ Markov parameters is given by

$$G(z) = \frac{H_{-1}z^{\mu+n-1} + \cdots + H_{\mu-2}z^n + \beta_{\mu,1}z^{n-1} + \cdots + \beta_{\mu,n}}{z^{\mu+n-1} + \alpha_{\mu,1}z^{n-1} + \cdots + \alpha_{\mu,n}}, \qquad (2.9)$$

and the *ARMARKOV time-domain representation* of $G(z)$ is given by

$$\begin{aligned} y(k) &= \sum_{j=1}^{n} -\alpha_{\mu,j}\, y(k-\mu-j+1) \\ &\quad + \sum_{j=1}^{\mu} H_{j-2} u(k-j+1) \\ &\quad + \sum_{j=1}^{n} \beta_{\mu,j}\, u(k-\mu-j+1). &(2.10)\end{aligned}$$

which involves the first μ Markov parameters $H_{-1},\ldots,H_{\mu-2}$, and where $\alpha_{\mu,1},\ldots,\alpha_{\mu,n} \in \mathcal{R}$ and $\beta_{\mu,1},\ldots,\beta_{\mu,n} \in \mathcal{R}$ are functions of the ARMA coefficients and Markov parameters. Note, however, that the ARMARKOV transfer function representation is not equivalent to an arbitrary nonminimal ARMA transfer function representation since the coefficients of $z^{\mu+n-2},\ldots,z^n$ in the denominator are constrained to be zero. Furthermore, note that the ARMA time-domain representation (2.6) is a specialized ARMARKOV time-domain representation (2.10) with $\mu = 1$.

Defining the *ARMARKOV regressor vector* $\Phi_\mu(k) \in \mathcal{R}^{2n+\mu}$ by

$$\Phi_\mu(k) \triangleq \begin{bmatrix} y(k-\mu) \\ \vdots \\ y(k-\mu-n+1) \\ u(k) \\ \vdots \\ u(k-\mu-n+1) \end{bmatrix}, \qquad (2.11)$$

it follows that

$$y(k) = W_\mu \Phi_\mu(k), \qquad (2.12)$$

where the *ARMARKOV weight matrix* W_μ is defined by

$$W_\mu \triangleq [\, -\mathcal{A}_\mu \quad H_{-1} \quad \cdots \quad H_{\mu-2} \quad \mathcal{B}_\mu\,], \qquad (2.13)$$

where

$$\mathcal{A}_\mu \triangleq [\,\alpha_{\mu,1} \quad \cdots \quad \alpha_{\mu,n}\,] \in \mathcal{R}^{1\times n},$$
$$\mathcal{B}_\mu \triangleq [\,\beta_{\mu,1} \quad \cdots \quad \beta_{\mu,n}\,] \in \mathcal{R}^{1\times n}.$$

Note that the ARMARKOV regressor vector (2.11) becomes the standard ARMA regressor vector and the ARMARKOV weight matrix (2.13) contains the ARMA coefficients when $\mu = 1$.

Henceforth, for convenience we omit the subscript μ and write W and $\Phi(k)$ for W_μ and $\Phi_\mu(k)$.

3. ARMARKOV/LS Identification Algorithm

Let $\widehat{W}$ denote an estimate of the ARMARKOV weight matrix and let $\widehat{y}(k)$ denote the *estimated output* defined by

$$\widehat{y}(k) \triangleq \widehat{W}\Phi(k). \tag{3.1}$$

Furthermore, define the *output error* $\varepsilon(k)$ by

$$\varepsilon(k) \triangleq y(k) - \widehat{y}(k), \tag{3.2}$$

and the *output error cost function* J by

$$J \triangleq \frac{1}{N}\sum_{k=1}^{N}\frac{1}{2}\varepsilon^2(k), \tag{3.3}$$

where N is the number of measurements. The following result provides the ARMARKOV/LS identification algorithm.

Proposition 3.1 Suppose $\sum_{k=1}^{N}\Phi(k)\Phi^{\mathrm{T}}(k)$ is nonsingular. Then $\widehat{W}$ is a strict minimizer of J if and only if

$$\widehat{W} = \left[\frac{1}{N}\sum_{k=1}^{N}\Phi^{\mathrm{T}}(k)y(k)\right]\left[\frac{1}{N}\sum_{k=1}^{N}\Phi(k)\Phi^{\mathrm{T}}(k)\right]^{-1}. \tag{3.4}$$

It can be seen that $\frac{1}{N}\sum_{k=1}^{N}\Phi(k)\Phi^{\mathrm{T}}(k)$ contains the covariance estimates of $u(k)$ and $y(k)$. Furthermore, if $\mu = 1$ then (3.4) yields the well known ARMA/LS identification algorithm.

4. Minimal Realizations from Markov Parameters

For positive integers r and s and for $j \geq -1$ the *Markov block Hankel matrix* $\mathcal{H}_{r,s,j} \in \mathcal{R}^{(r+1)l\times(s+1)m}$ is defined by

$$\mathcal{H}_{r,s,j} \triangleq \begin{bmatrix} H_j & \dots & H_{j+s} \\ \vdots & \ddots & \vdots \\ H_{j+r} & \dots & H_{j+r+s} \end{bmatrix}. \tag{4.1}$$

We first recall a well-known result concerning the rank of $\mathcal{H}_{r,s,0}$ [6, p. 442].

Lemma 4.1 Assume $G(z)$ has McMillan degree n, and let $r, s \geq n-1$. Then rank $\mathcal{H}_{r,s,0} = n$.

For convenience let 0_l and $0_{l\times m}$ denote the $l\times l$ and $l\times m$ zero matrices, respectively, and let I_l and $1_{l\times m}$ denote the $l\times l$ identity matrix and $l\times m$ ones matrix, respectively. Furthermore, let $E_{i,j} \triangleq \begin{bmatrix} I_j \\ 0_{ij\times j}\end{bmatrix}$. For $s \geq 1$ the *s-stage controllability* and *observability Gramians* $W_{\mathcal{C}_s}$ and $W_{\mathcal{O}_s}$ are defined by

$$W_{\mathcal{C}_s} \triangleq \sum_{k=0}^{s} A^k BB^{\mathrm{T}}A^{k\mathrm{T}}, \quad W_{\mathcal{O}_s} \triangleq \sum_{k=0}^{s} A^{k\mathrm{T}}C^{\mathrm{T}}CA^k.$$

The following result is the eigensystem realization algorithm [4] [5, pp. 133-137].

Proposition 4.1 Let $G(z)$ have McMillan degree n, and let $r, s \geq n-1$. Furthermore, let $P \in \mathcal{R}^{(r+1)l\times n}$, $\Sigma_{r,s} \in \mathcal{R}^{n\times n}$, and $Q \in \mathcal{R}^{n\times(s+1)m}$ satisfy

$$\mathcal{H}_{r,s,0} = P\Sigma_{r,s}Q, \tag{4.2}$$

where $P^{\mathrm{T}}P = QQ^{\mathrm{T}} = I$ and $\Sigma_{r,s} = \mathrm{diag}(\sigma_1^{r,s}, \dots, \sigma_n^{r,s})$, where $\sigma_1^{r,s} \geq \cdots \geq \sigma_n^{r,s} > 0$ are the singular values of $\mathcal{H}_{r,s,0}$. Then

$$G(z) \overset{\min}{\sim} \left[\begin{array}{c|c} \Sigma_{r,s}^{-1/2}P^{\mathrm{T}}\mathcal{H}_{r,s,1}Q^{\mathrm{T}}\Sigma_{r,s}^{-1/2} & \Sigma_{r,s}^{1/2}QE_{s,m} \\ \hline E_{r,l}^{\mathrm{T}}P\Sigma_{r,s}^{1/2} & H_{-1} \end{array}\right]. \tag{4.3}$$

Moreover, this realization is (r, s)-finitely balanced with

$$W_{\mathcal{C}_s} = W_{\mathcal{O}_r} = \Sigma_{r,s}. \tag{4.4}$$

For a system of McMillan degree n, ERA requires $r, s \geq n-1$. Note that for $r, s = n-1$ the highest indexed Markov parameter in $\mathcal{H}_{r,s,1}$ is $H_{1+r+s} = H_{2n-1}$. Therefore, ERA requires at least the first $2n+1$ Markov parameters in order to identify a system of McMillan degree n. Hence μ must be chosen to satisfy $\mu \geq 2n+1$ to guarantee that the ARMARKOV/LS identification algorithm produces a sufficient number of Markov parameters to apply Proposition 4.1.

In practical applications the rank of $\mathcal{H}_{r,s,0}$ will be greater than the system order due to measurement noise and other effects. In this case the magnitude of the singular values of $\mathcal{H}_{r,s,0}$ estimates the system order. Truncating the smallest singular values of $\mathcal{H}_{r,s,0}$ yields a reduced-order realization that retains the dominant dynamic characteristics of the system [4, 5].

5. Numerical Example

In this section we present a numerical example by which we compare the sensitivity of the eigenvalues of an ARMA representation to the sensitivity of the eigenvalues of a state space realization obtained from ERA based entirely upon the Markov parameters. The numerical example involves the continuous-time oscillator given by

$$G(s) = \frac{\omega_{\rm n}^2}{s^2 + 2\zeta\omega_{\rm n} s + \omega_{\rm n}^2}\,, \tag{5.1}$$

with a natural frequency $f_{\rm n} = 10$ Hz ($\omega_{\rm n} = 6.28$ rad/sec) and a damping ratio $\zeta = 1\%$. An equivalent zero-order-hold discrete-time ARMA transfer function representation of (5.1) with a sampling frequency of 100 Hz is given by

$$G(z) = \frac{1.9019\times 10^{-1} z + 1.8939\times 10^{-1}}{z^2 - 1.6079 z + 9.8751\times 10^{-1}}. \tag{5.2}$$

First we consider a 1% relative error in all of the coefficients of the ARMA representation (5.2). Let $\widehat{G}(z)$ denote the perturbed ARMA representation given by

$$\widehat{G}(z) = \frac{\widehat{b}_1 z + \widehat{b}_2}{z^2 + \widehat{a}_1 z + \widehat{a}_2}, \tag{5.3}$$

where

$$\widehat{a}_j = a_j\,(1 + 0.01 w_j)\ ,\ \ j = 1, 2, \tag{5.4}$$
$$\widehat{b}_j = b_j\,(1 + 0.01 w_{j+2})\ ,\ \ j = 1, 2, \tag{5.5}$$

and w_j, $j = 1, \ldots, 4$, are uncorrelated random variables uniformly distributed on $[-1, 1]$. The maximum and minimum natural frequency and damping ratio calculated from the poles of $\widehat{G}(z)$ for 100 trials are $f_{\rm n,min} = 9.6342$, $f_{\rm n,max} = 1.0341\times 10^1$, $\zeta_{\rm min} = -5.3988\times 10^{-3}$, and $\zeta_{\rm max} = 2.1972\times 10^{-2}$. Note that the minimum damping ratio is negative and thus some of the perturbed ARMA representations are unstable. Alternatively let $\widehat{H}_j$, $j \geq -1$, denote the Markov parameters that are obtained from the coefficients (5.4) and (5.5) of the perturbed ARMA representation (5.3) using (2.7) and (2.8). Applying ERA with these perturbed Markov parameters yields exactly the same results as above. Thus, generating perturbed Markov parameters from a perturbed ARMA representation and applying ERA to these perturbed Markov parameters provides no benefits over using the coefficients of the perturbed ARMA representation.

Next we consider a 1% relative error in all of the coefficients of an ARMARKOV transfer function representation. The perturbed Markov parameters appearing in the numerator of the perturbed ARMARKOV representation are used with ERA to construct a state space realization. Note that the first ten (unperturbed) Markov parameters of (5.2), which are given by

$$\begin{array}{llll}
H_{-1} = & 0\ , & H_4 = & 1.8594\times 10^{-1}\ ,\\
H_0 = & 1.9019\times 10^{-1}\ , & H_5 = & -1.8422\times 10^{-1}\ ,\\
H_1 = & 4.9520\times 10^{-1}\ , & H_6 = & -4.7983\times 10^{-1}\ ,\\
H_2 = & 6.0843\times 10^{-1}\ , & H_7 = & -5.8962\times 10^{-1}\ ,\\
H_3 = & 4.8931\times 10^{-1}\ , & H_8 = & -4.7423\times 10^{-1}\ ,
\end{array}$$

are of the same order of magnituded as the coefficients of the ARMA transfer function representation (5.2). Since increasing r and s improves the robustness of ERA to perturbed Markov parameters, r, s are chosen to be 50 and thus ERA requires the first 103 Markov parameters $\widehat{H}_{-1}, \ldots, \widehat{H}_{101}$. Now let $\widehat{G}(z)$ denote the perturbed ARMARKOV representation with $\mu = 103$ given by

$$\widehat{G}(z) = \frac{\widehat{H}_{-1} z^{104} + \cdots + \widehat{H}_{101} z^2 + \widehat{\beta}_1 z + \widehat{\beta}_2 z}{z^{104} + \widehat{\alpha}_1 z + \widehat{\alpha}_2},$$

where

$$\begin{aligned}
\widehat{\alpha}_j &= \alpha_j\,(1 + 0.01 w_j)\ ,\ \ j = 1, 2,\\
\widehat{H}_j &= H_j\,(1 + 0.01 w_{j+2})\ ,\ \ j = -1, \ldots, 101,\\
\widehat{\beta}_j &= \beta_j\,(1 + 0.01 w_{j+105})\ ,\ \ j = 1, 2,
\end{aligned}$$

and w_j, $j = 1, \ldots, 107$, are uncorrelated random variables uniformly distributed on $[-1, 1]$. The maximum and minimum natural frequency and damping ratio of second-order realizations constructed using ERA for 100 trials is $f_{\rm n,min} = 9.9994$, $f_{\rm n,max} = 1.0001\times 10^1$, $\zeta_{\rm min} = 9.8671\times 10^{-3}$, and $\zeta_{\rm max} = 1.0121\times 10^{-2}$. These numerical results show that the eigenvalues obtained from the Markov parameters of a perturbed ARMARKOV representation are much less sensitive than the eigenvalues obtained from the Markov parameters of a perturbed ARMA representation. Similar sensitivity results (not shown) are obtained based upon an absolute error of 0.01.

6. Identification of an Acoustic Duct

In this section the ARMA/LS identification algorithm and the ARMARKOV/LS/ERA identification algorithm are used to identify the dynamics of an acoustic duct. The acoustic duct is constructed from a 19.75 foot long 4 inch diameter PVC pipe with open-closed boundary conditions and a colocated microphone and speaker mounted on the side. The speaker input and microphone output were recorded at a sampling frequency of 1024 Hz with a time-record length of 4096 data points spanning 4.0 seconds. The input $u(k)$ was chosen to be white noise. The experimentally measured frequency response was obtained using a spectrum analyzer with the frequency range chosen to be 0 - 400 Hz with 1601 spectral lines of resolution. Hence the experimentally measured frequency response is an estimate of the frequency response of the duct at the discrete frequencies $0, 0.25, 0.5, \ldots, 399.5, 399.75, 400$ Hz.

First we used the ARMA/LS identification algorithm to obtain a transfer function representation of the dynamics of the acoustic duct. Based upon an analytical model of the acoustic duct [3] the system order was estimated to be approximately 40. The model order of the ARMA representation was varied from 20 to 50 and the relative degree was varied from 0 to 10. All of the ARMA representations with model order greater than 30 are unstable and have frequency responses that poorly match the experimentally measured frequency response. The best ARMA representation is 30th-order. The frequency response of the 30th-order ARMA representation and the experimental frequency response is shown in Figure 1. It can be seen that the 30th-order ARMA representation is a poor approximation of the duct dynamics above 250 Hz.

Next the ARMARKOV/LS/ERA identification algorithm was used to obtain a minimal realization of the dynamics of the acoustic duct. Since the system order was estimated to be approximately 40, μ was chosen to be 210 and $r, s = 100$. The singular value decomposition of $\mathcal{H}_{100,100,0}$ confirms the system order to be approximately 40, hence the choice of $r, s = 100$ is appropriate. The 210 estimated Markov parameters were used within ERA to obtain a minimal realization. The model order within ERA was varied from 30 to 50 with the 39th-order realization being the best with respect to its frequency response matching the experimentally measured frequency response. The frequency response of the 39th-order realization is shown in Figure 2. It can be seen that the magnitude of the frequency response of the 39th-order realization provides a good match of the experimentally measured magnitude. The residual phase error is an artifact of intersampling behavior.

7. Conclusions

ARMA least-squares time-domain identification has been generalized to ARMARKOV representations that directly estimate the Markov parameters of a system from input-output data. A numerical example involving a second-order lightly damped systems showed the eigenvalues obtained from the Markov parameters of a perturbed ARMARKOV representation are much less sensitive than the eigenvalues obtained from Markov parameters of a perturbed ARMA representation. Finally, using experimental data, the ARMA/LS identification algorithm and the ARMARKOV/LS/ERA identification algorithm were used to identify the dynamics of an acoustic duct.

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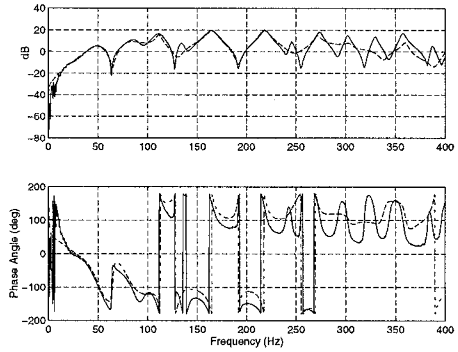

Figure 1: ARMA/LS identification algorithm, frequency response of 30th-Order transfer function (dashed line) and experimentally measured frequency response (solid line).

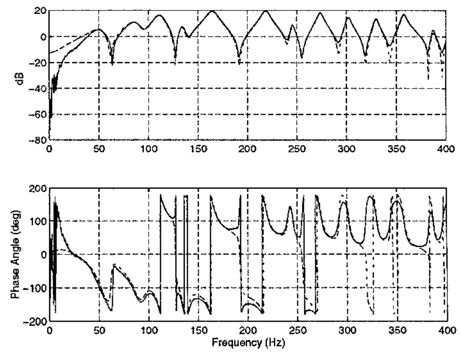

Figure 2: ARMARKOV/LS/ERA identification algorithm, $n = 40$, $\mu = 210$, $r, s = 100$, frequency response of 39th-order state space realization (dashed line) and experimentally measured frequency response (solid line).