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Adaptive Disturbance Rejection Using ARMARKOV/Toeplitz Models

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Abstract

In this paper we develop an adaptive disturbance rejection algorithm formulated in terms of an ARMARKOV/Toeplitz matrix system representation. The algorithm is applied to the problem of active noise suppression in an acoustic duct, and experimental results demonstrating tonal and broadband disturbance rejection are presented.

1. Introduction

An important objective of control system design is to minimize the effects of external disturbance signals. For applications such as active noise and vibration control, it is the primary focus. In cases where the system is time varying or difficult to identify, adaptive methods such as the feedforward LMS and RLMS algorithms are useful [1] - [4]. However, feedforward-type algorithms neglect the effect of the feedback (or secondary) path from control to measurement thus leading to poor performance and instability [5]. To remedy this problem, robust variations of the classical LMS algorithm have been proposed; see, for example [6].

This paper proposes a novel adaptive feedback disturbance rejection algorithm in which the system and the controller are represented in ARMARKOV weight matrix form [7]. A gradient-search algorithm that minimizes a performance cost function is used to update the entries of the controller weight matrix. The ARMARKOV representation of systems involves the Markov parameters of the system and relates windows of input and output data. In [8] it is shown that ARMARKOV models are less sensitive to noisy measurements than ARMA models. We also note that perturbations to ARMARKOV parameters have less impact on model behavior as compared to ARMA representations. In addition, adaptive algorithms that use a weight matrix representation have update laws based on windows of data rather than instantaneous measurements. Thus, the ARMARKOV weight matrix representation has a greater degree of robustness with respect to measurement noise and parameter uncertainity than the ARMA representation. Finally, Markov parameter based representations of systems provide a framework for direct controller synthesis based on input-output data [9].

The algorithm requires a measurement sensor and a performance sensor, as well as the Markov parameters and moving average coefficients that relate the performance to the control. These parameters are obtained by using the time domain identification algorithm of [7]. Since the ARMARKOV system representation is used for identification and since the controller is based upon this representation, the intermediate step of recovering a state space or frequency domain model of the system is eliminated.

2. Standard Problem Representation of Disturbance Rejection

Consider the n-th order linear discrete-time two-input, two-output (TITO) system. The *disturbance* $w(k)$, the *control* $u(k)$, the *measurement* $y(k)$ and the *performance* $z(k)$ are in $\mathcal{R}^{m_w}, \mathcal{R}^{m_u}, \mathcal{R}^{l_y}$ and $\mathcal{R}^{l_z}$, respectively. The system can be written in state space form as

$$x(k+1) = Ax(k) + Bu(k) + D_1 w(k), \quad (1)$$
$$z(k) = E_1 x(k) + E_2 u(k) + E_0 w(k), \quad (2)$$
$$y(k) = Cx(k) + Du(k) + D_2 w(k), \quad (3)$$

or equivalently in terms of LTI transfer matrices

$$z = G_{zw} w + G_{zu} u, \quad (4)$$
$$y = G_{yw} w + G_{yu} u. \quad (5)$$

The controller G_c generates the control signal $u(k)$ based on the measurement $y(k)$, that is,

$$u = G_c y. \quad (6)$$

The objective of the standard problem [10] is to determine a controller G_c that produces a control signal $u(k)$ based on the measurement $y(k)$ such that a performance measure involving $z(k)$ is minimized. In classical fixed-gain H_2 and H_∞ optimal control theory, the performance $z(k)$ is not required to be measured, but rather is used analytically for off-line controller design. Fixed-gain controller design methods for disturbance rejection also require knowledge of all four transfer matrices, namely, the *primary path* G_{zw}, the *secondary path* G_{zu}, the *reference path* G_{yw} and the *feedback path* G_{yu}, and the spectrum of the disturbance $w(k)$. This terminology is standard in the noise control literature.

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Unlike fixed-gain controller design methods, adaptive control techniques require on-line measurement of $z(k)$ for use in adaptation. If $z(k)$ is measured and used for control, we say that the *performance assumption* is satisfied. However, in contrast to standard fixed-gain methods, adaptive methods [1]-[3] often require that only the secondary path transfer matrix G_{zu} be known a priori. Other adaptive methods [6] identify G_{zu} on-line but require additional actuators and sensors.

3. ARMARKOV Representation of Systems

In this section we derive the ARMARKOV representation of a state space model. Consider the nth-order discrete-time finite-dimensional linear time-invariant system

$$x(k+1) = Ax(k) + Bu(k), \quad (7)$$
$$y(k) = Cx(k) + Du(k), \quad (8)$$

where $u(k) \in \mathcal{R}^{m_u}$ and $y(k) \in \mathcal{R}^{l_y}$. The Markov parameters $H_k \in \mathcal{R}^{l_y \times m_u}$ of this system are defined as

$$H_j \triangleq D, \quad j = -1, \quad (9)$$
$$\triangleq CA^j B, \quad j \geq 0, \quad (10)$$

and satisfy

$$G(z) \triangleq C(zI - A)^{-1}B + D = \sum_{j=-1}^{\infty} H_j z^{-(j+1)}. \quad (11)$$

We note that if $G(z)$ is strictly proper, that is, $D = 0$, then $H_{-1} = 0$. The transfer function $G(z)$ can be equivalently represented as

$$G(z) = \frac{1}{z^n + a_1 z^{n-1} + \cdots + a_n}(B_0 z^n + \cdots B_n), \quad (12)$$

where $\det(zI - A) = z^n + a_1 z^{n-1} + \cdots + a_n$ and $B_i \in \mathcal{R}^{l \times m}, i = 0, \ldots, n$. Equating (11) and (12), and multiplying both sides by $z^n + a_1 z^{n-1} + \cdots + a_n$ yields

$$\begin{bmatrix} B_0 \\ B_1 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} H_{-1} & 0 & \cdots & 0 \\ H_0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ H_{n-1} & \cdots & H_0 & H_{-1} \end{bmatrix} \begin{bmatrix} I_m \\ a_1 I_m \\ \vdots \\ a_n I_m \end{bmatrix}, \quad (13)$$

which provides recursive expressions for B_i in terms of H_i.

Now consider the ARMA representation of (12) given by

$$y(k) = -a_1 y(k-1) - \cdots - a_n y(k-n) + B_0 u(k) + \cdots + B_n u(k-n). \quad (14)$$

Replacing k with $k-1$ in (14) and substituting the resulting relation back into (14) yields

$$y(k) = (a_1^2 - a_2)y(k-2) + \cdots + a_1 a_n y(k-n-1) + B_0 u(k) + (B_1 - a_1 B_0)u(k-1) + \cdots - a_1 B_n u(k-n-1). \quad (15)$$

Noting from (13) that $H_{-1} = B_0$, $H_0 = B_1 - a_1 B_0$, and defining

$$\alpha_{2,i} \triangleq a_1 a_i - a_{i+1}, \quad i = 1, \ldots, n-1, \quad \alpha_{2,n} \triangleq a_1 a_n, \quad (16)$$
$$\mathcal{B}_{2,i} \triangleq B_{i+1} - a_1 B_i, \quad i = 1, \ldots, n-1, \quad \mathcal{B}_{2,n} \triangleq a_1 B_n, \quad (17)$$

(15) can be written as

$$y(k) = \alpha_{2,1} y(k-2) + \cdots + \alpha_{2,n} y(k-n-1) + H_{-1}u(k) + H_0 u(k-1) + \mathcal{B}_{2,1} u(k-2) + \cdots + \mathcal{B}_{2,n} u(k-n-1). \quad (18)$$

We note that (18) explicitly involves the first two Markov parameters H_{-1} and H_0, and thus is called an *ARMARKOV* representation. An ARMARKOV model whose weight matrix contains the first three Markov parameters can be obtained by substituting $y(k-2)$ given by (14) into (18). Repeating this procedure $\mu - 1$ times yields the μ-ARMARKOV time domain form of (7) and (8)

$$y(k) = \sum_{j=1}^{n} -\alpha_j y(k-\mu-j+1) + \sum_{j=1}^{\mu} H_{j-1} u(k-j+1) + \sum_{j=1}^{n} \mathcal{B}_j u(k-\mu-j+1), \quad (19)$$

where $\alpha_j \in \mathcal{R}$ and $\mathcal{B}_j \in \mathcal{R}^{l_y \times m_u}$, $j = 1, \ldots, n$. Equation (19) is an input-output relation which explicitly involves μ Markov parameters. In the case $\mu = 1$, (19) specializes to the usual ARMA model. The coefficients α_j and $\mathcal{B}_j$ are calculated recursively (in a manner similar to the way the coefficients are calculated in (18) by repeated substitution into (14) and by using (13).

Now, let p be a positive integer and define the output vector $Y(k) \in \mathcal{R}^{lp}$ and the ARMARKOV regressor vector $\Phi_{yu}(k) \in \mathcal{R}^{l_y(p+n-1)+m_u(\mu+p+n-1)}$ by

$$Y(k) \triangleq [y(k) \quad \cdots \quad y(k-p+1)]^{\mathrm{T}}, \quad (20)$$
$$\Phi_{yu}(k) \triangleq [y(k-\mu) \quad \cdots \quad y(k-\mu-p-n+2) \quad u(k) \quad \cdots \quad u(k-\mu-p-n+2)]^{\mathrm{T}}. \quad (21)$$

Using (19), the vectors $Y(k)$ and $\Phi_{yu}(k)$ are related by the ARMARKOV/Toeplitz time domain representation

$$Y(k) = W_{yu}\Phi_{yu}(k), \quad (22)$$

where the ARMARKOV weight matrix W_{yu} is defined by

$$W_{yu} \triangleq \begin{bmatrix} -\alpha_1 I_{l_y} & \cdots & -\alpha_n I_{l_y} & 0_{l_y} & \cdots & 0_{l_y} \\ 0_{l_y} & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & 0_{l_y} \\ 0_{l_y} & \cdots & 0_{l_y} & -\alpha_1 I_{l_y} & \cdots & -\alpha_n I_{l_y} \end{bmatrix} \quad (23)$$

$$
\begin{matrix}
H_{-1} & \cdots & H_{\mu-2} & \mathcal{B}_1 & \cdots & \mathcal{B}_n & 0_{l_y\times m_u} & \cdots & 0_{l_y\times m_u} \\
0_{l_y\times m_u} & \ddots & & \ddots & \ddots & & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & & \ddots & \ddots & & \ddots & 0_{l_y\times m_u} \\
0_{l_y\times m_u} & \cdots & 0_{l_y\times m_u} & H_{-1} & \cdots & H_{\mu-2} & \mathcal{B}_1 & \cdots & \mathcal{B}_n
\end{matrix}\right],
$$

where I_l denotes the $l\times l$ identity matrix. We note that (22) is a redundant representation of (19) and that we can recover a state space realization of the system from (19).

4. ARMARKOV Representation of TITO Systems

We now derive the ARMARKOV representation of the TITO system described in Section 2. First, the ARMARKOV form of (1) - (3) is

$$
\begin{aligned}
z(k) = {}& \sum_{j=1}^{n} -\alpha_j z(k-\mu-j+1) + \sum_{j=1}^{\mu} H_{zw,j-2} w(k-j+1) \\
&+ \sum_{j=1}^{n} \mathcal{B}_{zw,j} w(k-\mu-j+1) + \sum_{j=1}^{\mu} H_{zu,j-2} u(k-j+1) \\
&+ \sum_{j=1}^{n} \mathcal{B}_{zu,j} u(k-\mu-j+1), \quad (24)
\end{aligned}
$$

$$
\begin{aligned}
y(k) = {}& \sum_{j=1}^{n} -\alpha_j y(k-\mu-j+1) + \sum_{j=1}^{\mu} H_{yw,j-2} w(k-j+1) \\
&+ \sum_{j=1}^{n} \mathcal{B}_{yw,j} w(k-\mu-j+1) + \sum_{j=1}^{\mu} H_{yu,j-2} u(k-j+1) \\
&+ \sum_{j=1}^{n} \mathcal{B}_{yu,j} u(k-\mu-j+1), \quad (25)
\end{aligned}
$$

where $\alpha_j \in \mathcal{R}$, $\mathcal{B}_{zw,j}, H_{zw,j} \in \mathcal{R}^{l_z\times m_w}$, $\mathcal{B}_{zu,j}, H_{zu,j} \in \mathcal{R}^{l_z\times m_u}$, $\mathcal{B}_{yw,j}, H_{yw,j} \in \mathcal{R}^{l_y\times m_w}$ and $\mathcal{B}_{yu,j}, H_{yu,j} \in \mathcal{R}^{l_y\times m_u}$. Note that the system order n is the same in (24) and (25). Next, define the *extended performance vector* $Z(k)$ and the *extended control vector* $U(k)$ by

$$
Z(k) \triangleq [z(k) \ \cdots \ z(k-p+1)]^{\mathrm{T}}, \quad (26)
$$
$$
U(k) \triangleq [u(k) \ \cdots \ u(k-\mu-p-n+2)]^{\mathrm{T}}, \quad (27)
$$

the ARMARKOV regressor vector $\Phi_{zw}(k)$ by

$$
\begin{aligned}
\Phi_{zw}(k) \triangleq {}& [z(k-\mu) \ \cdots \ z(k-\mu-p-n+2) \quad (28) \\
& w(k) \ \cdots \ w(k-\mu-p-n+2)]^{\mathrm{T}} \quad (29)
\end{aligned}
$$

the block-Toeplitz ARMARKOV weight matrix W_{zw} by

$$
W_{zw} \triangleq \left[\begin{matrix}
-\alpha_1 I_{l_z} & \cdots & -\alpha_n I_{l_z} & 0_{l_z} & \cdots & 0_{l_z} \\
0_{l_z} & \ddots & & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & & \ddots & 0_{l_z} \\
0_{l_z} & \cdots & 0_{l_z} & -\alpha_1 I_{l_z} & \cdots & -\alpha_n I_{l_z}
\end{matrix}\right. \quad (30)
$$

$$
\left.\begin{matrix}
H_{zw,-1} & \cdots & H_{zw,\mu-2} & \mathcal{B}_{zw,1} & \cdots & \mathcal{B}_{zw,n} & 0_{l_z\times m_w} & \cdots & 0_{l_z\times m_w} \\
0_{l_z\times m_w} & \ddots & & \ddots & \ddots & & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & & \ddots & \ddots & & \ddots & 0_{l_z\times m_w} \\
0_{l_z\times m_w} & \cdots & 0_{l_z\times m_w} & H_{zw,-1} & \cdots & H_{zw,\mu-2} & \mathcal{B}_{zw,1} & \cdots & \mathcal{B}_{zw,n}
\end{matrix}\right],
$$

and the ARMARKOV control matrix $B_{zu} \triangleq$

$$
\left[\begin{matrix}
H_{zu,-1} & \cdots & H_{zu,\mu-2} & \mathcal{B}_{zu,1} & \cdots & \mathcal{B}_{zu,1} & 0_{l_z\times m_u} & \cdots & 0_{l_z\times m_u} \\
0_{l_z\times m_u} & \ddots & & \ddots & \ddots & & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & & \ddots & \ddots & & \ddots & 0_{l_z\times m_u} \\
0_{l_z\times m_u} & \cdots & 0_{l_z\times m_u} & H_{zu,-1} & \cdots & H_{zu,\mu-2} & \mathcal{B}_{zu,1} & \cdots & \mathcal{B}_{zu,n}
\end{matrix}\right]
$$

Then (24) can be written in the form

$$
Z(k) = W_{zw}\Phi_{zw}(k) + B_{zu}U(k). \quad (32)
$$

Similarly, (25) can be written as

$$
Y(k) = W_{yw}\Phi_{yw}(k) + B_{yu}U(k), \quad (33)
$$

where $Y(k)$ is defined as in (20) and Φ_{yw}, W_{yw} and B_{yw} are defined analogous to (27), (28) and (31) to yield the ARMARKOV weight matrix representation of (1) - (3). The length of the vector $U(k)$, $p_c = m_u(\mu+n+p-1)$.

5. Adaptive Disturbance Rejection Algorithm

In this section we formulate an adaptive disturbance rejection feedback algorithm for the TITO system represented in ARMARKOV form (32) and (33). We use a strictly proper ARMARKOV/Toeplitz form controller of order n_c with μ_c Markov parameters. The controller weight matrix $W_c(k) \triangleq$

$$
\left[\begin{matrix}
-\alpha_{c,1}I_{m_u} & \cdots & -\alpha_{n_c,c}I_{m_u} & 0_{m_u\times m_u} & \cdots & 0_{m_u\times m_u} \\
0_{m_u\times m_u} & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & 0_{m_u\times m_u} \\
0_{m_u\times m_u} & \cdots & 0_{m_u\times m_u} & -\alpha_{1,c}I_{m_u} & \cdots & -\alpha_{c,n_c}I_{m_u}
\end{matrix}\right.
$$

$$
\left.\begin{matrix}
H_{0,c} & \cdots & H_{\mu_c-2,c} & \mathcal{B}_{c,1} & \cdots & \mathcal{B}_{c,n_c} & 0_{m_u\times l_y} & \cdots & 0_{m_u\times l_y} \\
0_{m_u\times l_y} & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0_{m_u\times l_y} \\
0_{m_u\times l_y} & \cdots & 0_{m_u\times l_y} & H_{c,0} & \cdots & H_{c,\mu_c-2} & \mathcal{B}_{c,1} & \cdots & \mathcal{B}_{c,n_c}
\end{matrix}\right],
$$

and its nonzero entries are functions of k. Define the *estimated control vector* $\hat{U}(k)$ and the *estimated performance* as

$$
\hat{U}(k) = W_c(k)\Phi_{uy}(k), \quad (34)
$$
$$
\hat{Z}(k) = W_{zw}\Phi_{zw}(k) + B_{zu}\hat{U}(k), \quad (35)
$$

with

$$
\begin{aligned}
\Phi_{uy}(k) \triangleq {}& [u(k-\mu_c) \ \cdots \ u(k-\mu_c-n_c-p_c+2) \\
& y(k-1) \ \cdots \ y(k-\mu_c-n_c-p_c+2)]^{\mathrm{T}} \quad (36)
\end{aligned}
$$

Note that $\hat{Z}(k) = Z(k) + B_{zu}(\hat{U}(k) - U(k))$. Next, define the cost function

$$
J(k) \triangleq \frac{1}{2}\hat{Z}^{\mathrm{T}}(k)\hat{Z}(k). \quad (37)
$$

Using (32), (34) and (37), the constrained gradient of $J(k)$ with respect to $W_c(k)$ is given by [7]

$$\frac{\partial J(k)}{\partial W_c(k)} = U_c \circ B_{zu}^T \hat{Z}(k) \Phi_{uy}^T(k). \tag{38}$$

where "$\circ$" denotes the Hadamard product of two matrices and the constraint matrix U_c is defined by

$$U_c \triangleq \begin{bmatrix} \mathcal{I}_{m_u \times n} & 0_{m_u} & \cdots & 0_{m_u} \\ 0_{m_u} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0_{m_u} \\ 0_{m_u} & \cdots & 0_{m_u} & \mathcal{I}_{m_u \times n} \end{bmatrix} \tag{39}$$

$$\left. \begin{matrix} 1_{m_u \times (n+\mu-1)} & 0_{m_u \times l_y} & \cdots & 0_{m_u \times l_y} \\ 0_{m_u \times l_y} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0_{m_u \times l_y} \\ 0_{m_u \times l_y} & \cdots & 0_{m_u \times l_y} & 1_{m_u \times (n+\mu-1)} \end{matrix} \right],$$

with $\mathcal{I}_{m_u \times n} \in \mathcal{R}^{m_u \times n m_u} \triangleq [I_{m_u} \ \cdots \ I_{m_u}]$, and $1_{m_u \times (n+\mu-1)}$ denoting the $m_u \times (n+\mu-1)$ ones matrix. The Hadamard product of the constraint matrix and the gradient preserves the block zero structure of the weight matrix. Thus, if B_{zu} is known and the signals $z(k)$, $u(k)$ and $y(k)$ are available to construct the vectors $\hat{Z}(k)$ and $\Phi_{uy}(k)$, then we can use this gradient in the weight matrix update law

$$W_c(k+1) = W_c(k) - \eta(k) \frac{\partial J(k)}{\partial W_c(k)}, \tag{40}$$

where $\eta(k)$ is the adaptive step size given by

$$\eta(k) \triangleq \frac{1}{\|B_{zu}\|_F^2 \|\Phi_{uy}\|_2^2}, \quad k \geq 0. \tag{41}$$

This adaptive step size is motivated by the computationally efficient step size in the identification algorithm of [7] and disturbance attenuation using this step size is verified from simulation and experiment.

6. Controller Implementation

From the previous section, we note that (34) gives us the extended control vector $U(k)$ at each time step k while (40) is the update law for the weight matrix $W_{y_1}(k)$. Moreover, each row of (34) represents the ARMARKOV time domain relation

$$u(k) = \alpha_{c,1}(k)u(k-\mu_c) + \cdots + \alpha_{c,n_c}(k)u(k-\mu_c-n_c-p_c+2) \\ + H_{c,0}(k)y(k-1) + \cdots + \mathcal{B}_{c,n_c}(k)y(k-\mu_c-n_c-p_c+2).$$

However, note that the update law (40) with the constrained gradient does not preserve the block Toeplitz structure of the weight matrix but only the block zero structure. Hence, we average the parameters along the diagonals of the weight matrix $W_c(k)$ to generate the control signal using

$$u(k) = \bar{\alpha}_{c,1}(k)u(k-\mu_c) + \cdots + \bar{\alpha}_{c,n_c}(k)u(k-\mu_c-n_c-p_c+2) \\ + \bar{H}_{c,0}(k)y(k-1) + \cdots + \bar{\mathcal{B}}_{c,n_c}(k)y(k-\mu_c-n_c-p_c+2).$$

where the "-" superscript denotes the average value of the parameter.

We observe that of the four transfer matrices in the standard problem, G_{zw}, G_{zu}, G_{yw} and G_{yu}, the algorithm described above requires that we identify only one transfer matrix, namely, G_{zu}. The signals that we require to be measured are $y(k)$ and $z(k)$.

7. Experimental Results

Experimental demonstration of the ARMARKOV adaptive disturbance algorithm is performed on an acoustic duct of circular cross-section. The duct is 80 inches long and has a diameter of 4 inches. The disturbance speaker is located at one end of the duct and the measurement sensor (microphone) is located 4 inches in from the same end of the duct. The performance sensor is positioned 6 inches in from the other end while the control speaker is placed 16 inches in from that end of the duct.

The algorithm is tested on four types of disturbances, namely, a single-tone disturbance (139.65 Hz), a two-tone disturbance (135.74 Hz and 160.4 Hz), band-limited white noise (up to 390 Hz) and AM radio noise. The algorithm uses $n = 4$ and $\mu = 12$ for the matrix B_{zu} and for the identification of G_{uy}, and $n_c = 2$, $\mu_c = 10$ and $p = 2$ for the $W_c(k)$ matrix. The controller is implemented on a dSPACE ds1102 real time controller running a C30 DSP processor at a sampling frequency of 800 Hz. The microphone signals are passed through a low pass filter that rolls off at 315 Hz. Figure 1 shows the open-loop and closed-loop frequency domain performance with a single-tone disturbance. Disturbance attenuation of over 40 dB is achieved. Although the disturbance signal is a pure tone, speaker nonlinearities produce harmonics which appear on the frequency response plot along with ambient and measurement noise. The algorithm provides the same level of attenuation by adaptation when the frequency of the disturbance tone is changed. For the case of a dual-tone disturbance, attenuation of over 35 dB is observed as shown in Figure 2. Figure 3 shows the open-loop and closed-loop magnitude plots of the transfer function from disturbance to performance with a white noise disturbance, and noise suppression of up to 15 dB is observed over a frequency range from 0 - 300 Hz. Finally, Figure 4 shows the open-loop and closed-loop frequency response with an AM radio disturbance. Significant levels of noise reduction are observed over the frequency range 0 - 300 Hz. Thus, the algorithm is shown to be effective in rejecting both narrow-band and broad-band disturbances.

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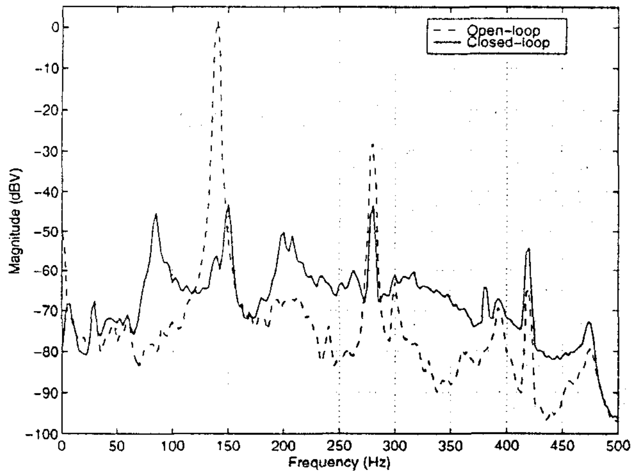

Figure 1: Open-loop and closed-loop frequency domain performance with a single-tone disturbance at 139.65 Hz

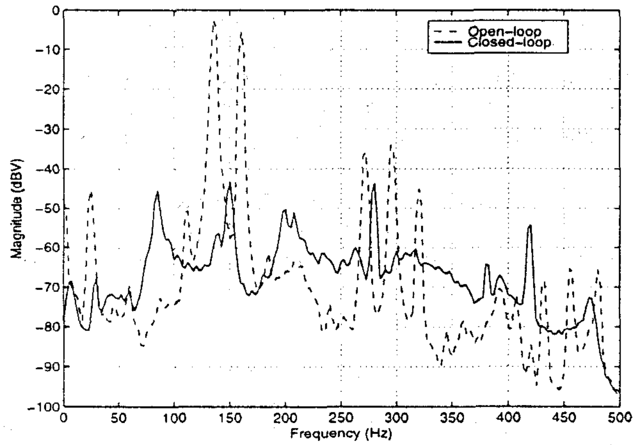

Figure 2: Open-loop and closed-loop frequency domain performance with a two-tone disturbance at 135.74 Hz and 160.4 Hz

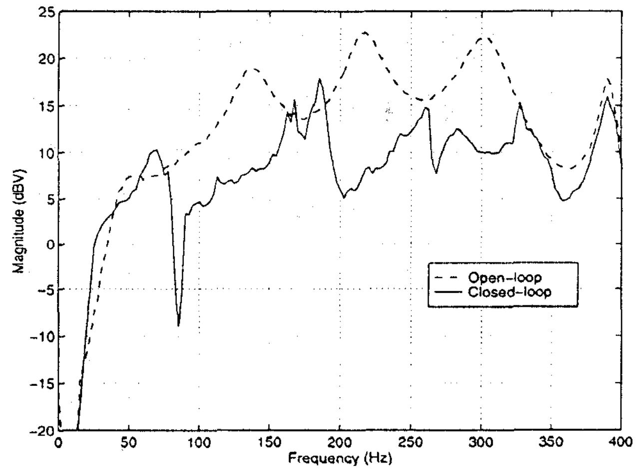

Figure 3: Open-loop and closed-loop performance with band-limited white noise

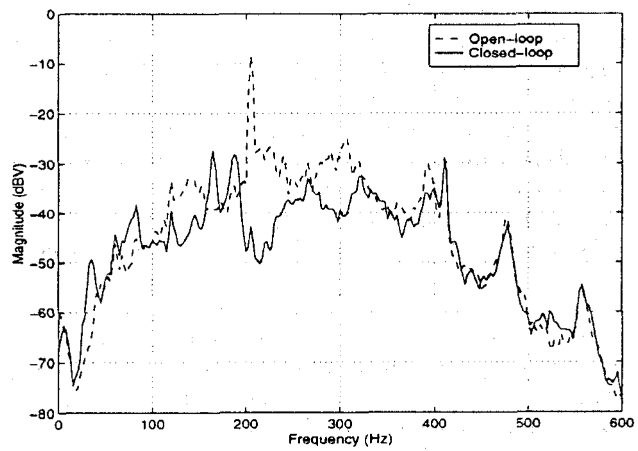

Figure 4: Open-loop and closed-loop performance with AM radio disturbance

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